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Intermittency in MHD turbulence: DNS and Lagrangian averaged modeling

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Subgrid modeling of MHD flows is still under development. Most LES for hydrodynamic turbulence are based upon self-similarity or universality, in that they assume a known power law of the energy spectrum. For MHD, the kinetic energy is not a conserved quantity, and this poses a problem for the extension of such techniques to this case. Additional difficulties arise from the fact that MHD has several regimes depending on the relative strengths of the magnetic and velocity fields, their degree of alignment, and whether mechanical or magnetic energy is injected into the flow. However, some LES have been developed for particular cases. There exists LES for MHD turbulence with some degree of alignment between the fields, dissipative LES which does not model the interactions between the two fields, and LES for low magnetic Reynolds number (see [1] for references). A more generally applicable subgrid model that would also handle transitional flows (*e.g.* dynamos) is then desirable. To this end, we investigate the Lagrangian averaged magneto-hydrodynamic alpha ($LAMHD-\alpha$) model. This model we have recently tested both in 2D and in 3D and it has been used to examine the onset of the dynamo instability when the magnetic Prandtl number is small. Most of these works compared the time evolution of ideal invariants for forced and free decaying turbulence, as well as the evolution of energy spectra. Also, some statistical comparisons were performed (*e.g.* studying the behavior of probability density functions). In this work, we apply a more stringent test to this model. Intermittency is a well known feature of turbulent flows, associated with the existence of strong events localized both in space and time. Intermittency can trigger large scale events, affect the transport coefficients, or give rise to corrections in the turbulent scaling. As a result, whether a subgrid model can capture the statistics of intermittent events is of utmost importance to model astrophysical or geophysical flows. This study also requires high order statistics, thereby extending comparisons between direct numerical simulations (DNS) and α-models.

The equations for $LAMHD-\alpha$ are

$$\begin{aligned} \partial_t \mathbf{u} + \mathbf{u}_s \cdot \nabla \mathbf{u} - \mathbf{B}_s \cdot \nabla \mathbf{B} + (\nabla \mathbf{u}_s)^T \cdot \mathbf{u} + (\nabla \mathbf{B})^T \cdot \mathbf{B}_s + \nabla \pi = \nu \Delta \mathbf{u}, \\ \partial_t \mathbf{B}_s + \mathbf{u}_s \cdot \nabla \mathbf{B}_s - \mathbf{B}_s \cdot \nabla \mathbf{u}_s = \eta \Delta \mathbf{B}\,, \end{aligned} \tag{1}$$

where $\mathbf{u}$ and $\mathbf{B}$ are the velocity and magnetic fields (both divergence free), ν is the viscosity, and η is the diffusivity. The subscript s denotes smoothing obtained by inverting the relations, $\mathbf{u} = (1 - \alpha^2 \Delta)\mathbf{u}_s$, $\mathbf{B} = (1 - \alpha_M^2 \Delta)\mathbf{B}_s$. We compare intermittency in (1) to that of DNS of MHD, regarded as true at a given Reynolds number. We define the longitudinal structure function of the Elsässer variable $\mathbf{z}^+ = \mathbf{u} + \mathbf{B}$ as $S_p^+(l) \equiv \langle |\delta z_L^+|^p \rangle$ where $\delta z_L^+ = (\mathbf{z}^+(\mathbf{x}+\mathbf{l}) - \mathbf{z}^+(\mathbf{x})) \cdot \mathbf{l}/l$ is the longitudinal increment of $\mathbf{z}^+$. Four sets of 2D simulations were computed with periodic boundary conditions, one set of MHD DNS with 1024^2 grid points, and three sets of $LAMHD-\alpha$ simulations, 512^2 and $\alpha = \alpha_M = 6/512$, 256^2 and $\alpha = \alpha_M = 6/256$, and 256^2 and $\alpha = \alpha_M = 6/128$. All simulations were identical with regards to dissipation ($\eta = \nu = 1.6 \times 10^{-4}$) and forcing (in the Fourier ring $k = [1, 2]$ with random phases in momentum and vector potential).

With the Extended Self-Similarity (ESS) hypothesis we determined the relative scaling exponents, ξ_p^+, from $S_p^+(l) \sim [L^+(l)]^{\xi_p^+}$ where $L^+ \propto l$ from the Kármán-Howarth theorem (see [1] for details). Figure 1 compares the scaling exponent, ξ_p^+, for the DNS runs and the three sets of *LAMHD*$-\alpha$ runs. In the figure, the She-Lévêque (SL) formula for MHD is shown as a reference, $\frac{\xi_p}{\xi_3} = \frac{p}{6} + 1 - \left(\frac{1}{2}\right)^{p/3}$. The $\alpha-$model captured the high-order statistics and the anomalous scaling of the longitudinal structure function exponents (to within the errors of our statistics), with a net gain in speed close to a factor of 16. For lower order structure functions, very little contamination of the scaling could be detected at scales larger than α.

In current sheets, where magnetic reconnection occurs, the magnetic field and the current rapidly change sign. To preserve reliable statistics of these events in subgrid models of MHD turbulence is of importance in many problems. In order to measure fast oscillations in sign of a field on arbitrary small scales, the cancellation exponent was introduced (see [2] for references). The signed measure for the current $j_z(\mathbf{x})$ on a set $Q(L)$ of size L is $\mu_i(l) = \int_{Q_i(l)} d\mathbf{x}\, j_z(\mathbf{x}) \,/ \int_{Q(L)} d\mathbf{x}\, |j_z(\mathbf{x})|$ where $\{Q_i(l)\} \subset Q(L)$ is a hierarchy of disjoint subsets of size l covering $Q(L)$. The partition function χ measures the cancellations at a given lengthscale l, $\chi(l) = \sum_{Q_i(l)} |\mu_i(l)|$. We can study the scaling behaviors of the cancellations defining the cancellation exponent κ, where $\chi(l) \sim l^{-\kappa}$. Positive κ indicates fast changes in sign on small scales. This exponent can also be related with the fractal dimension D of the structures, $\kappa = (d - D)/2$, where d is the number of spatial dimensions of the system. The evolution of the cancellation exponent as a function of time for free decaying simulations is shown in Fig. 2. The maximum of κ takes place slightly later than the maximum of magnetic dissipation. Note that the alpha-model captures the time evolution of the cancellation exponent, as well as the fractal structure of the problem as time evolves.

Future challenges will include implementation of *LAMHD*$-\alpha$ in domains with boundaries and the study of intermittency for magnetic Prandtl numbers besides unity.

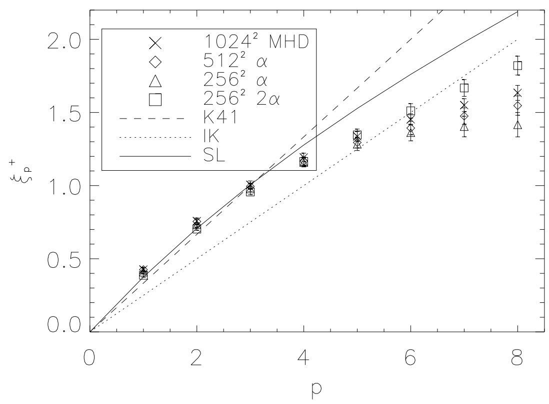

Figure 1: *Structure function scaling exponent:* ξ_p^+ *versus* p. 1024^2 *MHD are the pluses, for LAMHD*$-\alpha$ 512^2 *are the diamonds,* 256^2 *(*$\alpha = 6/256$*) are triangles, and* 256^2 *(*$\alpha = 6/128$*) are the squares. The error bars are the error to the least-squares fit.*

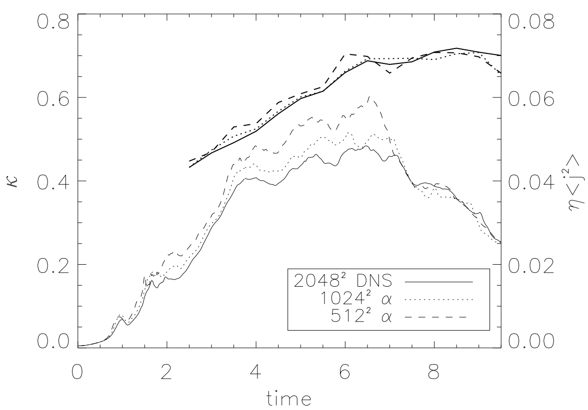

Figure 2: *Time history of the cancellation exponent (thick lines) for the three free decaying runs, and of* $\eta \left\langle j_z^2 \right\rangle$*, where the brackets denote spatial average.*

References

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[2] J. Pietarila Graham, P. D. Mininni, and A. Pouquet 2005. *Cancellation exponent and multifractal structure in 2D MHD: Direct numerical simulations and LAMHD*$-\alpha$, Phys. Rev. E **72**, 045301.