DART Tutorial Section 25:
A simple 1D advection model:
Tracer Data Assimilation
1D Simple Advection Model: Overview

Found in models/simple_advection

1. One dimensional periodic domain.
2. Chaotic/stochastic model for wind.
4. Stochastic model for tracer source can be added.
5. Diurnal cycle for tracer source can be added.
6. Model for phase offset of diurnal cycle can be added.
Model domain, timestepping, and namelist controls (in italics):

1. One-dimensional, periodic grid on domain $[0, 1]$. 

2. Equally-spaced gridpoints, number controlled by \textit{num\_grid\_points}. 

3. Location of gridpoints is $0, 1/N, 2/N,..., N-1/N$; $N=\textit{num\_grid\_points}$. 

4. Distance between gridpoints is \textit{grid\_spacing\_meters}. 

5. Time-stepping controlled by \textit{time\_step\_days}, \textit{time\_step\_seconds}. 
The wind model: Burger’s Equation

1. Wind equation is: \[ \frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} \]

2. Continuous solution forms shocks when \( u \) varies with \( x \).

3. Use upstream Lagrangian differencing:
   a. Let \( u_i \) be velocity at gridpoint \( i \) located at \( x_i \); \( \Delta t \) is timestep.
   b. Source location for wind is: \( x_s = x_i - u_i \Delta t \)
   c. Linearly interpolate \( u \) to \( x_s \); this is new wind at gridpoint \( i \).

4. This is heavy damping.

5. Generally won’t form shocks, even when analytic solution should.
1. Without something additional, damps to spatially constant.

2. Randomly perturb each gridpoint value of \( u \) every timestep:

\[
    u_i = u_i + \text{Normal}(0, W \Delta t), \; i = 1, ..., N
\]

\( W = \text{wind_random_amp} \)

3. Damp the spatial mean wind field to \( M = \text{mean_wind} \):

\[
    \bar{u}(t + \Delta t) = \bar{u}(t) - W_D \Delta t \left[ \bar{u}(t) - M \right]
\]

\( W_D \) is \( \text{wind_damping_rate} \)
cd models/simple_advection/work
csh workshop_setup.csh

1. Run `./perfect_model_obs` and use `ncview true_state.nc` to check out the winds (ignore the other fields for now).

2. Make sure you understand the spatial/temporal behavior of winds.

3. Play with `mean_wind`, `wind_damping_rate`, `wind_random_amp` to get different types of flows.

4. Having winds reverse will be interesting for some tracer cases.
1. A single passive (doesn’t affect wind) tracer is advected by the wind.

2. Have tracer concentration, \( C_i \), at each gridpoint.

3. Semi-lagrangian upstream time differencing (same as for wind).

4. There is a tracer source/sink rate, \( S_i \), at each gridpoint.

5. Also a background \( D = destruction\_rate \) (percent / second).

\[
C_i = C_i + S_i \Delta t - D \Delta t C_i
\]
An assimilation example

1. Observations (20 altogether):
   Ten equally-spaced co-located observations of $u$ and $C$. Located halfway between gridpoints (0.05, 0.15,..., 0.95).
   Forward observation operator is linear interpolation (mean).
   $u$ observations have error variance of 16.01.
   $C$ observations have error variance of 1000.2.
   (Apparently insignificant decimals make changing these easier)

2. Base namelist assimilates both types of observations.

3. Run 
   ./perfect_model_obs
   followed by
   ./filter.
   (Make sure to use base input.nml, etc.).

4. Use Matlab to examine behavior of the assimilation.

5. Keep track of errors for concentration and wind fields.
1. Look at impact of using subsets of the observations.
2. Observations being used are controlled in \texttt{&obs\_kind\_nml}. Default is:
   \begin{verbatim}
   assimilate\_these\_obs\_types = 'TRACER\_CONCENTRATION',
                                 'VELOCITY',
   evaluate\_these\_obs\_types   = 'TRACER\_SOURCE'
   \end{verbatim}

   Change to assimilating only wind observations:
   \begin{verbatim}
   assimilate\_these\_obs\_types = 'VELOCITY',
   evaluate\_these\_obs\_types   = 'TRACER\_CONCENTRATION',
                                 'TRACER\_SOURCE'
   \end{verbatim}

3. Try assimilating only wind, only concentration and not assimilating anything (delete \texttt{assimilate\_these\_obs\_types}). Record results and understand what’s going on.
1. So far, tracer source has been constant in time (not in space).

2. Make the source model a damped random walk at each gridpoint.
   
   Add random noise to each gridpoint at each timestep.
   Also damp to a time mean value.
   
   \[ S_i = S_i + Normal(0, S_R) - S_D (S_i - \bar{S}_i) \]

where \( S_R = source\_random\_amp\_frac \), \( S_D = source\_damping\_rate \) and \( \bar{S}_i \) is time mean value for source at this gridpoint.

\( \bar{S}_i \) is actually a 4th set of state variables (defined at each gridpoint). We will NOT work with this set of variables here.
Assimilation with time-varying tracer source

Use these settings:

```scheme
&model_nml
   source_random_amp_frac = 0.000002

&perfect_model_obs_nml
   input_state_files = 'perfect_input_source_noise.nc'

&filter_nml
   input_state_file_list = 'filter_input_list.txt'
```

*filter_input_list.txt contains a list of input files, for this model, there is only one file in the list. Edit the contents of* filter_input_list.txt *to contain* filter_input_source_noise.nc

1. Repeat the assimilations with different classes of observations. */perfect_model_obs* followed by */filter*

2. This time, also keep track of errors for the source field.

3. What happens if source_random_amp_frac = 0.00001?

Can you find ways to improve filter performance in these cases?
1. `source_diurnal_rel_amp` controls amplitude of diurnal component as fraction of source value.

2. Turn this on and try assimilation experiments again.

   ```
   &perfect_model_obs_nml
      ...
      input_state_files = "perfect_input_diurnal.nc"
   
   &model_nml
      ...
      source_diurnal_rel_amp = 0.05,
   ```

3. Change contents of `filter_input_list.txt` to `filter_input_diurnal.nc`
Impact of observation density

Try this:

&perfect_model_obs_nml

...  
  obs_seq_in_file_name = "obs_seq.in.half"

1. Repeat the different assimilation cases.
   Run ./perfect_model_obs followed by ./filter.

2. What happens if observations get even sparser? (You’ll have to do your own observation sequence design for this).

It is a good idea to svn revert set_def.out obs_seq.in after you are done with your own observation sequence design and experimentation to stay consistent with the tutorial material.
Using DA to learn about the source model

This is basically parameter estimation. Try this:

```nml
&model_nml
    source_phase_noise = 0.00001

&perfect_model_obs_nml
    obs_seq_in_file_name = 'obs_seq.in'
    input_state_files = 'perfect_input.nc'

&filter_nml
    input_state_file_list = 'filter_input_list.txt'
```

1. Change contents of `filter_input_list.txt` to `filter_input.nc`
2. Run `./perfect_model_obs` followed by `./filter`
DA for sources with lots of structure

Try this:

```plaintext
&model_nml
  source_phase_noise = 0.0

&perfect_model_obs_nml
  obs_seq_in_file_name = 'obs_seq.in'
  input_state_files = 'perfect_input_saw.nc'
```

1. Change contents of `filter_input_list.txt` to `filter_input_saw.nc`
2. Run `./perfect_model_obs` followed by `./filter`

The source has large mean value at points 1, 3 and 5, small elsewhere.

Can one resolve this spatial structure?
Problems to explore

1. Pick one of the configurations above. Suppose that wind observations cost $10 and concentration observations cost $20. If you have $200 to spend on observations (that MUST be located halfway between the gridpoints), what is the best observing/assimilation system you can design? The error variance of the observations is fixed as in the original cases. You can put multiple observations of any kind at an observing location. The metric of success is the smallest total error for the concentration. What if the metric is the smallest total error for the source?

2. Suppose you can only take observations once a day but the source has a fixed diurnal component. Is there any way to learn anything about the diurnal component? You can use as many observations as you want for this. Ask how to change the observation frequency.
3. For the saw tooth source pattern, what happens as the number of observing locations goes down in this case?

4. What happens to the quality of the assimilations in some of the cases above if the *destruction_rate* is made smaller or bigger (it’s already pretty big in some sense)?

5. We might be happier if tracer were not created or destroyed by the assimilation. Is the assimilation creating and destroying tracer? Is there any systematic component to this? Can you find ways to reduce creation/destruction by the assimilation scheme?

6. What happens if we use a fixed-lag smoother in this problem? How should the smoother be tuned? What happens to tracer destruction/creation in the smoother estimates?
7. Can you think of good ways to add model error into this system? If you do, what happens to the quality of the assimilations for various types of experiments? Simulating model error in a meaningful way is essential to predicting what will happen when one switches to real observations.
1. Filtering For a One Variable System
2. The DART Directory Tree
3. DART Runtime Control and Documentation
4. How should observations of a state variable impact an unobserved state variable?
   Multivariate assimilation.
5. Comprehensive Filtering Theory: Non-Identity Observations and the Joint Phase Space
6. Other Updates for An Observed Variable
7. Some Additional Low-Order Models
8. Dealing with Sampling Error
9. More on Dealing with Error; Inflation
10. Regression and Nonlinear Effects
11. Creating DART Executables
12. Adaptive Inflation
13. Hierarchical Group Filters and Localization
14. Quality Control
15. DART Experiments: Control and Design
16. Diagnostic Output
17. Creating Observation Sequences
18. Lost in Phase Space: The Challenge of Not Knowing the Truth
19. DART-Compliant Models and Making Models Compliant
20. Model Parameter Estimation
21. Observation Types and Observing System Design
22. Parallel Algorithm Implementation
23. Location module design (not available)
24. Fixed lag smoother (not available)
25. A simple 1D advection model: Tracer Data Assimilation