Abstract:

A new ensemble filter is proposed which is suitable for very large systems with spatially smooth states, such as numerical solutions of partial differential equations in numerical weather prediction, wildfire simulation, and other geophysical applications. The analysis step uses two ensembles: a weighted forecast ensemble, obtained by advancing the system in time, and a proposal ensemble obtained by a predictor. The forecast ensemble carries the statistics of the prior, while the proposal ensemble should have a good coverage of the support of the posterior and it is assumed to be a sample from some unknown probability distribution. The corrector step assigns importance weights to the proposal ensemble, thus recovering the correct statistics of the posterior. The use of suitable proposal ensembles avoids the degeneracy problem, which plagues particle filters, and the ensemble usually does not need to be resampled.

The key to the efficiency on the new ensemble filters is density estimation in suitable spaces of smooth functions, used to compute the importance weights from the ratio of the prior density and the proposal density. The density estimates are based on probability theory in infinitely dimensional spaces, and, unlike usual density estimates, they do not deteriorate in high dimension.

Predictors by the ensemble Kalman filter and by deterministic nudging to randomly perturbed observation data are considered. Numerical experiments confirm that the new filters combine the advantages of ensemble Kalman filters and of particle filters. Predictor-corrector ensemble filters use small ensembles and can handle large updates like ensemble Kalman filters, while they can handle non-gaussian distributions correctly like particle filters, which is important in order to fully exploit observational data in numerical weather prediction models.