GTP Workshop on
Modeling MHD Turbulence; Applications to Planetary and Stellar dynamos
at NCAR, 27-30 June, 2006, Boulder, CO, USA

Critical issues to get right about stellar dynamos

Axel Brandenburg
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Small scale versus large scale dynamos. A good definition of large scale and small scale dynamos is not available. For now, let us say that small scale dynamos have no mean flow ($\overline{\mathbf{U}} = 0$) and produce no mean field ($\overline{\mathbf{B}} = 0$). Here we reserve ourselves some freedom in the definition of meaningful averages (ensemble, time, or spatial averages over one or two coordinate directions, depending on the nature of the problem). Large scale dynamos produce a mean field ($\overline{\mathbf{B}} \neq 0$), but may or may not have a mean flow ($\alpha\Omega$ and $\overline{\mathbf{W}} \times \overline{\mathbf{J}}$ versus α^2 dynamos, for example). By this definition, dynamos in Taylor-Green flows [1] do have a finite time-averaged flow and would not be small scale dynamos.

Large scale dynamos. All known large scale dynamos ($\alpha\Omega$, $\overline{\mathbf{W}} \times \overline{\mathbf{J}}$, and α^2 dynamos) produce magnetic helicity, which reacts back on the dynamo. As a consequence, the mean field saturates at a low value, $\overline{\mathbf{B}}^2 \ll B_{\rm eq}^2 \equiv \langle \mu_0 \rho \mathbf{u}^2 \rangle$. It is demonstrated (Fig. 1) that, by allowing for magnetic helicity fluxes out of the domain, the large scale field is able to saturate at equipartition field strength.

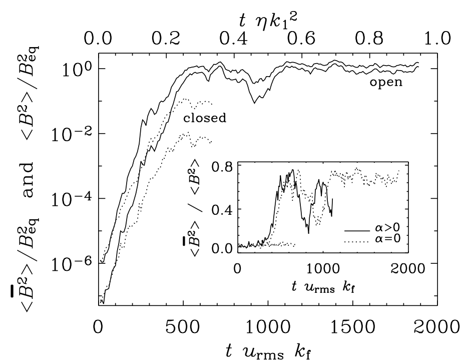

Figure 1: *Evolution of the energies of the total field $\langle \mathbf{B}^2 \rangle$ and of the mean field $\langle \overline{\mathbf{B}}^2 \rangle$, in units of $B_{\rm eq}^2$, for runs with non-helical forcing and open or closed boundaries; see the solid and dotted lines, respectively. The inset shows a comparison of the ratio $\langle \overline{\mathbf{B}}^2 \rangle / \langle \mathbf{B}^2 \rangle$ for nonhelical ($\alpha = 0$) and helical ($\alpha > 0$) runs. For the nonhelical case the run with closed boundaries is also shown (dotted line near $\langle \overline{\mathbf{B}}^2 \rangle / \langle \mathbf{B}^2 \rangle \approx 0.07$). Adapted from Ref. [2].*

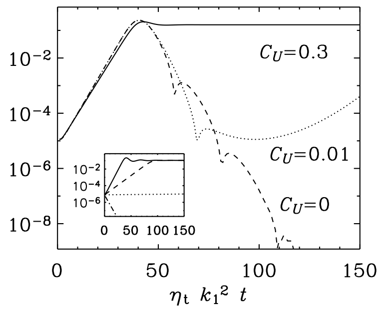

Figure 2: *Evolution of the field strength obtained by solving the mean field equations with vertical advection (solid line, $C_U = 0.3$) and without it (dashed line, $C_U = 0$). Here, $C_U = |\overline{\mathbf{U}}|_{\max}/(\eta_{\rm t} k_1)$ is a nondimensional measure of the strength of advection out of the dynamo domain. The dotted curve, obtained for $C_U \ll 1$, shows that even weak advection can affect the long-term evolution of magnetic field. The inset shows similar results for $C_U = 0.1$ (solid), 1.5 (dashed), 2 (dotted) and 3 (dash-dotted). Adapted from Ref. [3].*

The results of simulations are qualitatively, and in some cases also quantitatively, well reproduced by mean field models where the effect of magnetic helicity fluxes enters into the dynamical feedback formula for the magnetic alpha effect (even when there is no kinetic alpha effect!).

Magnetic helicity fluxes that are known to work include the shear-driven Vishniac-Cho flux [4, 5, 6], which can be written in the form $\overline{\mathbf{F}} \propto (\overline{\mathsf{S}}\,\overline{\mathbf{B}}) \times \overline{\mathbf{B}}$ and an advectively driven flux [3] of the form $\overline{\mathbf{F}} \propto \alpha_{\rm M} \overline{\mathbf{U}}$, where $\alpha_{\rm M}$ is the magnetic α effect. The former is the one operating predominantly in the simulations in Fig. 1, while the latter one operates in the mean field model shown in Fig. 2.

Small scale dynamos. An explanation is in order as to why simulations of dynamo action in spherical shells may not yet have shown strong large scale dynamos. The simulations of Brun et al. [7] show dynamo action at unit magnetic Prandtl number ($\mathrm{Pr}_M = 1$). As the value of Pr_M is decreased, one must increase the fluid Reynolds number Re at least by the same amount to maintain the same magnetic Reynolds number $R_{\rm m}$, but this is already prohibitively expensive. Solar-like simulations at $\mathrm{Pr}_M < 1$ have not yet been considered, but it is conceivable that the critical magnetic Reynolds number, $R_{\rm m,crit}$, increases with decreasing Pr_M, as is found for typical small scale dynamos with zero mean flow [8]. Thus, the tentative suggestion is that the simulations of Brun et al. show dynamo action that belongs to the class of small scale dynamos (even though they do have a mean flow). This type of dynamo action would go away for smaller value of Pr_M, provided they value of $R_{\rm m}$ is still not very large. At the same time, the large scale dynamo effect may still be sub-critical, i.e. shear and the effective α, or some other large scale effect, are still too weak, and the effective turbulent diffusivity is still too large.

Implications for LES. The indications are that, at low values of Pr_M, when the values of Rm are still small enough to allow a direct simulation of the induction equation, LES (including less advanced "tricks" such as Smagorinsky and hyper viscosity) for the momentum equation, produce accurate results [8] for the onset of dynamo action. However, similar approaches for the magnetic field are difficult and often not successful [9, 10]. Successful LESs for MHD would need to incorporate magnetic helicity fluxes (for large scale dynamos) and must somehow incorporate the fast growth at the Kazantsev (resistive) scale (for small scale dynamos).

References

[1] Y. Ponty, P. D. Mininni, D. C. Montgomery, J.-F. Pinton, H. Politano, A. Pouquet 2005. *Numerical Study of Dynamo Action at Low Magnetic Prandtl Numbers*, Phys. Rev. Lett. **94**, 164502.

[2] A. Brandenburg 2005. *The case for a distributed solar dynamo shaped by near-surface shear*, Astrophys. J. **625**, 539 – 547.

[3] A. Shukurov, D. Sokoloff, K. Subramanian and A. Brandenburg 2006. *Galactic dynamo and helicity losses through fountain flow*, Astron. Astrophys. **448**, L33 – L36.

[4] E. T. Vishniac and J. Cho 2001. *Magnetic helicity conservation and astrophysical dynamos*, Astrophys. J. **550**, 752 – 760.

[5] K. Subramanian and A. Brandenburg 2004. *Nonlinear current helicity fluxes in turbulent dynamos and alpha quenching*, Phys. Rev. Lett. **93**, 205001.

[6] K. Subramanian and A. Brandenburg 2006. *arXiv: astro-ph/0509392*, Magnetic helicity density and its flux in inhomogeneous turbulence.

[7] A. S. Brun, M. S. Miesch and J. Toomre 2004. *Global-scale turbulent convection and magnetic dynamo action in the solar envelope*, Astrophys. J. **614**, 1073 – 1098.

[8] A. A. Schekochihin, N. E. L. Haugen, A. Brandenburg, S. C. Cowley, J. L. Maron and J. C. McWilliams 2005. *Onset of small scale dynamo at small magnetic Prandtl numbers*, Astrophys. J. **625**, L115 – L118.

[9] A. Brandenburg and G. R. Sarson 2002. *The effect of hyperdiffusivity on turbulent dynamos with helicity*, Phys. Rev. Lett. **88**, 055003.

[10] N. E. L. Haugen and A. Brandenburg 2006. *arXiv: astro-ph/0412666*, Hydrodynamic and hydromagnetic energy spectra from large eddy simulations.