Critical issues to get right about stellar dynamos

Axel Brandenburg (Nordita, Copenhagen)

1. Small scale dynamo and LES
2. Pm=0.1 or less (to elim. SS dyn)
3. Magn. Helicity transport and loss

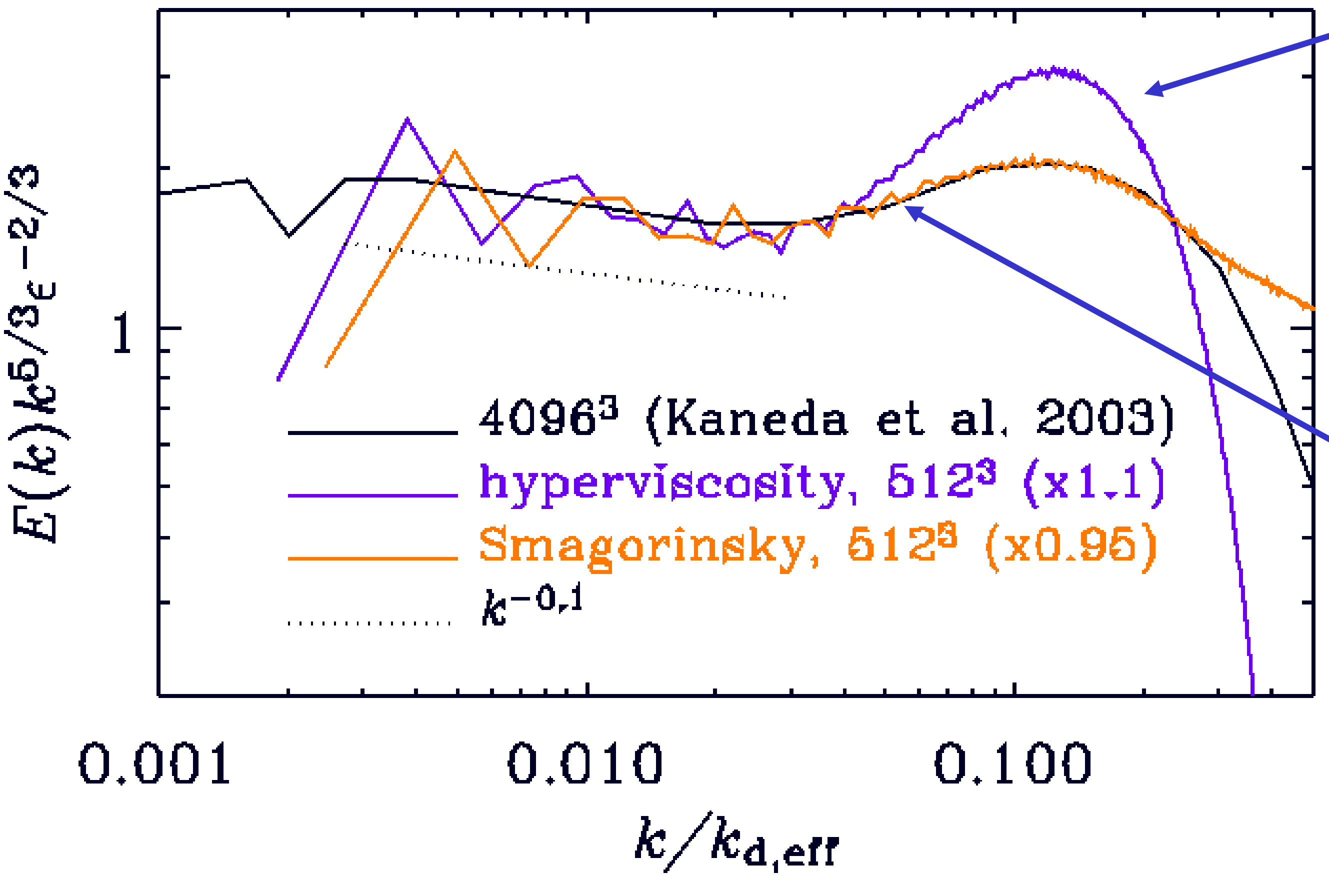
Shukurov et al. (2006, A&A **448**, L33)
Schekochihin et al. (2005, ApJ 625, L115)
Brandenburg & Subramanian (2005, Phys. Rep., 417, 1)
Brandenburg (2001, ApJ 550, 824; and 2005, ApJ 625, 539)

Long way to go: what to expect?

- Turbulent inertial range somewhere
- Departures from $k^{-5/3}$
  - $k^{-0.1}$ correction
  - Bottleneck effect
- Screw-up from MHD nonlocality?
  - $E_M(k)$ and $E_K(k)$ parallel or even overlapping?
  - Or peak of $E_M(k)$ at resistive scales?
  - Does small scale dynamo work for $P_m \ll 1$ and $R_m \gg 1$?
  - How is large scale dynamo affected by small scales?
- Implications for catastrophic α-quenching

Hyperviscous, Smagorinsky, normal

Haugen & Brandenburg (Phys. Fluids, astro-ph/041266)



height of bottleneck increased with hyper

onset of bottleneck at same position

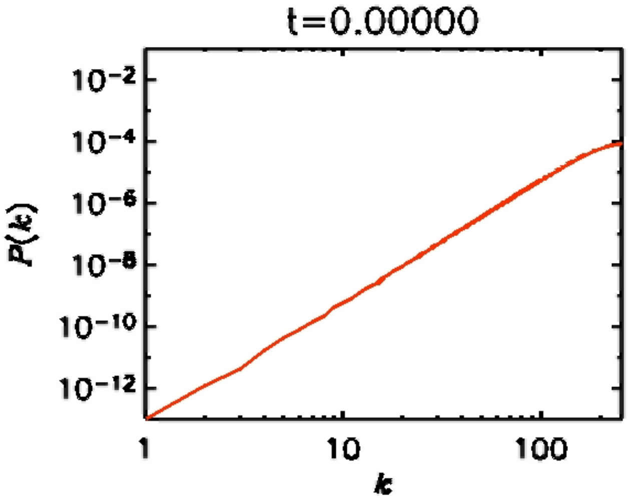
C_S=0.20

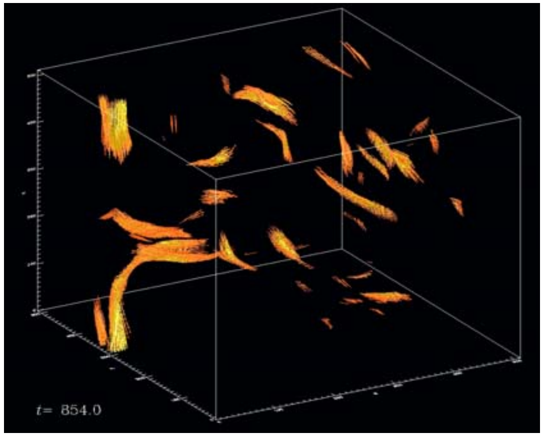
Inertial range unaffected by artificial diffusion

Allow for *B*: small scale dynamo action

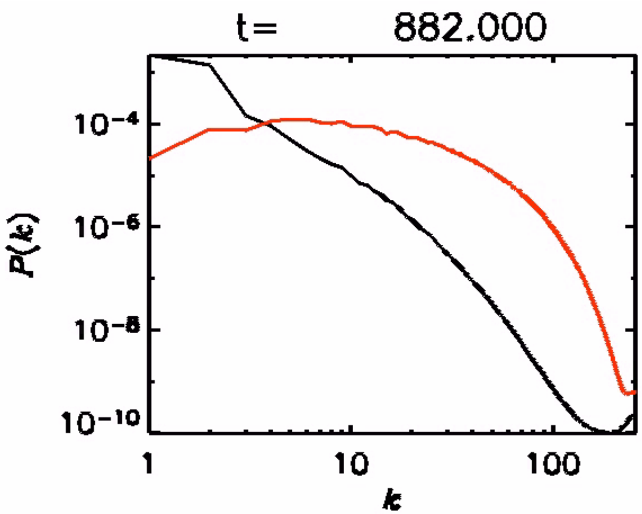
non-helically
forced turbulence

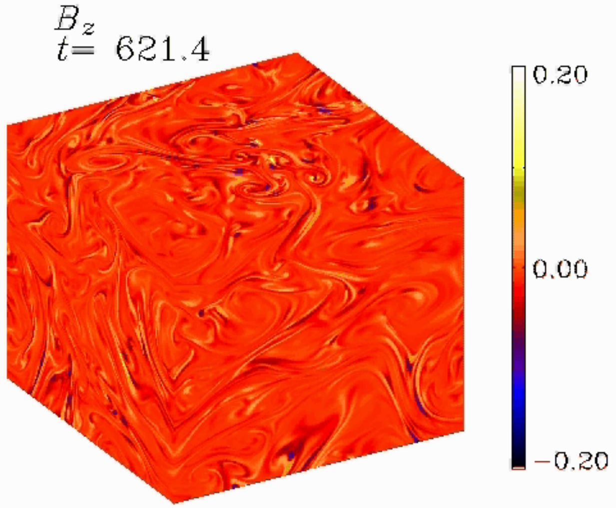
$Pr_M = \nu/\eta = 1$



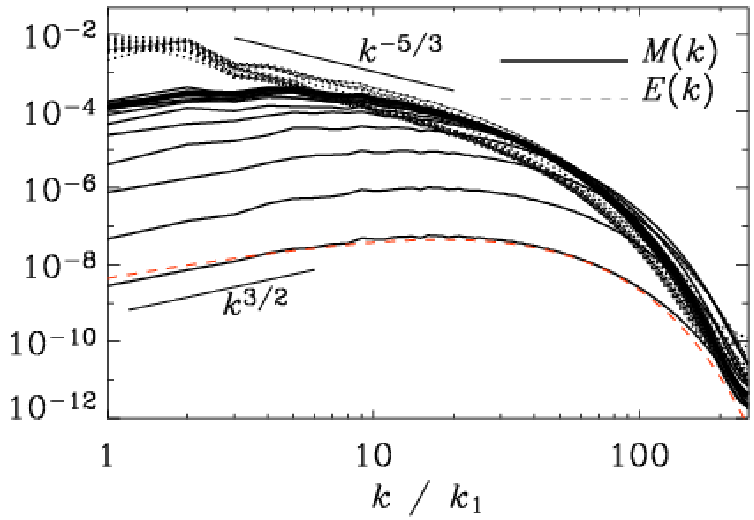


$Pr_M = \nu/\eta = 50$





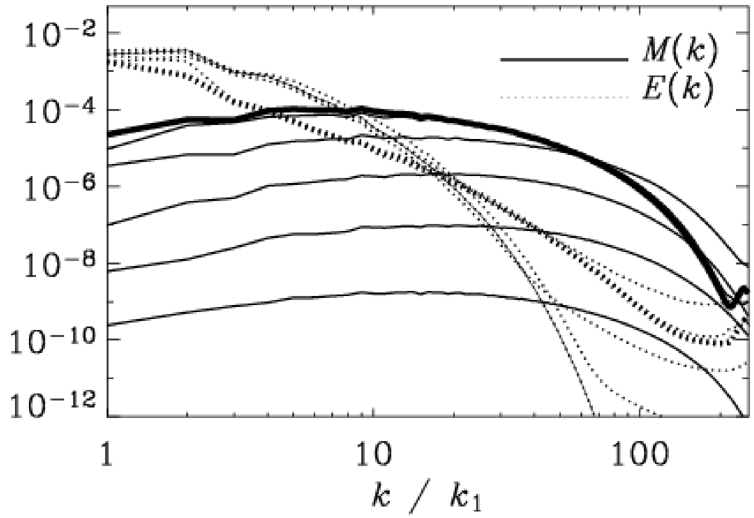
Peaked at small scales?



$Pr_M = \nu/\eta = 1$

$k^{+3/2}$ Kazantsev spectrum, even for $P_m = 1$

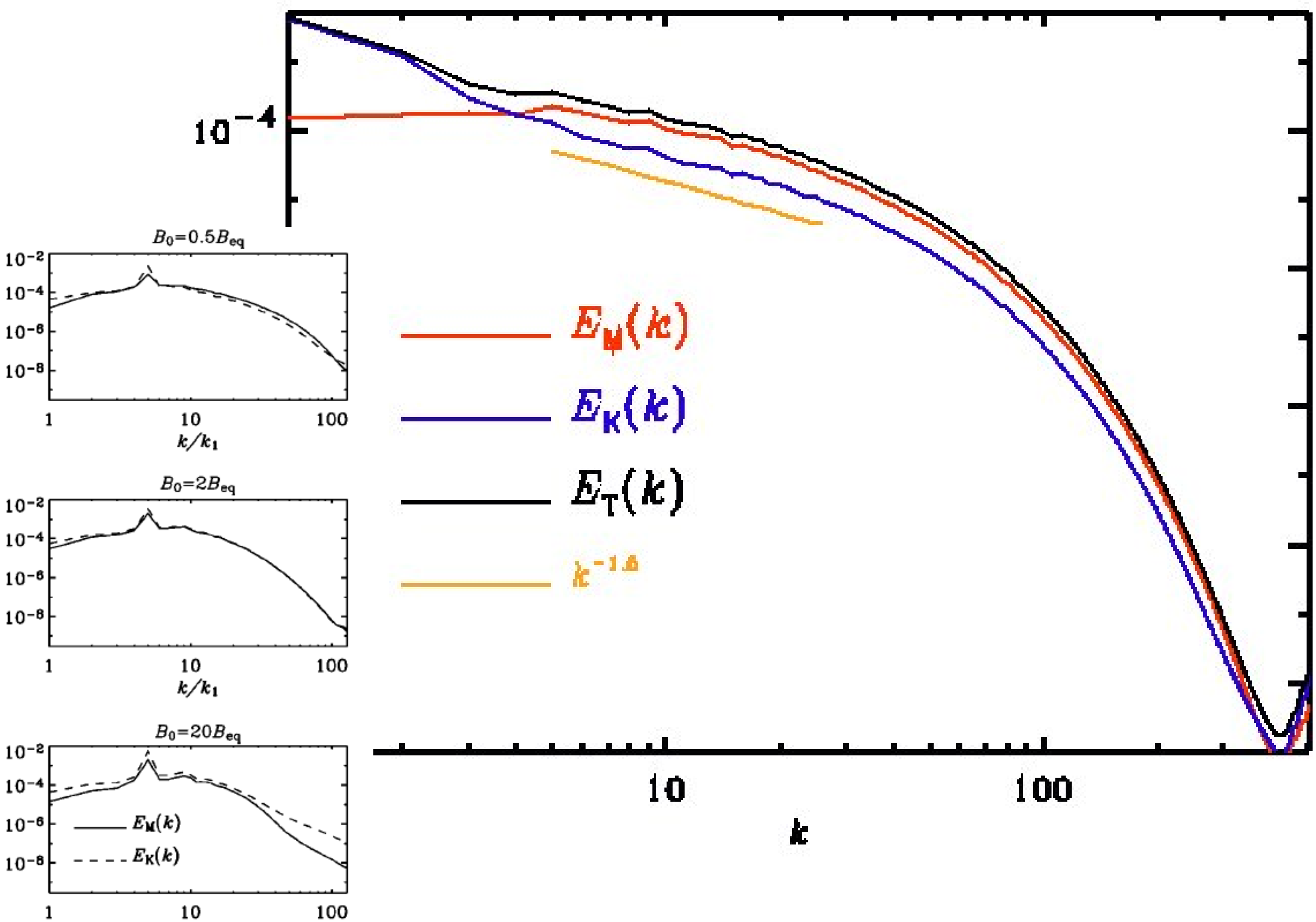
During saturation peak moves to smaller k.



$Pr_M = \nu/\eta = 50$

Can we expect inertial range at (much) larger resolution?

Looks like $k^{-3/2}$ at 1024^3

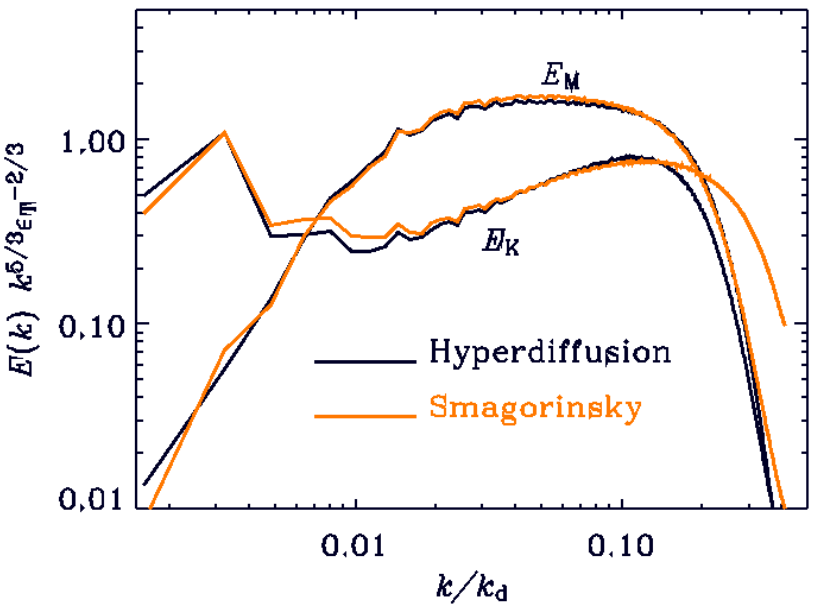


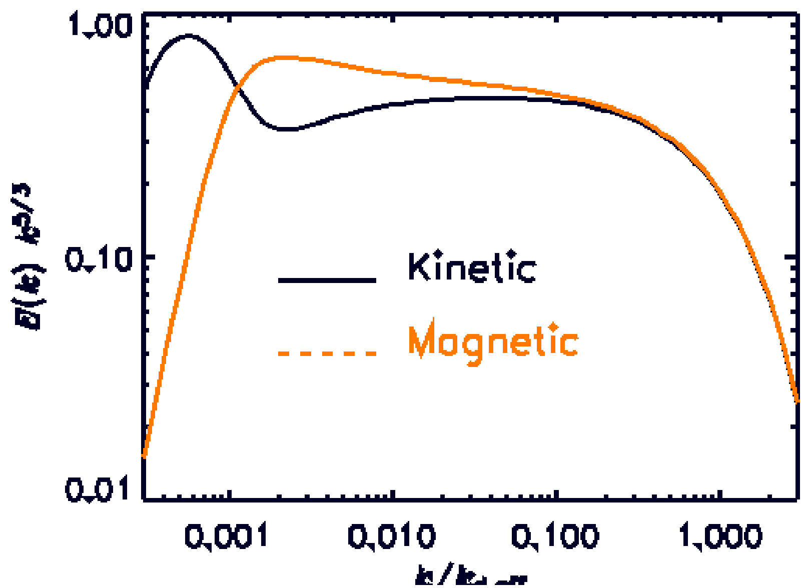
Spectra not on top of each other??

➔ Different from case with imposed field!

Still not large enough?!

Help from LES and theory



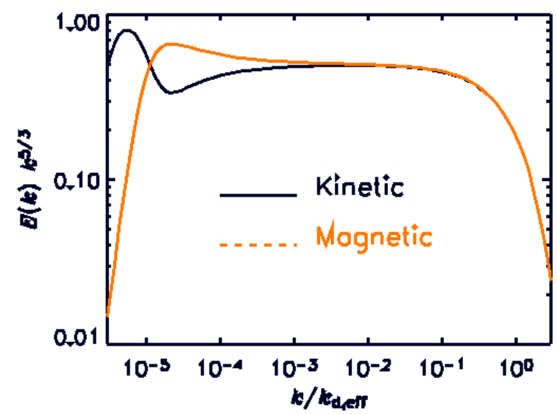


➔ converging spectra at large k ??

Can be reproduced with prediction
by Müller & Grappin (2005, PRL) :

$$|E_M(k)-E_K(k)| \sim k^{-7/3}$$

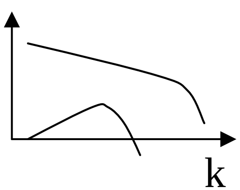
$$E_M(k)+E_K(k) \sim k^{-5/3}$$

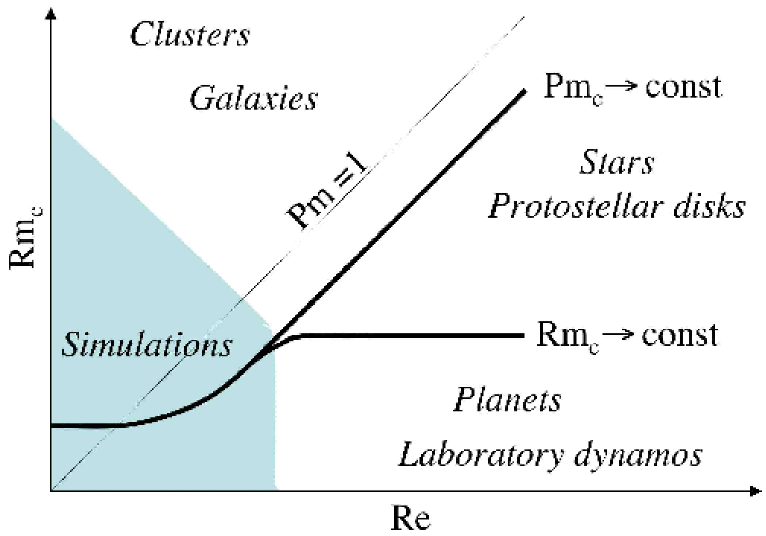


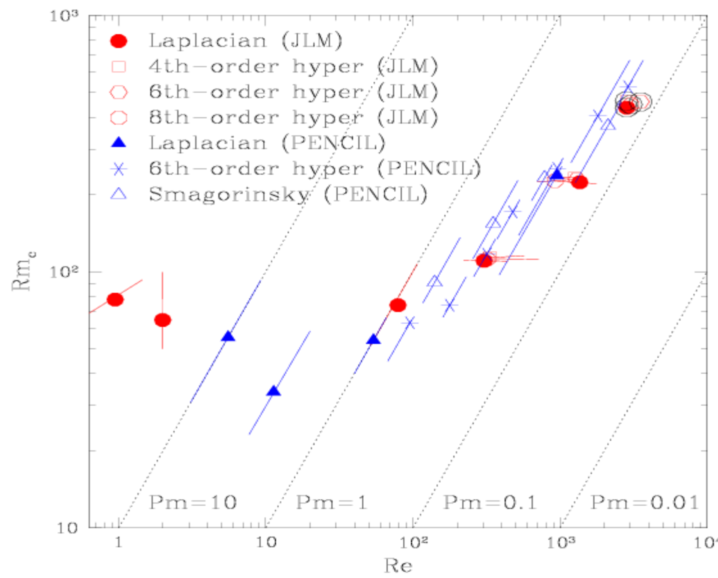
Maybe no small scale “surface” dynamo?

Small $Pr_M = \nu/\eta$: stars and discs around NSs and YSOs

Schekochihin
Haugen
Brandenburg
et al (2005)







Here: non-helically forced turbulence

When should we think of extrapolating to the sun?
Implications for global models (w/strong SS field)

Large scale dynamos

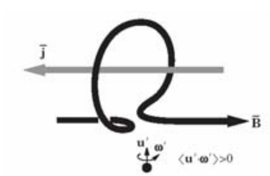
- Dynamo number for α^2 dynamo $C_\alpha = \frac{\alpha}{\eta_t k_1} = \frac{\frac{1}{3}\varepsilon_f \tau \overline{\boldsymbol{\omega}\cdot\mathbf{u}}}{\frac{1}{3}\tau \overline{\mathbf{u}^2} k_1} = \varepsilon_f \frac{k_f}{k_1}$
- May C_α and/or $C_\alpha C_\omega$ not big enough
- Catastrophic quenching
  - Suppression of lagrangian chaos?
  - Suffocation from small scale magnetic helicity?
    - Applies also to Babcock-Leighton
    - Most likely solution: magnetic helicity fluxes

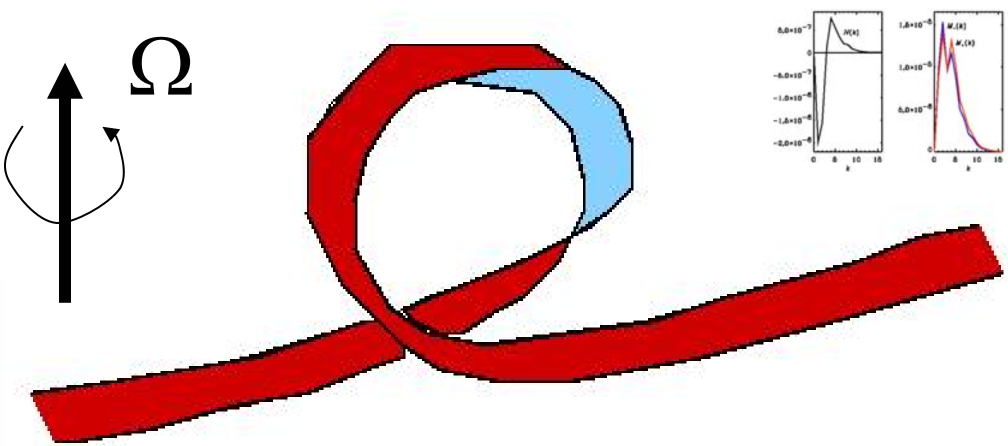
Penalty from α effect:
writhe with *internal* twist as by-product

$$\frac{d\overline{\mathbf{B}}}{dt} = \nabla \times \left(\alpha \overline{\mathbf{B}} - \eta_{\mathrm{T}} \overline{\mathbf{J}} \right)$$

α effect produces helical field

$$\overline{\mathbf{J}} \cdot \overline{\mathbf{B}} = \frac{\alpha}{\eta_{\mathrm{T}}} \overline{\mathbf{B}}^2$$





clockwise tilt (right handed)

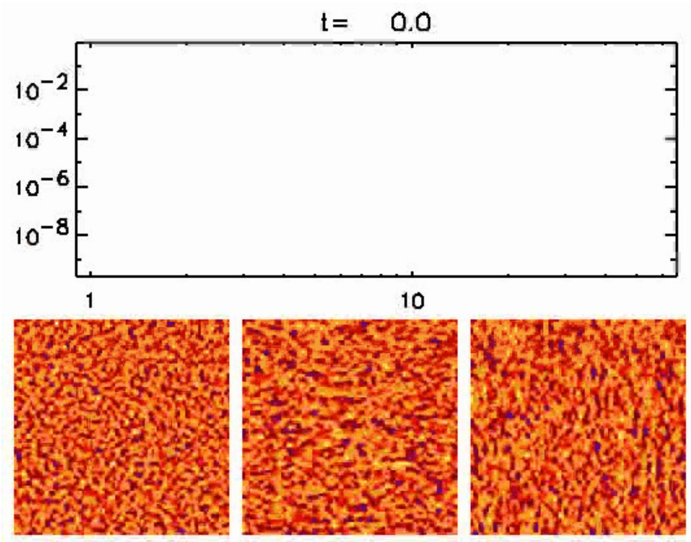
→ left handed internal twist

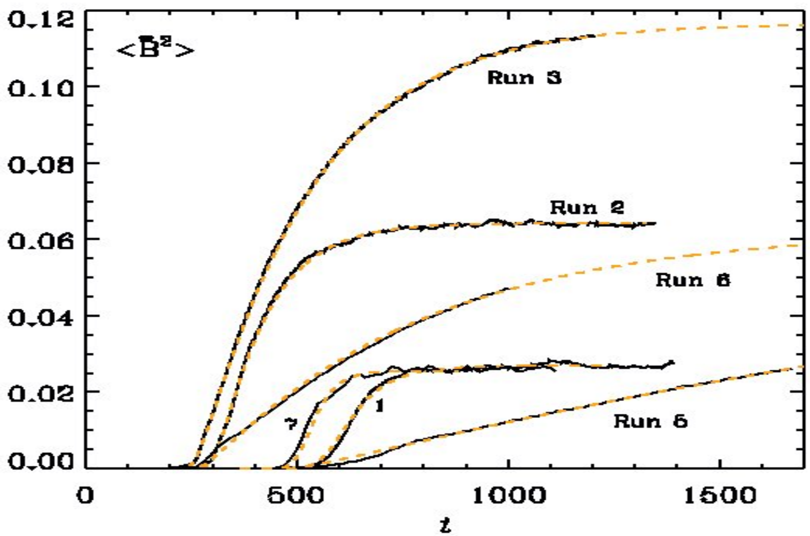
$$\alpha = -\tfrac{1}{3} \tau \left(\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle - \langle \mathbf{j} \cdot \mathbf{b} \rangle / \rho_0 \right)$$

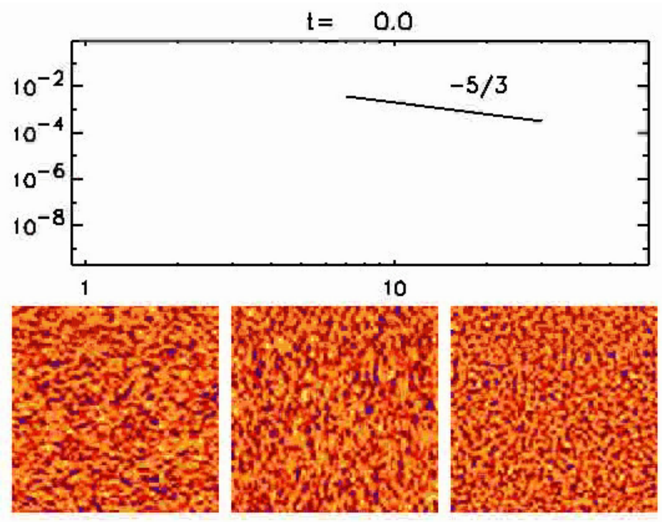
both for thermal/magnetic buoyancy

Slow saturation

Brandenburg (2001, ApJ 550, 824)







$$\overline{\mathbf{B}}^2 = \overline{\mathbf{b}^2}\,\frac{k_\mathrm{f}}{k_1}\left(1 - e^{-2\eta k_1^2 (t - t_\mathrm{s})}\right)$$

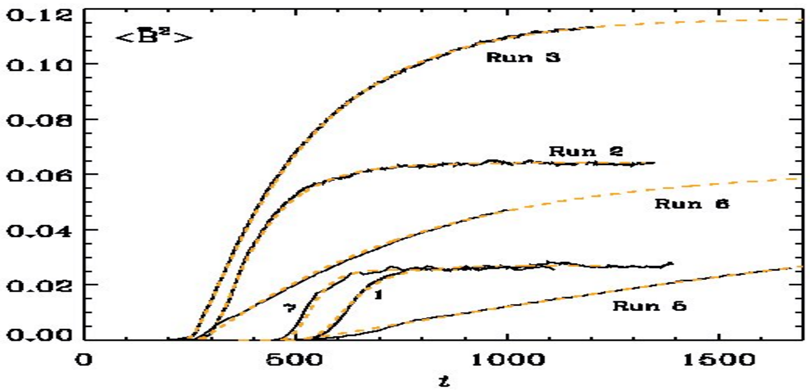
1. Excellent fit formula
2. Microscopic diffusivity

Slow-down explained by magnetic helicity conservation

$$\frac{d}{dt}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

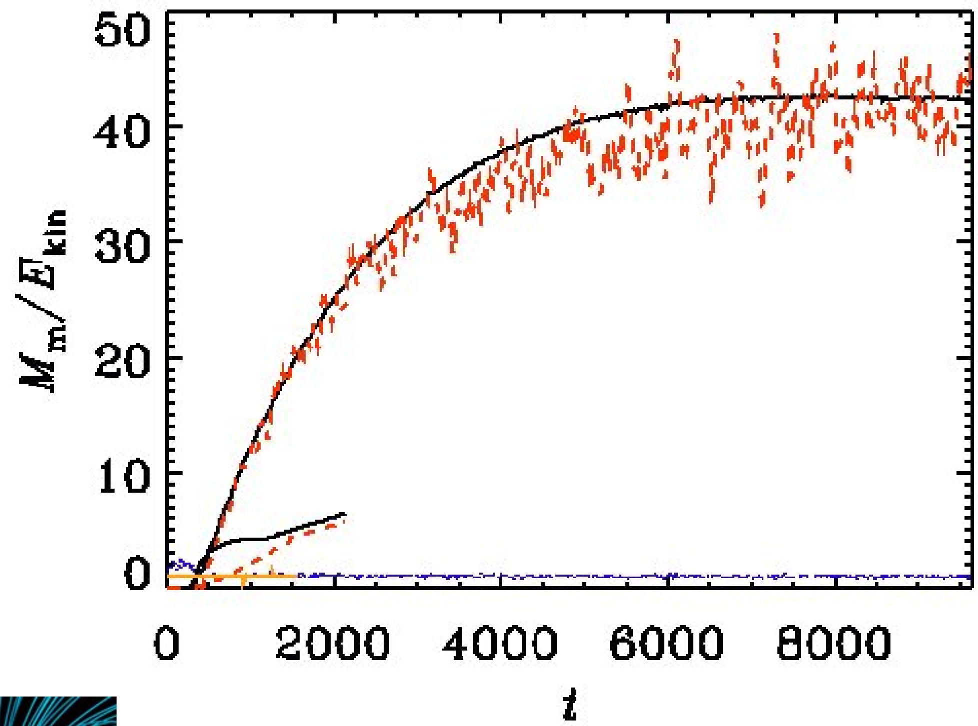
$$k_1^{-1}\frac{d}{dt}\left\langle \overline{\mathbf{B}}^2\right\rangle = -2\eta k_1\left\langle \overline{\mathbf{B}}^2\right\rangle + 2\eta k_\mathrm{f}\left\langle \mathbf{b}^2\right\rangle$$

$$\longrightarrow \quad \left\langle \overline{\mathbf{B}}^2\right\rangle = \left\langle \mathbf{b}^2\right\rangle \frac{k_f}{k_1}\left[1 - e^{-2\eta k_1^2 (t-t_\mathrm{s})}\right]$$



molecular value!!

Helical dynamo saturation with hyperdiffusivity



PRL 88, 055003

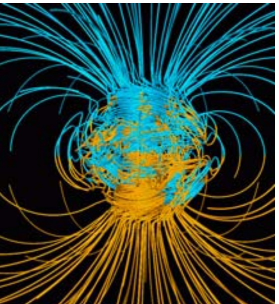
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

$$k_1^3\left\langle \overline{\mathbf{B}}^2\right\rangle = k_f^3\left\langle \mathbf{b}^2\right\rangle$$

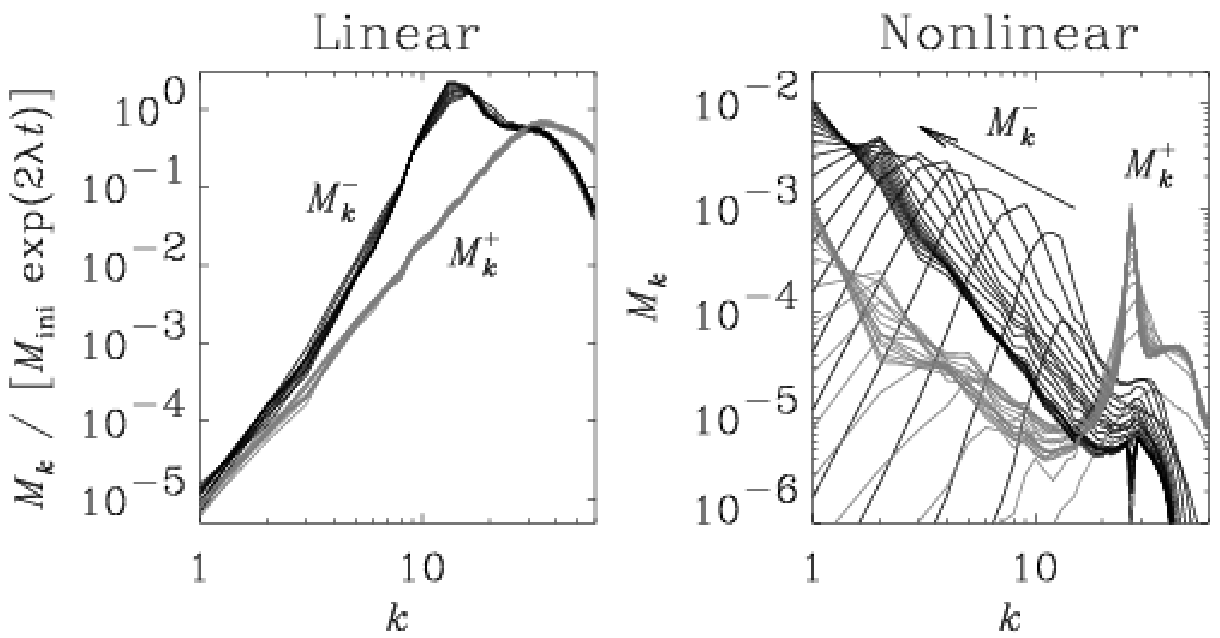
for ordinary hyperdiffusion $\propto \eta_2 k^4$

$$k_1\left\langle \overline{\mathbf{B}}^2\right\rangle = k_f\left\langle \mathbf{b}^2\right\rangle$$

ratio 5^3=125 instead of 5



Scale separation: inverse cascade



Position of the peak compatible with

$$k_{peak} = \frac{\alpha}{2\eta_{\rm t}}$$

No inverse cascade in kinematic regime

Decomposition in terms of
Chandrasekhar-Kendall-Waleffe functions

$$\mathbf{A}_{\mathbf{k}} = a_{\mathbf{k}}^{+}\mathbf{h}_{\mathbf{k}}^{+} + a_{\mathbf{k}}^{-}\mathbf{h}_{\mathbf{k}}^{-} + a_{\mathbf{k}}^{0}\mathbf{h}_{\mathbf{k}}^{0}$$

$$\overline{\mathbf{B}} = \begin{pmatrix} \cos z \\ \sin z \\ 0 \end{pmatrix}$$

LS field: force-free Beltrami

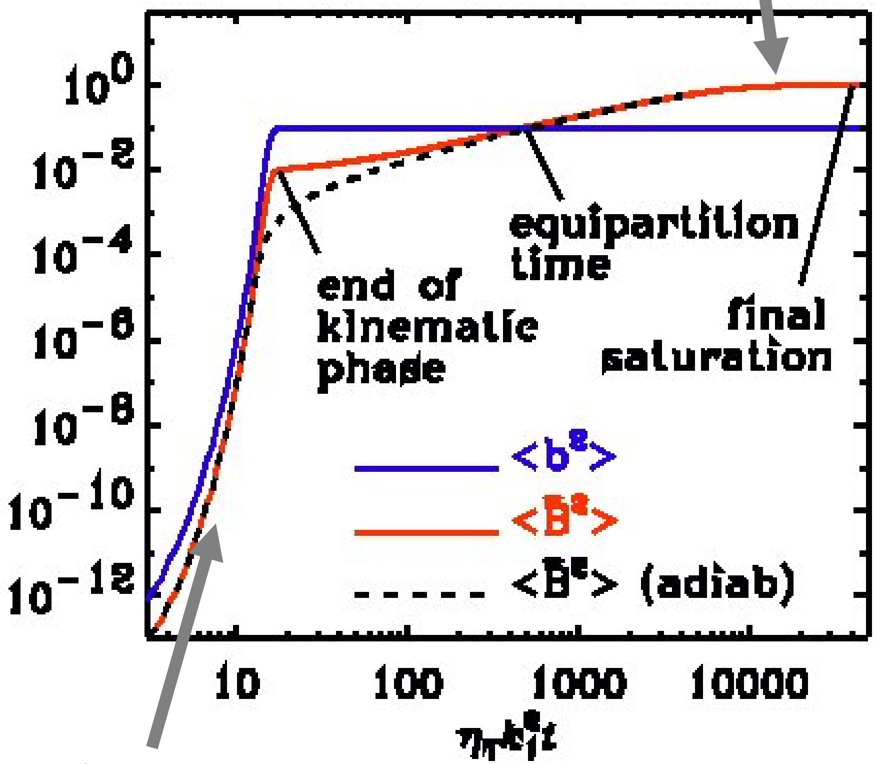
Periodic box, no shear: resistively limited saturation

Brandenburg & Subramanian
Phys. Rep. (2005, **417**, 1-209)

$$\frac{d}{dt}\langle \mathbf{A}\cdot\mathbf{B}\rangle = -2\eta\langle \mathbf{J}\cdot\mathbf{B}\rangle$$

$$k_1\langle \overline{\mathbf{B}}^2\rangle - k_f\langle \mathbf{b}^2\rangle = 0$$

$$\langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle + \langle \mathbf{j}\cdot\mathbf{b}\rangle = 0$$



Significant field already after kinematic growth phase

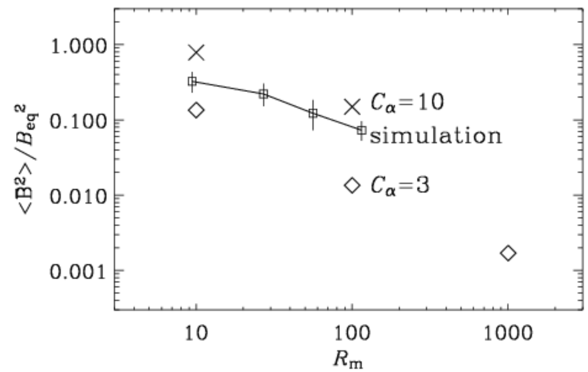
followed by slow resistive adjustment

$$\langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle + \langle \mathbf{a}\cdot\mathbf{b}\rangle = 0$$

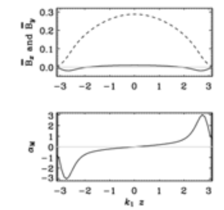
$$k_1^{-1}\langle \overline{\mathbf{B}}^2\rangle - k_f^{-1}\langle \mathbf{b}^2\rangle = 0$$

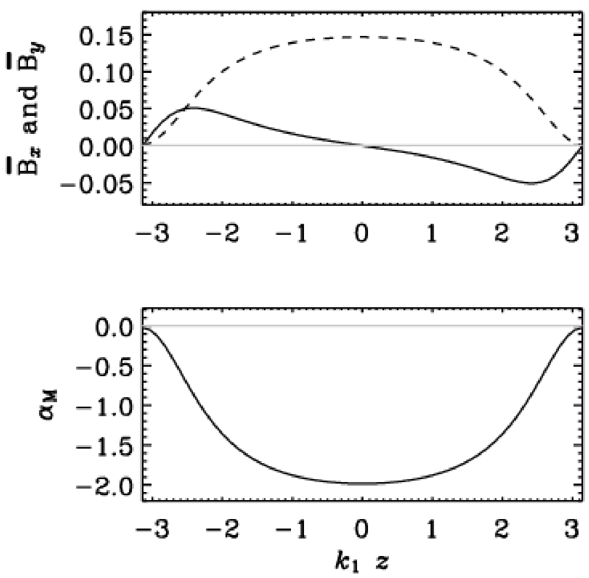
Blackman & Brandenburg (2002, ApJ 579, 397)

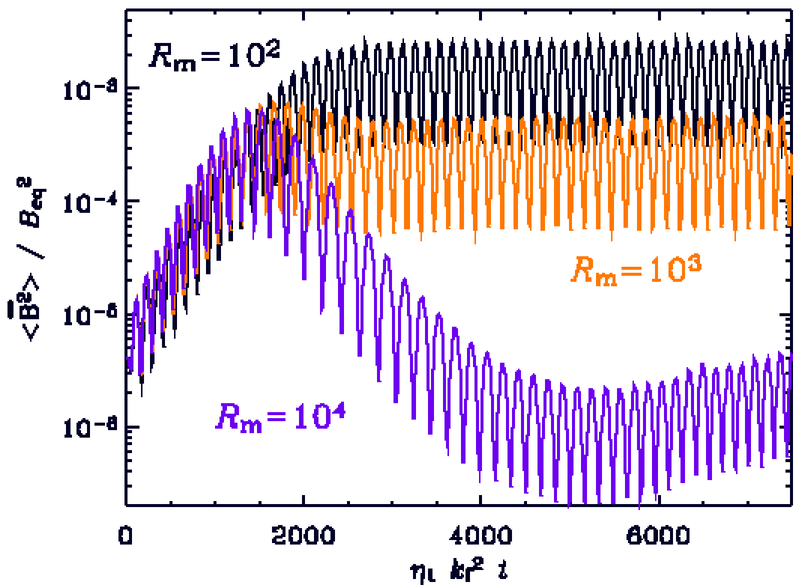
Boundaries instead of periodic



$$\text{b.c.:} \quad B_x = B_y = 0$$







Brandenburg & Subramanian
(2005, AN 326, 400)

Revised nonlinear dynamo theory

(originally due to Kleeorin & Ruzmaikin 1982)

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathbf{A}\cdot\mathbf{B}\rangle=-2\eta\langle\mathbf{J}\cdot\mathbf{B}\rangle$$

$$\overline{\mathcal{E}}=\alpha\overline{\mathbf{B}}-\eta_{\mathrm{t}}\overline{\mathbf{J}}$$

Two-scale assumption

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle=+2\langle\overline{\mathcal{E}}\cdot\overline{\mathbf{B}}\rangle-2\eta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathbf{a}\cdot\mathbf{b}\rangle=-2\langle\overline{\mathcal{E}}\cdot\overline{\mathbf{B}}\rangle-2\eta\langle\mathbf{j}\cdot\mathbf{b}\rangle$$

Dynamical quenching

$$\frac{\mathrm{d}}{\mathrm{d}t}\alpha_M=-2\eta k_f^2\left(R_m\frac{\langle\overline{\mathcal{E}}\cdot\overline{\mathbf{B}}\rangle}{B_{eq}^2}+\alpha_M\right)$$

$$\alpha=-\tfrac{1}{3}\tau\left(\langle\boldsymbol{\omega}\cdot\mathbf{u}\rangle-\langle\mathbf{j}\cdot\mathbf{b}\rangle/\rho_0\right)$$

Kleeorin & Ruzmaikin (1982)

Steady limit
→ algebraic quenching:

$$\alpha=\frac{\alpha_0+\eta_t R_m\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}/B_{eq}^2}{1+R_m\overline{\mathbf{B}}^2/B_{eq}^2}$$

(→ selective decay)

General formula with current helicity flux

Advantage over magnetic helicity

1) <**j.b**> is what enters α effect

2) Can define helicity density

$$\overline{\mathsf{F}}_C^{SS} = 2\overline{\mathbf{e}\times\mathbf{j}} + \overline{(\nabla\times\mathbf{e})\times\mathbf{b}}$$

$$\frac{\partial}{\partial t}\overline{\mathbf{j}\cdot\mathbf{b}} = -2\overline{\mathbf{e}\cdot\mathbf{c}} - \nabla\cdot\overline{\mathsf{F}}_C^{SS}$$

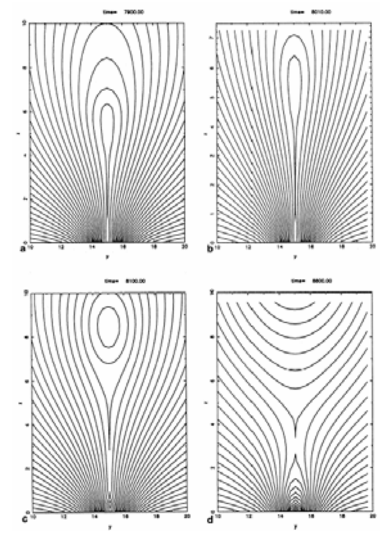
$$\mathbf{c} = \nabla\times\mathbf{j}$$

R_m also in the numerator

$$\alpha = \frac{\alpha_K + R_m\left[\left(\overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \tfrac{1}{2}k_\mathrm{f}^{-2}\nabla\cdot\overline{\mathsf{F}}_\mathrm{C}^\mathrm{SS}\right)/B_{eq}^2 - \dfrac{\partial\alpha/\partial t}{2\eta_\mathrm{t}k_\mathrm{f}^2}\right]}{1+R_m\overline{\mathbf{B}}^2/B_{eq}^2}$$

Significance of shear

- α → transport of helicity in *k*-space
- Shear → transport of helicity in *x*-space
  - Mediating helicity escape (→ plasmoids)
  - Mediating turbulent helicity flux



Schnack et al.

Expression for current helicity flux
(first order smoothing, tau approximation)

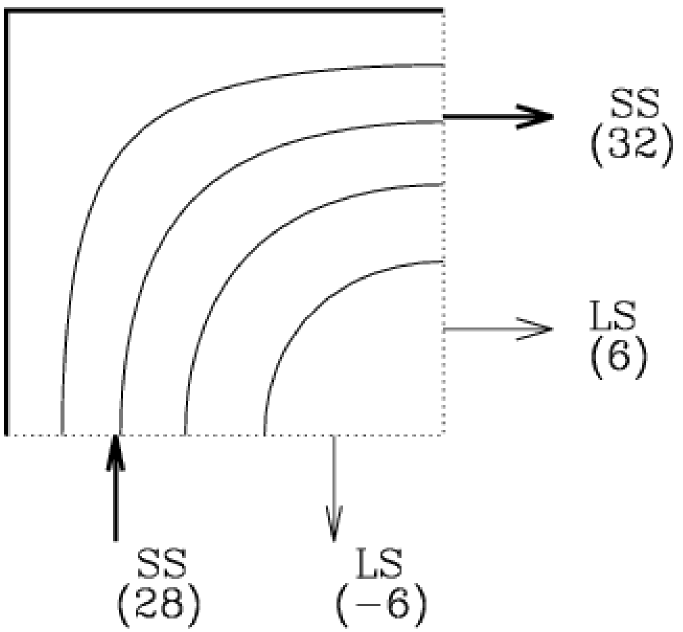
$$\overline{F}_i^{\mathrm{C}} = -4\tau\overline{\omega_j u_{k,i}}\,\overline{B}_j\overline{B}_k$$

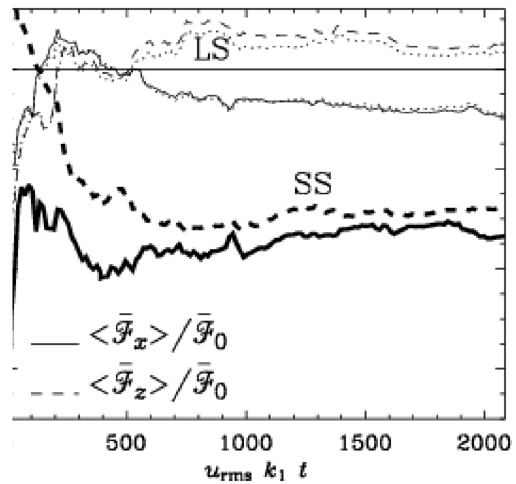
Vishniac & Cho (2001, ApJ 550, 752)
Subramanian & Brandenburg (2004, PRL 93, 20500)

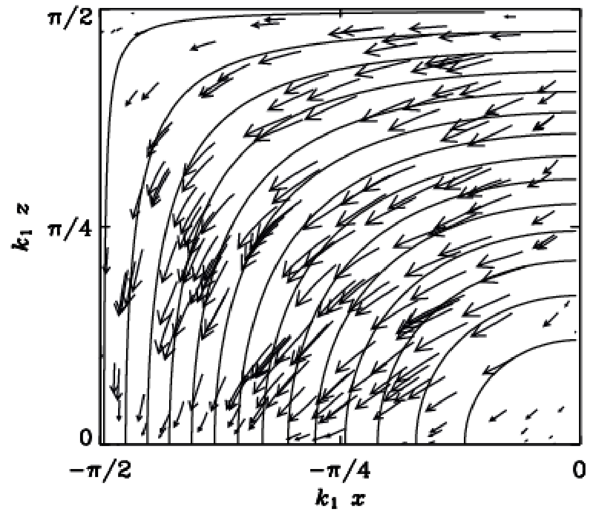
Expected to be finite on when there is shear

Arlt & Brandenburg (2001, A&A 380, 359)

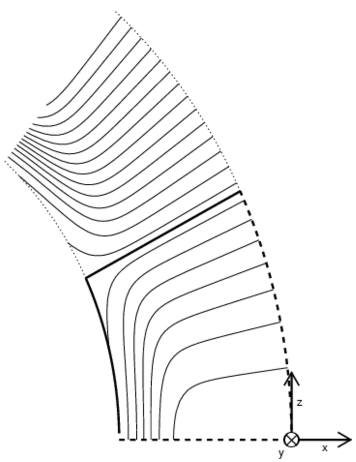
Helicity fluxes at large and small scales







Negative current helicity:
oduction in northern hemisphere



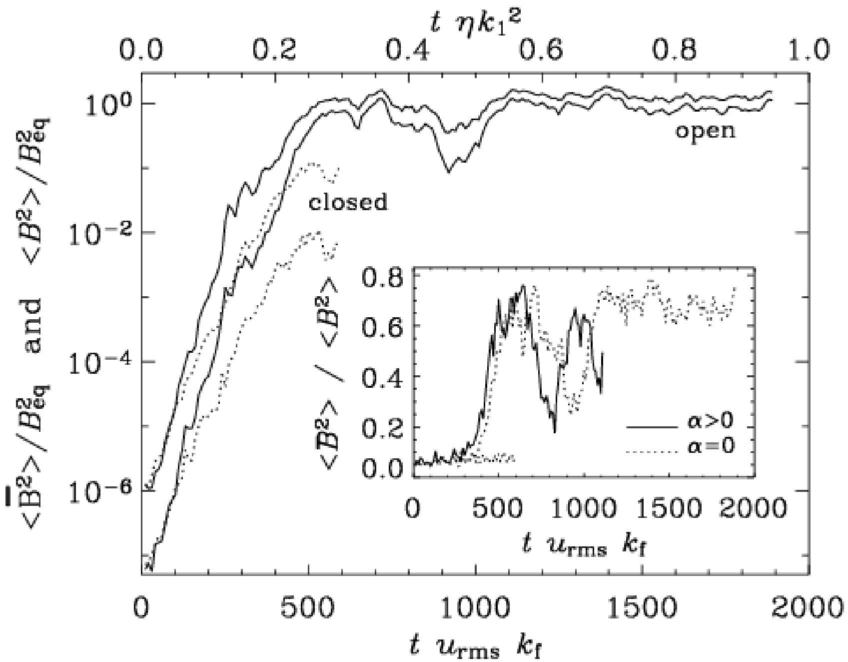
10^{46} Mx^2/cycle

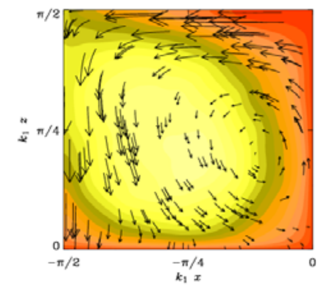
$$2\oint(\mathbf{e}\times\mathbf{j})\cdot d\mathbf{S} \qquad 2\oint(\overline{\mathbf{E}}\times\overline{\mathbf{J}})\cdot d\mathbf{S}$$

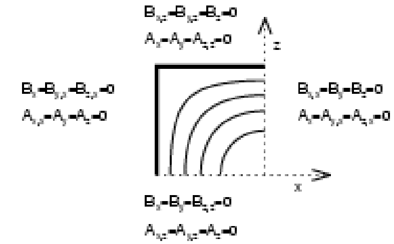
Brandenburg & Sandin (2004, A&A 427, 13)

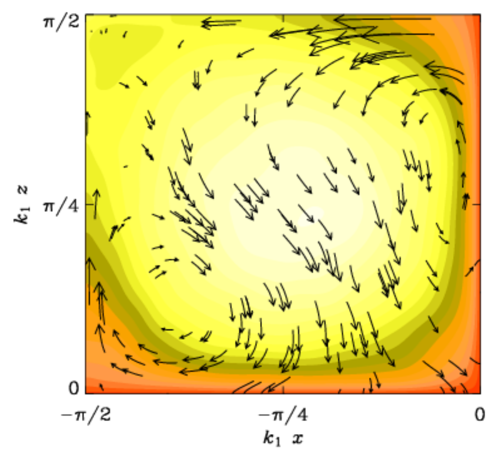
Helicity fluxes from shear: Vishniac & Cho (2001, ApJ 550, 752)
Subramanian & Brandenburg (2004, PRL 93, 20500)

Forced LS dynamo with ***no*** stratification









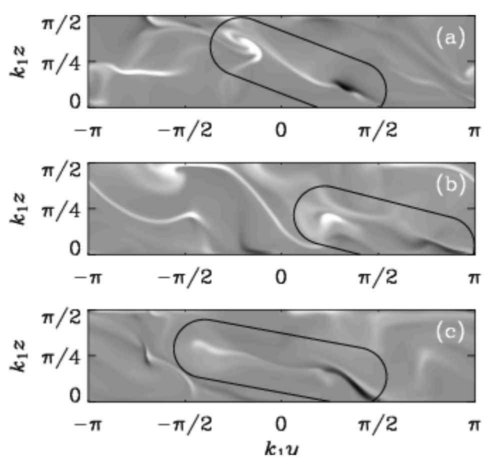
azimuthally averaged

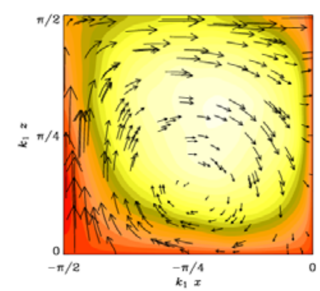
no helicity, e.g.

Rogachevskii & Kleeorin (2003, 2004)

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \left(a\overline{\mathbf{B}} + \tfrac{1}{\ell^2}\,\overline{\mathbf{W}} \times \overline{\mathbf{J}} + \ldots \right)$$

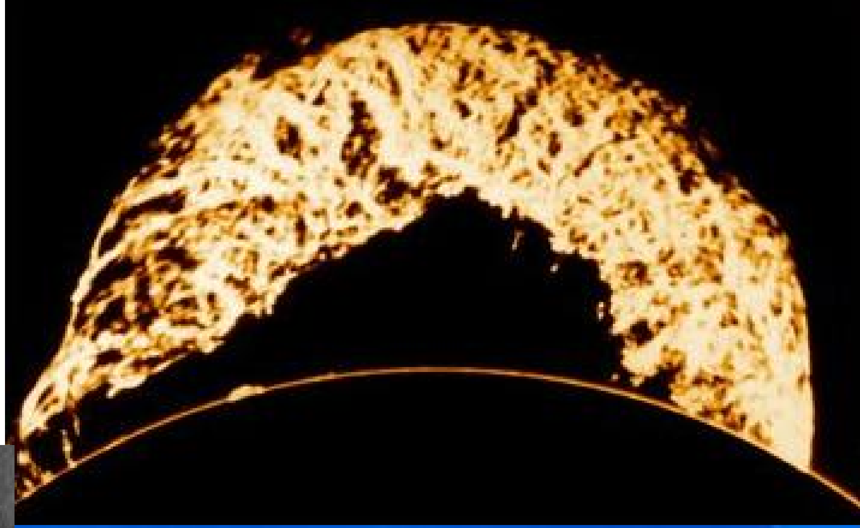
geometry here relevant to the sun

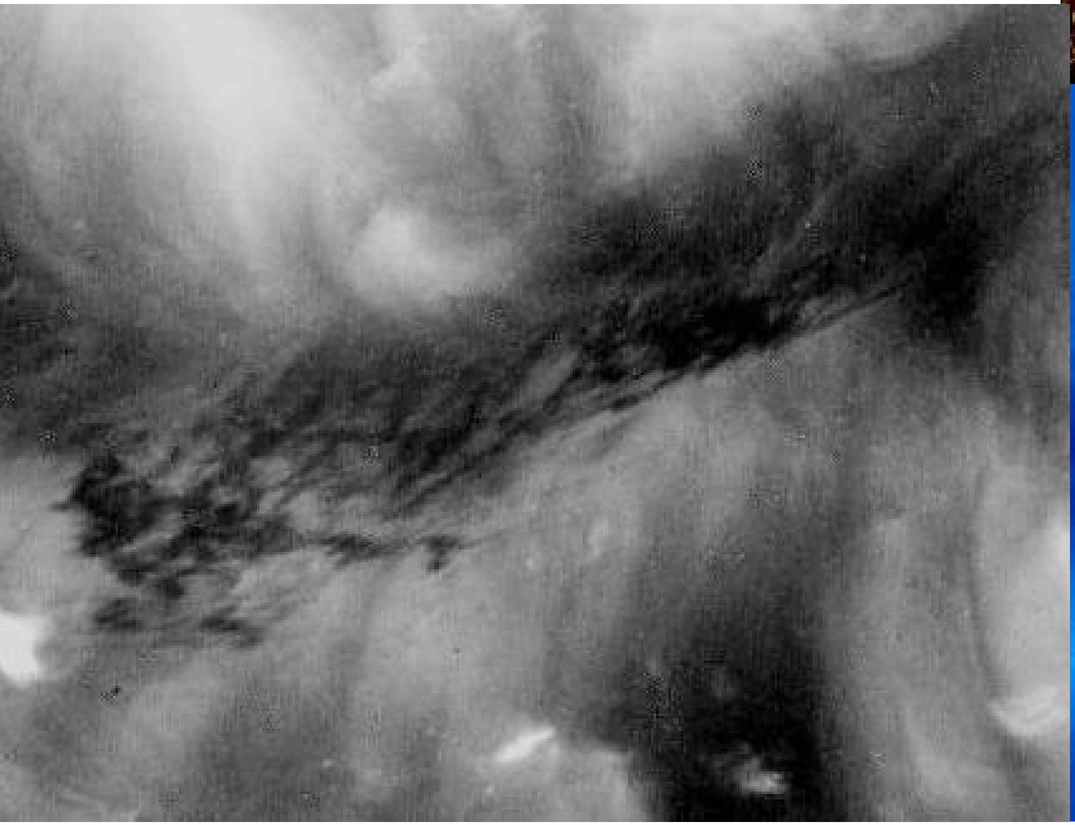


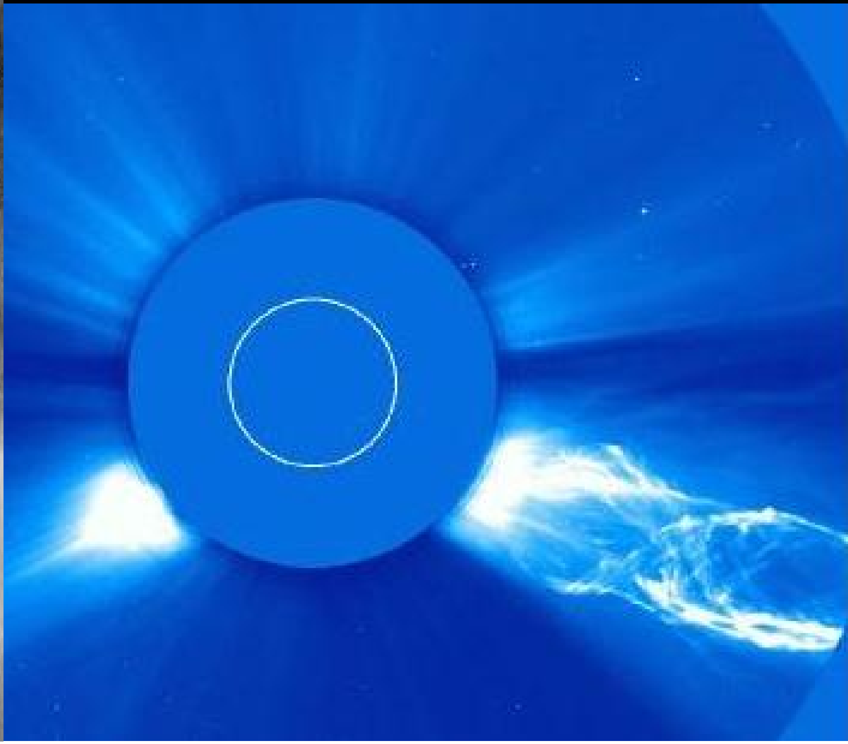


neg helicity (northern hem.)

Examples of helical structures







Mean field model with advective flux

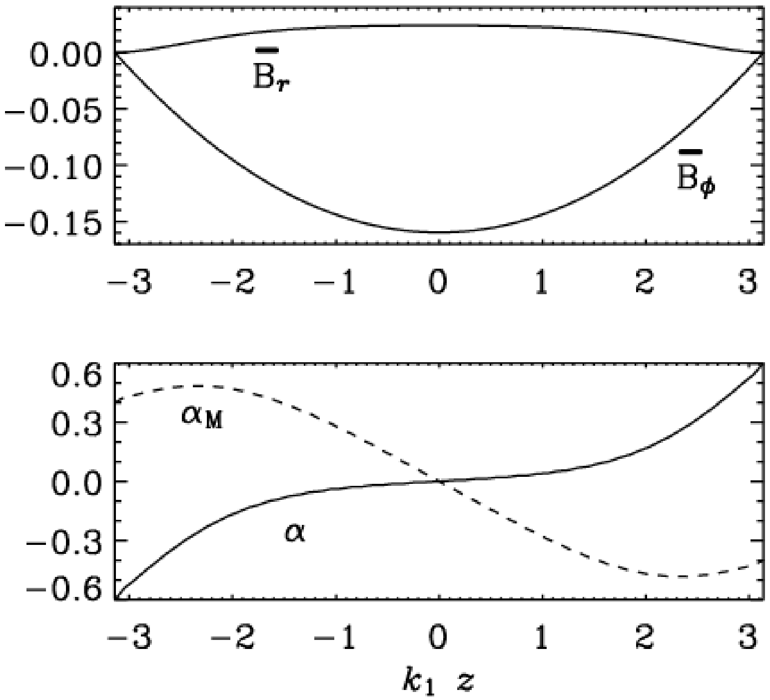
$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z \overline{B}_r + \mathcal{E}_\phi\right) + \eta \frac{\partial^2 \overline{B}_r}{\partial z^2},$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z \overline{B}_\phi - \mathcal{E}_r\right) + \eta \frac{\partial^2 \overline{B}_\phi}{\partial z^2} + q\Omega_0 \overline{B}_r,$$

$$\boldsymbol{\mathcal{E}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \overline{\boldsymbol{J}}$$

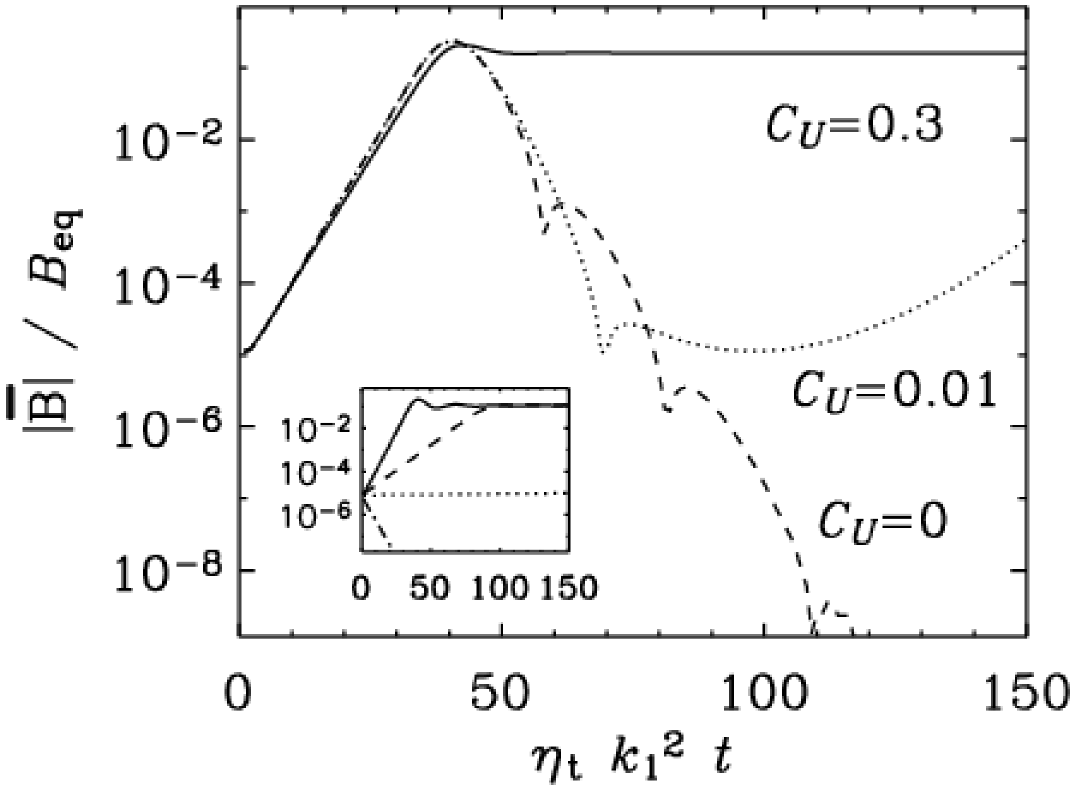
$$\alpha = \alpha_{\rm K} + \alpha_{\rm m}$$

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\eta_{\rm t} k_0^2 \left(\frac{\boldsymbol{\mathcal{E}} \cdot \overline{\boldsymbol{B}}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_{\rm m}}\right) - \boldsymbol{\nabla} \cdot \left(\alpha_{\rm m} \overline{\boldsymbol{U}}\right).$$



Shukurov et al. (2006, A&A 448, L33)

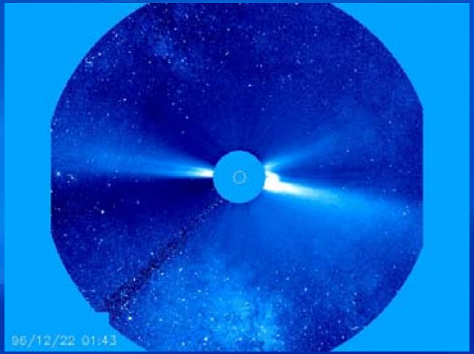
Saturation behavior with advective flux

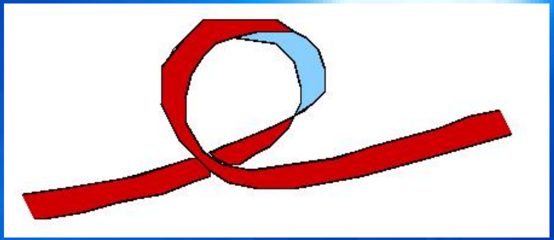


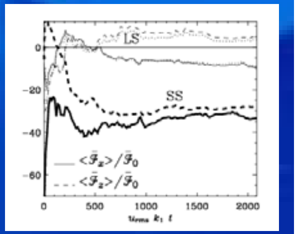
Shukurov et al. (2006, A&A 448, L33)

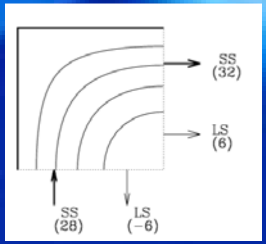
Conclusions

- LES & DNS ➔ $E_M(k)$ and $E_K(k)$ overlap (?)
- SS dynamo may not work in the sun
- Only LS dynamo, if excited
  - and if CMEs etc.









10^{46} Mx^2/cycle