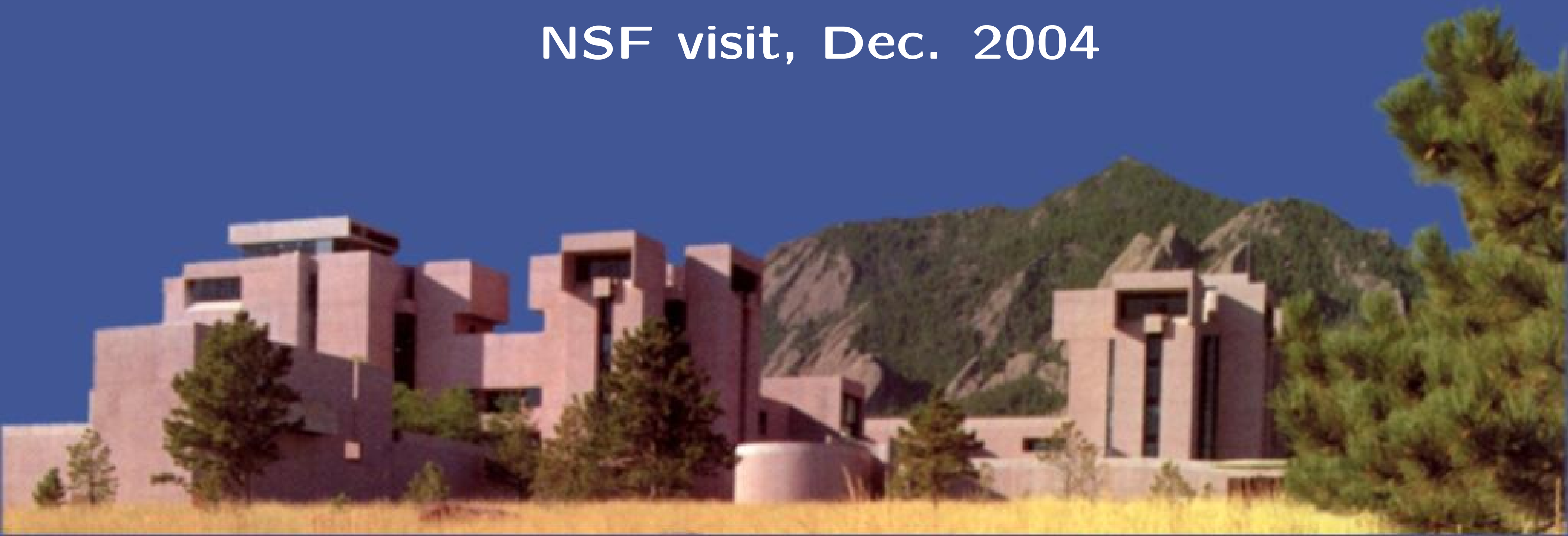
Overview of three Projects During my PostDoc Life

NSF visit, Dec. 2004



Introduction

- Tapering in ensemble Kalman filter

  Thomas Bengtsson (Berkeley), Jeff Anderson (NCAR)

- Tapering in spatial prediction

  Doug Nychka (NCAR), Marc Genton (Texas A&M)

- Bayes model for climate projections

  Tom Wigley (NCAR), Steve Sain (CU Denver)

Tapering in Ensemble KF

Let the state-space model be

$$\begin{aligned} \mathbf{y}_t &= \mathbf{H}_t \mathbf{x}_t + \mathbf{w}_t & \mathbf{w}_t &\sim \mathcal{N}_r(\mathbf{0}, \mathbf{R}_t) \\ \mathbf{x}_t &= \mathbf{G}_t \mathbf{x}_{t-1} + \mathbf{v}_t & \mathbf{v}_t &\sim \mathcal{N}_q(\mathbf{0}, \mathbf{Q}_t) \end{aligned}$$

and assume $[\mathbf{x}_t | \mathbf{y}_{t-1}, \dots, \mathbf{y}_0] \sim \mathcal{N}_q(\mathbf{x}_t^f, \mathbf{P}_t^f)$.

"Filter" the state $\mathbf{x}_t$ given the new observation $\mathbf{y}_t$:

$$[\mathbf{x}_t | \mathbf{y}_t, \dots, \mathbf{y}_0] \sim \mathcal{N}_q(\mathbf{x}_t^a, \mathbf{P}_t^a)$$

$$\begin{aligned} \mathbf{x}_t^a &= \mathbf{x}_t^f + \mathbf{K}_t(\mathbf{y}_t - \mathbf{H}_t \mathbf{x}_t^f) \\ \mathbf{P}_t^a &= (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t^\mathsf{T}) \mathbf{P}_t^f \\ \mathbf{K}_t &= \mathbf{P}_t^f \mathbf{H}_t^\mathsf{T} (\mathbf{H}_t \mathbf{P}_t^f \mathbf{H}_t^\mathsf{T} + \mathbf{R}_t)^{-1} \end{aligned}$$

Instead, update a sample from the forecast distribution.

Tapering in Ensemble KF

Let $\{\mathbf{x}_i\}$, $i = 1, \ldots, n$, iid $\mathcal{N}_q(\mathbf{0}, \boldsymbol{\Sigma})$ with n small and q huge. Denote $\mathbf{S}$ the sample covariance matrix.

For a positive definite matrix $\mathbf{C}$, consider $\mathbf{S} \circ \mathbf{C}$.

With an appropriate matrix norm describe the dependence of

$$\left\| \boldsymbol{\Sigma} - \widetilde{\boldsymbol{\Sigma}} \right\| \qquad\qquad \left\| (\boldsymbol{\Sigma} + \mathbf{I})^{-1} - (\widetilde{\boldsymbol{\Sigma}} + \mathbf{I})^{-1} \right\|$$

with

$$\widetilde{\boldsymbol{\Sigma}} = \mathbf{S} \qquad \text{or} \qquad \widetilde{\boldsymbol{\Sigma}} = \mathbf{S} \circ \mathbf{C}$$

on ensemble size n, state dimension q, covariance matrix $\boldsymbol{\Sigma}$ and taper $\mathbf{C}$.

Tapering in Prediction

Suppose a spatial process Z with $\mathsf{E}(Z) = \mathbf{0}$ and $\mathsf{Cov}(Z) = \mathbf{C}$.

Objective: predict Z at many locations
given the observations $\mathbf{Z} = \big(Z(\mathbf{x}_1), \ldots, Z(\mathbf{x}_n)\big)^{\mathsf{T}}$.

The kriging predictor (BLUP) is

$$\widehat{Z}(\mathbf{x}_0) = \mathbf{c}^{\mathsf{T}}\mathbf{C}^{-1}\mathbf{Z}$$

with $\mathbf{c}_i = \mathsf{Cov}\big(Z(\mathbf{x}_0), Z(\mathbf{x}_i)\big)$.
Approach:

$\boxed{\text{Introduce sparseness in } \mathbf{C} \text{ to gain computational advantages}}$

Tapering in Prediction

Taper Condition: *Let f_θ be the spectral density of the taper covariance, C_θ, and for some $\epsilon > 0$ and $M < \infty$*

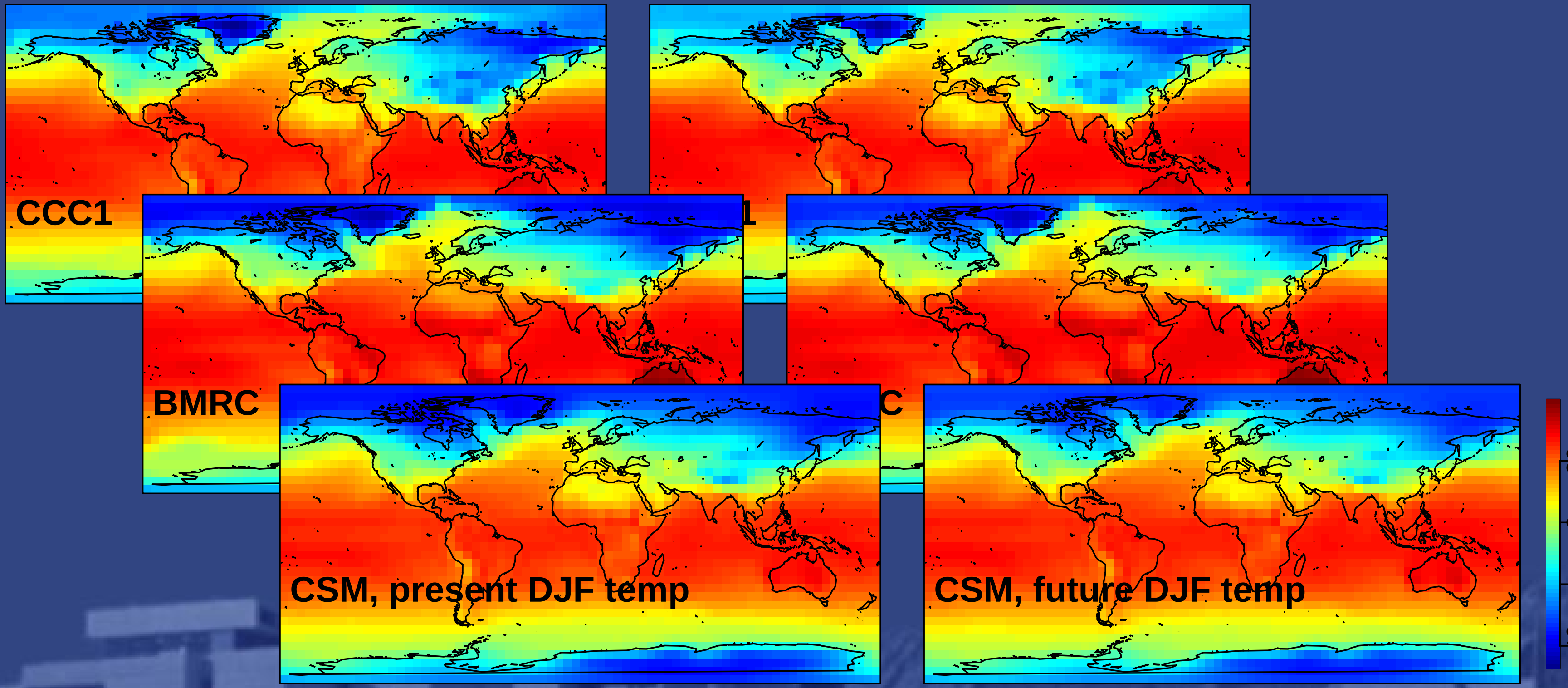
$$f_\theta(\rho) < \frac{M}{(1+\rho^2)^{\nu+d/2+\epsilon}}$$

Taper Theorem: *Assume that $C_{\alpha,\nu}$ is a Matérn covariance with smoothness parameter ν and that the Taper Conditions and ... hold. Then*

$$\lim_{n\to\infty} \frac{\mathrm{MSE}(\mathbf{x}^*, C_{\alpha,\nu}C_\theta)}{\mathrm{MSE}(\mathbf{x}^*, C_{\alpha,\nu})} = 1 \qquad \lim_{n\to\infty} \frac{\varrho(\mathbf{x}^*, C_{\alpha,\nu}C_\theta)}{\mathrm{MSE}(\mathbf{x}^*, C_{\alpha,\nu})} = 1$$

Models for Climate Projections

AOGCMs: Atmospheric-Ocean General Circulation Models



Goal: Probabilistic description of modeled climate change

Models for Climate Projections

"Large" linear fixed effects model

$$\mathbf{X}_0 = \boldsymbol{\mu}_x + \boldsymbol{\varepsilon} \qquad \text{(observed present climate)}$$

$$\mathbf{X}_i = \boldsymbol{\mu}_x + \mathbf{u}_i + \boldsymbol{\sigma}_i \qquad \text{(simulated present climate)}$$

$$\mathbf{Y}_i = \boldsymbol{\mu}_y + \mathbf{v}_i + \boldsymbol{\nu}_i \qquad \text{(simulated future climate)}$$

Models for Climate Projections

"Large" linear fixed effects model

$$\begin{aligned}
\mathbf{X}_0 &= \boldsymbol{\mu}_x + \varepsilon && \text{(observed present climate)}\\
\mathbf{X}_i &= \boldsymbol{\mu}_x + \mathbf{u}_i + \boldsymbol{\sigma}_i && \text{(simulated present climate)}\\
\mathbf{Y}_i &= \boldsymbol{\mu}_y + \mathbf{v}_i + \boldsymbol{\nu}_i && \text{(simulated future climate)}
\end{aligned}$$

$$\begin{array}{lll}
\boldsymbol{\mu}_x = \mathbf{M}_c\theta & \varepsilon \sim \mathcal{N}_n(\mathbf{0}, \mathbf{S}) & \\
\boldsymbol{\mu}_x = \mathbf{M}_c\theta & \mathbf{u}_i = \mathbf{M}_b\beta_i & \boldsymbol{\sigma}_i \sim \mathcal{N}_n(\mathbf{0}, \Sigma_i) \\
\boldsymbol{\mu}_y = \mathbf{M}_c\eta & \mathbf{v}_i = \mathbf{M}_b\gamma_i & \boldsymbol{\nu}_i = \boldsymbol{\omega}_i + \rho_i\boldsymbol{\sigma}_i \\
 & & \quad \boldsymbol{\omega}_i \sim \mathcal{N}_n(\mathbf{0}, \Omega_i) \quad \boldsymbol{\omega}_i \perp \boldsymbol{\sigma}_i
\end{array}$$

Use a Gibbs sampler to obtain posterior climate changes.

Opportunity

- Collaboration with scientists

  Ammann, Anderson, Baker, Berner, Cassou, Collins, Fournier, Mahowald, Mearns, Meehl, Saravanan, Schimel, Snyder, Tribbia, Wigley, . . .

- Pursue cutting edge research

  KriSp, IPCC, . . .

- Establishing a scientific network

  “nobody ever leaves NCAR” . . .