Planetary and stellar dynamos likely result from turbulent motions in magnetofluids with kinematic viscosities that are small compared to their magnetic diffusivities. Laboratory experiments are in progress to produce similar dynamos in liquid metals. Plasmas in stellar interiors and conducting fluids in planetary cores and liquid sodium experiments are characterized by a magnetic Prandtl number $P_M$ (the ratio of the kinematic viscosity $\nu$ to the magnetic diffusivity $\eta$) much smaller than one. As a few examples, the magnetic Prandtl number in the solar convective region is estimated to be $P_M \approx 10^{-5} - 10^{-6}$, and in the Earth’s core $P_M \approx 10^{-5}$.

While numerical simulations of dynamo action in these objects are available, the large values of the kinetic ($R_V$) and magnetic ($R_M$) Reynolds numbers forbid a study using realistic values of $P_M$. Simulations of the geodynamo or the solar convective region are often done for $P_M \sim 1$. While the proper separation of the kinetic and magnetic dissipation scales cannot be achieved in these simulations, values of $P_M$ much smaller than one can be reached under more idealized conditions. Pseudospectral methods in periodic boxes give an excellent tool to study the behavior of magnetohydrodynamic (MHD) turbulence in the regime $P_M < 1$.

In this talk, we review recent results from simulations of helical and non-helical dynamos at $P_M < 1$ using pseudospectral codes in periodic boxes [1, 2, 3, 4]. To extend the range in $P_M$ in the simulations, subgrid scale (SGS) models of MHD turbulence were used. We discuss some of these models with particular emphasis in the Lagrangian Average MHD (LAMHD) equations [5, 6, 7]. To validate results from SGS models, direct numerical simulations (DNS) of MHD turbulence with resolutions up to $1024^3$ grid points are discussed.

Three flows are considered: the flow resulting from Taylor-Green forcing [1, 2], the result of Roberts forcing [3], and the result of Arn’old-Beltrami-Childress (ABC) forcing [4]. The first case corresponds to a flow with no net helicity that gives amplification of magnetic energy in large and small scales, while the second forcing gives a helical flow where only dynamo action in small scales is permitted by introducing mechanical energy in the largest available scale. Finally, ABC forcing at intermediate scales gives an example of a helical case where large scale magnetic amplification is allowed. For this forcing, values of $P_M$ down to $5 \times 10^{-3}$ are reached. The results obtained for such a low value of $P_M$ are expected to be of relevance for astrophysical and geophysical applications, as well as for laboratory dynamos.

For all cases studied, dynamo action is observed to persist at the smallest values of $P_M$ that can be reached. Moreover, for values of $P_M$ smaller than $\sim 0.1$ an independence of the threshold in the magnetic Reynolds number $R_M$ with $P_M$ is observed (Figure 1). While for the Taylor-Green (non-helical) forcing and the Roberts forcing (helical, but with magnetic amplification only at small scales) a sharp increase in the critical parameter is observed before reaching the asymptotic regime, in the ABC case almost no such increase is found.

For simulations with magnetic Reynolds numbers $R_M > R_M^*$, the saturation and nonlinear behavior of dynamos with $P_M < 1$ is discussed. In helical flows, as magnetic energy saturates, a large scale magnetic field develops (at scales larger than the forcing scale). It is of interest to know what happens with the amplitude
Figure 1: Critical magnetic Reynolds $R_M^c$ to have dynamo action as a function of $R_V$ ($P_M = R_M/R_V$) for different forcing functions: Taylor-Green (crosses), Roberts (squares), and ABC (diamonds). Points connected with solid lines were obtained from DNS, while points connected with dashed lines were obtained using the LAMHD equations.

of the magnetic field as the value of $P_M$ is decreased. However, different results are obtained when the space of parameters is explored keeping $R_M$ constant and increasing $R_V$, or keeping $R_V$ constant and decreasing $R_M$ as another way to decrease $P_M$. As the value of $P_M$ is decreased, if $R_V$ is kept constant and $R_M$ (and thus $P_M$) decreases, the saturation of the dynamo takes place for lower values of the magnetic energy. But if $R_M$ is kept constant and $R_V$ is increased (and thus $P_M$ decreases), first a decrease in the saturation value of the magnetic energy is observed, followed by a regime where its amplitude seems to be roughly independent of $P_M$.

References


