Experimental and numerical studies of the role of turbulence on current generation and magnetic field self-excitation in the Madison Dynamo Experiment

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The Madison Dynamo experiment is investigating the role of turbulence on current generation and self-excitation of magnetic fields. The geometry, a 1 meter diameter spherical vessel with a flow driven by two counter-rotating internal impellers, is motivated to a large degree by the two vortex flow proposed by Dudley and James. The geometry is shown in Fig. 1. In this talk will report on an effort to compare the results from the experiment with simulations of a similar geometry using a 3D numerical solution of the MHD equations.

The numerical model is a pseudo-spectral code using spherical harmonic basis functions in the azimuthal and polar directions and finite difference in the radial direction. An Adams-Bashforth predictor corrector technique for the advancement of the non-linear terms. A simple impeller model has been developed which drives a flow quantitatively similar to that observed in water experiments (in a geometry dimensionally identical to the sodium experiment). These flows can be dynamos, depending upon the value of the magnetic Reynolds number $Rm = \mu_0 \sigma V a$ and the fluid Reynolds number $Re = V a/\nu$ of the flow. For $Re < 420$ the flow is laminar and the dynamo transition is governed by a simple threshold in $Rm > 100$, above which a growing magnetic eigenmode is observed that is primarily of a dipole field transverse to axis of symmetry of the flow. In saturation the Lorentz force slows the flow such that the magnetic eigenmode becomes marginally stable. For $Re > 420$ and $Rm \sim 100$ the flow becomes turbulent and the dynamo eigenmode is suppressed. The mechanism of suppression is due to a combination of a time varying large-scale field and the presence

Figure 1: Photograph and schematic of the Madison Dynamo Experiment. The sphere is 1 meter in diameter. It is filled with 105–110°C liquid sodium and a flow is created by two counter-rotating impellers. Two sets of coils, one coaxial with and one transverse to the drive shafts, are used to apply various magnetic field configurations. The magnetic field induced by the flow is measured using Hall-effect sensors both on the surface of the sphere and within tubes that extend into the flow.
of fluctuation driven currents (such as those predicted by the mean-field theory) which effectively enhance the magnetic diffusivity. For higher \( Rm \) a dynamo reappears, however the structure of the magnetic field is often different from the laminar dynamo; it is dominated by a dipolar magnetic field aligned with the axis of symmetry of the mean-flow which is apparently generated by fluctuation-driven currents.

In the experiment, a fully self-sustained dynamo has not yet been observed, although there is evidence for an intermittently excited magnetic field which has structure similar. There may be evidence for intermittent self-excitation in the simulations, but the limited duration of the runs makes it difficult to determine this for certain.

The experiments have been focused on understanding the magnetic fields generated by the turbulent flows when a weak seed field is applied which shares a symmetry axis with the mean flow. Clear evidence for the presence of fluctuation driven currents is present. The EMF generated by the mean-flow and mean-magnetic field lead to currents which are unable to account for the detailed structure of the mean-magnetic field. In the experiment, there is a dipole observed in the experiment which cannot be explained by the axisymmetric mean flows, and the magnitude of the predicted fields are much larger than those observed. Similar behavior is also seen in numerical simulations of turbulent flows (subcritical for dynamo excitation) with externally applied magnetic fields.

Finally, the spectrum of the velocity field and magnetic field fluctuations are discussed. In the experiment, clear evidence for an inertial range and a dissipation scale are observed on single point measurements of the magnetic field using a hall probe and of the velocity field using LDV in the water experiment. The dissipation scale for the magnetic field moves to higher frequencies as \( Rm \) is increased. Simulations at \( Re \sim 1000 \) predict qualitatively similar behavior.

References