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Rapidly Rotating Convection and the Geodynamo

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Rapidly rotating convection in spherical geometry has been explored using the quasi-geostrophic approximation [1]. This is a reasonable model of convection between rotating spherical shells outside the tangent cylinder that touches the inner core. This approximation assumes a simple z-dependence and solves the two-dimensional nonlinear fluid equations in s, the distance from the axis, ϕ the azimuthal coordinate, and time t. Attention is focussed on the heat transport and the azimuthal zonal flow. We find that the local Peclet number, the product of the typical convective velocity and local convective length scale divided by the thermal diffusivity, is helpful for understanding the dynamics of rapidly rotating convection.

For small $R/R_c - 1$, R being the Rayleigh number and R_c its linear critical value, the Nusselt number varies linearly with $R/R_c - 1$, with a slope that diminishes rapidly as the Prandtl number $P = \nu/\kappa \to 0$. At larger values of $R/R_c - 1$ the Nusselt number becomes less dependent on P, and eventually increases more slowly with $R/R_c - 1$ as thin thermal boundary layers develop. At small $R/R_c - 1$, the zonal flow $U_\phi^0 \sim \hat{U}_c^2$, where $\hat{U}_c$ is the convective velocity, but as $R/R_c - 1$ increases saturation occurs and the exponent is reduced to $U_\phi^0 \sim \hat{U}_c^{4/3}$ approximately. Some possible reasons for this exponent will be discussed. The zonal flow sometimes exhibits a multiple jet structure, and sometimes has a simple radial structure. Factors affecting multiple jet formation will be considered.

We compare our results with the inertial scaling, [2, 3], developed to study rapidly rotating convection, which predicts that $\hat{U}_c \sim {R_Q}^{2/5}(EP)^{1/5}$, where R_Q is the flux Rayleigh number, $R_Q = R(Nu - 1)$, and E is the Ekman number, $\nu/\Omega d^2$, d being the gap between the inner and outer core. The scalings for R_Q and E are in reasonable agreement with our numerical solutions, but the Prandtl number scaling is poor. It appears that the viscous length scale at onset, $dE^{1/3}$, is still relevant even at Rayleigh numbers 50 times critical.

When a dynamo generated magnetic field is present, Christensen and Aubert [4] find that the scaling $\hat{U}_c \sim {R_Q}^{2/5}$ still holds (the Rayeigh number here being defined in terms of the rotation time rather than the dissipation time), while Starchenko and Jones [5] argued that $\hat{U}_c \sim {R_Q}^{1/2}$. In the Earth's core, velocities are so low that inertia is negligible except on very small length scales. The conditions that inertia is negligible in spherical dynamo models have been investigated recently by Sreenivasan and Jones [6].

The vorticity equation can be written

$$-2(\Omega \cdot \nabla)\mathbf{u} = \nabla \times g\alpha T\hat{\mathbf{r}} + \frac{1}{\rho}\nabla \times (\mathbf{j} \times \mathbf{B}), \tag{1}$$

suggesting $2\Omega\hat{U}_c/L_z \sim g\alpha T/L_x$. The temperature perturbation T can be eliminated using the convective heat flux per square metre $F \sim \rho c_p \hat{U}_c T$, to give

$$\hat{U}_c \sim \left(\frac{g\alpha F}{\rho c_p \Omega}\right)^{1/2} \frac{L_z}{2L_x}, \tag{2}$$

or $\hat{U}_c \sim {R_Q}^{1/2}$ if the ratio L_z/L_x is constant. It is therefore the asymptotic behaviour of L_z/L_x which is crucial. In the presence of a magnetic field, the zonal flow is much reduced, and more interest attaches to the strength of the generated magnetic field. Ohmic dissipation balances the buoyancy work, since viscous dissipation is small, so

$$\eta\mu\mathbf{j}^2 \sim \frac{g\alpha F}{c_p}. \tag{3}$$

The length scale δ_B over which the magnetic field varies comes from the induction equation

$$(\mathbf{B}\cdot\nabla)\mathbf{u} \sim \eta\nabla^2\mathbf{B} \rightarrow \delta_B \sim Rm^{-1/2}d \tag{4}$$

if flux ropes with thickness δ_B are created, [7]. Then setting $|\mu\mathbf{j}| \sim |\mathbf{B}|/\delta_B$ and using (3),

$$B \sim \mu^{1/2}d^{1/2}\left(\frac{g\alpha F}{c_p}\right)^{1/2}\frac{1}{\hat{U}_c^{1/2}}. \tag{5}$$

The scaling for $\hat{U}_c$, (2), can then be used to estimate the typical field strength. This field strength scaling implies that the Lorentz force is primarily balanced by pressure in the flux ropes where it is created. To obtain the magnetic field strength directly from the vorticity equation (1) we must note that in the magnetic flux tube configuration the current varies only slowly along the (long) flux tube.

These scalings can be applied to obtain estimates of the typical velocity and magnetic field strength of the planets. For a planet to actually have an active dynamo obeying these scalings, the total heat flux must exceed the heat flux that can be conducted down the adiabat at least somewhere in the core. Also, the magnetic Reynolds number predicted by (2) must be sufficient for dynamo action to occur.

References

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