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Energy transfers in anisotropic MHD turbulence

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Navier-Stokes as well as magnetohydrodynamic (MHD) turbulence has long been described in terms of energy spectra and cascades of ideal quadratic invariants such as energy or enstrophy. The concept of energy transfer between eddies is closely related to the energy cascade picture and a number of numerical studies have been devoted to the characterization of nonlinear energy exchanges between different scales in fully developed turbulence. To the best of our knowledge, former studies have focused on isotropic turbulence [1, 2, 3, 4, 5]. In that case, the Fourier representation of the velocity and magnetic field variables ($u_i(\vec{k})$ and $b_i(\vec{k})$) is commonly adopted and the Fourier space is often split into shells of wave vectors s_n that contain all the wave vectors satisfying $k_n \leq ||\vec{k}|| < k_{n+1}$. The study of energy exchanges between the modes that belong to these shells allows to determine the direction of cascade as well as the intensity of energy fluxes [6, 7, 8, 9, 10].

In MHD, especially in applications to planetary and stellar dynamos, turbulence can however rarely be considered as isotropic. Indeed, external magnetic fields and rotation definitively impose an anisotropy direction and have a strong influence on the energy cascade. For instance, it is well known that rotation tends to inhibit the energy cascade towards the small scales. Similarly, a strong magnetic field tends to produce quasi two-dimensional turbulence in which the cascade mechanisms are significantly different from those observed in fully developed three-dimensional turbulence. The present workshop is thus the perfect occasion to adapt the study of energy transfers to systems for which a clear direction of anisotropy is observed. Such systems are far from being exceptional. Beside the planetary and stellar dynamos problems, anisotropy is also commonly observed in various hydrodynamic and magneto-hydrodynamic turbulent systems such as liquid metal flows submitted to an external magnetic field, atmospheric turbulence influenced by the Earth rotation as well as interstellar magnetized plasmas.

In all these cases, the statistical distribution of energy between velocity and/or magnetic field modes depends not only on the norm of the wave vector $k = ||\vec{k}||$ but also of the angle between this wave vector and the direction of anisotropy described by the vector $\vec{A}$, $\cos\theta = \vec{k}\cdot\vec{A}/(|\vec{k}|\,|\vec{A}|)$. Typically, $\vec{A}$ represents a rotation vector or a magnetic field, though it could also represent an electric field or the acceleration vector due to gravitation. The use of shell decomposition of the Fourier space is thus no longer appropriate to describe these systems and a more refined decomposition is required. In the analysis that will be reported at the workshop, we have adopted a partition of the Fourier space that remains based on the shell decomposition. However, each shell is itself split into subsets $s_{n,\alpha}$ defined by a range of angles ($\theta_\alpha \leq \theta < \theta_{\alpha+1}$) as well as by a range of wave vector amplitudes ($k_n \leq ||\vec{k}|| < k_{n+1}$). The subsets $s_{n,\alpha}$ have the shape of a ring in the Fourier space and the energy exchanges between these sets are referred to as ring-to-ring transfers.

The complete knowledge of ring-to-ring transfers yields a fairly detailed picture of the possible cascade mechanisms in anisotropic turbulence. The traditional shell-to-shell transfers (radial flux of energy) are easily derived from the ring-to-ring transfers by summing over α. Quantities that are normally not considered in isotropic turbulence, such as the angular transfers of energy, can also be computed from the ring-to-ring transfers after summation over n.

The analysis of these ring-to-ring energy transfers will be presented for two types of systems. First, fully non-linear MHD equations are solved for decaying and forced turbulence in presence of an external magnetic field. For zero external magnetic field, the ring-to-ring energy transfer structure must exhibit some symmetry properties that are shown in the analysis of an isotropic decaying turbulence run. The increasing influence of the magnetic field on the energy transfer is shown by considering the deviation from these symmetry properties for larger and larger external magnetic fields. Second, the quasi-static MHD equations valid in the low magnetic Reynolds number range in presence of an external magnetic field are also solved. The influence of the external field is characterized by the so-called interaction parameter. Results will be presented for various values of this parameter.

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