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Magnetic dynamo calculations inside a sphere

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This presentation describes some recent computational efforts to demonstrate magnetic dynamo action inside a sphere that is filled with an incompressible electrically conducting fluid, avoiding rectangular periodic boundary conditions. The motivation is ultimately directed toward planetary and laboratory dynamos, but our first concern is to identify and understand the physical processes involved at the simplest level consistent with the magnetohydrodynamic (MHD) equations. The idea is to compute the simplest dynamo situations first, and put in the imaginable complications (thermal convection, irregularities on the inner surface of the Earth's mantle, variable fluid mass density, a differentially rotating inner core, for examples) one at a time. We are not putting a high priority on realistic numbers at this point.

The system studied is a sphere with a weightless, rigid, perfectly conducting shell at a radius $r = R$. The shell is assumed to be coated on the inside with a very thin layer of insulating dielectric, so that the normal components of the magnetic field and current density vanish there. The normal components of the velocity field and vorticity are also assumed to vanish at $r = R$. These conditions are implied by, but do not imply, no-slip boundary conditions on the velocity field. In addition to being difficult to implement, there are conceptual difficulties associated with no-slip boundary conditions that remain unresolved (e.g., [1] and [2]) and controversial, and are better engaged with in simpler situations than this one.

Inside the sphere, $r < R$, the equations of viscous, resistive, incompressible MHD are assumed to govern the dynamics. We have chosen the sphere to be either stationary or rotating with a constant angular velocity in the latter case by introducing Coriolis and centrifugal terms into the equation of motion. In the induction equation for the magnetic field, no corrections for the fact that the coordinate frame may be rotating and non-inertial are deemed necessary, since the rotation velocity is very small compared to the speed of light.

The method of solution is wholly spectral; all of the solenoidal fields are expanded as series of Chandrasekhar-Kendall (C-K) orthonormal eigenfunctions of the curl. A similar program was used some years ago in cylindrical geometry [3] for nonlinear MHD computations. The C-K functions are believed to be complete for solenoidal fields, but a proof has been given only for the cylindrical case [4]. The boundary conditions are all built into the expansion functions themselves, and do not require attention again in the computation. The dynamical variables are the time-dependent complex coefficients in the expansions, which are advanced by a set of nonlinear ordinary differential equations. The known quadratic ideal invariants are very well conserved over many eddy-turnover times, for the initial-value problem with zero viscosity and resistivity. We return to configuration space only for graphical purposes. The price paid for this scheme is the absence of fast transforms that make pseudospectral computation in periodic boundary conditions economical, so that the convolution sums become unwieldy at resolutions achievable by FFT-based codes. The advantages are that the boundary conditions are automatically satisfied and the expansion functions are physically natural to the geometry: far fewer of them are necessary to represent the MHD structures that arise than would be necessary, for example, in a rectangular Fourier series representation.

Mechanical forcing is introduced as an inhomogeneous term on the right hand side of the equation of motion. The forcing, too, is represented in terms of C-K functions and can be chosen to mimic such processes as thermal convection or irregular boundaries on the inner surface.

The code can be run, of course, as a purely hydrodynamic code by deleting the magnetic terms. Doing so reveals, for the rotating case, flow patterns characteristic of Ekman pumping and internal wave motion in which the inertial terms in the equation of motion are not neglected and no geostrophic approximations are made. Wide variations in behavior are observed depending upon Rossby number, Reynolds number, Ekman number, and the scale of the forcing terms. Fully exploring the possible parameter space will be a lengthy task. Each corner of parameter space shows its own peculiarities.

Dynamo actions with and without rotation are very different. In both cases, the technique is to force a mechanical flow pattern which may be time dependent but which has ceased to evolve systematically and may or may not be turbulent. Then a small seed magnetic field is introduced and allowed to evolve according to the full set of MHD equations. At the early stages, the magnetic energy is observed either to amplify or decay, and at this stage we may be considered to be solving the kinematic dynamo problem. For the case of amplifying magnetic fields, they may be followed on into the saturation regime, where the Lorentz force is no longer negligible in the equation of motion. Both laminar and disordered magnetic fields can be observed in different parameter regimes, and magnetic dipole moments may be computed. For the former, flips from one dipolar orientation to another are observed in some cases. For the latter, essentially stochastically varying small-scale magnetic fields are possible. Some details appear in Ref. [5].

Future plans involve the inclusion of a differentially rotating inner solid core, and the replacement of the conducting shell by a mechanically impenetrable insulator, so that the generated magnetic field can penetrate the vacuum region outside.

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