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Lagrangian evolution, non-Gaussianity, and statistical geometry in intermittent hydrodynamic turbulence

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Recent theoretical and numerical results on intermittency in hydrodynamic turbulence and scalar transport are described, with special emphasis on the Lagrangian evolution. First, we derive the advected delta-vee system. This simple dynamical system deals with the Lagrangian evolution of two-point velocity and scalar increments in turbulence [1, 2]. It shows that ubiquitous trends of three-dimensional turbulence such as exponential or stretched exponential tails in the probability density functions of transverse velocity increments, as well as negatively skewed longitudinal velocity increments, emerge quite rapidly and naturally from initially Gaussian ensembles. Further extensions of the system are shown to provide simple explanations for other known intermittency trends in turbulence: (i) that transverse velocity increments tend to be more intermittent than longitudinal ones, (ii) that in two dimensions, vorticity increments are intermittent while velocity increments are not, (iii) that scalar increments typically become more intermittent than velocity increments and, finally, (iv) that velocity increments in four-dimensional turbulence are more intermittent than in three dimensions. While the origin of these important trends can thus be elucidated qualitatively, predicting quantitatively the statistically steady-state levels and dependence on scale remains an open problem that would require including the neglected effects of pressure, inter-scale interactions and viscosity.

Next, we describe recent efforts to incorporate a new model for the anisotropic part of the pressure Hessian into the Lagrangian dynamics. A stochastic model for the full velocity gradient tensor is proposed, based on a closure in which spatial gradients of pressure and the viscous Laplacian term are expressed in terms of the material deformation tensor (this is also related to the tetrad model of Ref. [4]). Here the deformation tensor is modeled based on the assumption that the velocity gradient tensor's autocorrelation along its Lagrangian history is strong over a Kolmogorov time-scale, and is uncorrelated for longer times[3]. The model reproduces important geometric trends such as vorticity-strain rate alignments, joint PDFs in the so-called "R-Q" plane, as well as nearly lognormal statistics for the dissipation rate.

Finally, we describe the implications of these findings on the problem of generating synthetic 3D vector fields that mimic non-Gaussian turbulence statistics, and that may be used as initial or inlet boundary conditions for simulations. Inspired by the advected delta-vee system, a simple method is proposed based on the minimal Lagrangian map, by which an initial Gaussian field generated using random-phase Fourier modes is deformed[5]. The deformation is achieved by moving fluid particles of a sequence of low-pass filtered fields at their fixed velocity for some scale-dependent time-interval, interpolating onto a regular grid, and imposing the divergence-free condition. Statistical analysis shows that the resultant non-Gaussian field displays many properties commonly observed in turbulence, ranging from skewed and intermittent velocity gradient and increment probability distributions, preferential alignment of vorticity with intermediate strain-rate, and non-trivial vortex stretching statistics. Differences begin to appear only when interrogating the data with measures associated with intense vortex tubes that are conspicuously absent in the synthetic field. To explore the dynamical implications of these observations, the synthetic non-Gaussian fields are used as initial conditions in DNS and LES of decaying isotropic turbulence, and results are compared with initializations using Gaussian fields. The non-Gaussian synthetic fields yield more realistic results with significantly shortened initial adjustment periods.

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