Intermittency in MHD turbulence: DNS and Lagrangian averaged modeling

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Interesting Properties of the $\alpha-$model

- Lots of names: Camassa-Holm Eqs, Lagrangian averaged model, α-model, Lagrangian averaged magnetohydrodynamics (LAMHD), MHD α-model
- regularization of fluid equations
- conserves Kelvin's Thm (HD), Alfvén's Thm (MHD), and ideal invariants
- Kármán-Howarth theorem
- no need to run DNS and then switch on
- generally applicable MHD LES? (instabilities, dynamo, ...)

MHD-α

$$\mathbf{u_s} = (1 - \alpha_K^2 \nabla^2)^{-1} \mathbf{u} \qquad \mathbf{b_s} = (1 - \alpha_M^2 \nabla^2)^{-1} \mathbf{b}$$

$$\partial_t \mathbf{u} + \omega \times \mathbf{u}_s = -\nabla P + \mathbf{j} \times \mathbf{b}_s + \nu \nabla^2 \mathbf{u} + \mathcal{F}_\mathbf{K}$$

$$\partial_t \mathbf{b}_s = \nabla \times (\mathbf{u}_s \times \nabla \mathbf{b}_s) + \eta \nabla^2 \mathbf{b} + \mathcal{F}_\mathbf{M}$$

$$\nabla \cdot \mathbf{u_s} = 0 \qquad \nabla \cdot \mathbf{b_s} = 0$$

Decay Laws

$$\frac{dE}{dt} = \frac{d}{dt} \langle \frac{1}{2} (\mathbf{u} \cdot \mathbf{u}_s + \mathbf{b} \cdot \mathbf{b}_s) \rangle = -\nu \langle \omega \cdot \omega_s \rangle - \eta \langle j^2 \rangle$$

$$\frac{dH_C}{dt} = \frac{d}{dt} \langle \frac{1}{2} \mathbf{u} \cdot \mathbf{b}_s \rangle = -\frac{1}{2} \eta \langle \omega \cdot \mathbf{j} \rangle - \frac{1}{2} \nu \langle \omega \cdot \mathbf{j_s} \rangle$$

$$(2D) \frac{d\mathcal{A}}{dt} = \frac{d}{dt} \langle \frac{1}{2} a_{s_z}^2 \rangle = -\eta \langle \mathbf{b} \cdot \mathbf{b}_s \rangle$$

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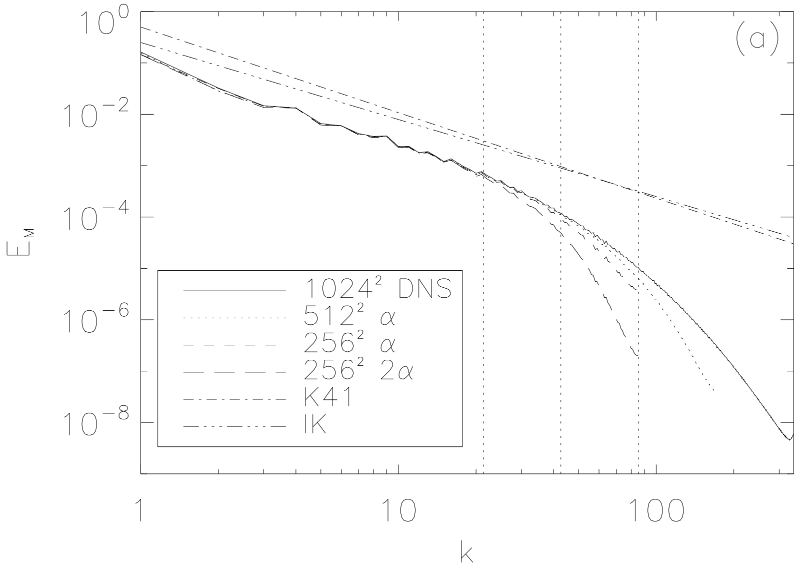
Previous Tests of the MHD α−model

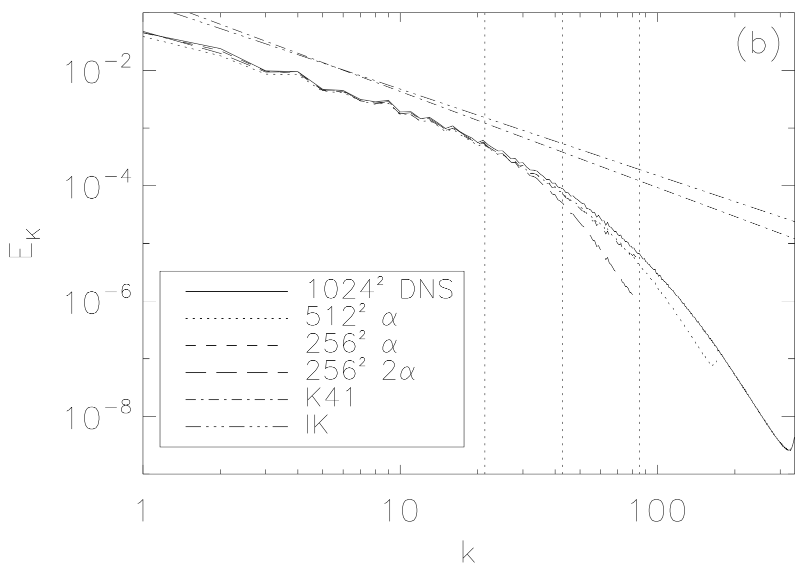
2D†	time evolution of energies	$\checkmark$
	time evolution of cross-helicity	$\approx$
	energy spectra	$+$
	dynamic alignment	$\approx$
	PDFs	except tails
	inverse cascade of vector potential	$<$
3D‡	time evolution of energies	$\checkmark$
	time evolution of magnetic helicity	$\approx$
	energy spectra	$\checkmark$
	dynamic alignment	$<$
	inverse cascade of magnetic helicity	$<$
	dynamo	$\checkmark$

† Mininni et al. *Phys. Fluids* **17**, 035112 (2005). ‡ Mininni et al. *Phys. Rev. E* **71**, 046304 (2005), Ponty et al. *Phys. Rev. Lett.* **94**, 164502 (2005).

Results - Spectra



magnetic



kinetic

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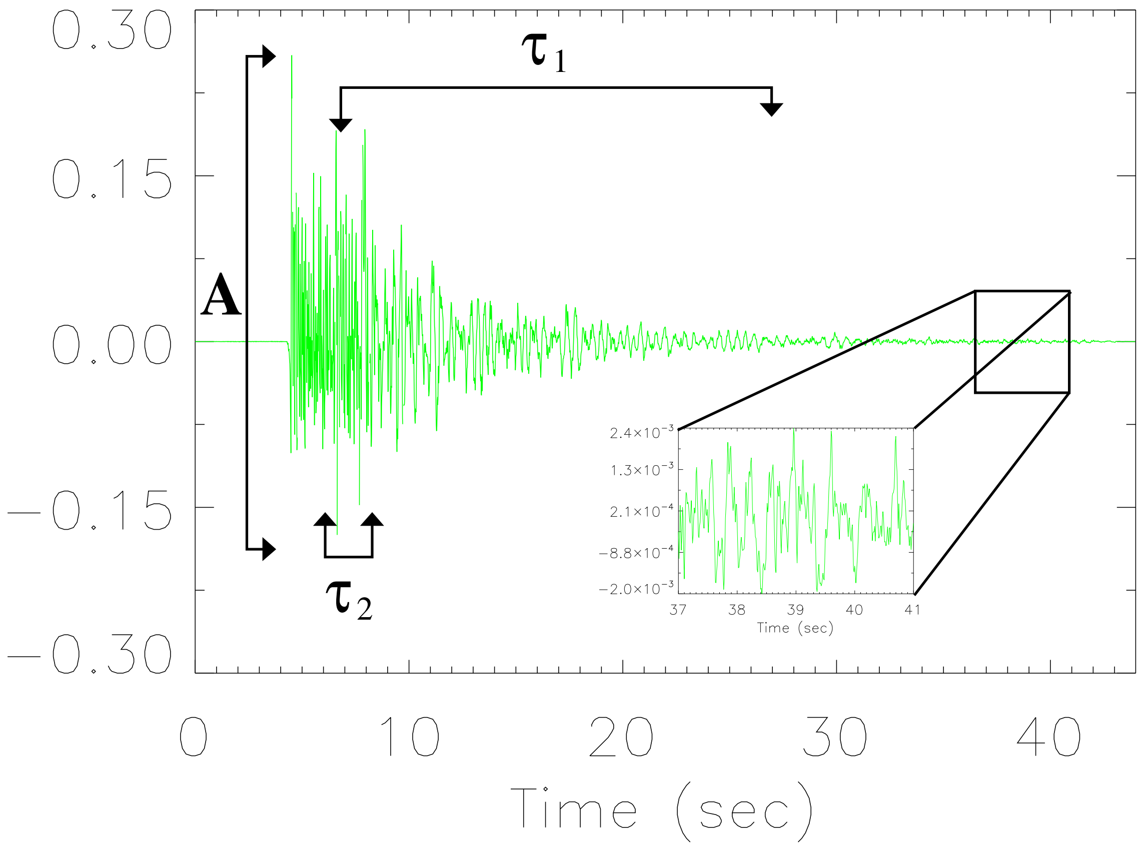
Goals of this work

More difficult (still *a posteriori*) tests

- rapid sign changes $\longrightarrow$ cancellation exponent
  - current sheets
  - magnetic reconnection
- intermittency (non-Gaussian statistics) $\longrightarrow$ anomalous scaling
  - reversals of Earth's magnetic field?
  - phase & amplitude variations of solar cycle
  - possible explanation for Maunder-like minima

EXPOSITION - Increments of Intermittent Signal



EXPOSITION - Structure Function Scaling for Self-Similarity

$$\delta f(\tau) \equiv f(t+\tau) - f(t) \tag{1}$$

$$\mathfrak{S}_p^f(\tau) \equiv \langle |\delta f(\tau)|^p \rangle^1 \tag{2}$$

$$\delta f(\lambda T) = \lambda^h \delta f(T) \tag{3}$$

self-similarity

$$\tau \equiv \lambda T$$

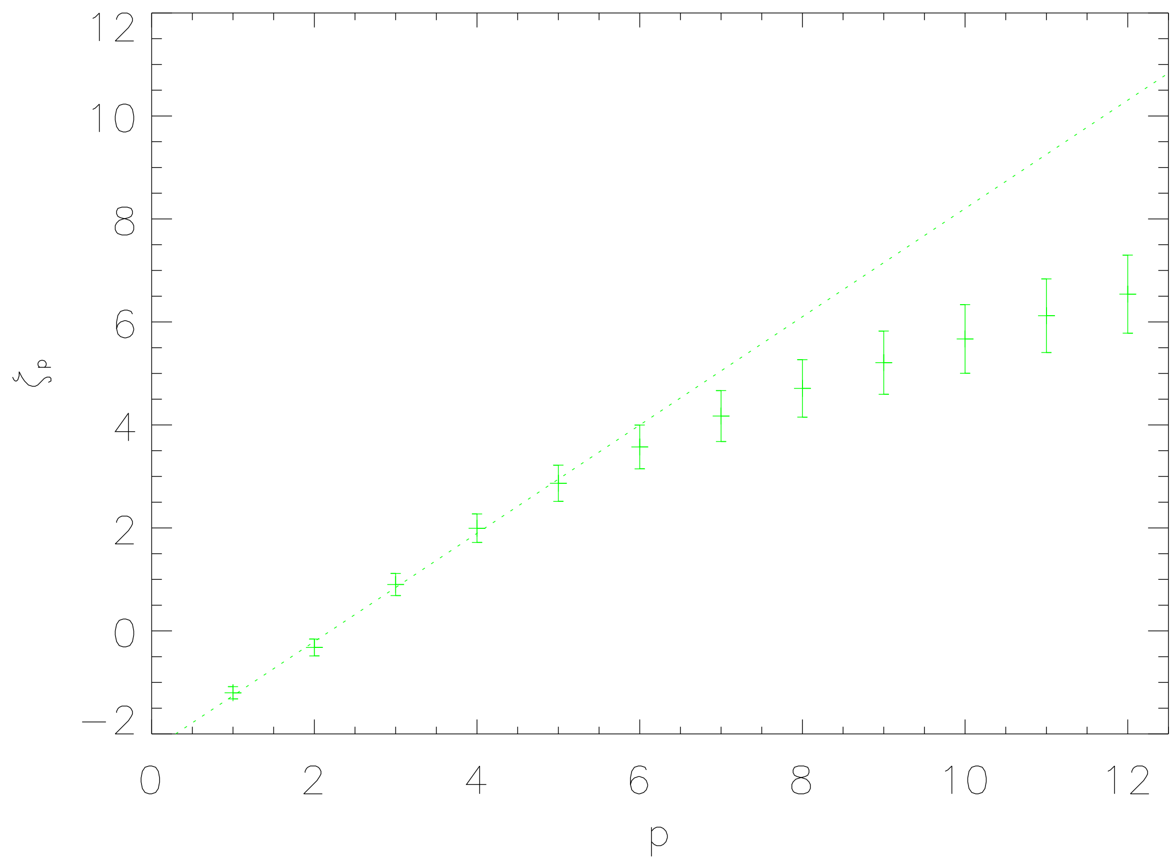
$$\mathfrak{S}_p^f(\tau) = \langle |\lambda^h \delta f(T)|^p \rangle = \lambda^{h \cdot p} \langle |\delta f(T)|^p \rangle \sim \tau^{\zeta_p^f}$$

$$\zeta_p^f = h \cdot p \tag{4}$$

If the statistical features of the system are independent of spatial scale, it is described as self-similar and it's scaling will be linear, $\zeta_p^f \sim p$.

[1] Angle brackets, $\langle \cdot \rangle$, denote integration over the entire domain.

EXPOSITION - Anomalous Scaling of Exponents



$$S_p(\tau) \sim \tau^{\zeta_p}$$

Numerical Tests

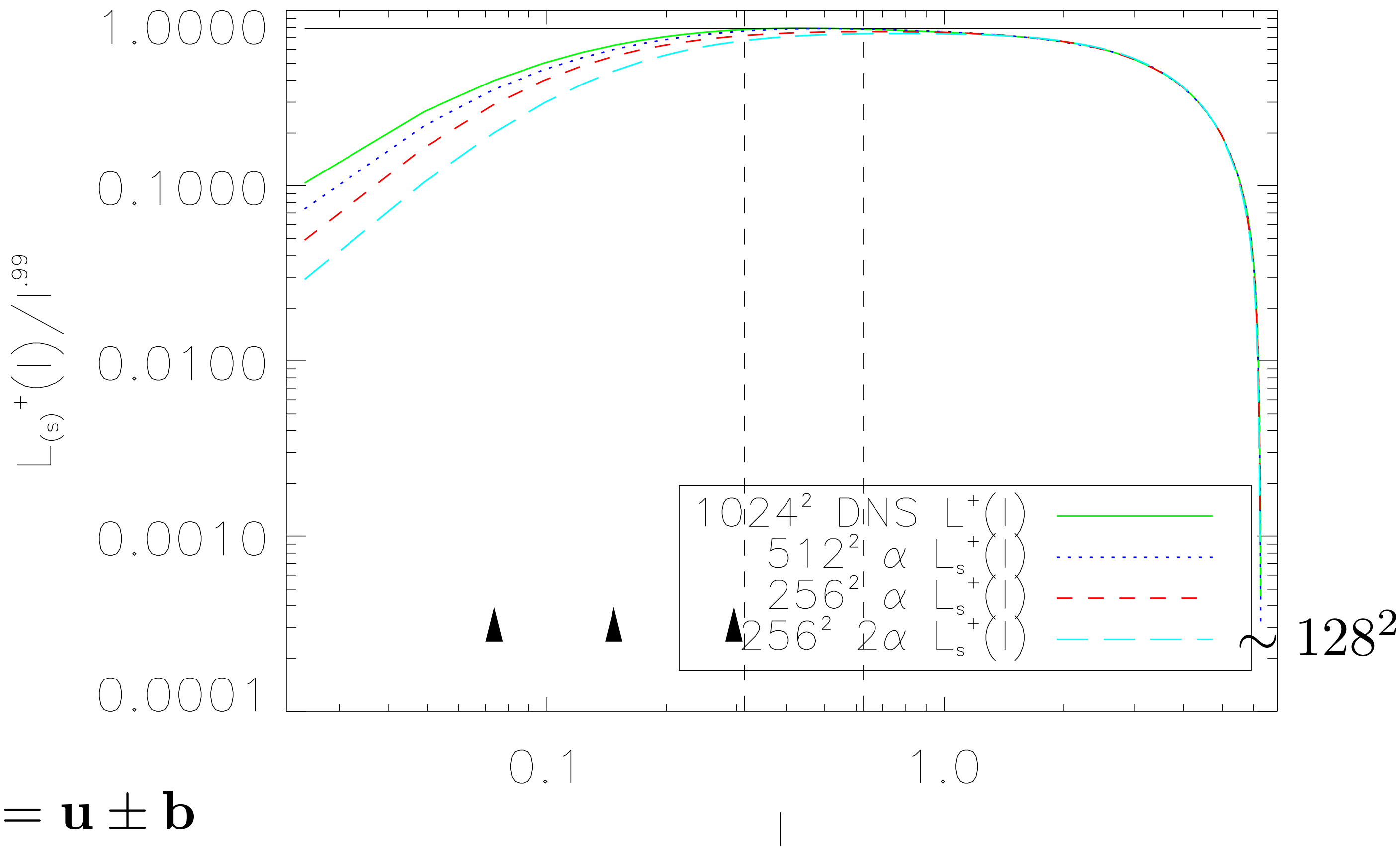
Table 1: Resolution, N, model parameter, α, viscosity, $\nu = \eta$, forcing, $\mathcal{F}_M, \mathcal{F}_K$, initial conditions, $u_o = b_0$, Taylor Reynolds number, R_λ,†

Run	N	$\alpha \cdot N$	$\nu = \eta$	$\mathcal{F}_M$	$\mathcal{F}_K$	$u_o = b_0$	R_λ
a	2048	0 (DNS)	10^{-4}	0	0	1	1500
b	1024	6	10^{-4}	0	0	1	1700
c	512	6	10^{-4}	0	0	1	1700
d	1024	0 (DNS)	$1.6 \cdot 10^{-4}$	0.2	0.45	0	1600
e	512	6	$1.6 \cdot 10^{-4}$	0.2	0.45	0	1300
f	256	6	$1.6 \cdot 10^{-4}$	0.2	0.45	0	1100
g	$256 \sim 128$	12	$1.6 \cdot 10^{-4}$	0.2	0.45	0	1000

$^\dagger R_\lambda = \frac{\lambda v_{rms}}{\nu}$, at peak of the dissipation, $t \approx 6.5$, for freely decaying runs (a-c) and averaged over $t = [50, 150] \times 9$ for forced runs (d-g).

$$\alpha^{-1} = k_{max}/2$$

Kármán-Howarth Thm Scaling - Numerical Results

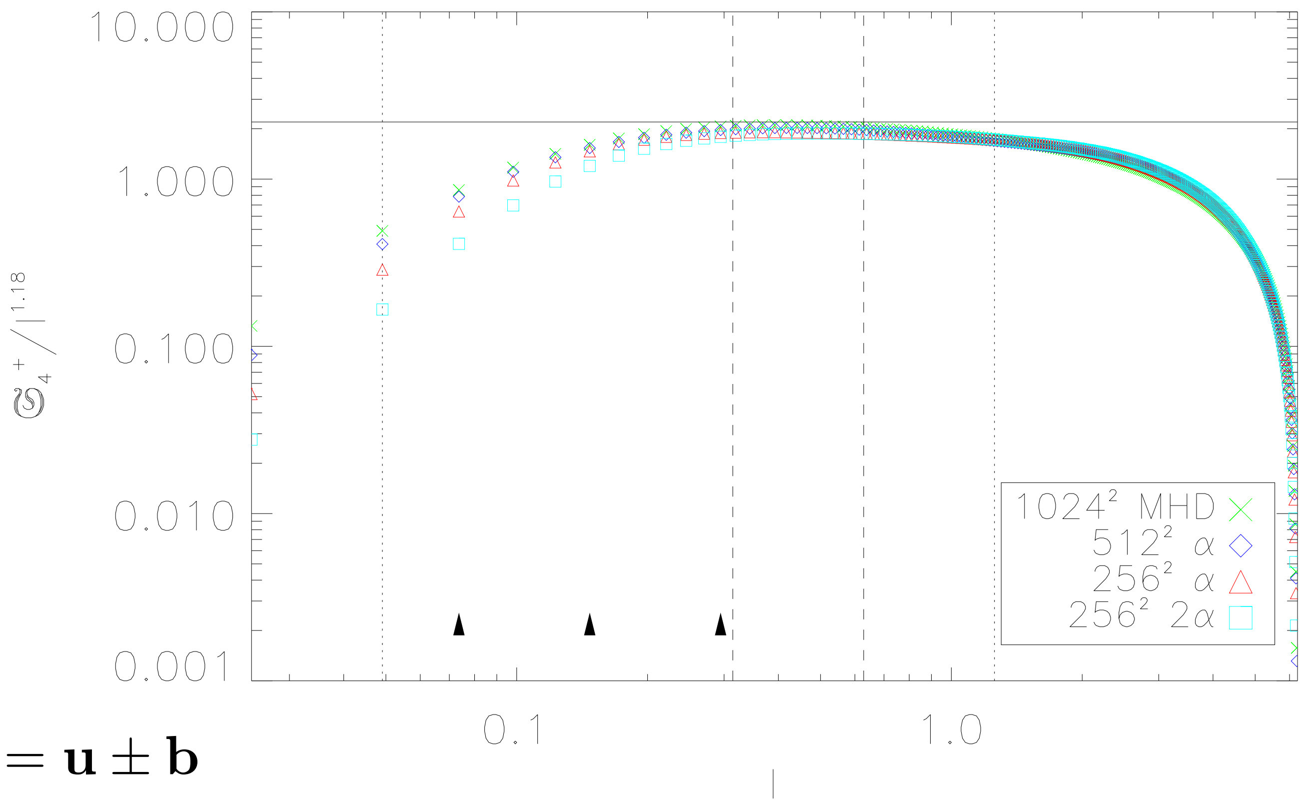


$$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$$

$$L^{+}(l) \equiv \langle |\delta z_L^{-}||\delta z^{+}|^2 \rangle \propto l$$

$$L_s^{+}(l) = \langle |\delta z_{s_L}^{-}||\delta z^{+}||_{\alpha}^{2} \rangle \propto l$$

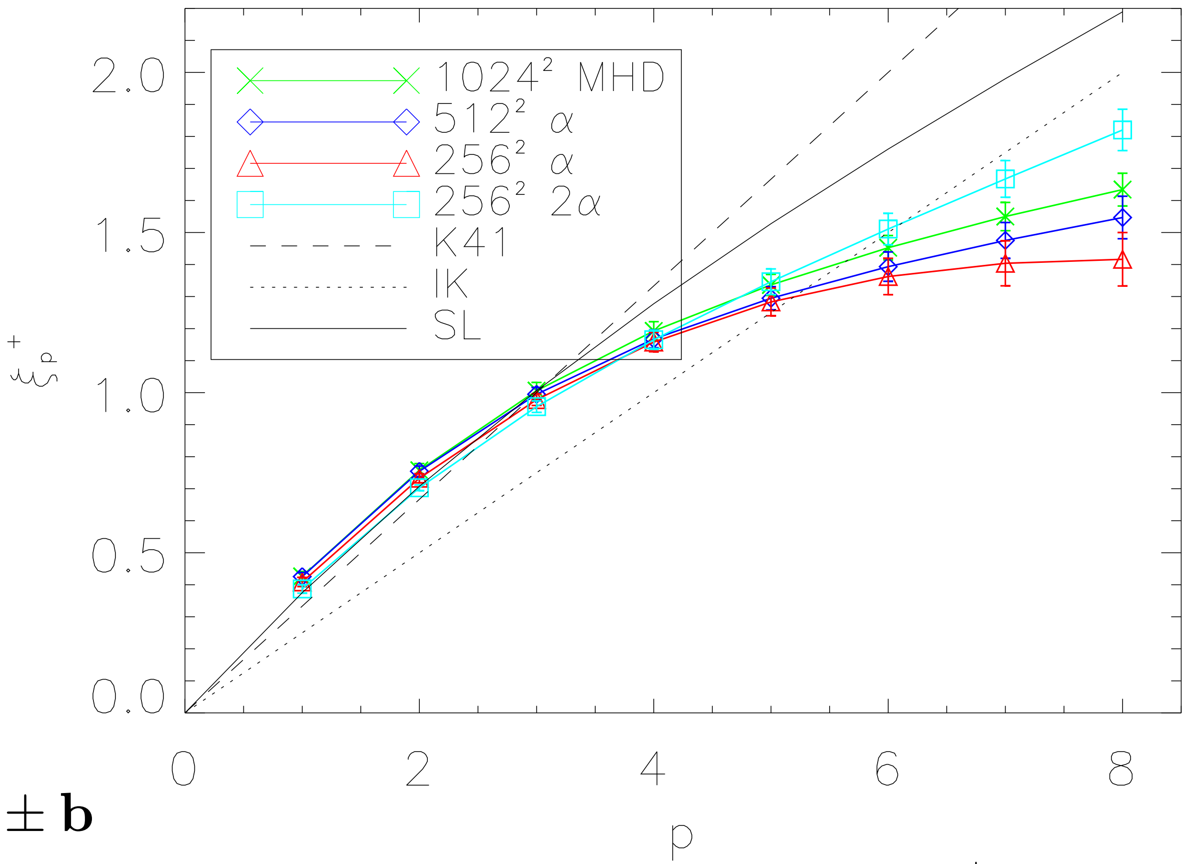
Scaling - Forced Numerical Results



$$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$$

$$\mathfrak{S}_4^+(l) \equiv \langle |\delta z_L^+|^4 \rangle$$

Anomalous Scaling Exponents - Forced



$$\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$$

$$\mathfrak{S}_p^+(l) \equiv \langle |\delta z_L^+|^p \rangle \sim [L_{(s)}^+(l)]^{\xi_p^+}$$

EXPOSITION - Cancellation Exponent

$$\mu_i(l) = \int_{Q_i(l)} d\mathbf{x}\, j_z(\mathbf{x}) \Big/ \int_{Q(L)} d\mathbf{x}\, |j_z(\mathbf{x})|$$

Phys. Plasmas, Vol. 9, No. 1, January 2002 Analysis of cancellation in two-dimensional . . . 93

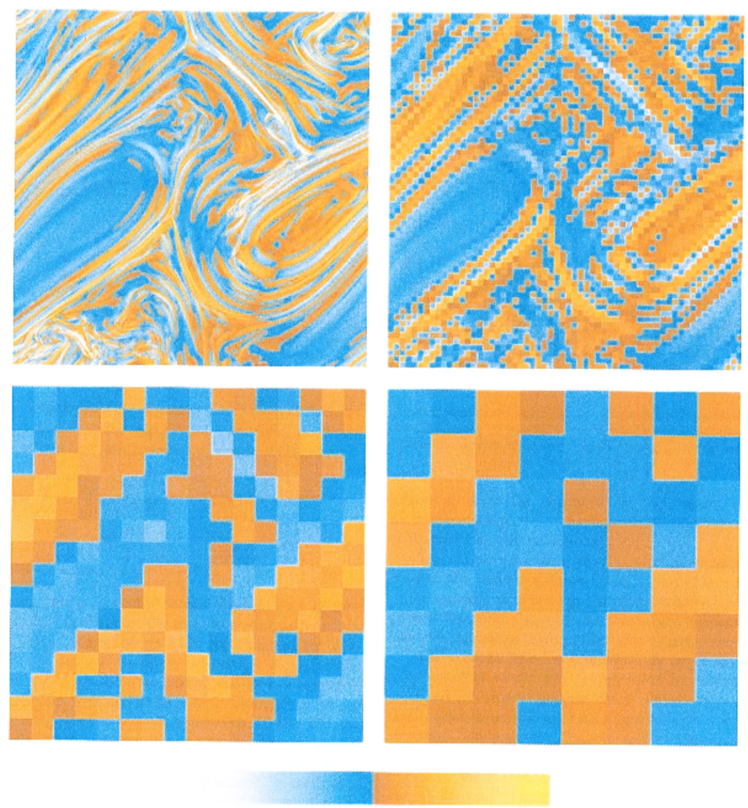


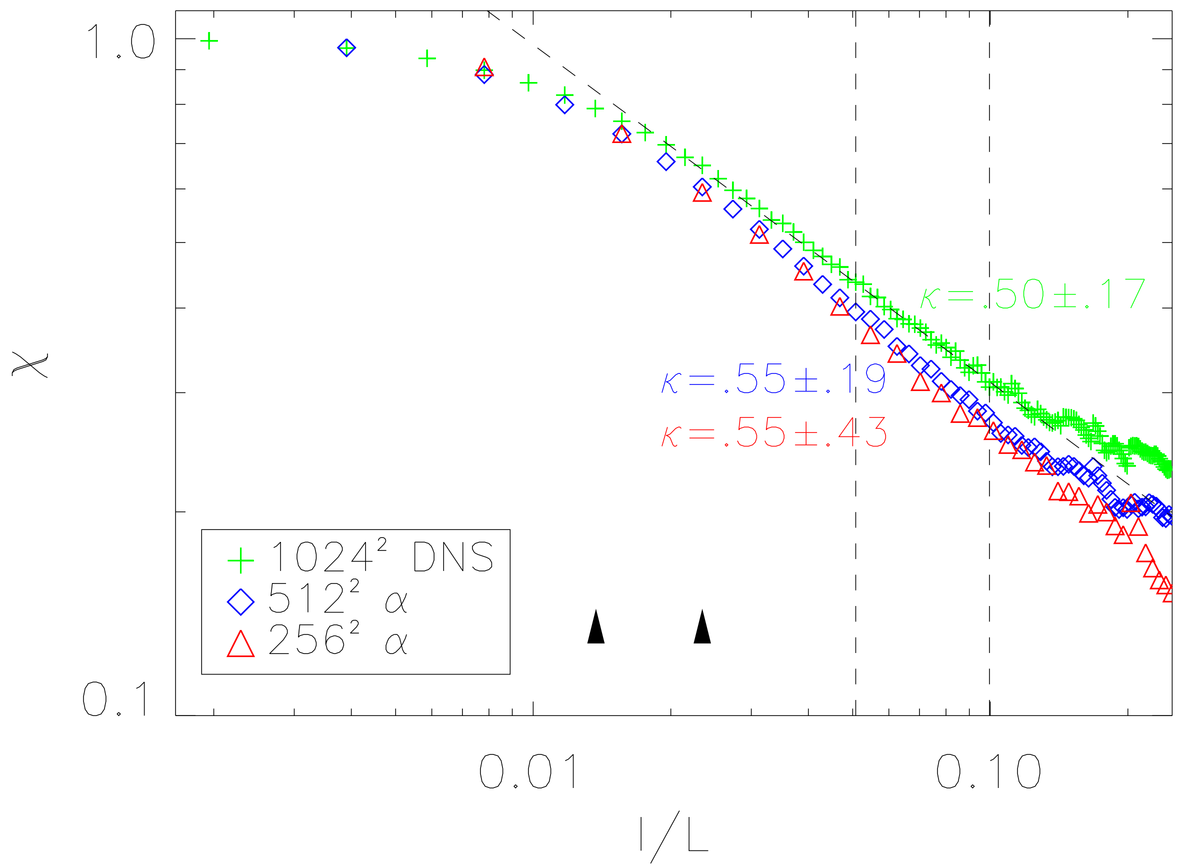
FIG. 3. (Color) The coarse-grained signed measure of the current J at time $t=7.3$ for four different box sizes, namely $l/L=0.001$, $l/L=0.016$, $l/L=0.059$, $l/L=0.12$, from top to bottom. Colors range from cyan for negative J values to yellow for positive ones, going through blue and brown. Cancellations at large scales are responsible for the decrease in magnitude of the measure.

$$\chi(l) = \sum_{Q_i(l)} |\mu_i(l)|$$

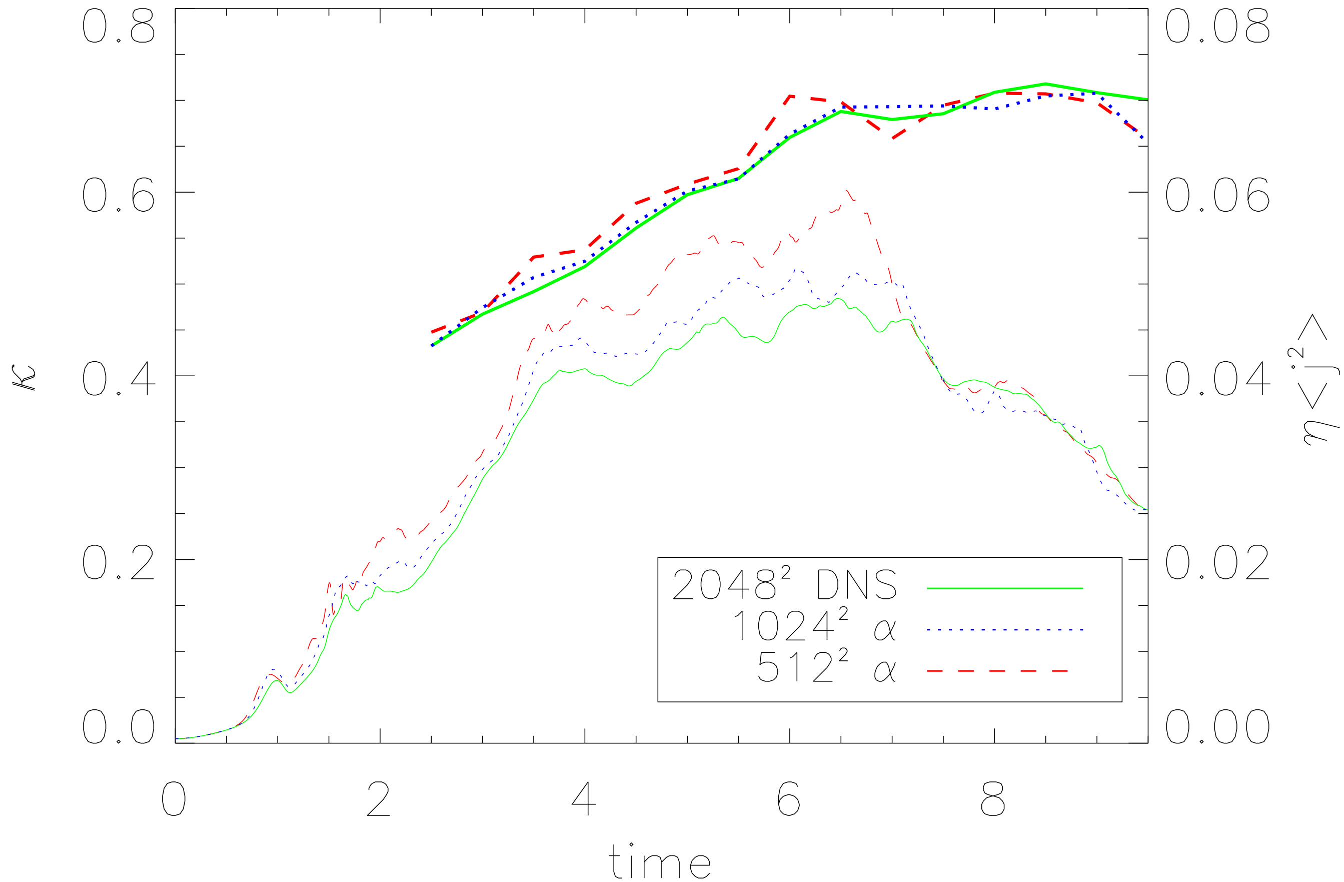
$$\chi(l) \sim l^{-\kappa}$$

$$\kappa = (d - D)/2$$

Results - Partition Function



Results - Cancellation Exponent



Summary & Future Directions

- α-model satisfies scalings of MHD
- reproduces intermittent features of turbulent flows
- alters small scales, but preserves some stats
- Future Directions
  - BC's (and geometries)
  - $P_M \neq 1$
  - α-model and related regularizations: Leray & Clark for 3D HD