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Turbulent dynamos at low magnetic Prandtl number

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Planetary and stellar dynamos likely result from turbulent motions in magnetofluids with kinematic viscosities that are small compared to their magnetic diffusivities. Laboratory experiments are in progress to produce similar dynamos in liquid metals. Plasmas in stellar interiors and conducting fluids in planetary cores and liquid sodium experiments are characterized by a magnetic Prandtl number P_M (the ratio of the kinematic viscosity ν to the magnetic diffusivity η) much smaller than one. As a few examples, the magnetic Prandtl number in the solar convective region is estimated to be $P_M \approx 10^{-5} - 10^{-6}$, and in the Earth's core $P_M \approx 10^{-5}$.

While numerical simulations of dynamo action in these objects are available, the large values of the kinetic (R_V) and magnetic (R_M) Reynolds numbers forbid a study using realistic values of P_M. Simulations of the geodynamo or the solar convective region are often done for $P_M \sim 1$. While the proper separation of the kinetic and magnetic dissipation scales cannot be achieved in these simulations, values of P_M much smaller than one can be reached under more idealized conditions. Pseudospectral methods in periodic boxes give an excellent tool to study the behavior of magnetohydrodynamic (MHD) turbulence in the regime $P_M < 1$.

In this talk, we review recent results from simulations of helical and non-helical dynamos at $P_M < 1$ using pseudospectral codes in periodic boxes [1, 2, 3, 4]. To extend the range in P_M in the simulations, subgrid scale (SGS) models of MHD turbulence were used. We discuss some of these models with particular emphasis in the Lagrangian Average MHD (LAMHD) equations [5, 6, 7]. To validate results from SGS models, direct numerical simulations (DNS) of MHD turbulence with resolutions up to 1024^3 grid points are discussed.

Three flows are considered: the flow resulting from Taylor-Green forcing [1, 2], the result of Roberts forcing [3], and the result of Arn'old-Beltrami-Childress (ABC) forcing [4]. The first case corresponds to a flow with no net helicity that gives amplification of magnetic energy in large and small scales, while the second forcing gives a helical flow where only dynamo action in small scales is permitted by introducing mechanical energy in the largest available scale. Finally, ABC forcing at intermediate scales gives an example of a helical case where large scale magnetic amplification is allowed. For this forcing, values of P_M down to 5×10^{-3} are reached. The results obtained for such a low value of P_M are expected to be of relevance for astrophysical and geophysical applications, as well as for laboratory dynamos.

For all cases studied, dynamo action is observed to persist at the smallest values of P_M that can be reached. Moreover, for values of P_M smaller than ~ 0.1 an independence of the threshold in the magnetic Reynolds number R_M^c with P_M is observed (Figure 1). While for the Taylor-Green (non-helical) forcing and the Roberts forcing (helical, but with magnetic amplification only at small scales) a sharp increase in the critical parameter is observed before reaching the asymptotic regime, in the ABC case almost no such increase is found.

For simulations with magnetic Reynolds numbers $R_M > R_M^c$, the saturation and nonlinear behavior of dynamos with $P_M < 1$ is discussed. In helical flows, as magnetic energy saturates, a large scale magnetic field develops (at scales larger than the forcing scale). It is of interest to know what happens with the amplitude

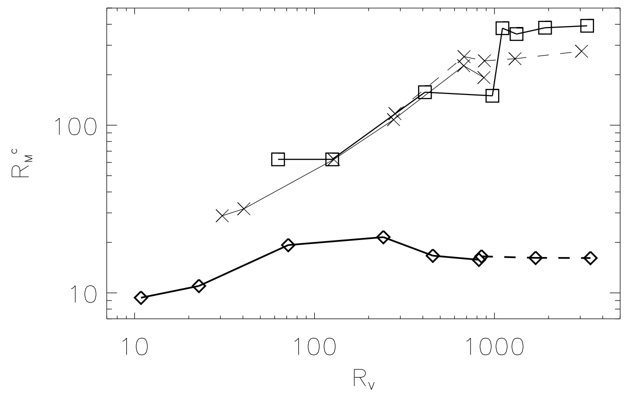

Figure 1: *Critical magnetic Reynolds R_M^c to have dynamo action as a function of R_V ($P_M = R_M/R_V$) for different forcing functions: Taylor-Green (crosses), Roberts (squares), and ABC (diamonds). Points connected with solid lines were obtained from DNS, while points connected with dashed lines were obtained using the LAMHD equations.*

of the magnetic field as the value of P_M is decreased. However, different results are obtained when the space of parameters is explored keeping R_M constant and increasing R_V, or keeping R_V constant and decreasing R_M as another way to decrease P_M. As the value of P_M is decreased, if R_V is kept constant and R_M (and thus P_M) decreases, the saturation of the dynamo takes place for lower values of the magnetic energy. But if R_M is kept constant and R_V is increased (and thus P_M decreases), first a decrease in the saturation value of the magnetic energy is observed, followed by a regime where its amplitude seems to be roughly independent of P_M.

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