
*Analysis of Ensemble Methods*

Shree P. Khare

skhare@samsi.info

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Adaptive observations ...

1. Atmospheric models
2. A (biased) view of sequential data assimilation in a perfect model setting
3. The adaptive observations problem
4. A general Bayesian solution
5. Using ensemble forecasts I - particle filter implementation of the general solution
6. Using ensemble forecasts II - an EnKF approach
7. Experiments in an atmospheric ‘toy model’
Coupled, nonlinear set of pde’s governs the flow of the atmosphere (wind, temperature, surface pressure ... )

\[ \frac{dx}{dt} = M(x, t) \] - state vector order $10^7$
Atmospheric models: east-west winds

![Wind Speed Diagram](image)

- **LATITUDE**
- **SIGMA**
- **[U] (m/s)**
Atmospheric models: mid-level temperature
Atmospheric models: surface pressure (PS)
Atmospheric models: PS Day 2/20

latitude (degrees_north)  longitude (degrees_east)
Atmospheric models: PS Day 3/20

Latitude (degrees_north) vs Longitude (degrees_east)
Atmospheric models: PS Day 6/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 7/20

Latitude (degrees_north)

Longitude (degrees_east)
Atmospheric models: PS Day 8/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 11/20

- latitude (degrees_north)

- longitude (degrees_east)
Atmospheric models: PS Day 12/20
Atmospheric models: PS Day 13/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 14/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 15/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 16/20
Atmospheric models: PS Day 17/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 18/20

longitude (degrees_east)

latitude (degrees_north)
Atmospheric models: PS Day 19/20

latitude (degrees_north)

longitude (degrees_east)
Atmospheric models: PS Day 20/20
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A (biased) view of sequential data assimilation

Objective - estimate the state $x_{t_i}$ given observations

$$y_t = H(x_t) + e_t$$
A (biased) view of sequential data assimilation

Observations of atmospheric quantities are uncertain and incomplete

\[ y_t = H(x_t) + e_t \]
A (biased) view of sequential data assimilation

- Adopt a probabilistic point of view - solve for conditional probabilities

\[ y_t = H(x_t) + e_t \]
A (biased) view of sequential data assimilation

Assume that at $t_i$ we have some pdf for the system state $p(x_{t_i})$
A (biased) view of sequential data assimilation

\[ y_t = H(x_t) + e_t \]

**Time evolution - Liouville equation**
A (biased) view of sequential data assimilation

\[
y_t = H(x_t) + e_t
\]

Updating - at $t_{i+1}$ update system pdf using Bayes rule
A (biased) view of sequential data assimilation

\[ y_t = H(x_t) + e_t \]

- Problem! x is order $10^7$
A (biased) view of sequential data assimilation

Time evolution - evolve an ensemble of state estimates - samples of the pdf

\[ y_t = H(x_t) + e_t \]
A (biased) view of sequential data assimilation

- Practical applications of interest - ensemble size $<<$ degrees of freedom
A (biased) view of sequential data assimilation

\[ y_t = H(x_t) + e_t \]

In practice - approximations are introduced in update algorithm - Gaussianity (KF)
Within subspace spanned by the ensemble - mean and covariance updated consistent with Kalman Filter update equations (EnKF)
A (biased) view of sequential data assimilation

Deterministic Ensemble Kalman Filters - we use EAKF (Anderson 2001, 2003)
A (biased) view of sequential data assimilation

\[ y_t = H(x_t) + e_t \]

Damping sampling error is critical - can handle sampling error systematically with EnKF algorithms
A (biased) view of sequential data assimilation

\[ y_t = H(x_t) + e_t \]

- The ability to handle sampling error → algorithms can be applied to systems with many d.o.f.
A (biased) view of sequential data assimilation

Fully nonlinear sequential filters have achieved marginal success in systems with many d.o.f.
A (biased) view of sequential data assimilation

- Can we interpret ensembles generated via approximate data assimilation schemes probabilistically?
A (biased) view of sequential data assimilation

- Imperfect models - difficult to implement - even harder to interpret probabilistically

\[ y_t = H(x_t) + e_t \]
Next ...

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The adaptive observations problem

- Routine observational network
  \[ y_{t, \text{routine}} = H_{t, \text{routine}}(x_t) + \epsilon_{t, \text{routine}} \]

- Adaptive observations - supplement to the routine network
The adaptive observations problem: an example

Today's global observing network

Operator: \( y_t = H(x_t) + e_t \)
The adaptive observations problem: an example

Intense LOW developing over Pacific and is expected to make landfall 4 days from now.
The adaptive observations problem: an example

LOW is poorly observed...
Supplement with Adaptive Observations to improve prediction
The adaptive observations problem: an example

What is the optimal FLIGHT PATH of an airplane equipped to drop dropsondes?
Operator: \[ y_t = [H_t; H_t]x_t + e_t \]
The adaptive observations problem: an example

Adaptive Observations: important source of information for Hurricane prediction
The adaptive observations problem: importance

- Provide supplemental observations in data sparse regions (hurricanes, winter storms)
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- Data denial
The adaptive observations problem: importance

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- Data denial
- Recent field study in Atlantic (THORPEX) shows *positive benefits*
The adaptive observations problem: importance

- Provide supplemental observations in data sparse regions (hurricanes, winter storms)
- Data denial
- Recent field study in Atlantic (THORPEX) shows positive benefits
- Winter Storm Reconnaissance Program (WSRP) operational at NCEP since 1999 - uses ensemble based methods
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A general Bayesian solution: specific problem

\[ F(t_v | H\text{-adaptive},t_a) \]

\[ H\text{-routine},t_a + H\text{-adaptive},t_a \]
A general Bayesian solution: analytically

- Perfect, deterministic dynamics $\frac{dx}{dt} = M(x, t)$
A general Bayesian solution: analytically

- Perfect, deterministic dynamics $\frac{dx}{dt} = M(x, t)$
- Apply to one adaptive observation time - can extend

Entire calculation takes place at $t_i$. 

- Trial observing network: $H$ routine $t_a$, $H$ adaptive $t_a$
A general Bayesian solution: analytically

- Perfect, deterministic dynamics $dx/dt = M(x, t)$
- Apply to one adaptive observation time - can extend
- Assume all distributions can be obtained analytically (may need MC approximations in practice)
A general Bayesian solution: analytically

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- Apply to one adaptive observation time - can extend
- Assume all distributions can be obtained analytically (may need MC approximations in practice)
- For a given \( H_{ta}^{\text{adaptive}} \), compute \( F(t_v|H_{ta}^{\text{adaptive}}) \)
A general Bayesian solution: analytically

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- For a given \( H_{ta}^{\text{adaptive}} \), compute \( F(t_v|H_{ta}^{\text{adaptive}}) \)
- Idea: try many different \( H_{ta}^{\text{adaptive}} \) to minimize \( F(t_v|H_{ta}^{\text{adaptive}}) \) (flight paths)
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- Trial observing network: $H_{ta}^{\text{routine}}, H_{ta}^{\text{adaptive}}$
A general Bayesian solution: analytically

- Perfect, deterministic dynamics $dx/dt = M(x, t)$
- Apply to one adaptive observation time - can extend
- Assume all distributions can be obtained analytically (may need MC approximations in practice)
- For a given $H_{t_a}^{\text{adaptive}}$, compute $F(t_v | H_{t_a}^{\text{adaptive}})$
- Idea: try many different $H_{t_a}^{\text{adaptive}}$ to minimize $F(t_v | H_{t_a}^{\text{adaptive}})$ (flight paths)
- Trial observing network: $H_{t_a}^{\text{routine}}, H_{t_a}^{\text{adaptive}}$
- Entire calculation takes place at $t_i$
Given $p(x_{ti})$
A general Bayesian solution: analytically

- Given $p(x_{ti})$
- Integrate Liouville equation to obtain $p(x_{ta})$
A general Bayesian solution: analytically

- Given $p(x_{ti})$
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- Let $y_{ta}$ be some specified set of observations consistent w/ trial observing network
A general Bayesian solution: analytically

1. Given $p(x_{ti})$
2. Integrate Liouville equation to obtain $p(x_{ta})$
3. Let $y_{ta}$ be some specified set of observations consistent w/ trial observing network
4. $p(x_{ta} | y_{ta}) = \frac{p(x_{ta})p(y_{ta} | x_{ta})}{p(y_{ta})}$
A general Bayesian solution: analytically

- Given \( p(x_{t_i}) \)
- Integrate Liouville equation to obtain \( p(x_{t_a}) \)
- Let \( y_{t_a} \) be some specified set of observations consistent w/ trial observing network

\[
p(x_{t_a} | y_{t_a}) = \frac{p(x_{t_a})p(y_{t_a} | x_{t_a})}{p(y_{t_a})}
\]

- Integrate Liouville equation to obtain \( p(x_{tv} | y_{t_a}) \)
A general Bayesian solution: analytically

- Given $p(x_{t_i})$
- Integrate Liouville equation to obtain $p(x_{t_a})$
- Let $y_{t_a}$ be some specified set of observations consistent w/ trial observing network
- $p(x_{t_a} | y_{t_a}) = \frac{p(x_{t_a})p(y_{t_a} | x_{t_a})}{p(y_{t_a})}$
- Integrate Liouville equation to obtain $p(x_{t_v} | y_{t_a})$
- Apply some norm to $p(x_{t_v} | y_{t_a})$ - to measure uncertainty
A general Bayesian solution: analytically

- Given \( p(x_{ti}) \)
- Integrate Liouville equation to obtain \( p(x_{ta}) \)
- Let \( y_{ta} \) be some specified set of observations consistent w/ trial observing network
- \[ p(x_{ta} | y_{ta}) = \frac{p(x_{ta})p(y_{ta} | x_{ta})}{p(y_{ta})} \]
- Integrate Liouville equation to obtain \( p(x_{tv} | y_{ta}) \)
- Apply some norm to \( p(x_{tv} | y_{ta}) \) - to measure uncertainty
- Denote \( L(p(x_{tv} | y_{ta})) \)
A general Bayesian solution: analytically

Example: \( L(p(x_{tv} | y_{ta})) = \int (x_{tv} - \overline{x}_{tv})^2 p(x_{tv} | y_{ta}) dx_{tv} \)
A general Bayesian solution: analytically

Example: \[ L(p(x_{tv} | y_{ta})) = \int (x_{tv} - \bar{x}_{tv})^2 p(x_{tv} | y_{ta}) dx_{tv} \]

A reasonable method of assigning a number to \[ F(t_v | H_{ta}^{\text{adaptive}}) \] ...
A general Bayesian solution: analytically

Example: \( L(p(x_{tv} | y_{ta})) = \int (x_{tv} - \bar{x}_{tv})^2 p(x_{tv} | y_{ta}) dx_{tv} \)

A reasonable method of assigning a number to \( F(t_v | H_{ta}^{\text{adaptive}}) \) ...

Compute expectation of \( L(p(x_{tv} | y_{ta})) \) over all possible observations consistent with trial network
A general Bayesian solution: analytically

Example: \[ L(p(x_{tv} | y_{ta})) = \int (x_{tv} - \bar{x}_{tv})^2 p(x_{tv} | y_{ta}) dx_{tv} \]

A reasonable method of assigning a number to \( F(t_v | H_{ta}^{\text{adaptive}}) \) ...

Compute expectation of \( L(p(x_{tv} | y_{ta})) \) over all possible observations consistent with trial network

\[ F(t_v | H_{ta}^{\text{adaptive}}) = \int L(p(x_{tv} | y_{ta})) p(y_{ta}) dy_{ta} \]
A general Bayesian solution: analytically

- Example: \( L(p(x_{tv} | y_{ta})) = \int (x_{tv} - \bar{x}_{tv})^2 p(x_{tv} | y_{ta}) dx_{tv} \)

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- Compute expectation of \( L(p(x_{tv} | y_{ta})) \) over all possible observations consistent with trial network

- \( F(t_v | H_{ta}^{\text{adaptive}}) = \int L(p(x_{tv} | y_{ta})) p(y_{ta}) dy_{ta} \)

- \( p(y_{ta}) = \int p(x_{ta}) p(y_{ta} | x_{ta}) dx_{ta} \)
A general Bayesian solution: analytically

Example: \( L(p(x_{tv}|y_{ta})) = \int (x_{tv} - \bar{x}_{tv})^2 p(x_{tv}|y_{ta}) dx_{tv} \)

A reasonable method of assigning a number to \( F(t_v|H_{ta}^{\text{adaptive}}) \) ...

Compute expectation of \( L(p(x_{tv}|y_{ta})) \) over all possible observations consistent with trial network

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F(t_v|H_{ta}^{\text{adaptive}}) = \int L(p(x_{tv}|y_{ta}))p(y_{ta}) dy_{ta}
\]

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p(y_{ta}) = \int p(x_{ta})p(y_{ta}|x_{ta}) dx_{ta}
\]

Generally not possible analytically - revert to MC approximations for all above calculations
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General solution: particle filter approach

$H$-routine, $t_i$
General solution: particle filter approach

H-routine, \( t_i \)

Generate ensemble forecast out to - \( t_v \)
General solution: particle filter approach

H-routine,t_i

Trial network at t_a -

H-routine,t_a ‘+’ H-adaptive,t_a
General solution: particle filter approach

Forecast for $t_a - x_{i,t_a}^f$ with $i = 1, ..., N$ and weights $w_{i,t_a}^f$
General solution: particle filter approach

- Forecast for $t_a - x_{i,t_a}^f$ with $i = 1, ..., N$ and weights $w_{i,t_a}^f$
- Generate $j = 1, ..., J$ random samples of
  \[ p(y_{t_a}) = \int p(x_{t_a})p(y_{t_a}|x_{t_a})dx_{t_a} \]
General solution: particle filter approach

- Forecast for $t_a - x_{i,t_a}^f$ with $i = 1, ..., N$ and weights $w_{i,t_a}^f$

- Generate $j = 1, ..., J$ random samples of
  $p(y_{t_a}) = \int p(x_{t_a})p(y_{t_a}|x_{t_a})dx_{t_a}$

- For $j^{th}$ sample, re-weight: $w_{i,t_a,j}^u \sim e^{-[y_{t_a,j} - H_{t_a}^{\text{adaptive}}(x_{i,t_a}^f)]^T R_{t_a}^{-1} [y_{t_a,j} - H_{t_a}^{\text{adaptive}}(x_{i,t_a}^f)]/2} w_{i,t_a}^f$
General solution: particle filter approach

- Forecast for \( t_a \) - \( x_{i,t_a}^f \) with \( i = 1, \ldots, N \) and weights \( w_{i,t_a}^f \)
- Generate \( j = 1, \ldots, J \) random samples of
  \[
p(y_{t_a}) = \int p(x_{t_a}) p(y_{t_a} | x_{t_a}) dx_{t_a}
  \]
- For \( j^{th} \) sample, re-weight: \( w_{i,t_a,j}^u \sim \)
  \[
e^{-[y_{t_a,j} - H_{t_a}^{\text{adaptive}}(x_{i,t_a}^f)]^T R_{t_a}^{-1} [y_{t_a,j} - H_{t_a}^{\text{adaptive}}(x_{i,t_a}^f)]/2} w_{i,t_a}^f
  \]
- Updated weights apply for \( t_v \) - compute \( j^{th} \) value for norm using \( x_{i,t_v}^f \) and \( w_{i,t_a,j}^u \)
General solution: particle filter approach

- Forecast for $t_a - x_{i,t_a}^f$ with $i = 1, \ldots, N$ and weights $w_{i,t_a}^f$

- Generate $j = 1, \ldots, J$ random samples of
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  p(y_{t_a}) = \int p(x_{t_a})p(y_{t_a} | x_{t_a})dx_{t_a}
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  \]

- Updated weights apply for $t_v$ - compute $j^{th}$ value for norm using $x_{i,t_v}^f$ and $w_{i,t_a,j}^u$

- Take expectation over $J$ samples approximates
  \[
  F(t_v | H_{t_a}^{\text{adaptive}}) = \int L(p(x_{t_v} | y_{t_a})) p(y_{t_a}) dy_{t_a}
  \]
General solution: particle filter approach

- Forecast for $t_a - x_{i,t_a}^f$ with $i = 1, \ldots, N$ and weights $w_i^{f,t_a}$

- Generate $j = 1, \ldots, J$ random samples of
  \[ p(y_{t_a}) = \int p(x_{t_a})p(y_{t_a}|x_{t_a})dx_{t_a} \]

- For $j^{th}$ sample, re-weight:
  \[ w_{i,t_a,j}^{u} \sim e^{(-[y_{t_a,j}-H_{ta}^{adaptive}(x_{i,t_a}^f)]^T R_{t_a}^{-1}[y_{t_a,j}-H_{ta}^{adaptive}(x_{i,t_a}^f)]/2)} w_i^{f,t_a} \]

- Updated weights apply for $t_v$ - compute $j^{th}$ value for norm using $x_{i,t_v}^f$ and $w_{i,t_a,j}^{u}$

- Take expectation over $J$ samples approximates
  \[ F(t_v|H_{ta}^{adaptive}) = \int L(p(x_{t_v}|y_{t_a}))p(y_{t_a})dy_{t_a} \]

- Problems with sampling errors for small ensembles?
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Approximate solution: EnKF approach

H-routine, $t_i$
Approximate solution: EnKF approach

H-routine, $t_i$

Generate ensemble forecast out to $t_v$
Approximate solution: EnKF approach

H-routine, t_i

Trial network at t_a -

H-routine, t_a ‘+’ H-adaptive, t_a
Approximate solution: EnKF approach

H-routine, t_i

Idea: Use an EnKF based algorithm to update the ensemble forecast
Approximate solution: EnKF approach

H-routine,t_i

This generates an estimate of $F(t_v \mid H\text{-adaptive},t_a)$
Approximate solution: EnKF approach

H-routine, t_i

*** The KEY - the ensemble forecast need only be generated once ***
Approximate solution: EnKF approach

- $t_i$
- $t_a$
- $t_v$

H-routine, $t_i$

Can evaluate many H-adaptive, $t_a$ without repeatedly integrating the forecast model - Comp. Efficient
Approximate solution: EnKF approach

- Interpret ensemble forecast from $t_i$ as a sample of a PRIOR $p(x_{t_a}, x_{t_v})$

- Let the observation values at $t_a$ be:
  $$y_{t_a} = [H_{t_a}^{\text{routine}}, H_{t_a}^{\text{adaptive}}] \vec{x}_{t_a}^f$$

- The updated joint state distribution, given the hypothetical network at $t_a$:

  $$p(x_{t_a}, x_{t_v} | y_{t_a}) = \frac{p(x_{t_a}, x_{t_v})p(y_{t_a})}{\text{Normalization}}$$  \hspace{1cm} (0)

- Update PRIOR using and EnKF (Gaussian specifications)
A computationally efficient scheme: theory

- Use updated ensemble to compute covariance for $x_{tv}$ - can then quantify $F(H_{ta}^{adaptive})$

- Theory for Deterministic EnSRF implementations with no covariance localization

- Without covariance localization - DEnSRF implementations are equivalent NCEP’s operational scheme (Bishop et al. 2001)

- No repeated integrations of forecast model in evaluating many $H_{ta}^{adaptive}$ - operationally feasible
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Experiments in an atmospheric ‘toy model’

\[ \frac{dx_j}{dt} = -x_{j-2}x_{j-1} + x_{j-1}x_{j+1} - x_j + F \]

- Non-linear advection, forcing, linear dissipation, energy conservation \( E = (x_1^2 + \ldots + x_{40}^2) \)
- \( F = 8 \) sensitive dependence on IC’s, \( j = 1, \ldots, 40 \), cyclic
- Climatology \( \sqrt{\sigma_{climate}} = 3.6 \) and \( \bar{x} = 2.3 \)
- Look at small perturbations to steady state solution, \( x_j = F \),
  \[ \delta x_j = \sum_k \exp(-i\omega t)\exp(ikj) \]

\[ \omega(k) = -[\sin(k) + \sin(2k)]F + i[(\cos(k) - \cos(2k))F - 1] \]
Experiments in an atmospheric ‘toy model’

Lorenz 1996 (F = 8, j = 1, ..., 40)

dT = 0.05 units = 6 ‘hours’

Look at results from 10000 consecutive experiments

Find optimal location of one accurate observation at each t_a

Verification region - x1

N = 20, EAKF assimilation with inflation and GC localization

T_v - T_a = T_a - T_i = L(dT)
Experiments in an atmospheric ‘toy model’

Ensemble size 020

- OPTIMAL
- RANDOM
- ETKF

mean absolute forecast error for x1

lead time
Experiments in an atmospheric ‘toy model’
Experiments in an atmospheric ‘toy model’

\[ T_i \]

\[ T_t \]

\[ X_1 \]

\[ X_j \]

\[ X_j \]
Experiments in an atmospheric ‘toy model’

Ensemble size 020

mean absolute forecast error for x1

lead time

OPTIMAL
RANDOM
ETKF
SPACE–TIME–LOC
Experiments in an atmospheric ‘toy model’
Summary and conclusions

- Outlined general theory in perfect model setting - particle filter implementation
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- Computationally efficient approximation - ETKF scheme used operationally at NCEP
Summary and conclusions

- Outlined general theory in perfect model setting - particle filter implementation
- Computationally efficient approximation - ETKF scheme used operationally at NCEP
- Key result - handling sampling errors crucial to improving operational ETKF scheme
Key future areas in adaptive observations

- Using optimization - non-trivial - flight paths, air traffic control, physical restrictions ...
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- Data denial (THORPEX)
Key future areas in adaptive observations

- Using optimization - non-trivial - flight paths, air traffic control, physical restrictions ...

- Data denial (THORPEX)

- How do we quantify uncertainty under influence of $H_{ta}^{\text{adaptive}}$ in the imperfect model case?