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Intermittent magnetic field excitation by a turbulent flow of liquid sodium

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Determining the onset conditions for magnetic field growth in magnetohydrodynamics is fundamental to understanding how astrophysical dynamos such as the Earth, the Sun, and the galaxy self-generate magnetic fields. These onset conditions are now being studied in laboratory experiments using flows of liquid metals [1]. The Madison Dynamo Experiment, currently the largest of the devices, is used to study a flow composed of two counter-rotating helical vortices predicted to produce a growing magnetic field for sufficiently fast flow speeds [2]. The flow is generated by impellers in a 1 m diameter sphere filled with liquid sodium. Liquid metals generally have a low rate of viscous diffusion compared with the rate of resistive diffusion, *e.g.* the Prandtl number for liquid sodium is $Pr = \mu_0 \sigma \nu \sim 10^{-5}$ where σ is the conductivity and ν is the viscosity. Due to the low viscosity, the flows generated in the experiment tend to be quite turbulent. One of the goals of the experiment is to address the effect of turbulence on the threshold conditions for a dynamo.

The threshold of magnetic field generation due to the dynamo instability is governed by the magnetic Reynolds number $Rm = \mu_0 \sigma a v_0$ where a is the radius of the sphere and v_0 is a characteristic flow speed [3]. The magnetic Reynolds number increases for larger impeller rotation rates, and hence larger mean flow speeds. The flow is predicted to generate a magnetic field for $Rm > Rm_{\text{crit}}$, where Rm_{crit} is calculated from a model of the mean velocity field constructed from velocity measurements in a full scale water model of the sodium experiment [4]. The structure of the magnetic field generated by the flow is predicted to be a dipolar with an orientation perpendicular to the symmetry axis of the flow as seen in Fig. 1. The growth rate of the field as a function of Rm, shown by the solid curve in Fig. 2, becomes positive for $Rm \geq 190$.

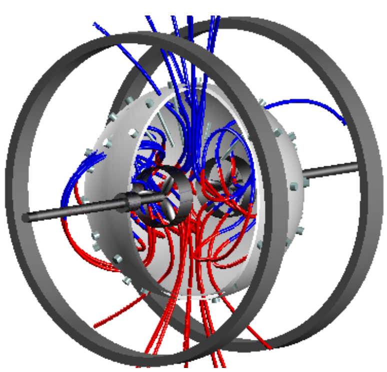

Figure 1: A schematic of the Madison Dynamo Experiment with superimposed magnetic field lines of the theoretically predicted dominant magnetic field.

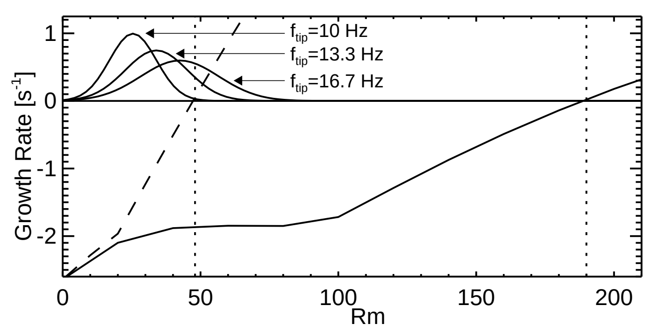

Figure 2: Growth rate of the transverse dipole field versus Rm for the mean flow (solid) and for a slightly different flow geometry (dashed). The vertical lines identify Rm_{crit} for each case. The PDFs of Rm for flows with three different impeller rotation rates are shown to demonstrate the increasing overlap of the ranges of Rm and Rm_{crit}.

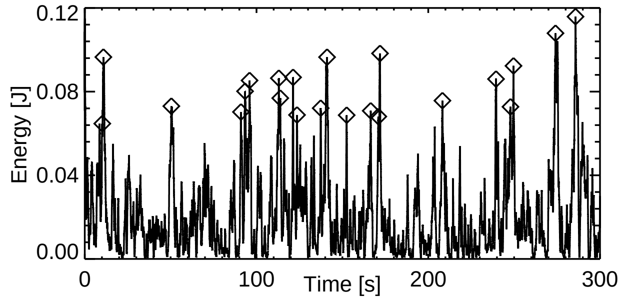

Figure 3: Time series of the energy in the transverse dipole field for an impeller rotation rate of 10 Hz. The diamonds mark the peak of a burst where the energy exceeds 50% of its maximum value.

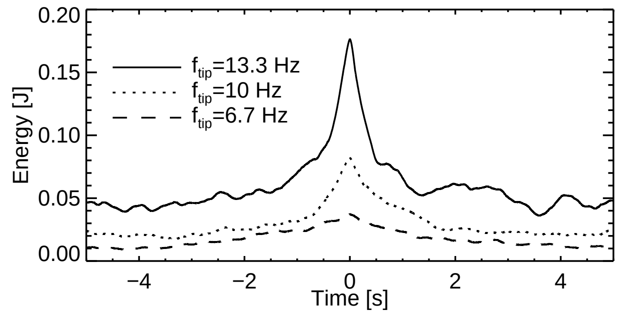

Figure 4: The ensemble average of bursts from three time series. The averaged burst is used to calculate the growth rate.

The mean flow, however, is not the most efficient flow geometry for exciting a magnetic field. A flow with a slightly different geometry has a much lower Rm_{crit} as seen from the dashed curve in Fig. 2 demonstrating that the threshold for field generation is extremely sensitive to the flow geometry.

Due to the simply-connected geometry of the experiment, turbulent eddies range from the viscous dissipation scale (on the order of 1 mm) up to the largest scale of the flow. The large-scale eddies can change the peak flow speed effectivly varying Rm. An estimate of the variation in Rm based on the measured velocity fluctuations is shown in Fig. 2. The eddies can also change the flow geometry which can cause significant variations of Rm_{crit}. Thus, although the the mean flow may be subcritical, there can be times for which the instantaneous flow satisfies $Rm > Rm_{\text{crit}}$. The magnetic field momentarily grows while this condition is satisfied and then decays. Hence, the magnetic field is expected to have intermittent bursts.

These bursts are observed in the sodium experiment. Figure 3 shows a time series of the energy in the transverse dipole component of the measured magnetic field. The bursts are ensemble averaged to determine typical characteristics. A burst is defined to occur when the energy in the transverse dipole field exceeds a certain threshold. For this analysis, the threshold is 50% of the maximum energy of the time series. This threshold is sufficiently small to capture a large number of bursts yet significantly larger than the mean energy (about two standard deviations above the mean energy for each time series). The bursts are averaged together and the growth rate is determined by an exponential fit to the curves shown in Fig. 4. The bursts become more frequent and have faster growth rates at larger values of Rm. They become stronger in amplitude but shorter in duration as the turnover time of the large eddies decreases.

The results presented in this paper demonstrate how turbulence in a simply-connected geometry changes the onset conditions of the dynamo. Rather than a smooth transition from damped to growing fields, the transition is characterized by intermittent magnetic field bursts which may be relevant to some dynamo models [5].

References

[1] A. Gailitis, O. Lielausis, E. Platacis, G. Gerbeth, and F. Stefani, Rev. Mod. Phys. **74**, 973 (2002).

[2] M. L. Dudley and R. W. James, Proc. R. Soc. London, Ser. A **425**, 407 (1989).

[3] H. K. Moffatt, (Cambridge University Press, Cambridge, England, 1978).

[4] M. D. Nornberg, E. J. Spence, R. D. Kendrick, C. M. Jacobson, and C. B. Forest, Phys. Plasmas **13**, 055901 (2006).

[5] C. M. Ko and E. N. Parker, Astrophys. J. **341**, 828 (1989).