GTP Workshop on
Modeling MHD Turbulence; Applications to Planetary and Stellar dynamos
at NCAR, 27-30 June, 2006, Boulder, CO, USA

Fluctuations of magnetic induction in von Kármán swirling flows, applications to unconstrained dynamo effect.

Philippe Odier, Romain Volk, and Jean-François Pinton
École Normale Supérieure de Lyon, 46 allée d'Italie, 69364 Lyon Cedex 07, FRANCE

The von Kármán flows generated in the gap between counter-rotating impellers have been considered by several groups as a possible candidate [1, 2, 3] in the search for a laboratory demonstration of a homogeneous fluid dynamo that would be less constrained than the Riga and Karlsruhe designs. In these experiments, the fluid's magnetic diffusivity is many orders of magnitude larger than its hydrodynamic viscosity. Therefore the flow Reynolds number needs to be very large in order for non-linearities to develop in the magnetic induction. As a consequence, the hydrodynamic flow is very turbulent, and many questions arise concerning the influence of turbulence on the bifurcation threshold and the dynamics in an eventual saturated regime.

This problem is very complex, and many studies so far have focused on the dynamo capacity of the average flow engineered in the von Kármán (VK) geometry. Here, 'average' has the meaning of 'time-average'. A time-averaged flow field is derived from measurements, where the averaging time is much longer than the one characteristic of the forcing of the flow (estimated, for instance, as the period of rotation of the driving impellers). This stationary flow profile is then usually inserted into a kinematic numerical solver, in which the induction equation is solved with the velocity field kept constant in time [4]. This type of studies have shown the possibility of dynamo action in the average VK flows, and the underlying induction processes have been measured in sodium and gallium flows [3, 5] and analyzed in details [6]. The helicity and differential rotation present in the von Kármán mean flow cooperate to generate a self-sustained dynamo. The existence of a kinematic dynamo threshold for the VK flows, together with the possibility to bring its value within experimental reach (in terms of power requirements) has motivated the sodium experiments in Maryland, Cadarache and Wisconsin. However, due to the strong fluctuations, the instantaneous mean flow structure can differ significantly from the time-averaged flow.

The purpose of the present study was to relate the fluctuations in time of magnetic induction to the large scale fluctuations in the geometry of VK flows. We use a VK flow of liquid Gallium, and we study the magnetic induction in the presence of an externally applied field. The magnetic Reynolds number in Gallium flows reaches values of order 1: induction effects are measurable and the non-linearities are not yet strong so that the induced magnetic field is directly related to the velocity gradients in the flow.

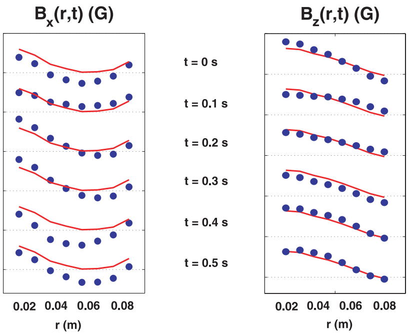

Figure 1: Induction profiles, axial applied field. (left) transverse induced field; (right) axial induced field. The dots are the instantaneous profiles and the solid line is the time averaged profile.

The induction measurements use a probe made of a line of magnetic field sensors. It samples simultaneously one component of the magnetic field at several locations on a radius of the cylindrical vessel, in the median plane of the flow. It thus allows to study the time evolution of radial induction profiles. Figure 1 shows examples of such profiles. For an axial applied field, the transverse induced field (left figure) is due to the differential rotation imposed by the counter-rotating disks (the Ω effect), and the axial induced field (right figure) is due to the stretching of the applied field by the stagnation point in the recirculation flow. The profiles are shown at time intervals equal to the period of rotation of the disks, and compared to the mean profile (solid line), averaged over 100 rotation periods of the disks. One observes significant fluctuations with respect to the time-averaged curve. In addition, the points in the profiles tend to vary as a block. This was evidenced by computing the correlation function between different sensors, that showed a typical correlation length of the order of the cylinder radius. In order to caracterize more precisely these fluctuations, we used two different approaches :

1. One can compute a global distance between the mean flow induction profile and a given instantaneous realisation. Using the $\mathcal{L}_2$ norm, we define:

$$E_k(t) = \sqrt{\frac{1}{N}\sum_{i=1}^{N=8}\left(B_k(r_i,t) - \langle B_k(r_i)\rangle\right)^2}\ , \tag{1}$$

   where k denotes a given composent of the filed and i a given sensor, N being the number of sensors in the profile. In units of the applied magnetic field, we measure $\langle E\rangle \sim 0.07\ B_0$ for situations in which the maximum of the induced field $(\max_{r_i}\langle B\rangle(r_i))$ is also of the order of $0.1\ B_0$. These observations indicated that instantaneous induction profiles differ significantly from the time averaged computation. In addition, we also observe that the distance E has large fluctuations away from its mean value, in particular in the case of the induction due to differential rotation.

2. Because of the strong correlation in the signal measured by successive elements in the magnetic array, the induction profiles are smooth and are well described by polynomials of order three. Using the symmetries associated to the von Kármán flows, certain coefficients of the polynoms can be simply related to specific induction mechanisms (Ω effect, streching or compression of field lines). A study of the statistics of these coefficients enabled us to show that the fluctuations of the induction profiles denote not only a change in the amplitude of the poloidal and toroidal components of the flow but also in its symmetries and its overall structure: we computed that the flow doesn't spend more than 50% of the time in its mean von Kármán configuration.

In conclusion, one can say that these slow global changes in the geometry of the flow may not favor dynamo action. All configurations may not be consistent with self generation – in the sense that any instantaneous velocity field may not lead to a positive growth rate when inserted in the induction equation for a kinematic dynamo computation. This indicates that a stable flow configuration is desirable for the self-generation of a stationary dynamo. As a result, depending on the forcing and large scale hydrodynamic evolution of the flow, one may have to be cautious with approaches that estimate the dynamo threshold from mean flow geometries. The observed slow dynamics is associated with important changes in the flow topology, and a mean field kinematic simulation may underestimate the threshold. The results briefly presented here are decribed in details in the Ph-D thesis of Romain Volk [7] defended in 2005 and will be published in [8].

References

[1] N.L. Peffley, A.B. Cawthrone, D.P. Lathrop 2000. Phys. Rev. E **61**, 5287.
[2] R. O'Connell et al. 2000. *Dynamo and dynamics, a mathematical Challenge, (NATO)* **vol.26**, 59 – 66.
[3] M. Bourgoin et al. 2002. Phys. Fluids **14**, 3046 – 3058.
[4] F. Ravelet, A. Chiffaudel, F. Daviaud, J. Léorat 2005. Phys. Fluids **17**, 117104.
[5] M. Bourgoin et al. 2004. Magnetohydrodynamics **1**, 13 – 31.
[6] M. Bourgoin, P. Odier, J.-F. Pinton, Y. Ricard 2004. Phys. Fluids **16**, 2529 – 2547.
[7] R. Volk, Ph-D thesis, École Normale Supérieure de Lyon 2005.
[8] R. Volk, P. Odier, J.-F. Pinton 2006. to appear in Phys. Fluids. (http://arxiv.org/abs/physics/0511204)