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Preliminary results from a new incompressible spectral element MHD solver in the Geophysical-astrophysical spectral-element adaptive mesh (GASpAR) code

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Accurate and efficient simulation of strongly turbulent flows is a prevalent challenge in many atmospheric, oceanic, and astrophysical applications. New simulation codes are being developed to investigate such flows in the parameter regimes that interest the scientific communities corresponding to these application areas.

In the case of nonmagnetized fluids, nonlinearities prevail when the Reynolds number Re is large. The number of degrees of freedom in three dimensions increases as $Re^{9/4}$ as Re tends to infinity in the Kolmogorov 1941 framework. For geophysical and astrophysical flows, often $Re \gg 10^8$. Computations of turbulent flows must contain enough scales to encompass the energy-containing and dissipative scale ranges distinctly. Three-dimensional compressible flow simulations show that in order to achieve the desired scale ranges, uniform grids must contain at least 2048^3 cells [5], a feat which, today, can barely be accomplished. Indeed, a pseudo-spectral Navier-Stokes code on a grid of 4096^3 uniformly spaced points has been run on the Earth Simulator [2], with a Taylor Reynolds number ($\propto \sqrt{Re}$) of ≈ 1200, still far from what is required for most geophysical and astrophysical flows.

We have been engaged in the development of a high-order code for modeling turbulence in a variety of systems. Our code, the geophysical and astrophysical spectral-element adaptive refinement (GASpAR) code, is an object-oriented framework for solving PDEs using high-order adaptive methods. Like most spectral-element codes, GASpAR combines finite-element efficiency with spectral-method accuracy. It is designed to be flexible enough for a range of geophysics and astrophysics applications where turbulence or other complex multi-scale problems arise. The formalism accommodates both conforming and non-conforming elements, and it includes a new formulation of dynamic adaptive refinement (DARe) of non-conforming h-type [4], with the order of polynomials in each element kept fixed. The code has been tested thoroughly in two space dimensions, but is written in a modular fashion that can be extended readily to three dimensions.

One of the main goals of our development effort is to ask, if the significant structures of the flow are indeed sparse, so that their dynamics can be followed accurately even if they are embedded in random noise, then does dynamic adaptivity offer a means for achieving an otherwise unattainable large (effective) number of degrees of freedom? The figure represents an example of adaptivity for the merger of three vortices for two dimensional Navier–Stokes as also computed in [3].

A new spectral-element solver for incompressible magnetohydrodynamics (MHD) has recently been developed for the GASpAR code based on the Elsässer formulation [1]. This solver, like the existing ones, automatically takes advantage of the DARe capability offered by the code, as well as of other user-defined adminstration features.

In this talk, we will describe the MHD solver, and present some preliminary results, primarily with regard to validation in the laminar and turbulenct regimes. This description will be couched in a brief discussion of the code and of the DARe methodology. Some examples will be given. We will also discuss some of the issues involved in modeling MHD turbulence using spectral-element methods.

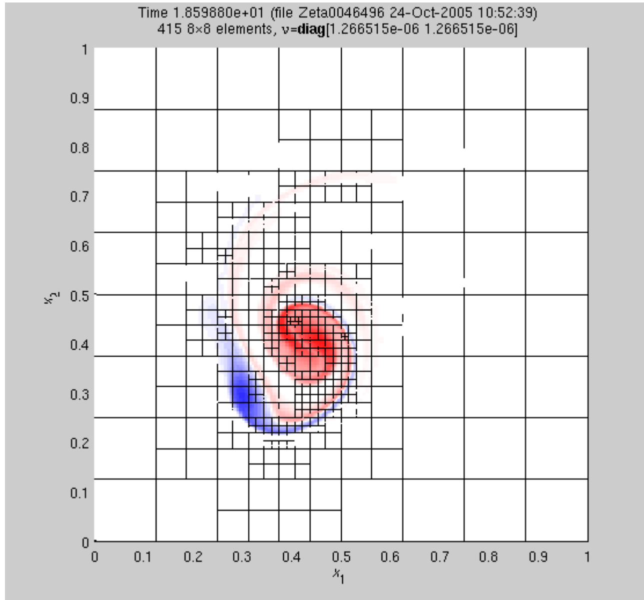

Figure 1: *3-vortex simulation showing merging of two positive (red) and one negative (blue) vortices. Four levels of refinement are used with order 7 in each element.*

References

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