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Multi-resolution (wavelet) Based Nonstationary Covariance Modeling

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in collaboration with

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INTRODUCTION

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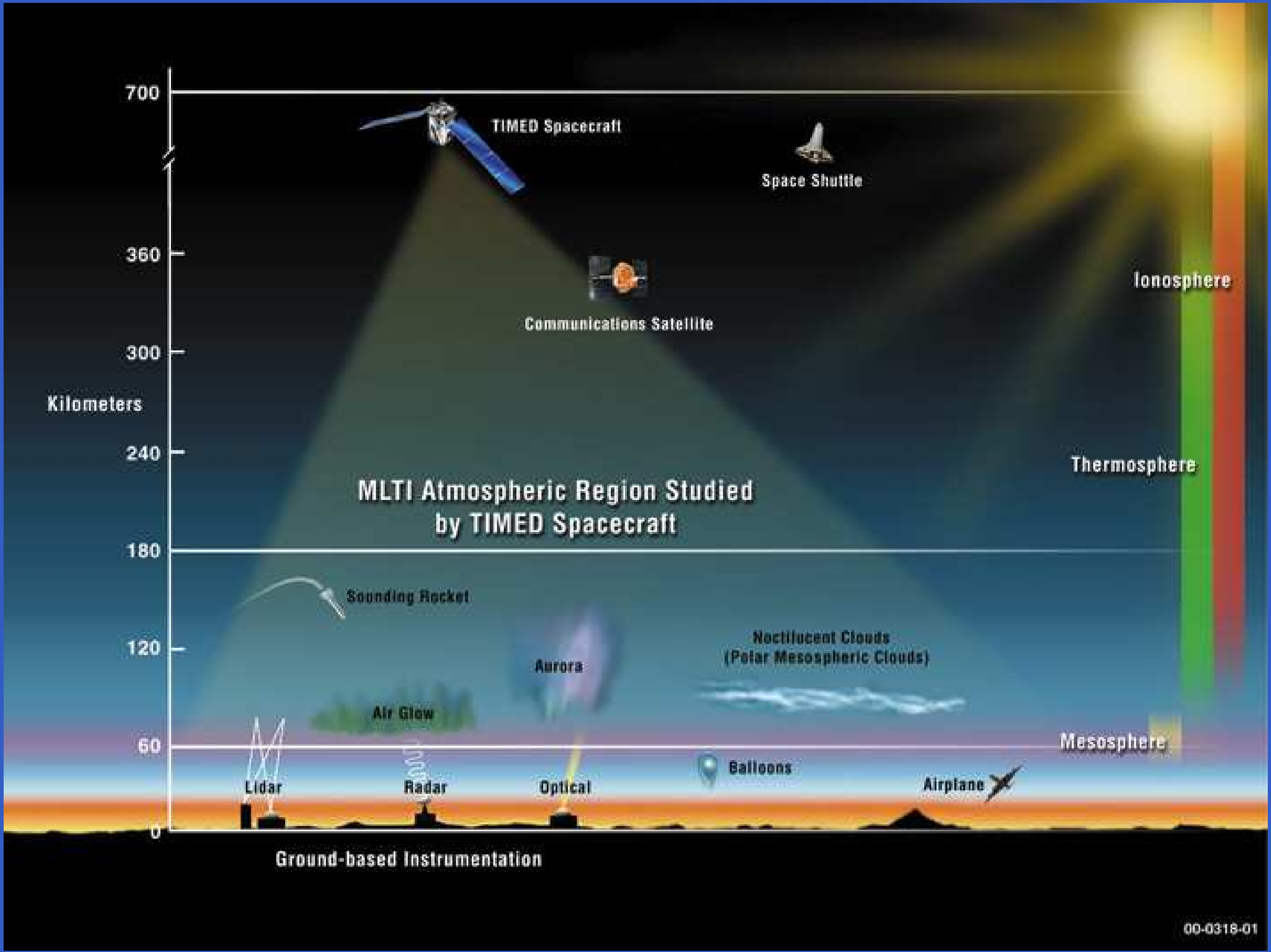

OUTLINE

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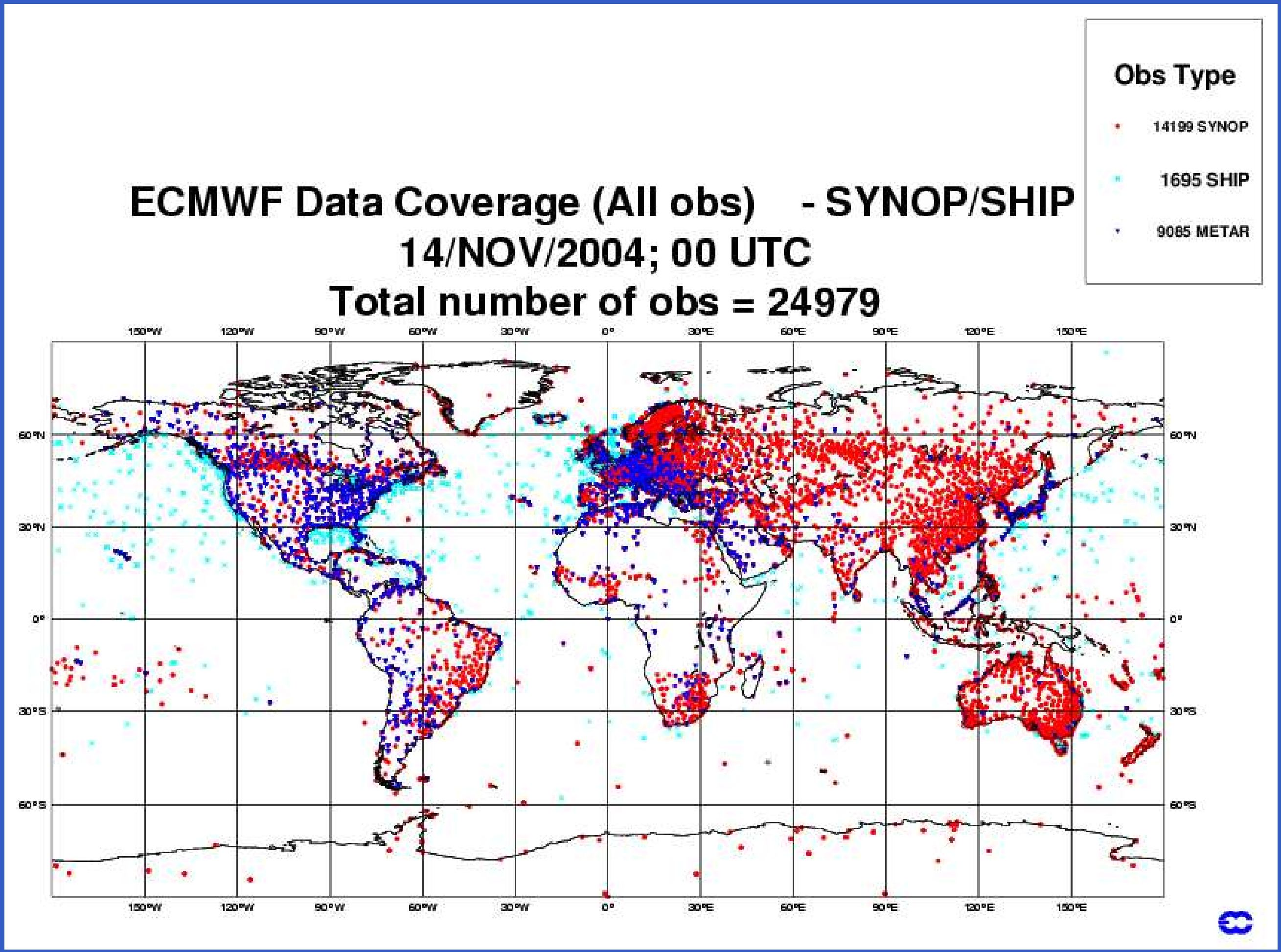
- Background:
  - Challenges of covariance modeling in data assimilation application
  - Observations
- Multi-resolution (wavelet) based covariance
  - Heuristic EM-type approach
  - Examples (ground-level ozone data)
- Summary and Future Work

Observations

- Numerous, Incomplete, Irregularly distributed



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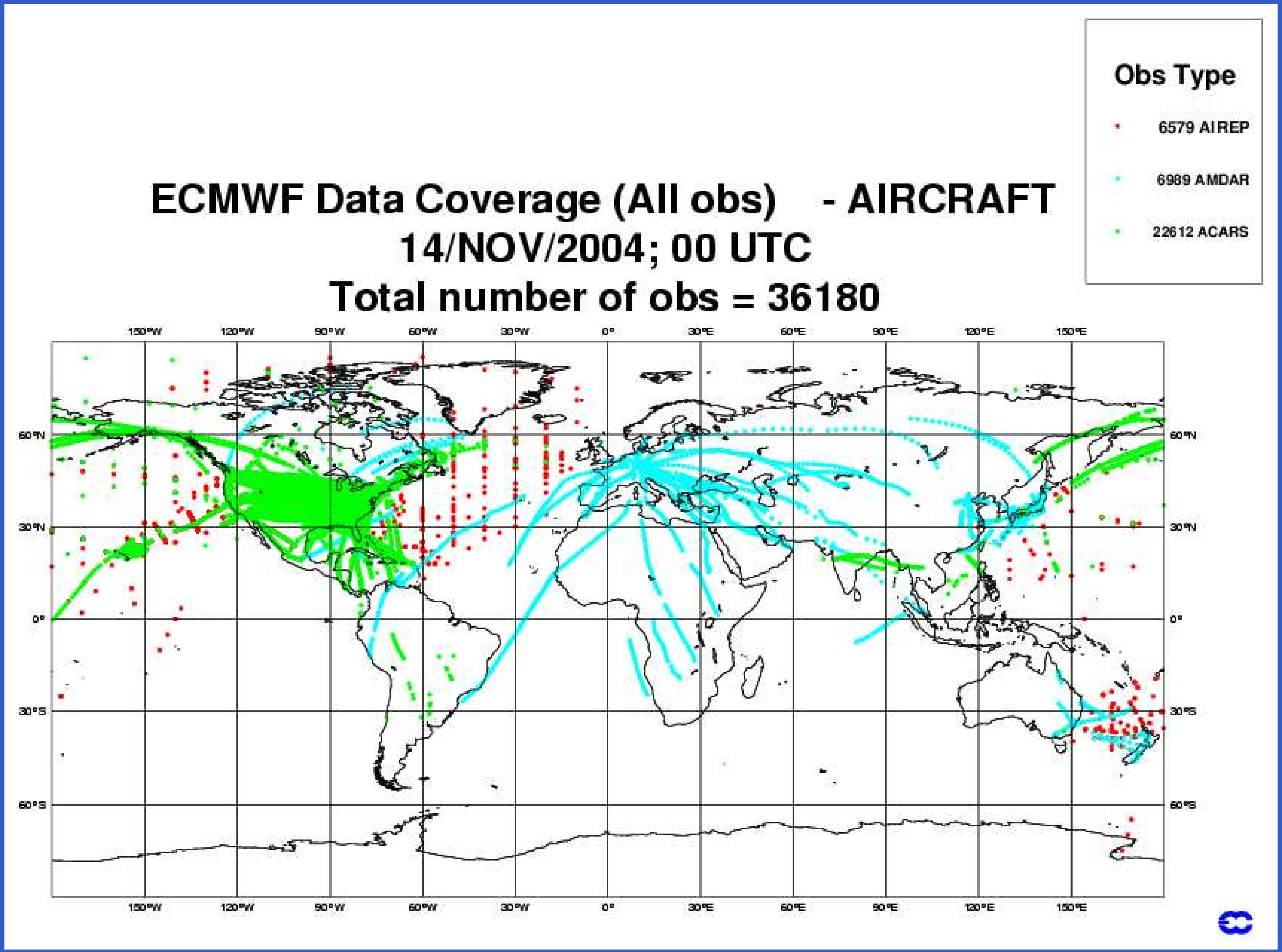

MOTIVATION and GOAL

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Spatial analysis of large nonstationary processes poses challenges in both modeling and computation.

- Need the flow-dependent nonstationary covariance
- Incomplete, irregularly distributed observational data
- Require computational efficiency

Multi-resolution based covariance

$$\Sigma = \mathcal{W}\hat{H}^2\mathcal{W}^T$$

- DWT:
  $h_{\mathcal{W}} = \mathcal{W}^{-1}\mathbf{x}$
- $\hat{H}^2 = cov(h_{\mathcal{W}})$
- Determine $\hat{H}$
- Enforce sparsity on $\hat{H}$

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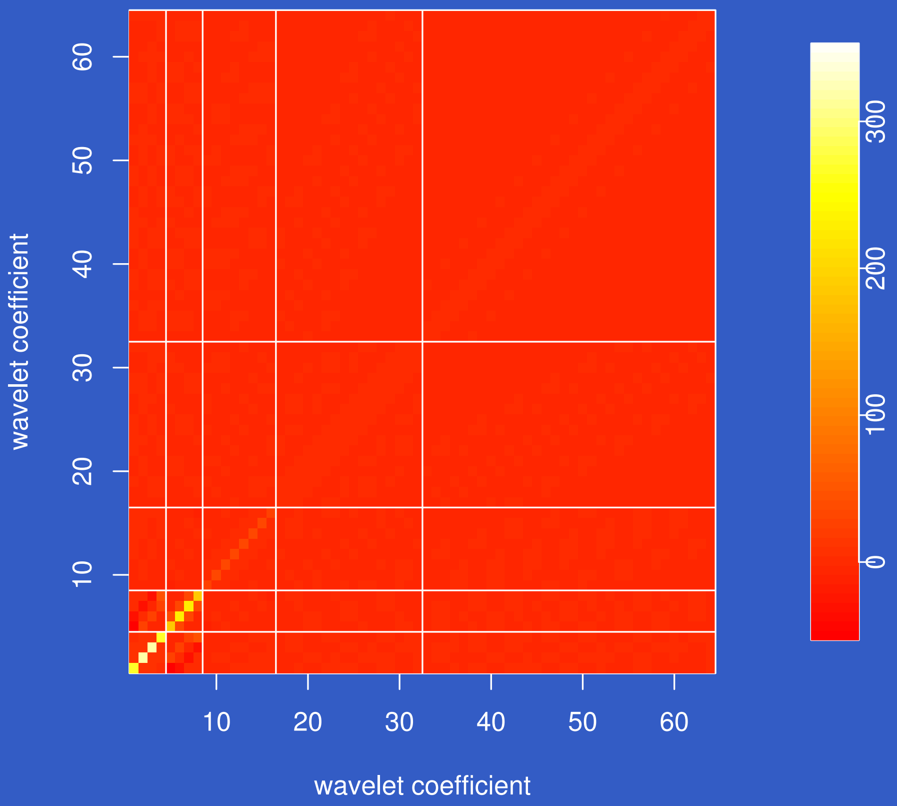

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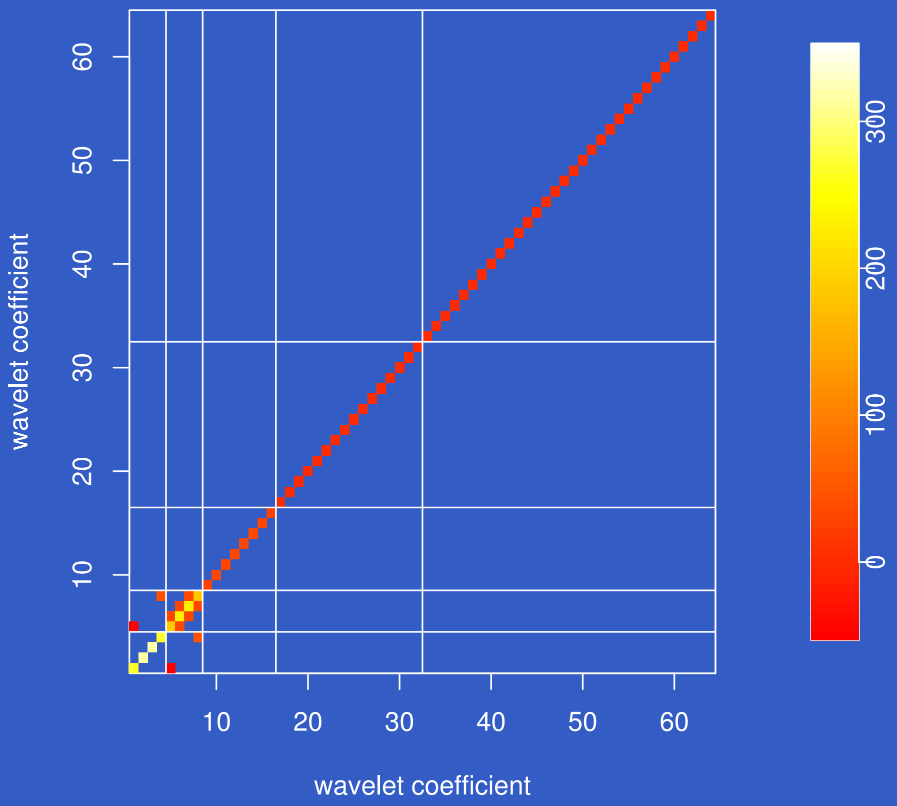
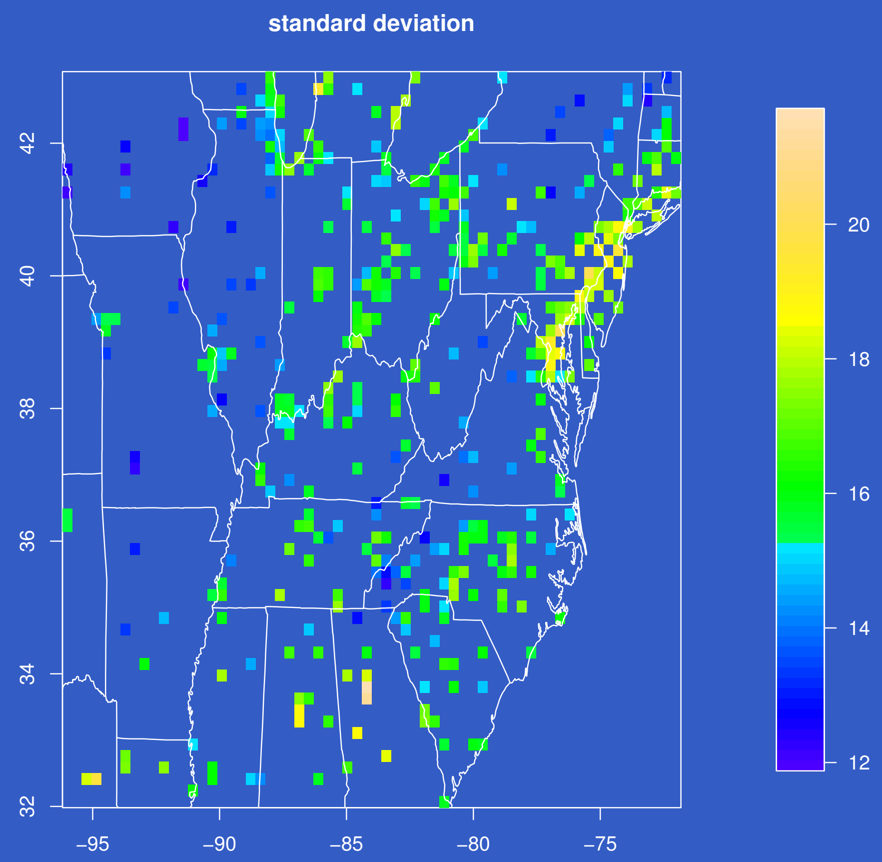


Example: Surface Ozone Data

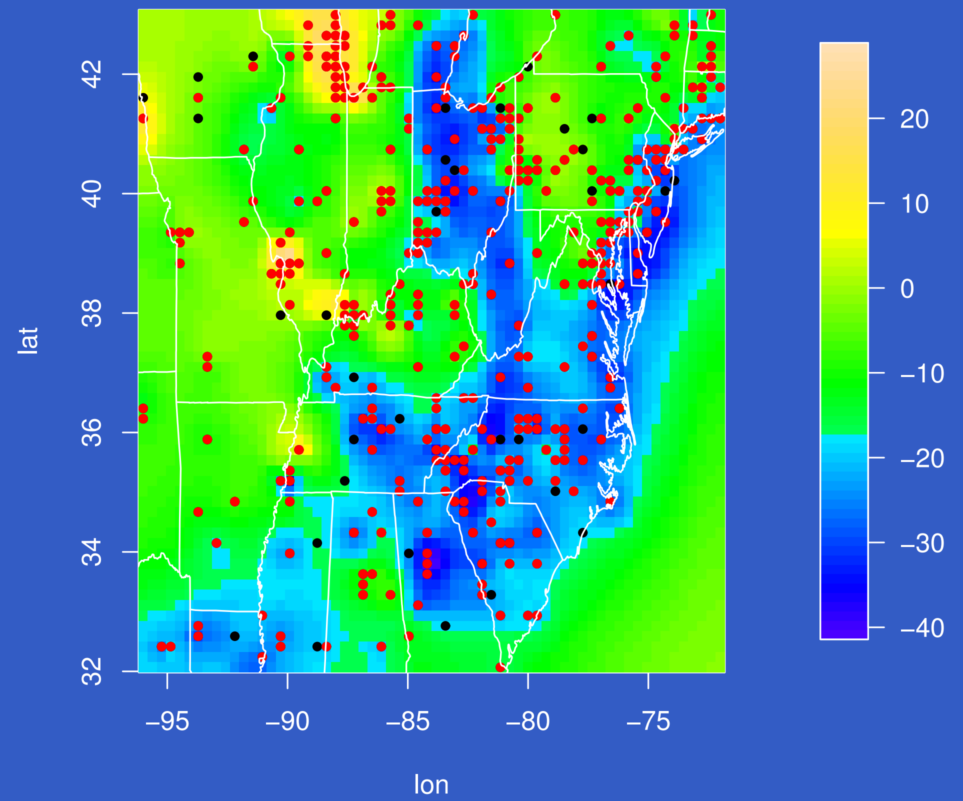


- 513 stations
- 930 realizations over 5 years
- 10% on 64-by-64 grid

EM-type approach

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_{obs} \\ \mathbf{x}_{mis} \end{pmatrix} \sim \mathcal{N}(0, \Sigma), \quad \Sigma = \mathcal{W}\hat{H}^2\mathcal{W}^T$$

- Predict $\mathbf{x}_{mis}$
  $[\,\mathbf{x}^{(k-1)} | \mathbf{x}_{obs}, \hat{H}^{(k-1)}\,]$
- $h_{\mathcal{W}} = \mathcal{W}^{-1}\mathbf{x}$
- $\ldots$
- $\hat{H}^{(k)}$



Results: Covariance Surface

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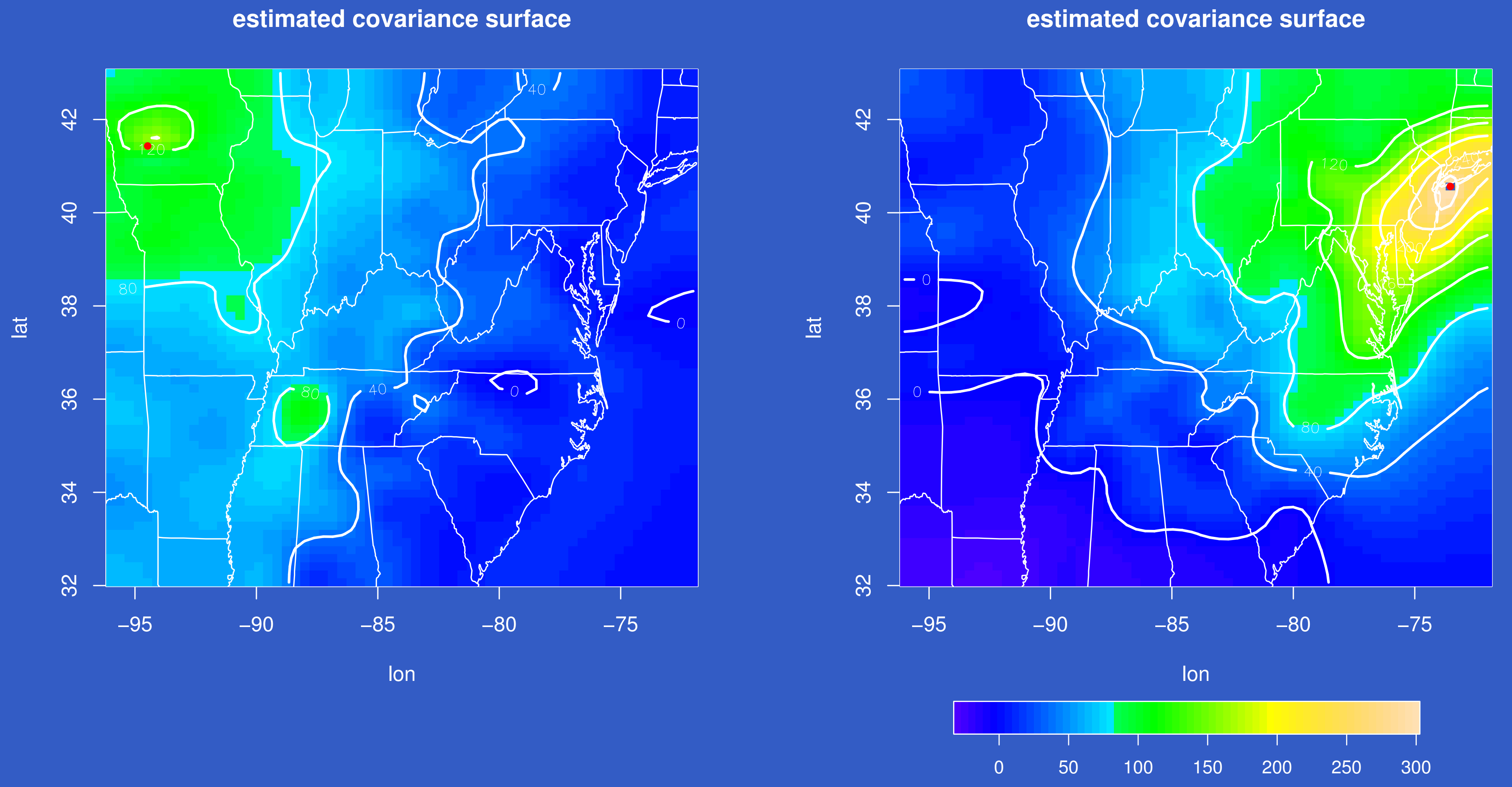
- After 5th iteration



Summary and Future Work

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  - Proof of concept: Heuristic approach
  - Parameterization: $\Sigma(\theta) = \mathcal{W} H^2(\theta) \mathcal{W}^T$

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- Application to a large data set:
  - Aurora Image data ($\sim$ 2K)
  - NCEP data ($\sim$ 100K)