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Relation between time spectrum of geomagnetic dipole moment and turbulent state in the Earth's core

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The Earth's magnetic field is generated by fluid motion in the liquid outer core, where highly turbulent flow is expected because of extremely low viscosity. Since the core is located deep inside the Earth and surrounded by weakly conductive and partially magnetized mantle and crust, it is inherently difficult to infer small-scale features of the core turbulence by direct observation. On the other hand, the geomagnetic dipole moment, which is nearly axial and intrinsically stronger than other multipoles, is a relatively robust index of the geomagnetic field and can be estimated by paleomagnetic observation throughout a long time range, giving the possibility that the turbulent nature is imprinted in its time series.

Consolini *et al.* [1] reported that the geomagnetic power spectral density was found to be proportional to $f^{-11/3}$ by using recent annual mean data at geomagnetic observatories, where f denotes frequency. They concluded that it should be related to turbulent motion at the surface of the core where a strong magnetic field is present. A spectrum of longer time scales can be estimated by historical, archeomagnetic and paleomagnetic data [2]. Recent estimation by Constable and Johnson [3] clearly shows that the power spectral density of the geomagnetic dipole moment has two corner frequencies, the first one being about 0.02 kyr^{-1} (50 kyr in period) that divides frequency ranges proportional to f^0 and $f^{-5/3}$, and the second one being about 2 kyr^{-1} (500 yr in period) from which a higher frequency range follows with a steeper spectral slope. The higher spectral range seems to connect to the one reported by Consolini *et al.* [1]. Although Constable and Johnson did not mention any implications of turbulence, their data seem to tell us much about magnetohydrodynamic motion hidden inside the Earth.

A rate of change of the axial magnetic dipole moment (m_z) can be expressed as the surface integral of the azimuthal electric current density (J_ϕ); that is,

$$\frac{dm_z}{dt} = -\frac{3}{2}\eta c^2 \oint_{r=c} J_\phi \sin^2\theta \, d\theta d\phi \equiv S(t), \tag{1}$$

provided that the Earth's core is spherical and surrounded by a solid insulator [4]. Here, (r, θ, ϕ) are the spherical polar coordinates and η and c are the magnetic diffusivity and the radius of the core, respectively. Once the power spectral density $|\tilde{m}_z(f)|^2$ is observed, it is possible to infer that of the surface current integral

$$|\tilde{S}(f)|^2 = (2\pi f)^2 |\tilde{m}_z(f)|^2, \tag{2}$$

implying that $|\tilde{S}|^2$ of the Earth's core would be characterized by three frequency ranges whose slopes are, from lower to higher frequencies, f^2, $f^{1/3}$ and $f^{-5/3}$, respectively. It is of particular interest that $|\tilde{S}|^2$ has a peak around $f = 2$ kyr^{-1} and a well-known Kolmogorov slope appears in the higher frequency range.

For further investigation, we carried out a computer simulation of three-dimensional, time-dependent, thermally driven spherical MHD dynamo. The model is almost the same as our previous geodynamo model [5], but the Ekman number is slightly lowered to 10^{-5} and artificial hyper-diffusivities are absent. The Prandtl numbers are all unity. The generated magnetic field is dominated by a quasi-stable axial dipole field. The relation (2) is confirmed by calculating $\tilde{S}$ and $\tilde{m}_z$ independently. The calculated power spectral density $|\tilde{S}|^2$ bears a remarkable resemblance to the observed one. Figure 1 shows that $|\tilde{S}|^2$ has a broad peak around the period of 5 kyr that divides the frequency ranges proportional to $f^{1/3}$ and $f^{-5/3}$.

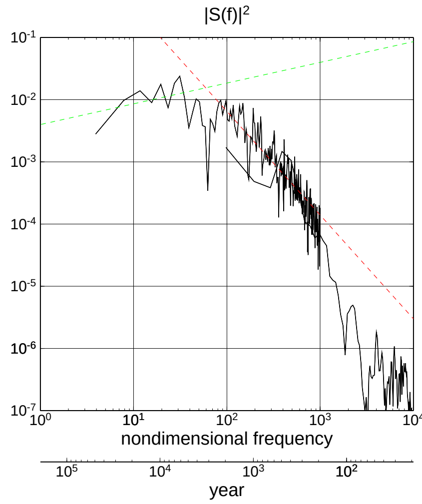

Figure 1: *A power spectral density of the surface current integral* $|\tilde{S}(f)|^2$ *estimated from the numerical model. Two data sets are used to connect lower and higher frequencies. Dotted lines represent* $f^{1/3}$ *and* $f^{-5/3}$ *slopes. Time is scaled by* $c^2/\eta = 192$ *kyr.*

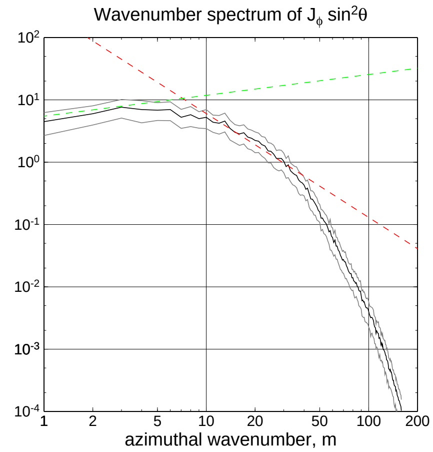

Figure 2: *Time-averaged squared Fourier coefficients of* $\int J_\phi \sin^2\theta \, d\theta$ *as a function of the azimuthal wavenumber* m*. Dotted lines represent* $m^{1/3}$ *and* $m^{-5/3}$ *slopes.*

The spatial pattern of the surface zonal current looks like a number of small-scale patches of either positive and negative signs. The shape of the patches is so elongated in the θ-direction that the sectorial components dominate when expanded by spherical harmonics. Considering a stochastic behavior of the patches in time and space, we could relate $\tilde{S}(f)$ to the time-averaged wavenumber spectrum of $J_\phi \sin\theta$. Figure 2 indeed shows similarity between them.

The surface electric current reflects the fluid motion inside the core, because an Ekman-Hartmann boundary layer forms beneath the core surface in which Coriolis and Lorentz forces are mutually related. Therefore, $|\tilde{S}|^2$ gives some information about turbulent spectrum of the core. In conclusion, it is suggested that the geomagnetic time spectrum reflects time-averaged small-scale structures of electric current and velocity inside the core. The difference of the peak frequencies in $|\tilde{S}|^2$ spectra between the Earth and the numerical model indicates that the time-averaged electric current and velocity fields in the Earth have typical wavenumber around $m = 50 \sim 100$ and their power decreases in proportion to $m^{-5/3}$ in higher wavenumbers.

References

[1] G. Consolini, P. Michelis and A. Meloni 2002. *Fluid motions in the Earth's core inferred from time spectral features of the geomagnetic field*, Phys. Rev. E **65**, 037303.

[2] R. T. Merrill, M. W. McElhinny and P. L. McFadden 1996. *The magnetic field of the Earth* (Academic Press).

[3] C. Constable and C. Johnson 2005. *A paleomagnetic power spectrum*, Phys. Earth Planet. Inter. **153**, 61–73.

[4] P. A. Davidson 2001. *An introduction to magnetohydrodynamics* (Cambridge Univ. Press).

[5] A. Sakuraba and M. Kono 1999. *Effect of the inner core on the numerical solution of the magnetohydrodynamic dynamo*, Phys. Earth Planet. inter. **111**, 105–121.