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Dynamic Sub-Grid Scale Modelling of Drift Wave Turbulence within Magnetohydrodynamics

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Modelling disparate scale interactions within MHD remains an ongoing theoretical and computational challenge. In order to facilitate computation, high Reynolds number systems are often described via the introduction of phenomenological dissipation coefficients as a means of modelling stresses exerted by the unresolved scales. The temporal and spatial evolution of these phenomenological coefficients are usually described via heuristic turbulence models. As the dynamics of the unresolved scales play a crucial role in the evolution of the overall system, especially in cases where inverse cascades are present, a simple dynamic sub-grid scale model for the unresolved turbulent scales, that is rigorously derivable from the original fluid equations, is clearly desirable.

In this work, we present a minimal self-consistent model of the multi-scale interaction of large scale MHD flows with small scale drift wave turbulence. Here we utilize the temporal and spatial scale separation between the large scale MHD flows and the small scale drift wave turbulence to separate the system into a set of resolved and unresolved variables. Wave kinetics and adiabatic theory are used to treat the feedback of the large scale MHD flows on the drift waves via shearing and advection. The stresses exerted by the self-consistently evolved drift wave population density on the MHD flows are calculated by mean field methods. This model has the advantage of being both systematically derivable from the original fluid equations without introducing any free parameters, as well as being simple to implement. The principal effect of the drift waves is to pump the resonant low-m mode via a negative viscosity, consistent with the classical notion of an inverse cascade in quasi-2D turbulence. This mechanism is similar to that by which drift wave turbulence drives zonal flows [1].

We study, two types of low-m, resonant structures. The first is a localized, electrostatic vortex mode, driven unstable by Reynolds stresses exerted by the unresolved scales. The width of the mode is set by resistively dissipated magnetic field line bending, and whose growth rate is given by $\gamma = \left(|\nu_T|^{2/3}/\eta^{1/3}\right)(v_A q_y/L_s)^{2/3}$, where ν_T is the turbulent viscosity. A unique feature of this mode, is that the inverse cascade is ultimately terminated via Ohmic heating as opposed to collisional damping as is the case of m=0 zonal flows.

The second mode is similar to the usual tearing mode as discussed by Furth, Killeen, and Rosenbluth [2], which matches the visco-resistive layer to an MHD exterior via Δ' . The calculation is complicated by the presence of a strong Reynolds stress term emanating from the background turbulence, which induces strong shear flows within the interior layer. In fact, we find that the magnitude of the turbulent stresses exerted by the drift waves are consistent with a gyro-Bohm diffusivity, and thus, usually exceed the magnitude of the inertia term within the tearing mode equations. Outgoing wave boundary conditions are imposed in order to effect the match with the exterior region. The growth rate in the turbulent viscosity dominated regime is given by $\gamma = \left(\eta^{5/6}/|\nu_T|^{1/6}\right)(q_y v_A/L_s)^{1/3}\Delta'$.

References

[1] P.H. Diamond, S.-I. Itoh, K. Itoh and T.S. Hahm 2005. *Zonal flows in plasma-a review*, Plasma Phys. Control. Fusion **47**, R35 – R161.

[2] H.P. Furth, J. Killeen and M.N. Rosenbluth 1963. *Finite-Resistivity Instabilities of a Sheet Pinch*, Phys. Fluids **6**, 459 – 484.