Part I: Adaptive Observations for Improved Prediction of Geophysical Fluid Flows

Analysis of Ensemble Methods

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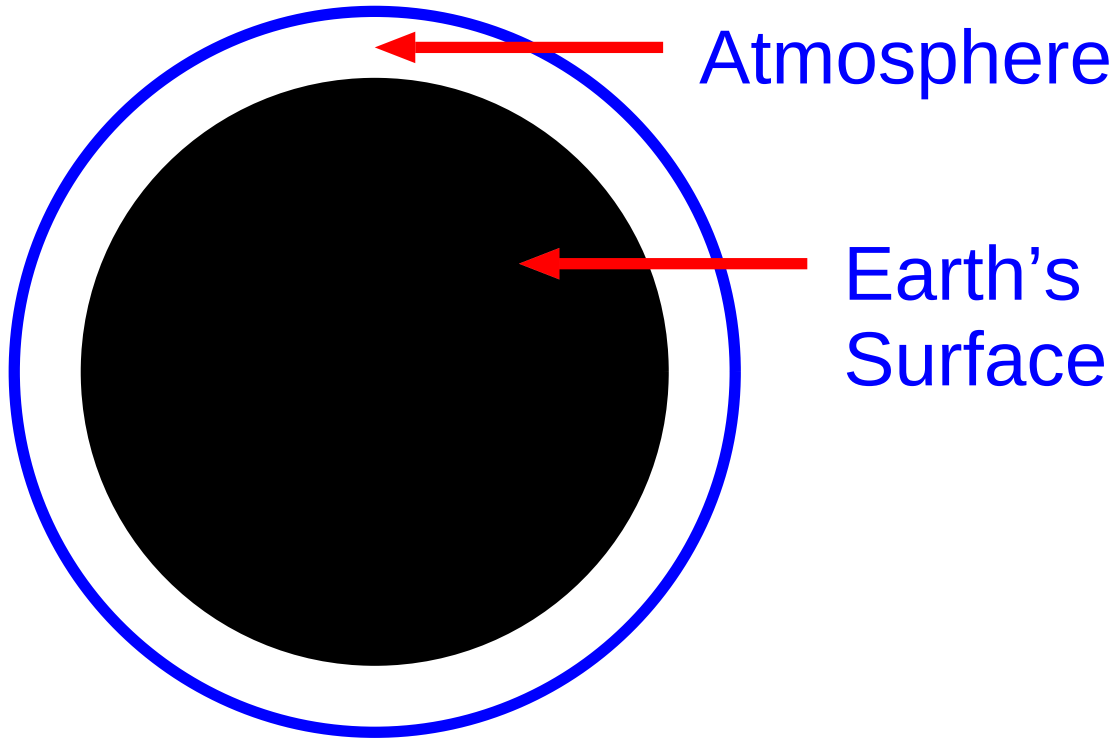
SAMSI-IMAGE Summer School, June 2005

Adaptive observations ...

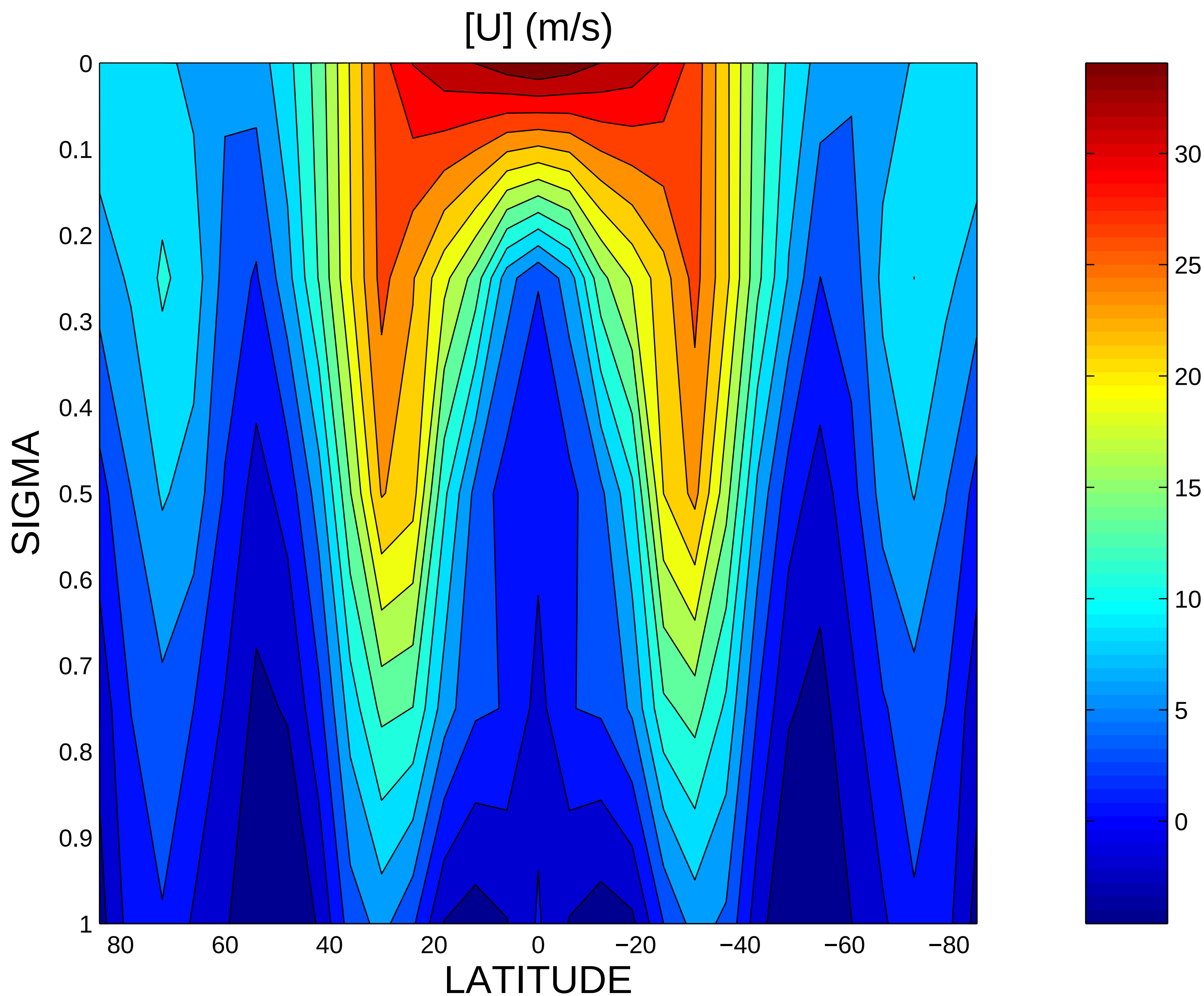
- 1. Atmospheric models
- 2. A (biased) view of sequential data assimilation in a perfect model setting
- 3. The adaptive observations problem
- 4. A general Bayesian solution
- 5. Using ensemble forecasts I - particle filter implementation of the general solution
- 6. Using ensemble forecasts II - an EnKF approach
- 7. Experiments in an atmospheric ‘toy model’

Atmospheric models

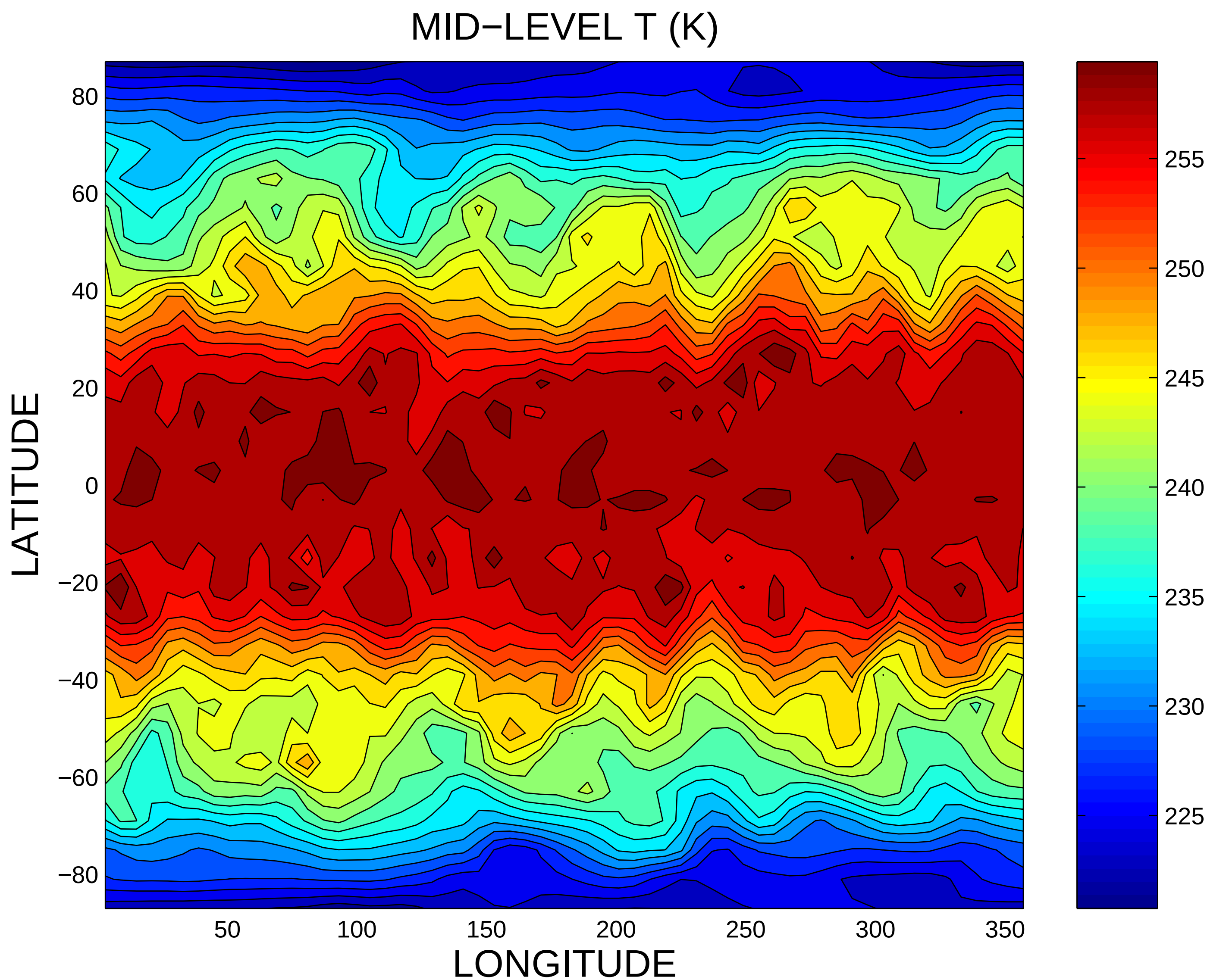


- Coupled, nonlinear set of pde's governs the flow of the atmosphere (wind, temperature, surface pressure ...)
- $d\mathbf{x}/dt = M(\mathbf{x}, t)$ - state vector order 10^7

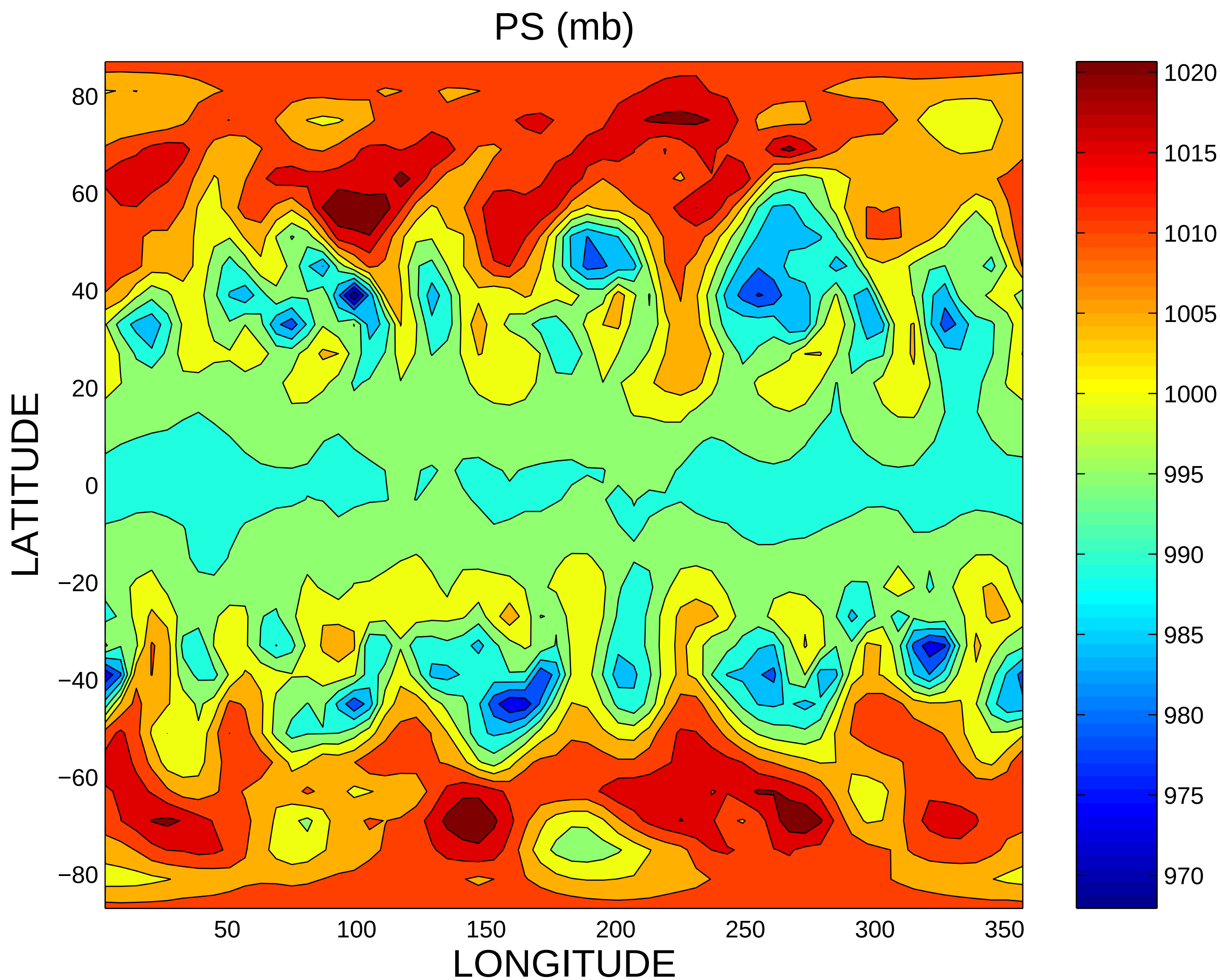
Atmospheric models: east-west winds



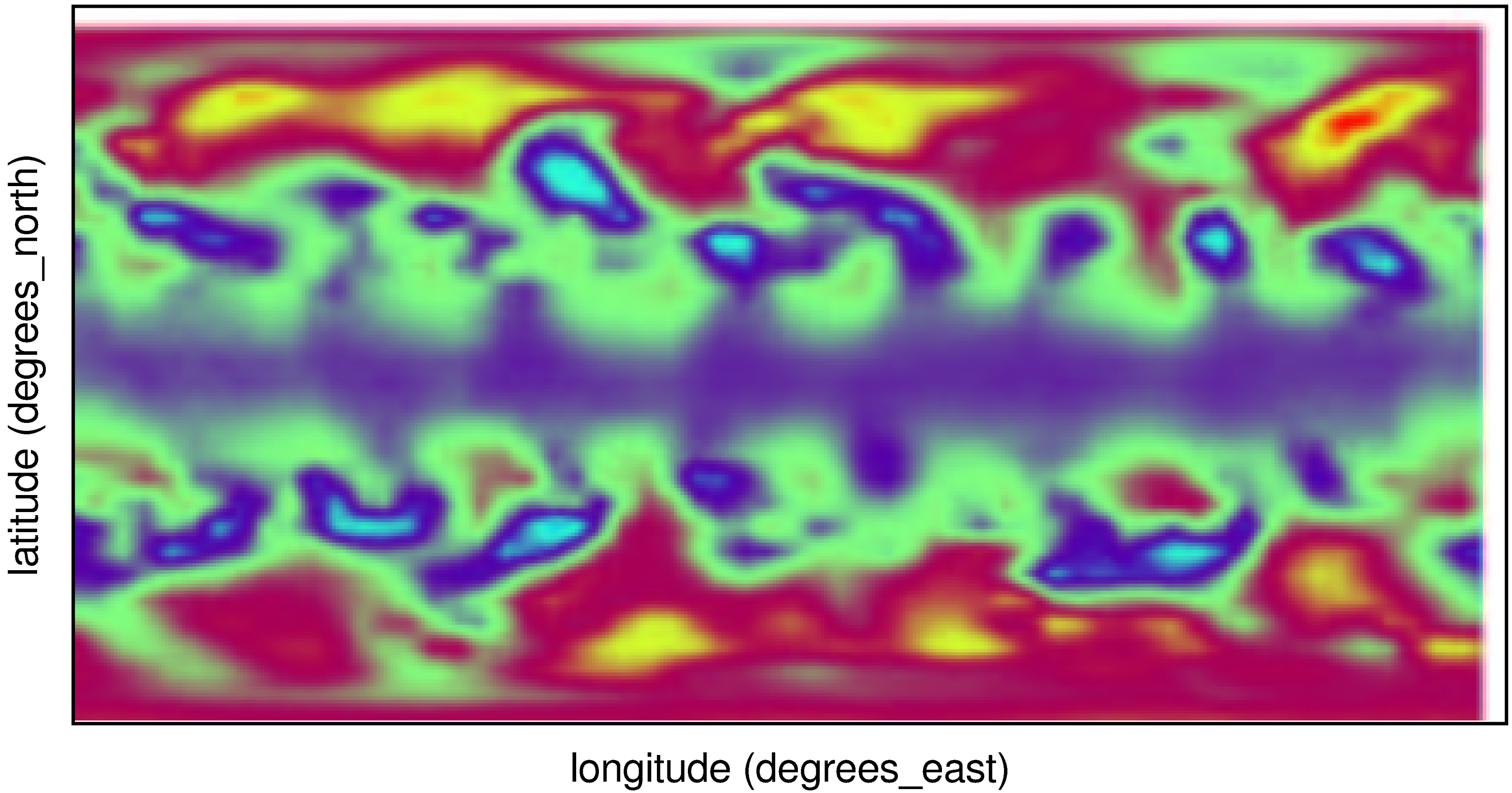
Atmospheric models: mid-level temperature



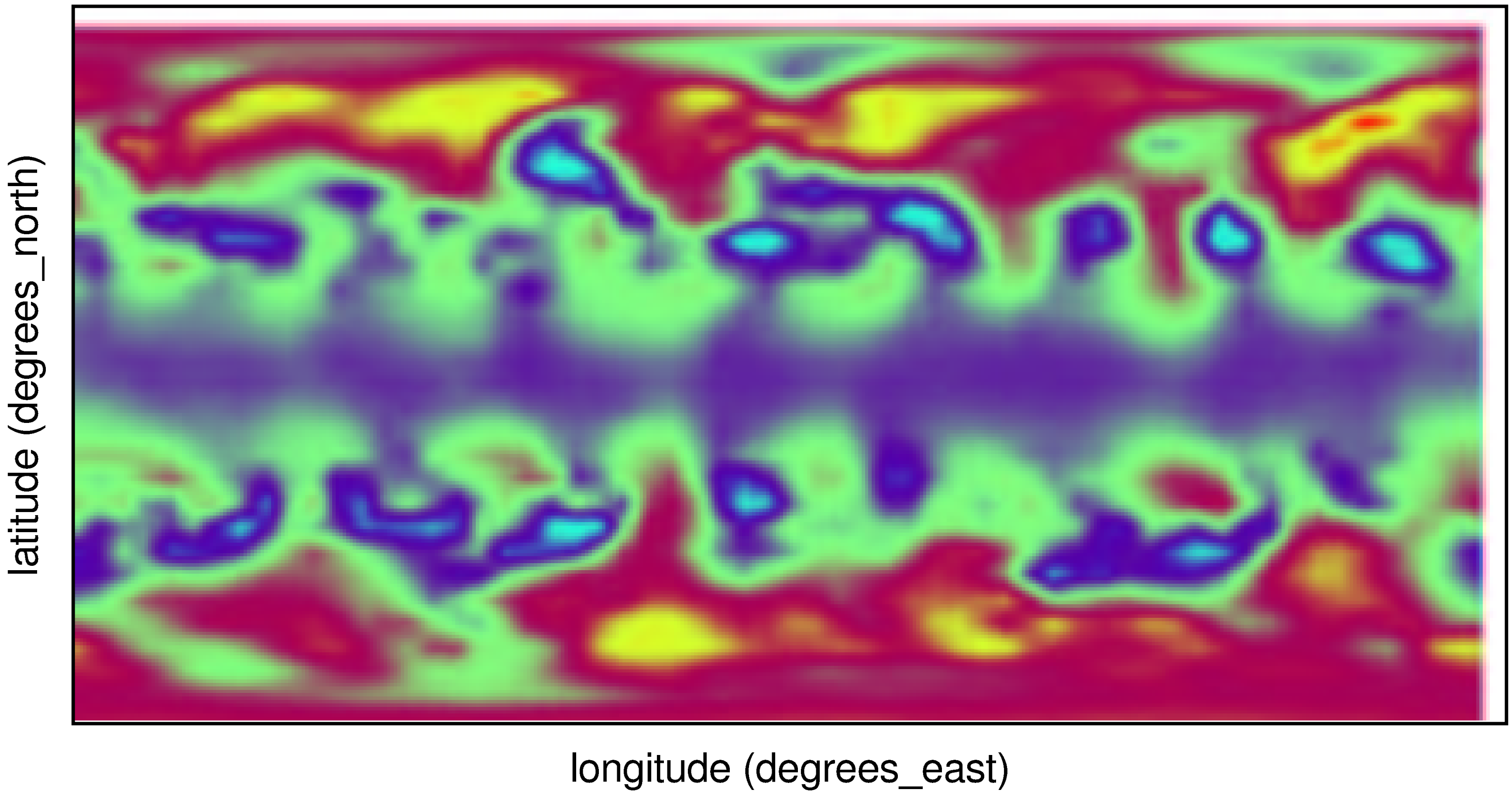
Atmospheric models: surface pressure (PS)



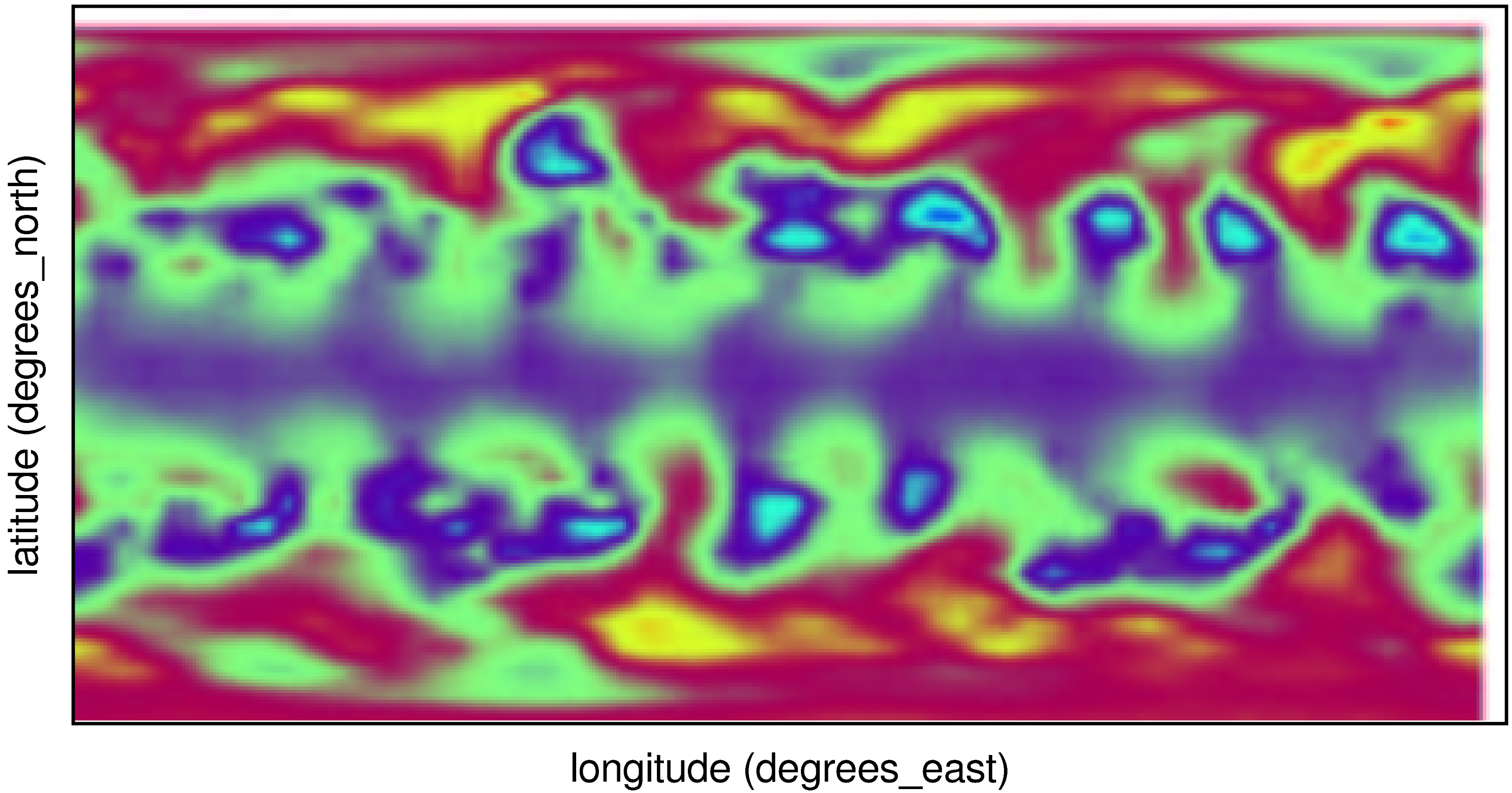
Atmospheric models: PS Day 1/20



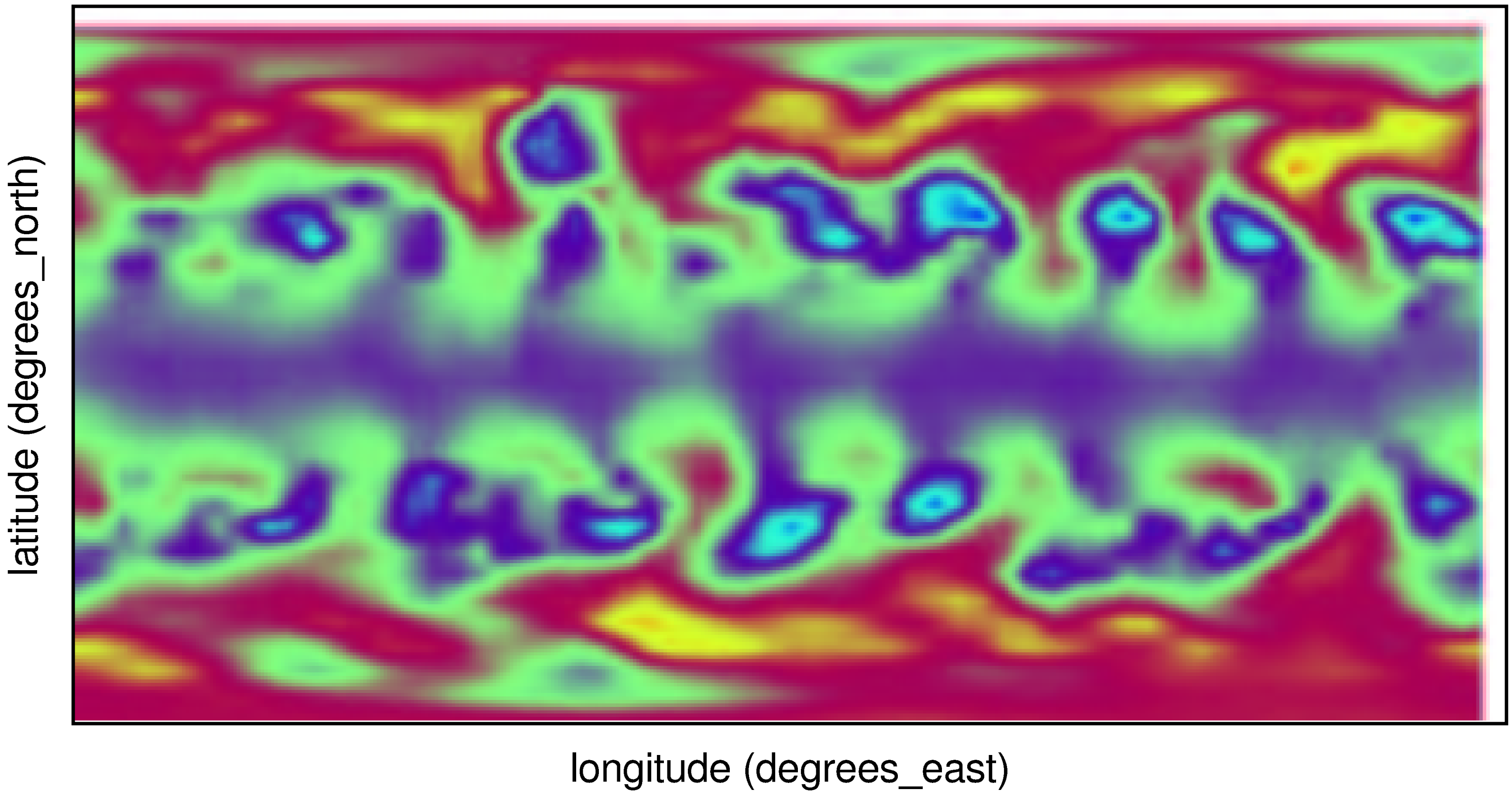
Atmospheric models: PS Day 2/20



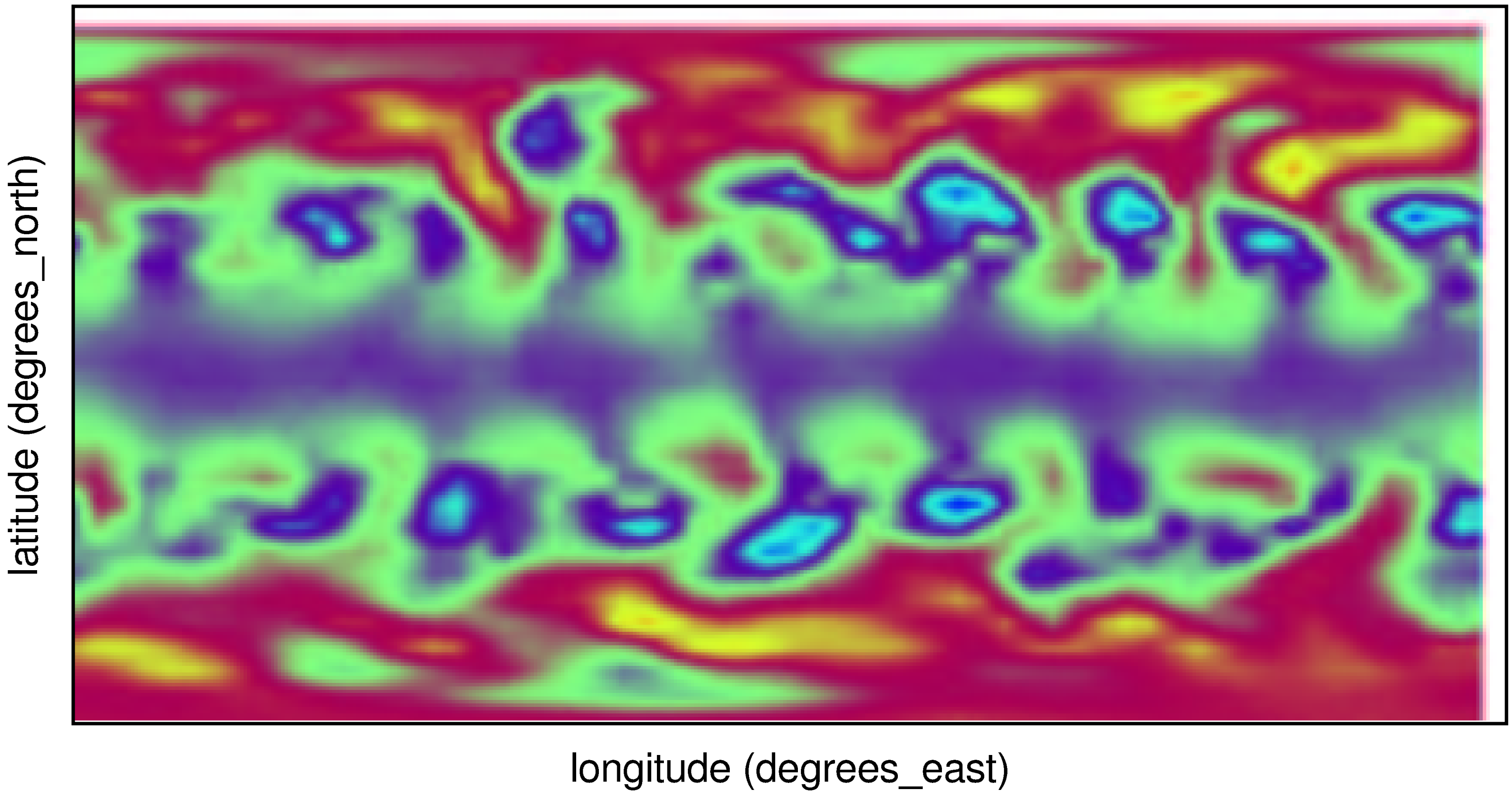
Atmospheric models: PS Day 3/20



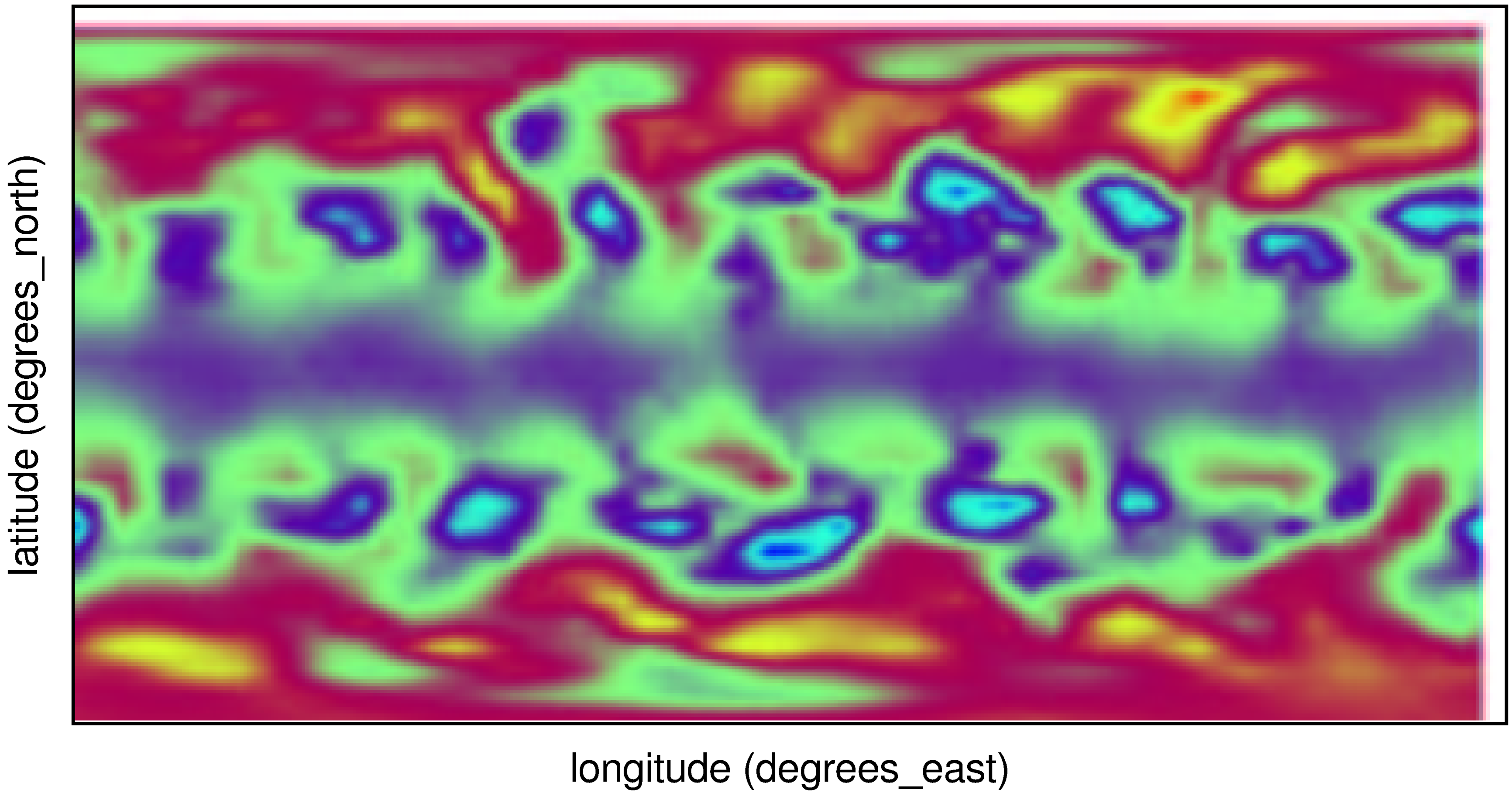
Atmospheric models: PS Day 4/20



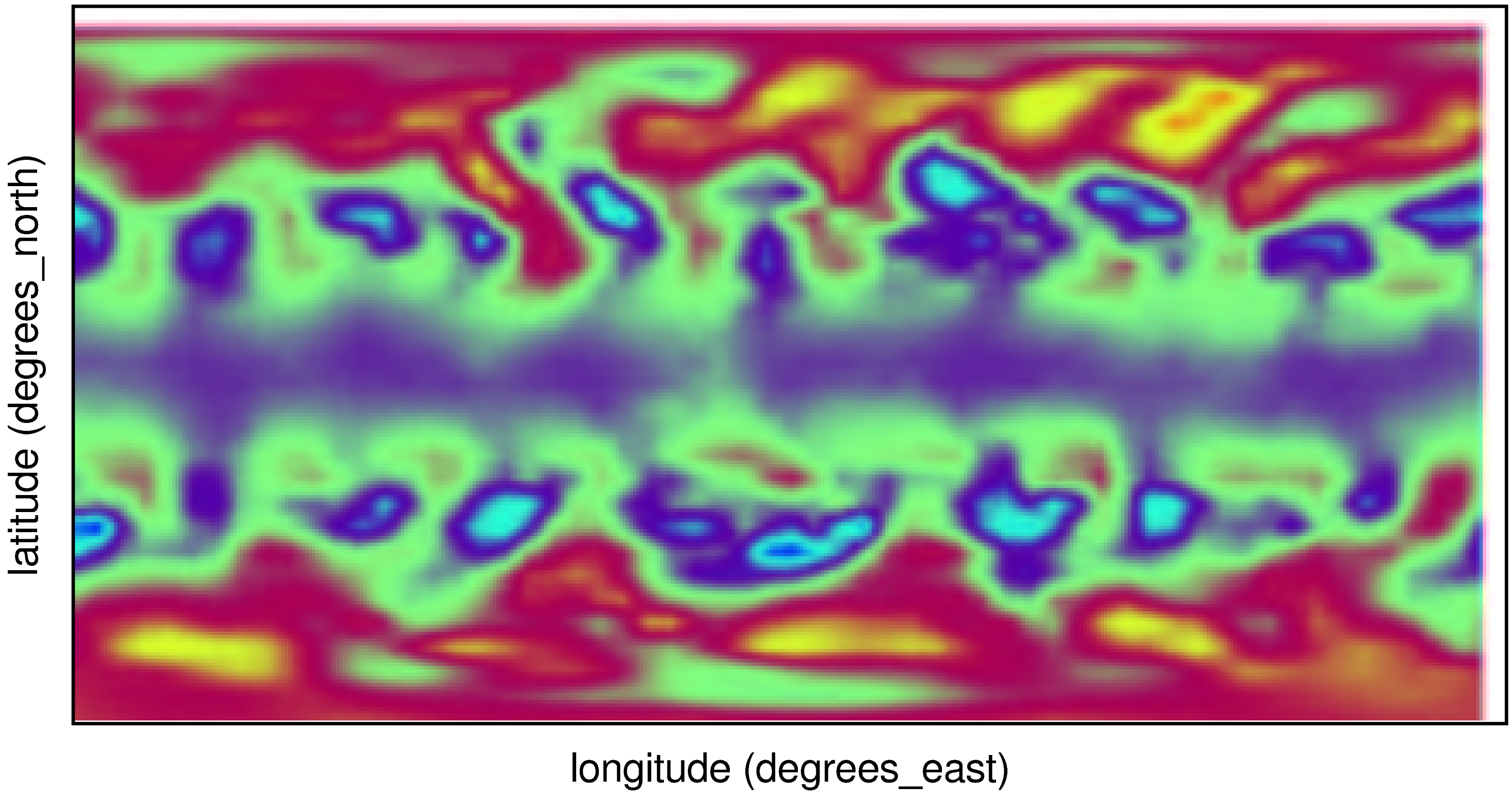
Atmospheric models: PS Day 5/20



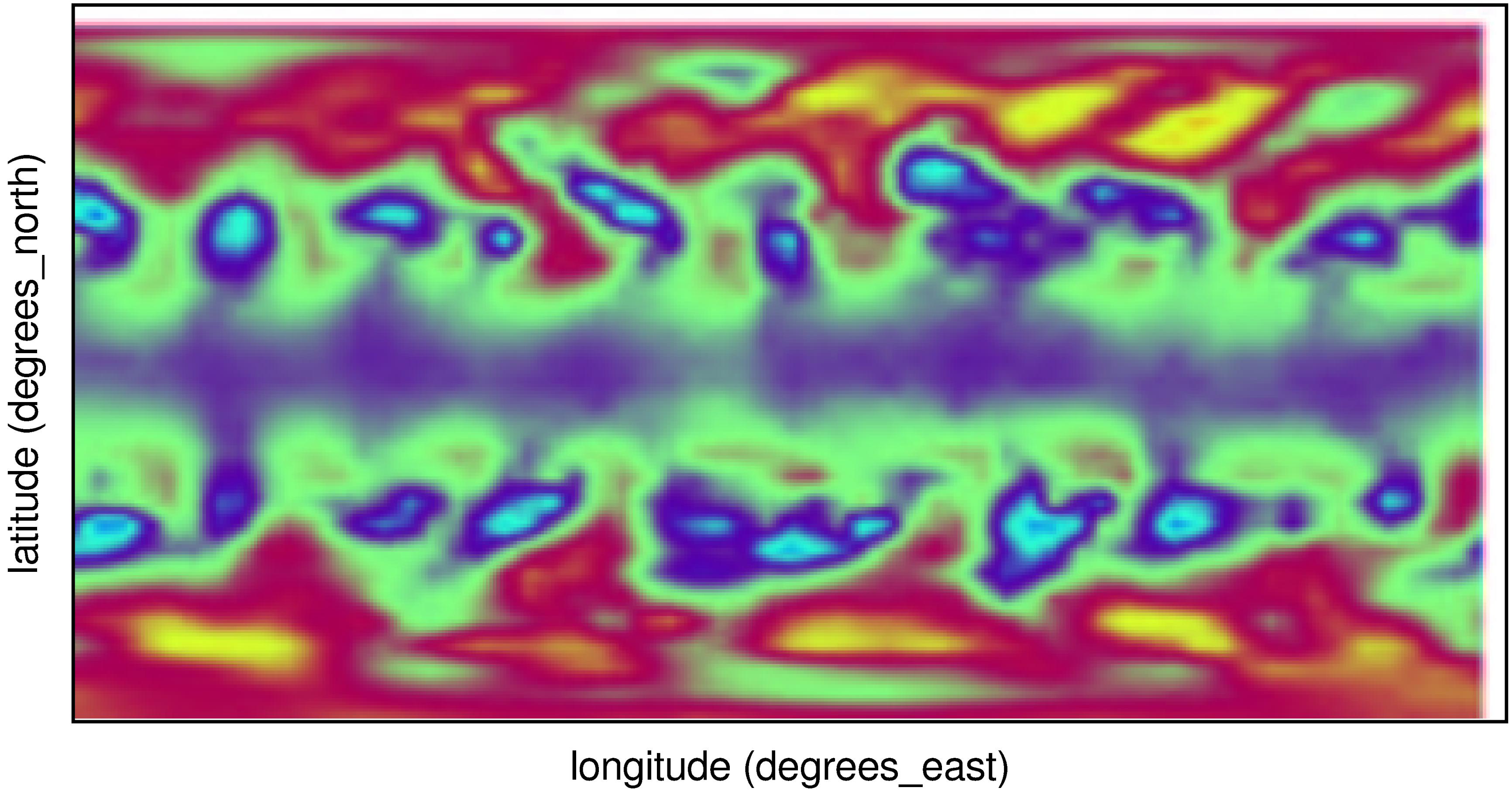
Atmospheric models: PS Day 6/20



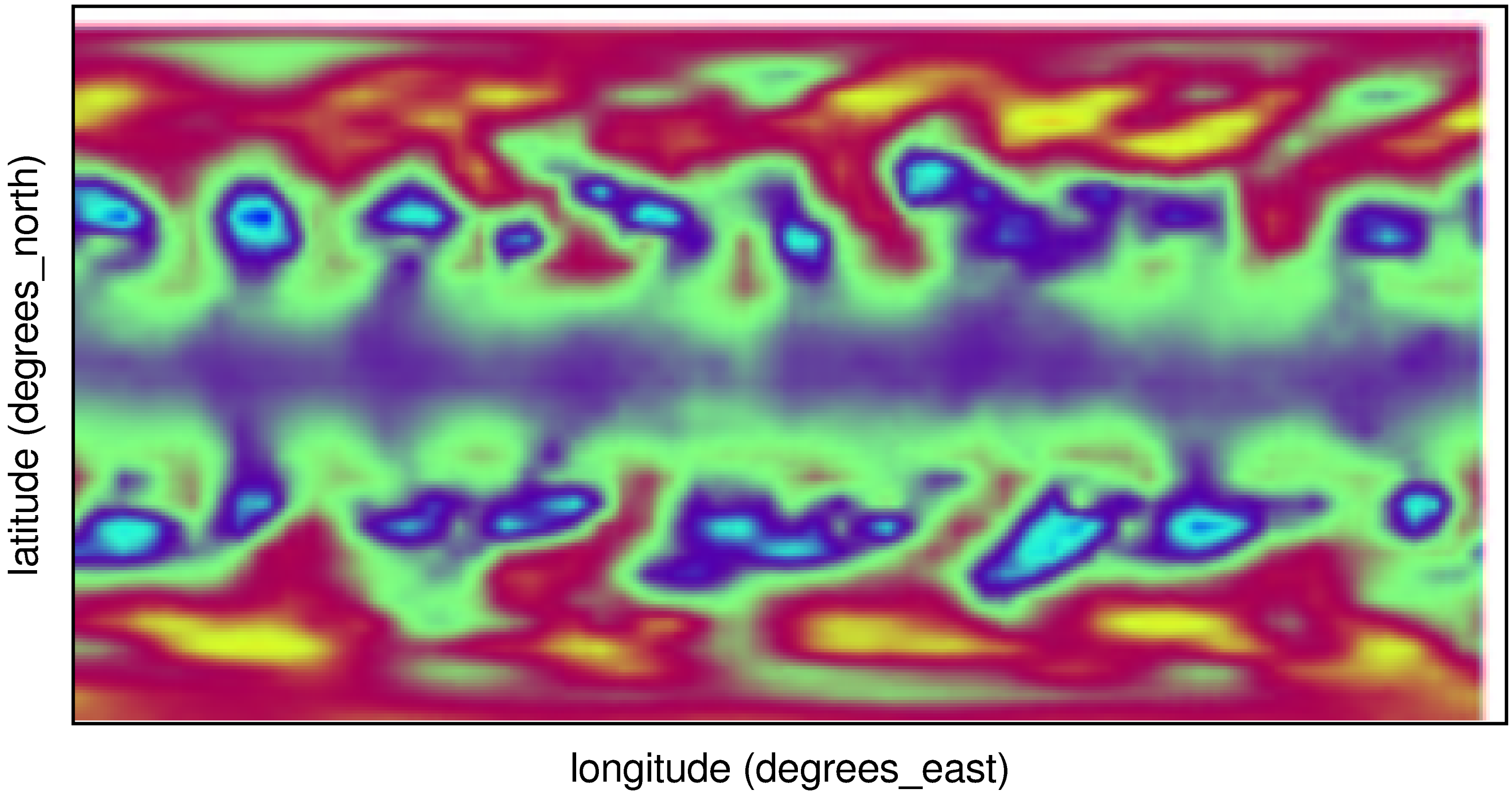
Atmospheric models: PS Day 7/20



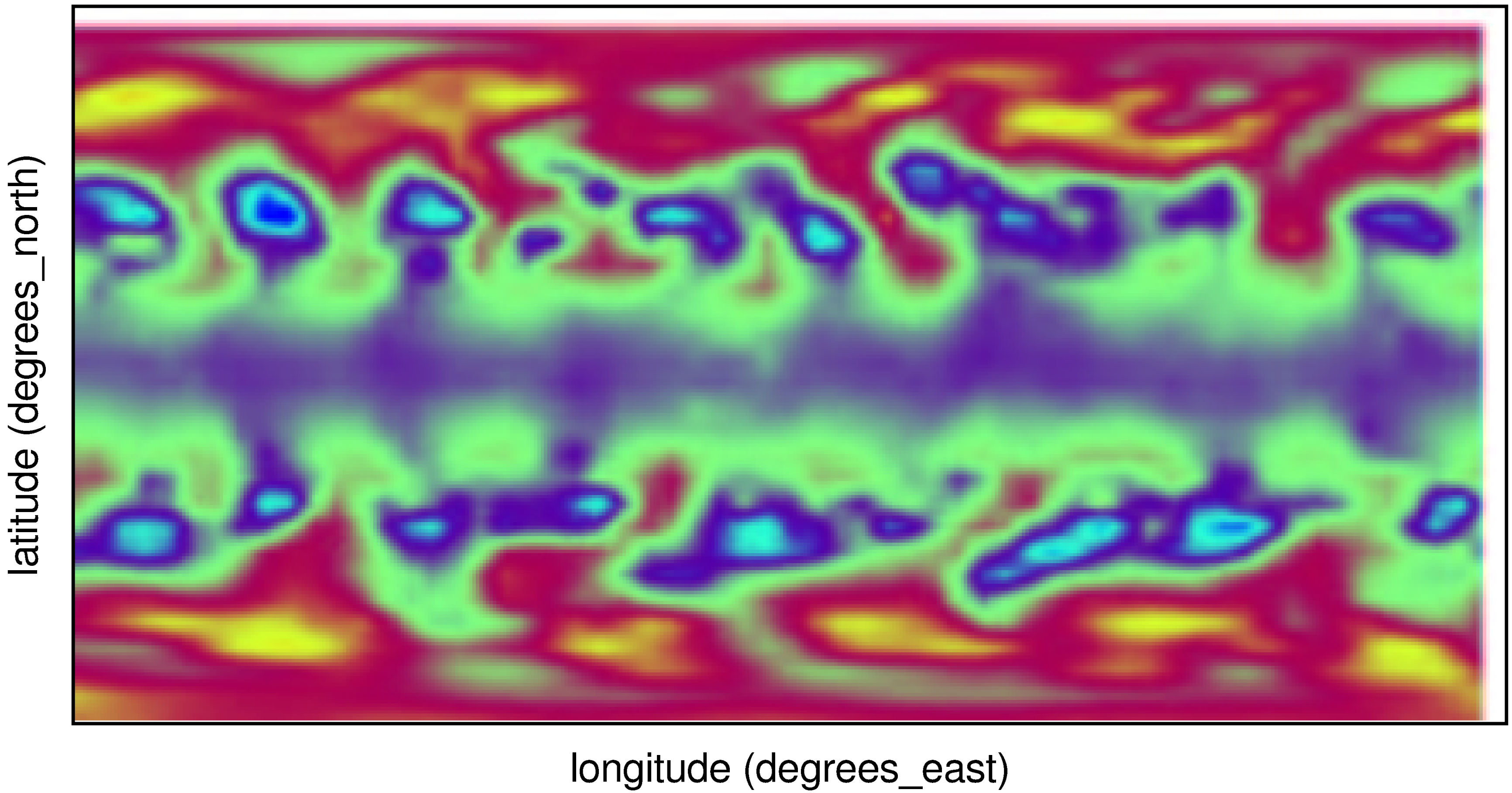
Atmospheric models: PS Day 8/20



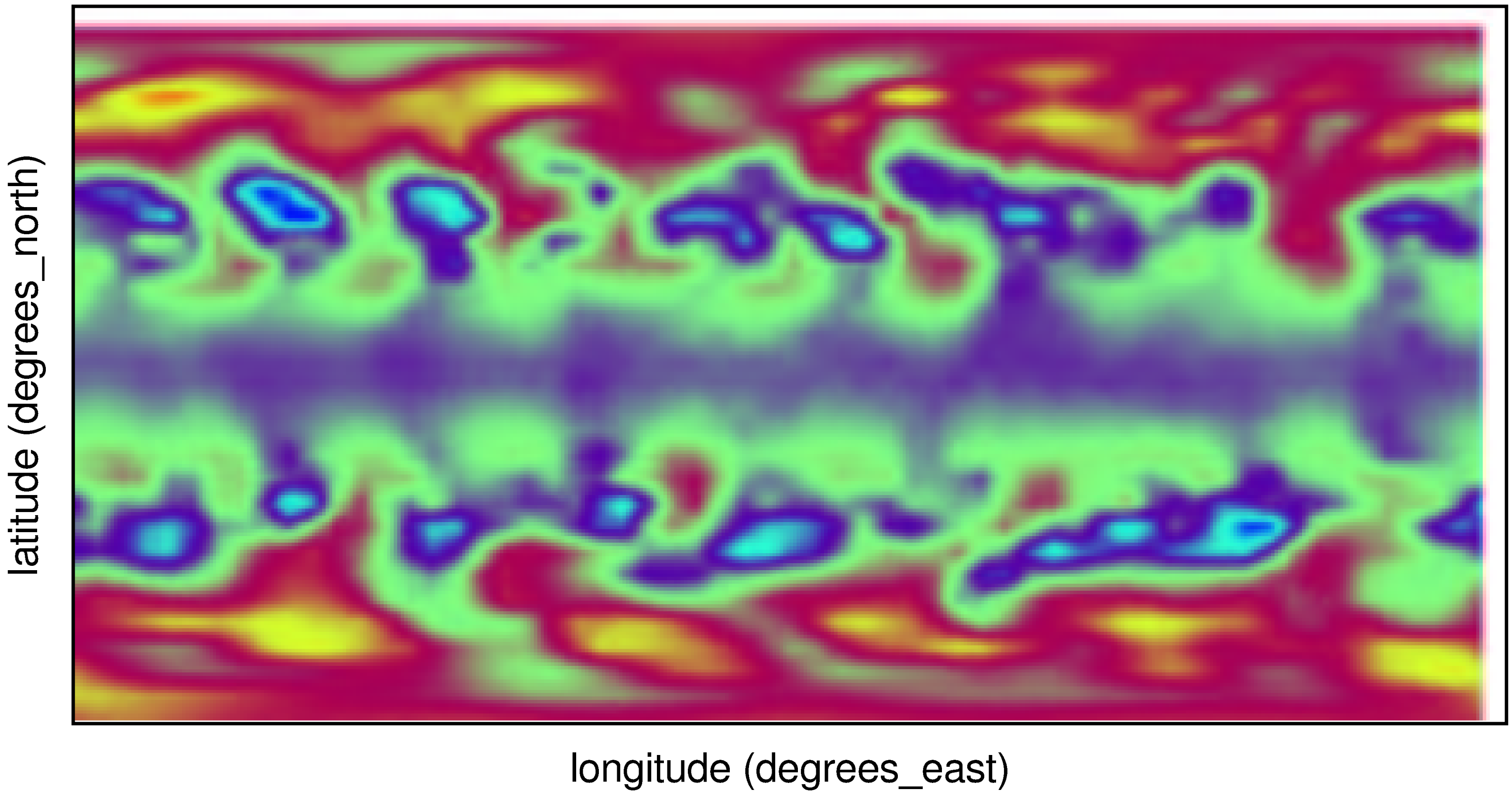
Atmospheric models: PS Day 9/20



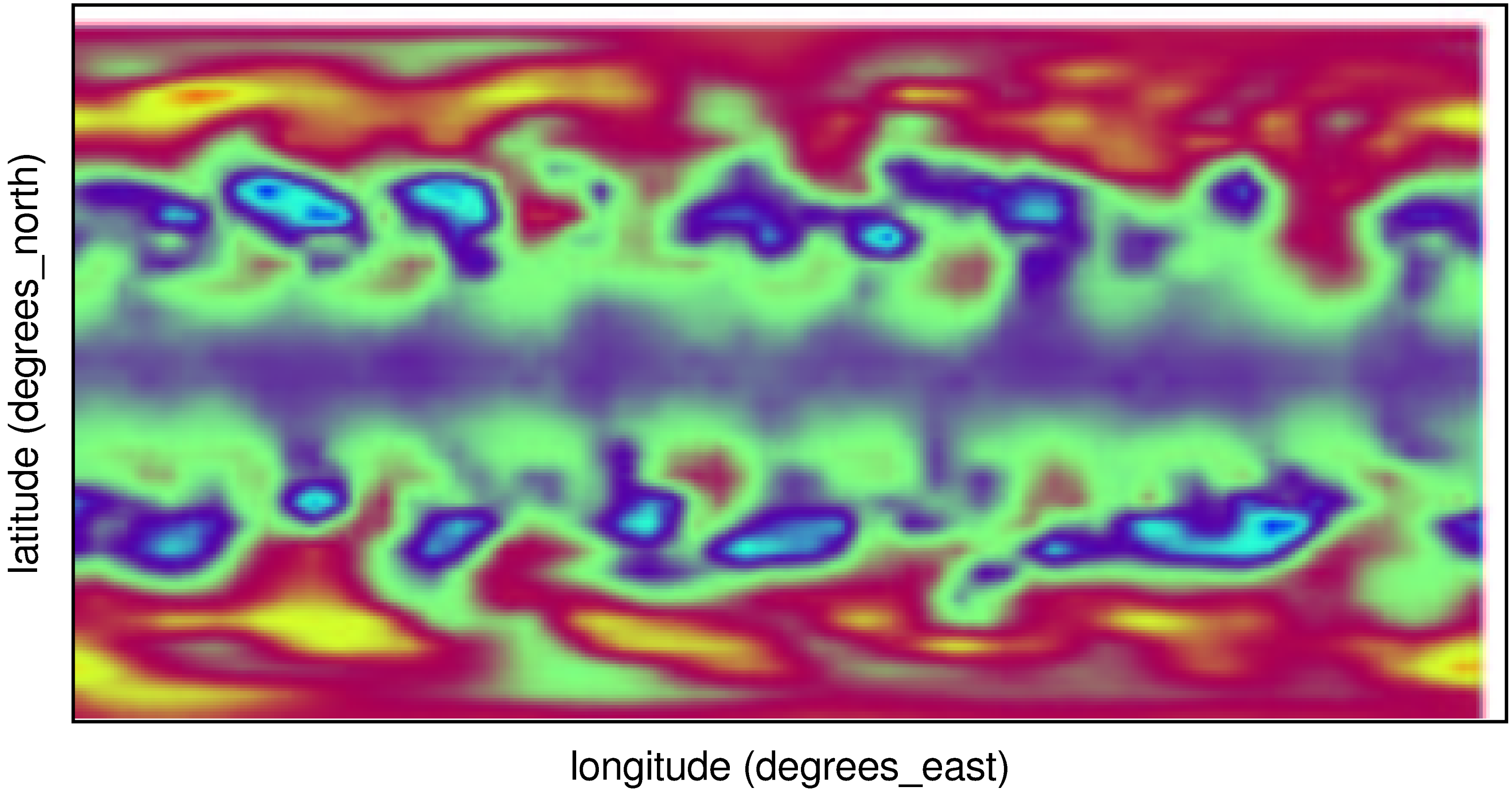
Atmospheric models: PS Day 10/20



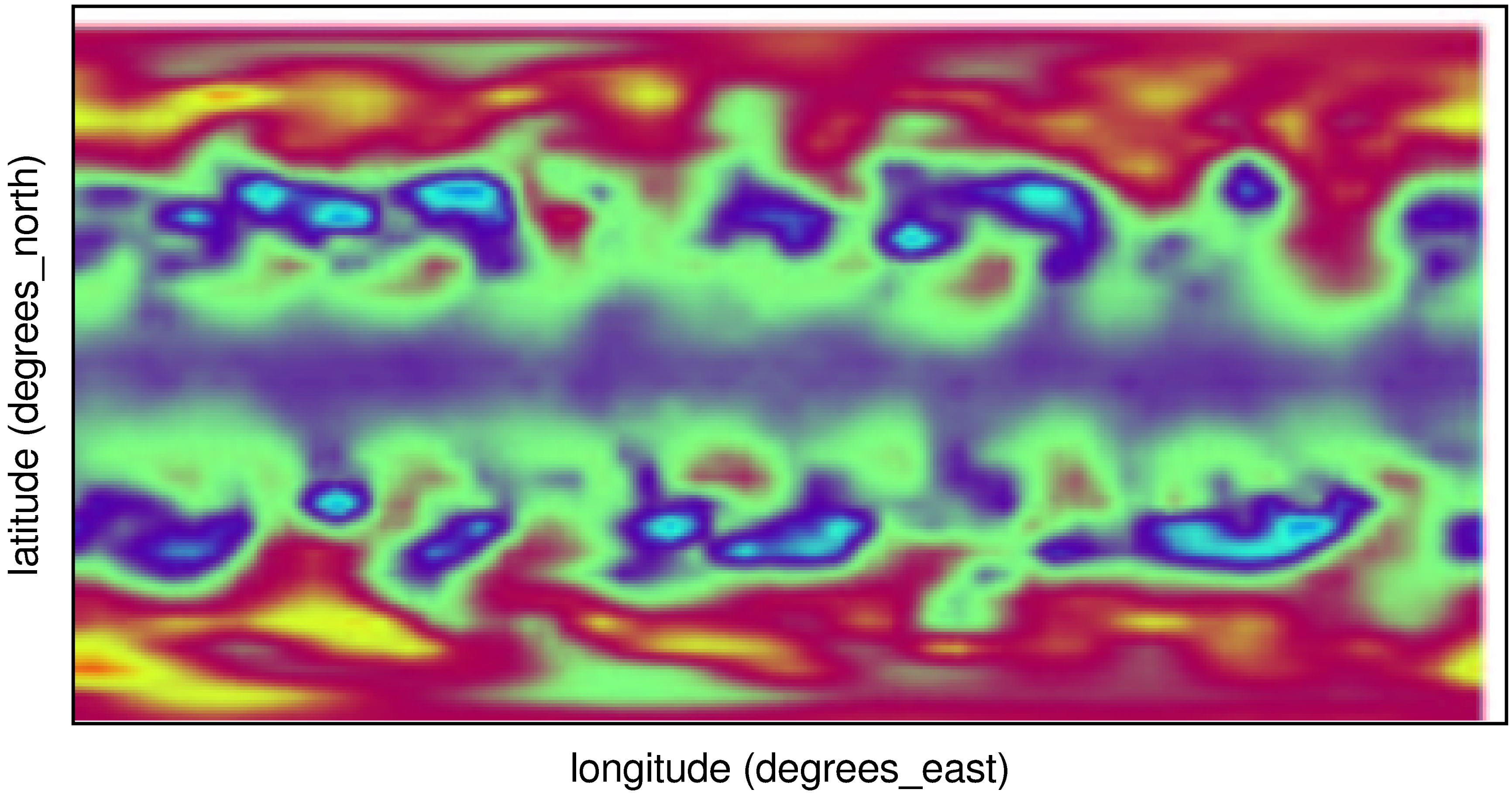
Atmospheric models: PS Day 11/20



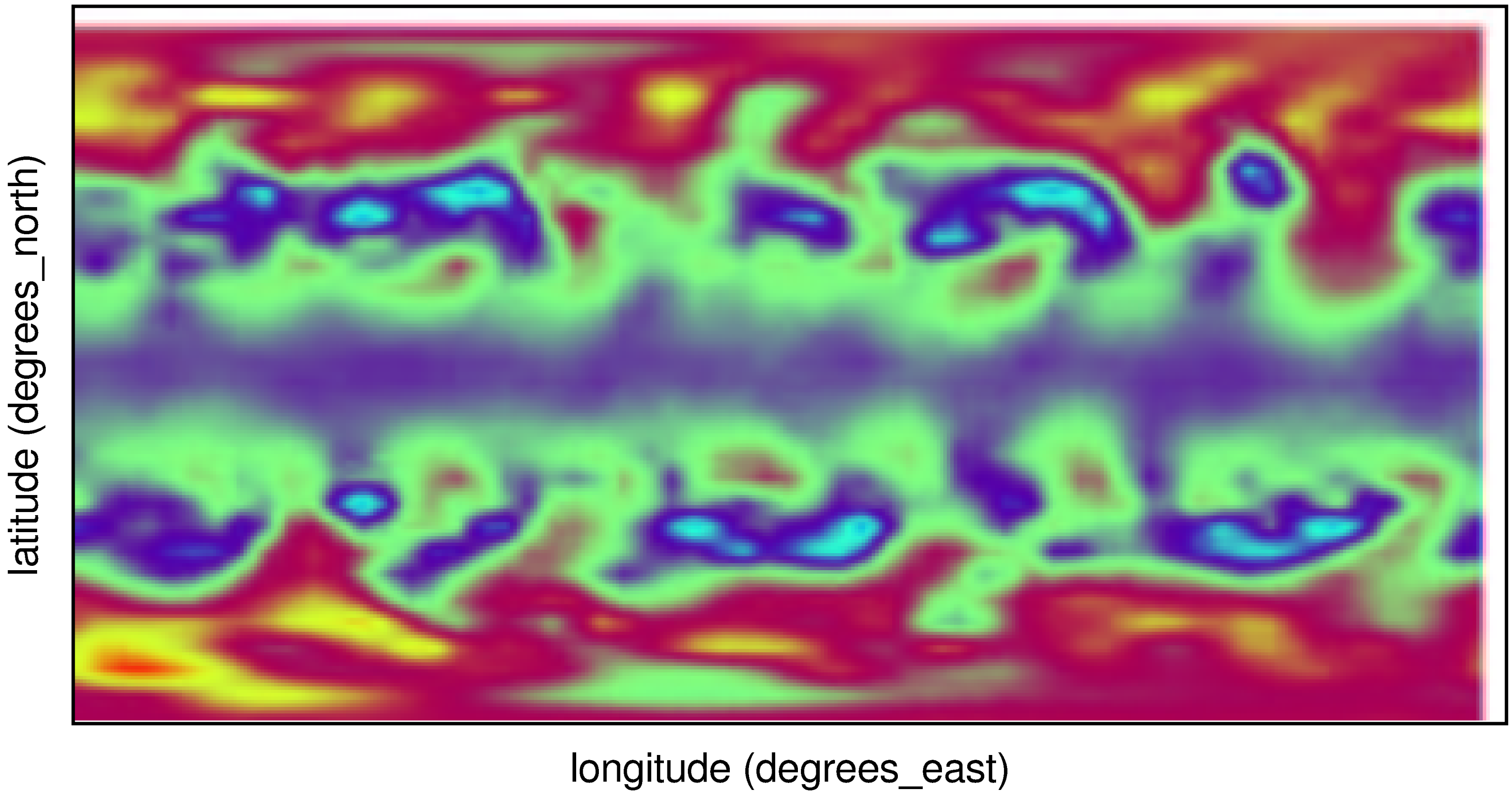
Atmospheric models: PS Day 12/20



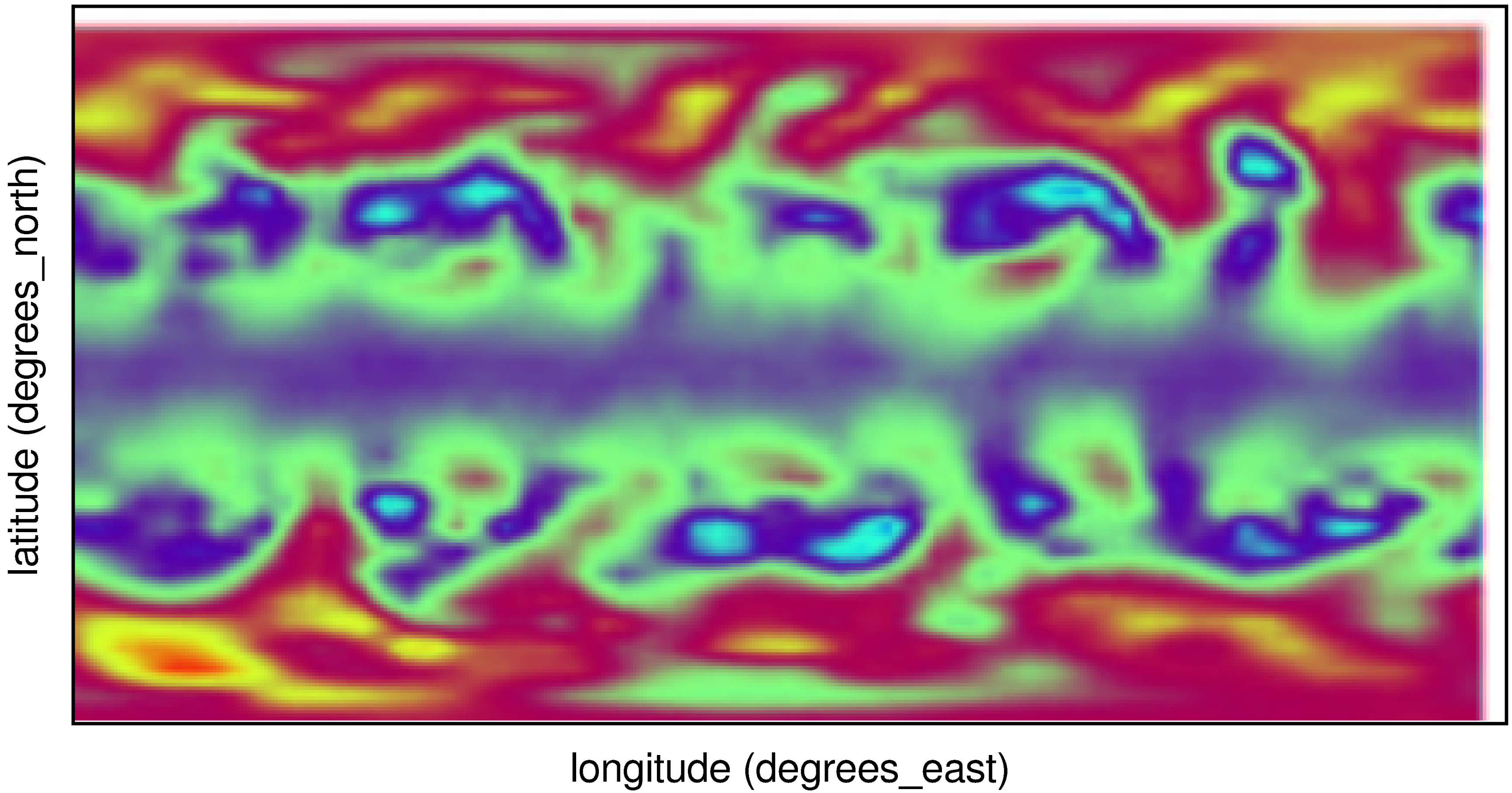
Atmospheric models: PS Day 13/20



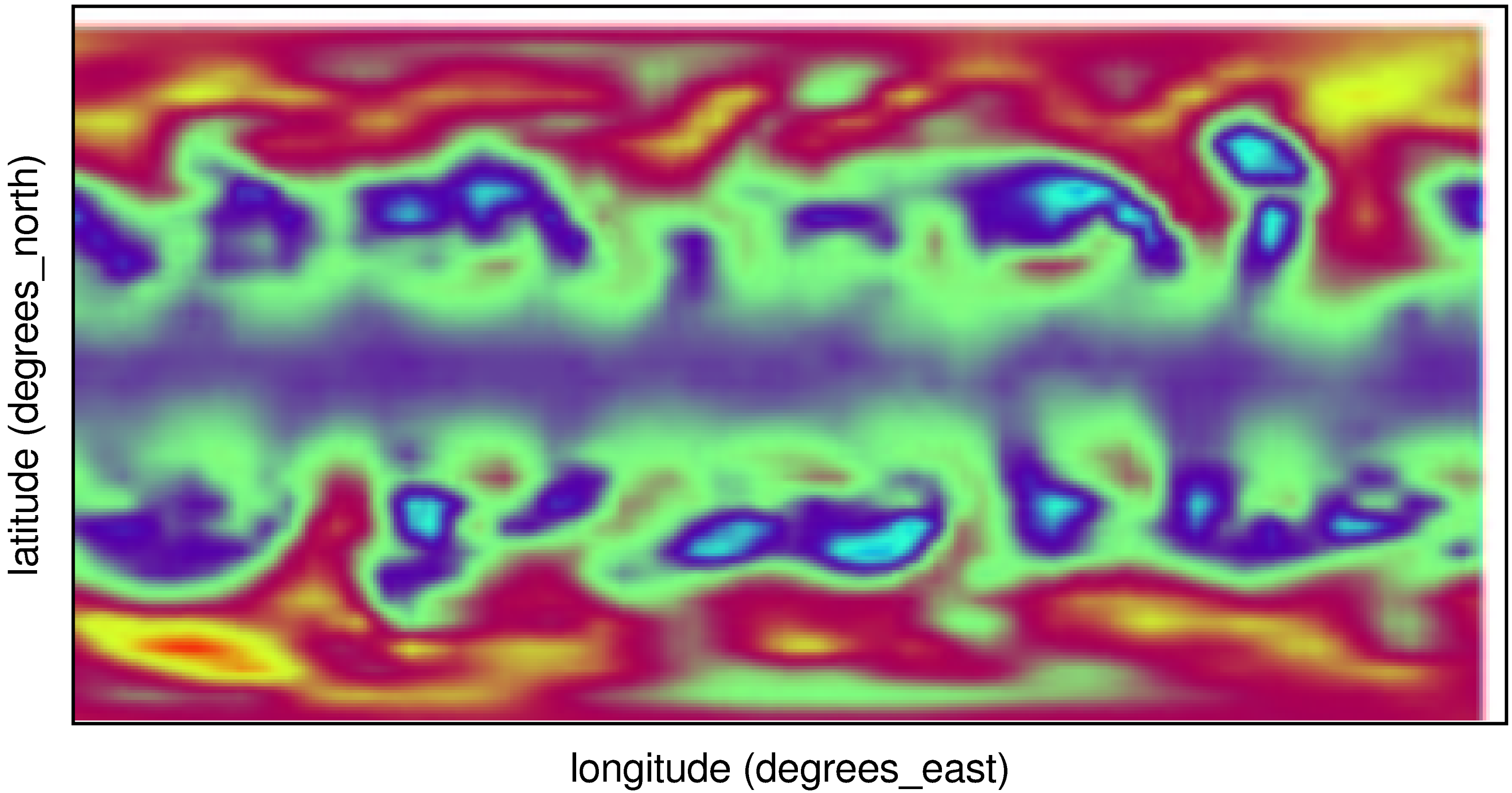
Atmospheric models: PS Day 14/20



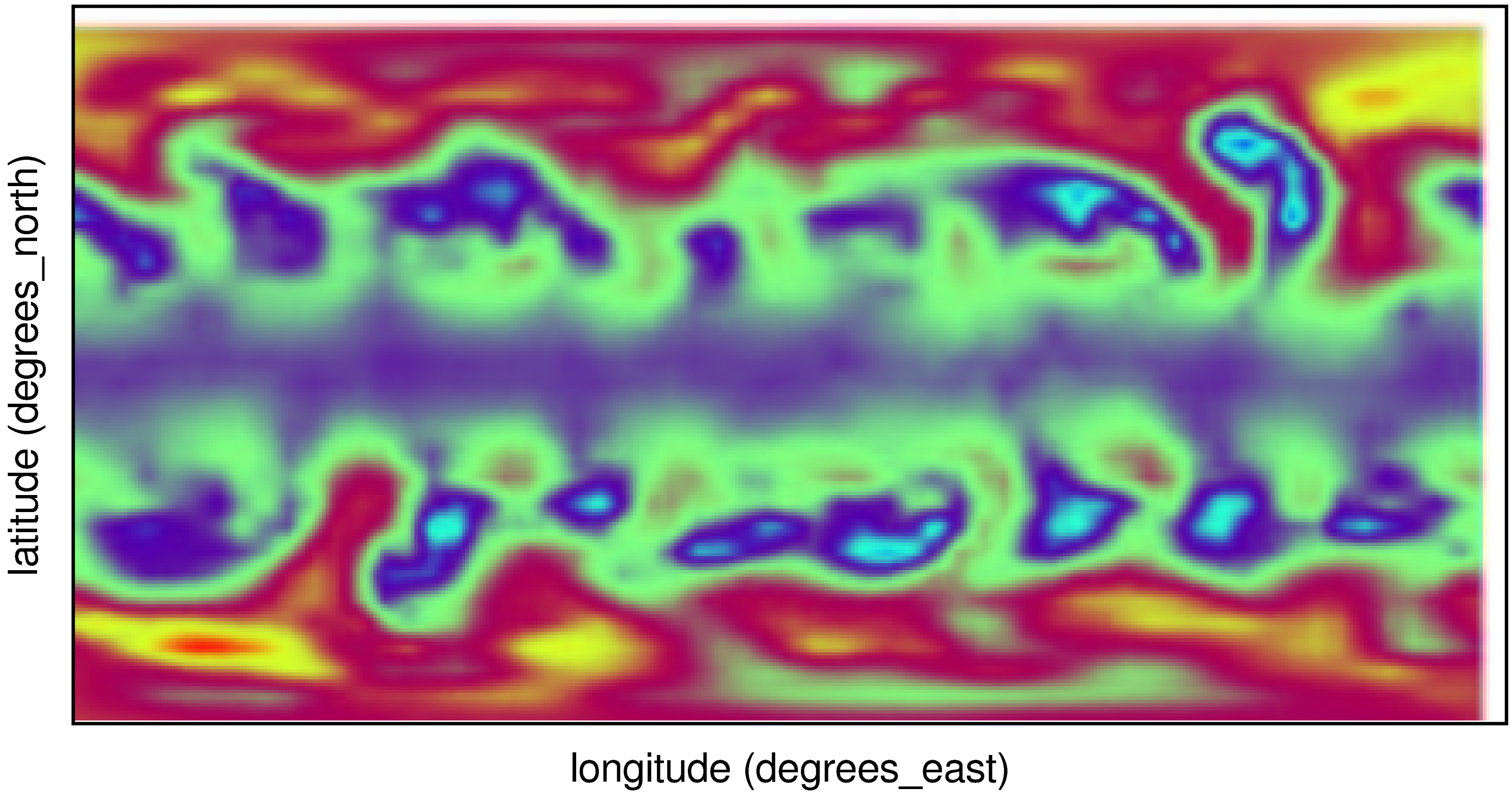
Atmospheric models: PS Day 15/20



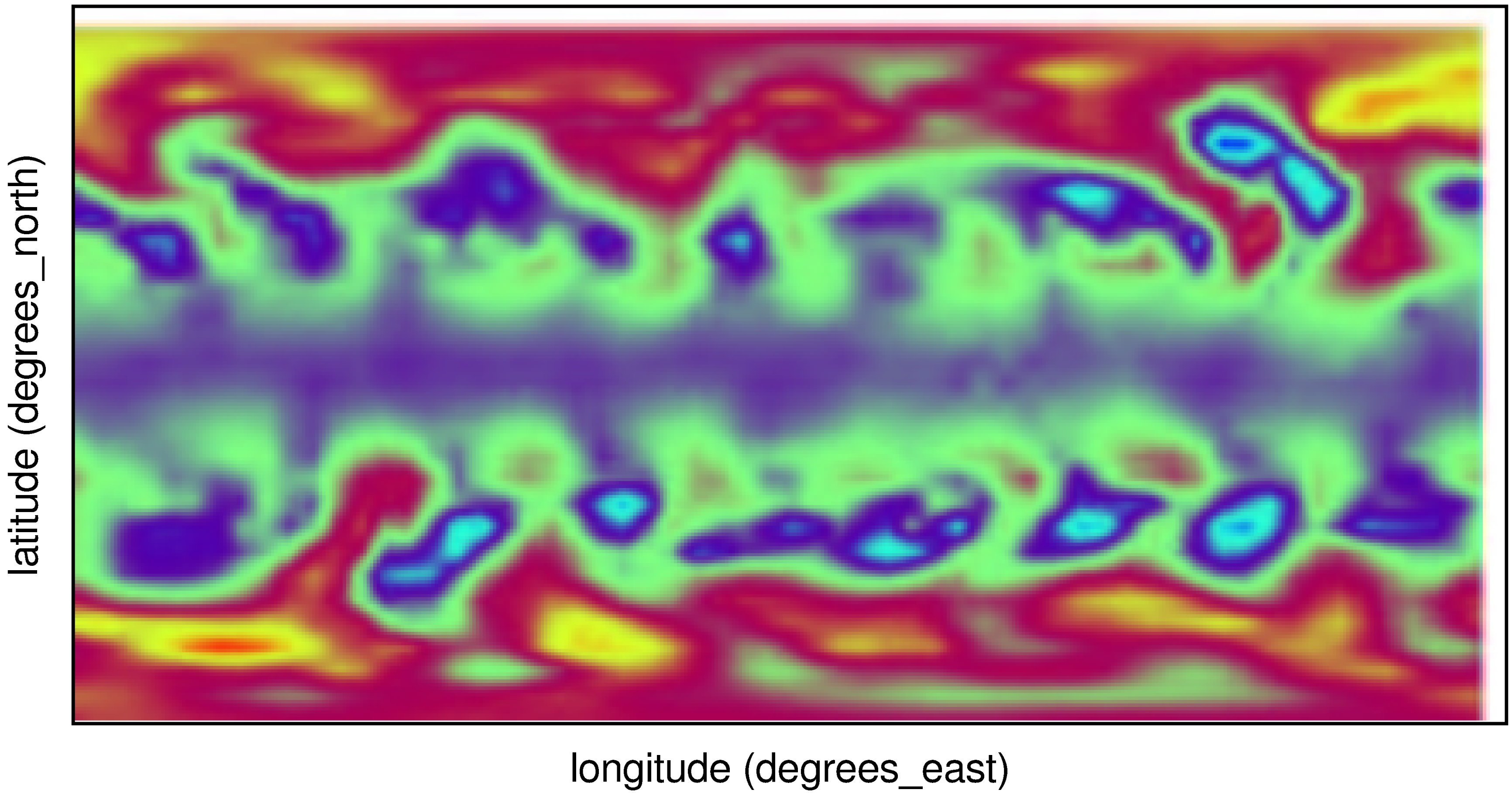
Atmospheric models: PS Day 16/20



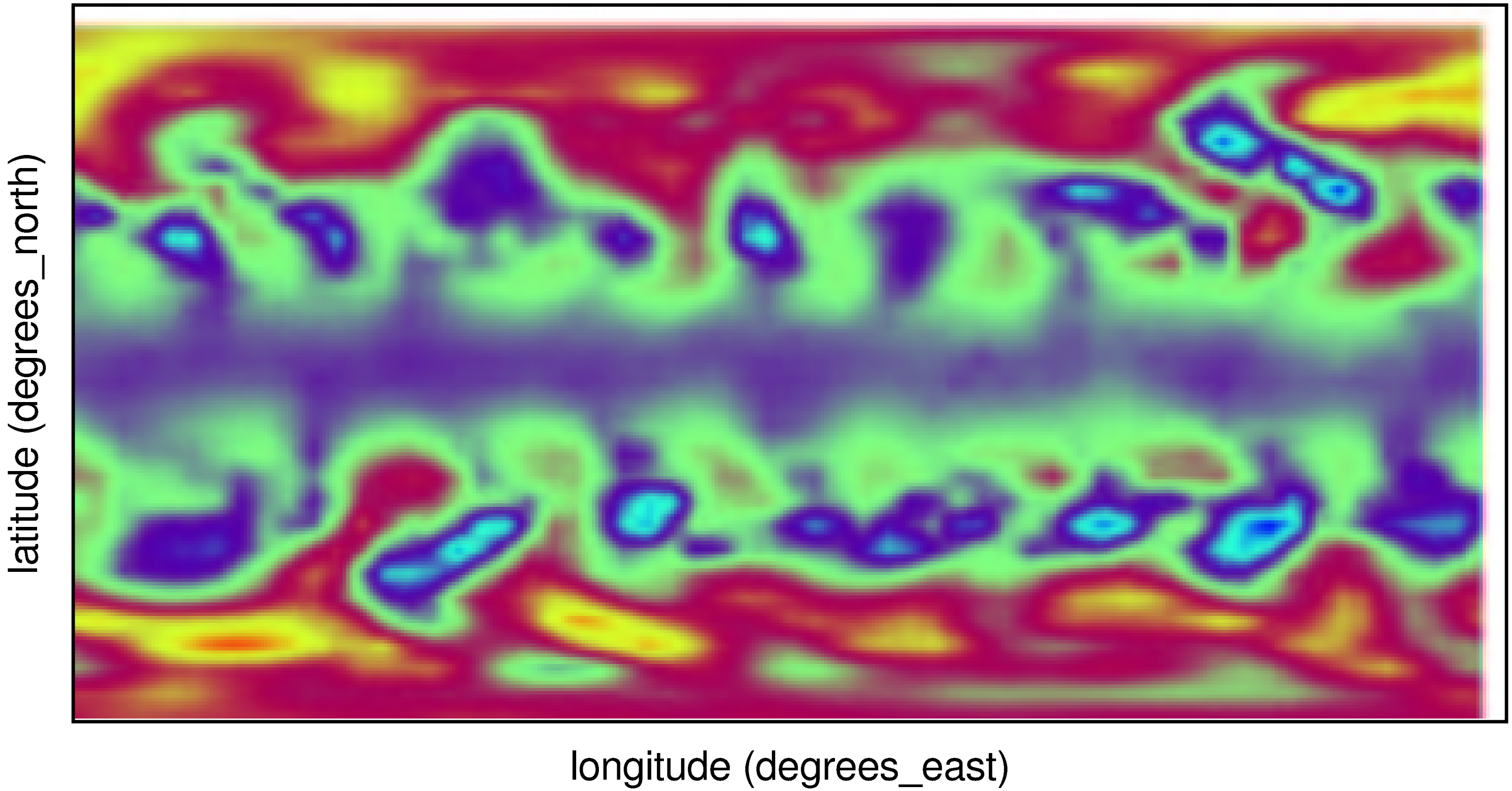
Atmospheric models: PS Day 17/20



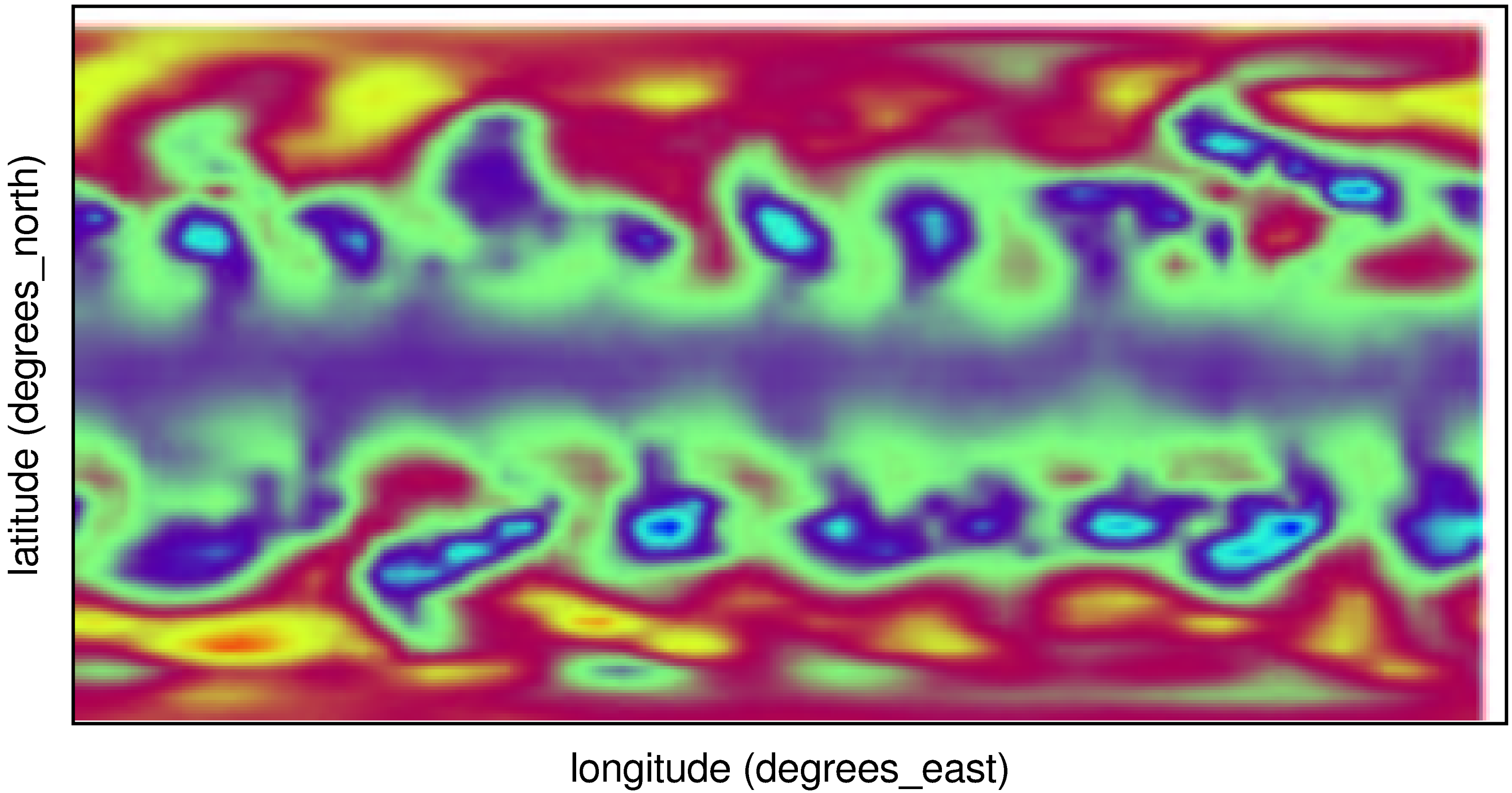
Atmospheric models: PS Day 18/20



Atmospheric models: PS Day 19/20



Atmospheric models: PS Day 20/20



Next ...

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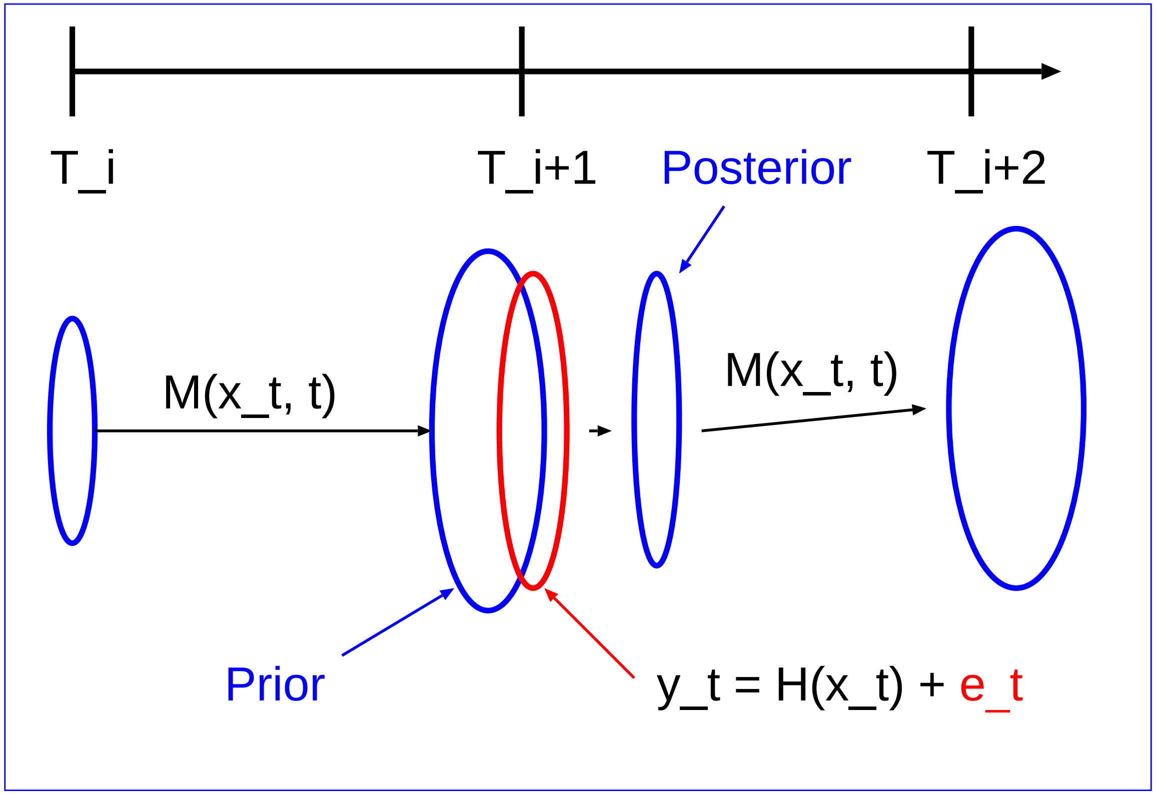
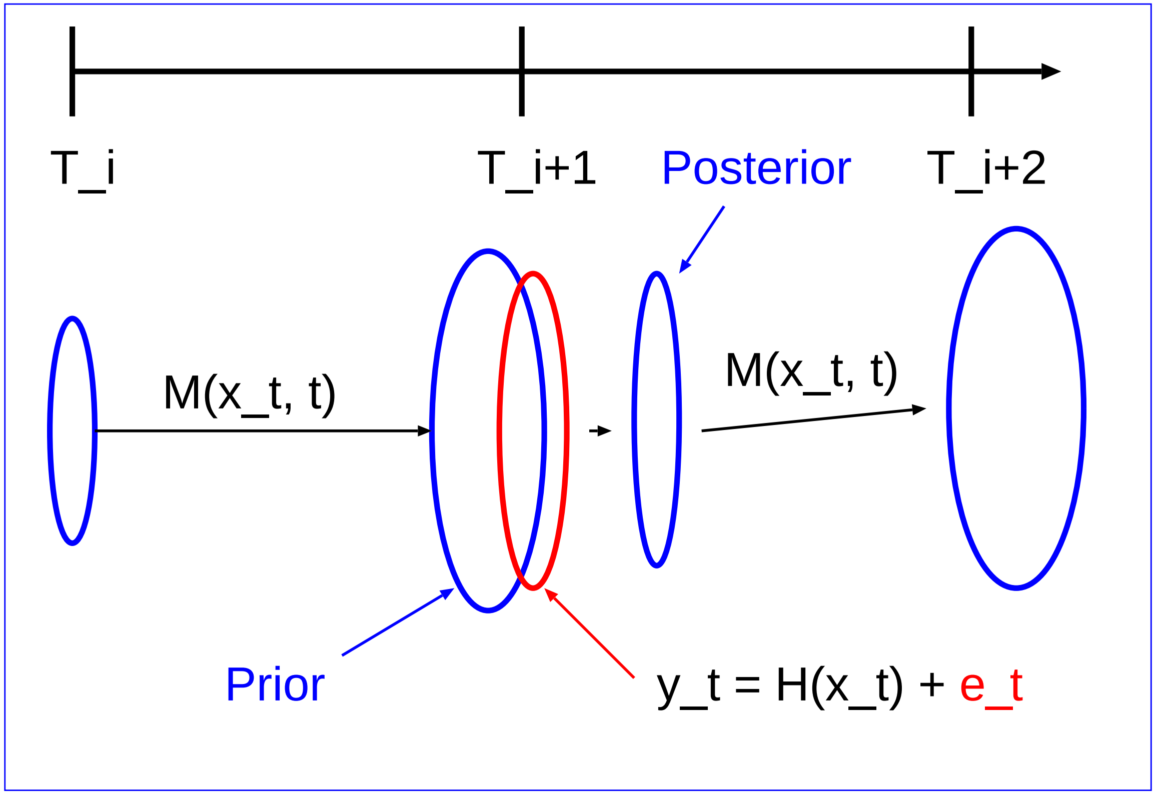
A (biased) view of sequential data assimilation



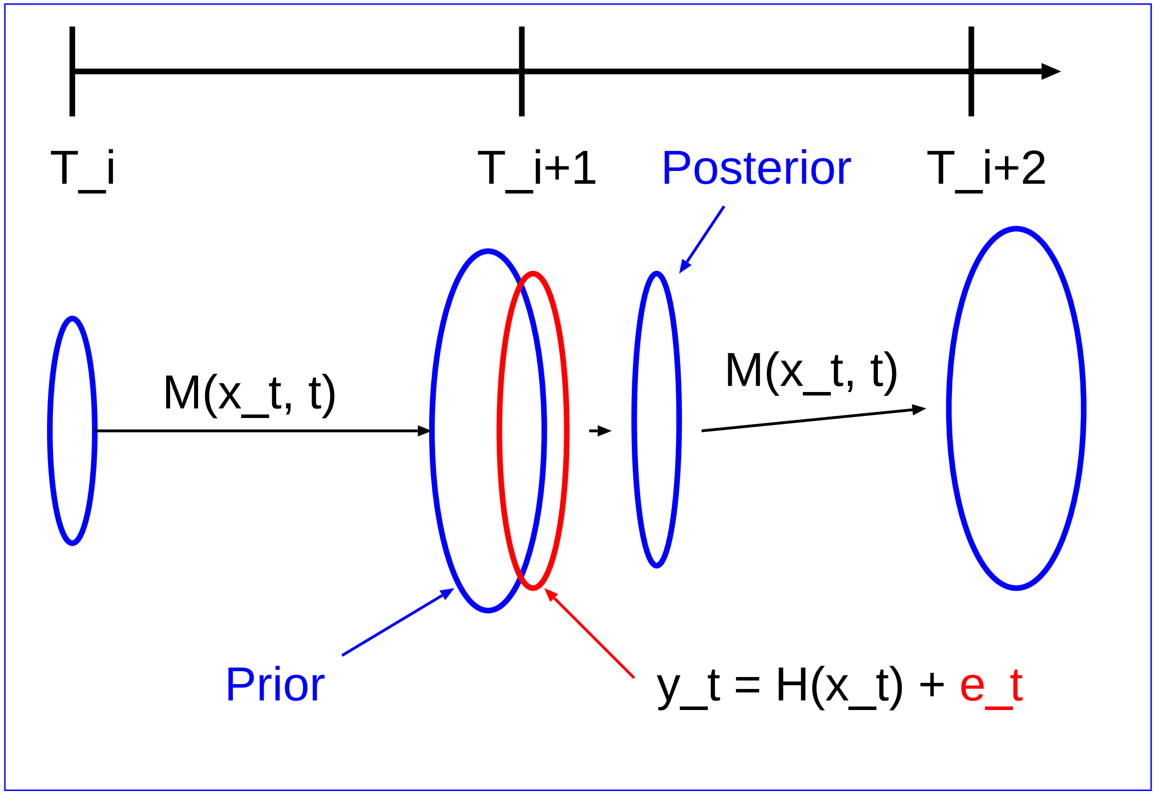
- Objective - estimate the state $\mathbf{x}_{t_i}$ given observations

A (biased) view of sequential data assimilation



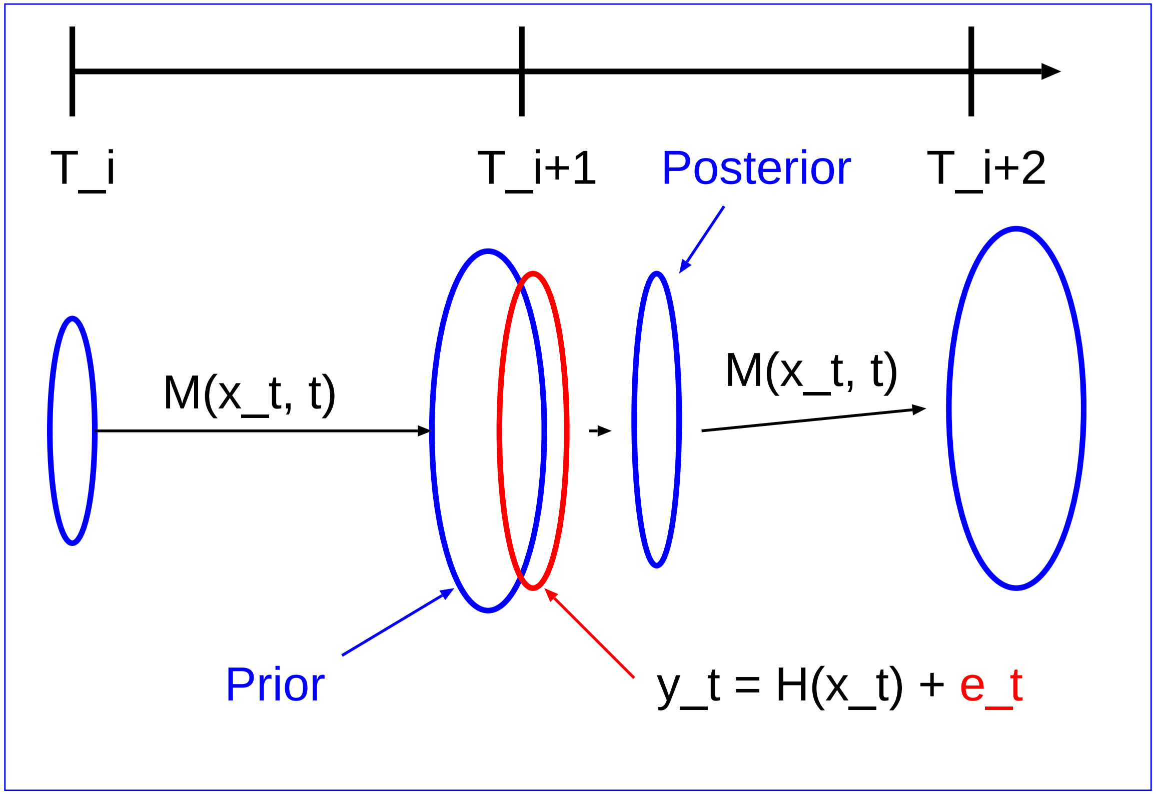
- Observations of atmospheric quantities are uncertain and incomplete

A (biased) view of sequential data assimilation



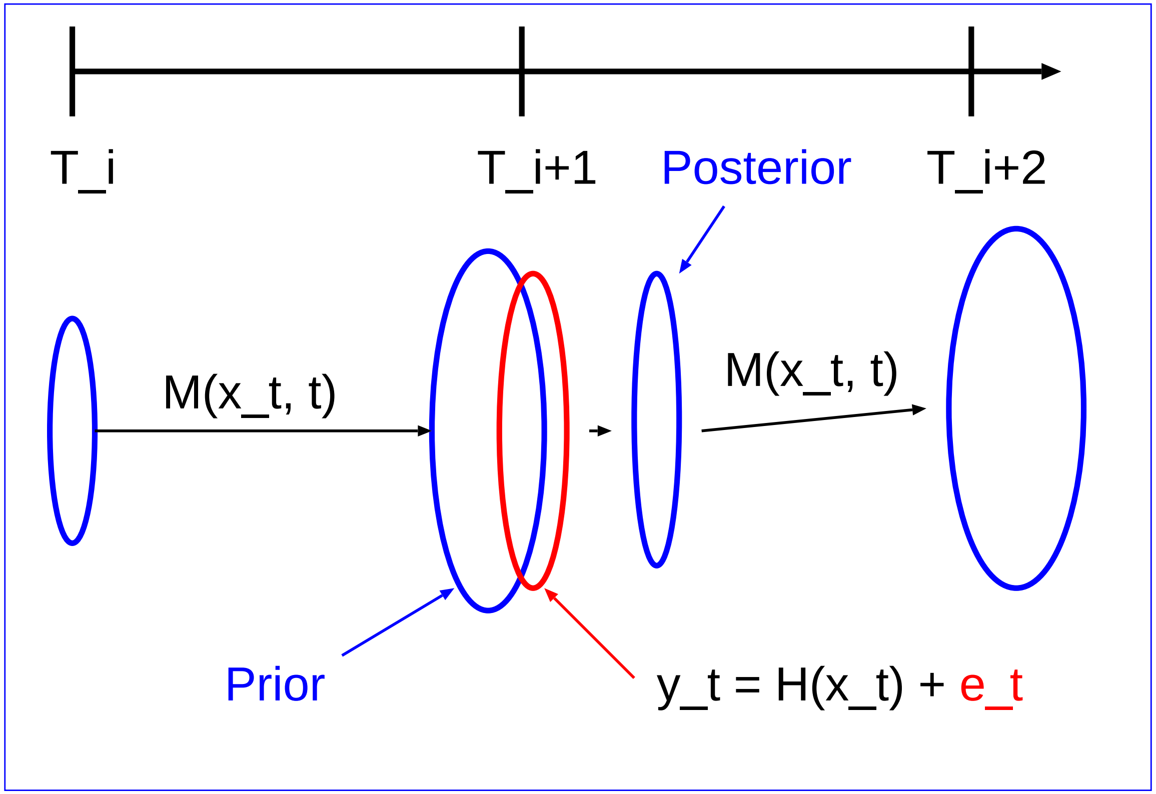
- Adopt a probabilistic point of view - solve for conditional probabilities

A (biased) view of sequential data assimilation



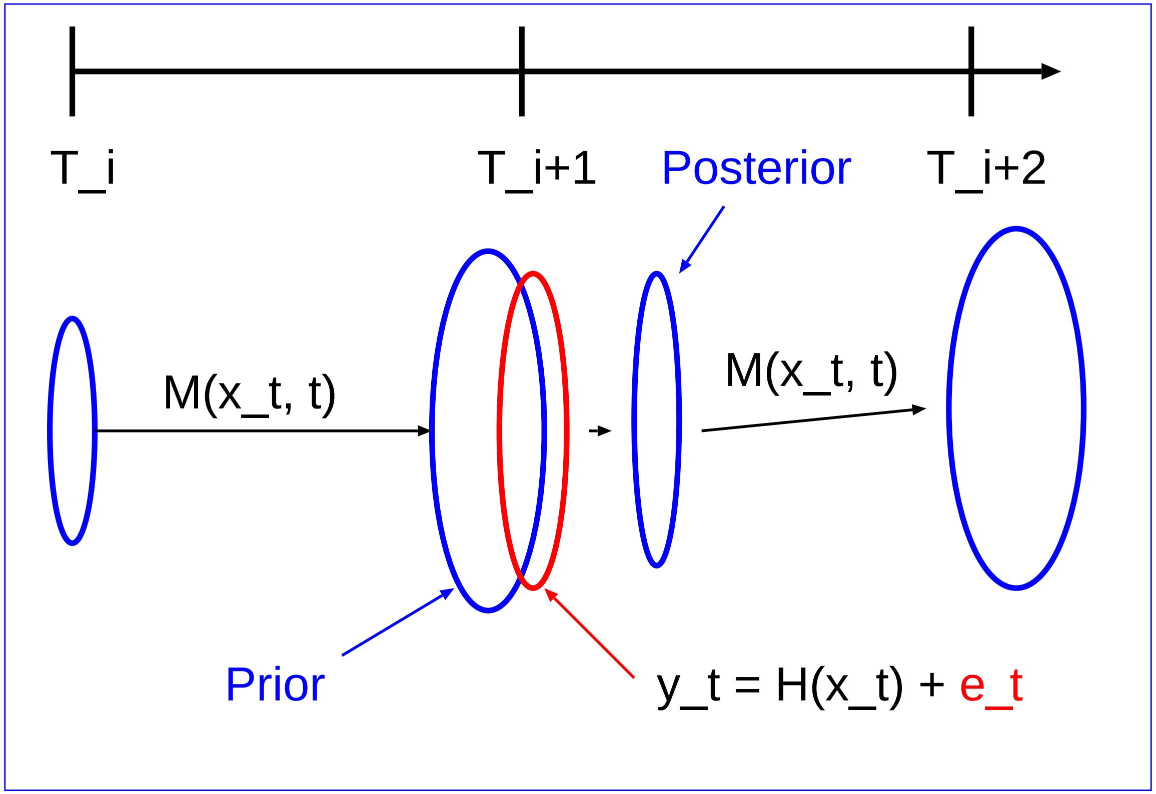
- Assume that at t_i we have some pdf for the system state $\mathbf{p}(\mathbf{x}_{t_i})$

A (biased) view of sequential data assimilation



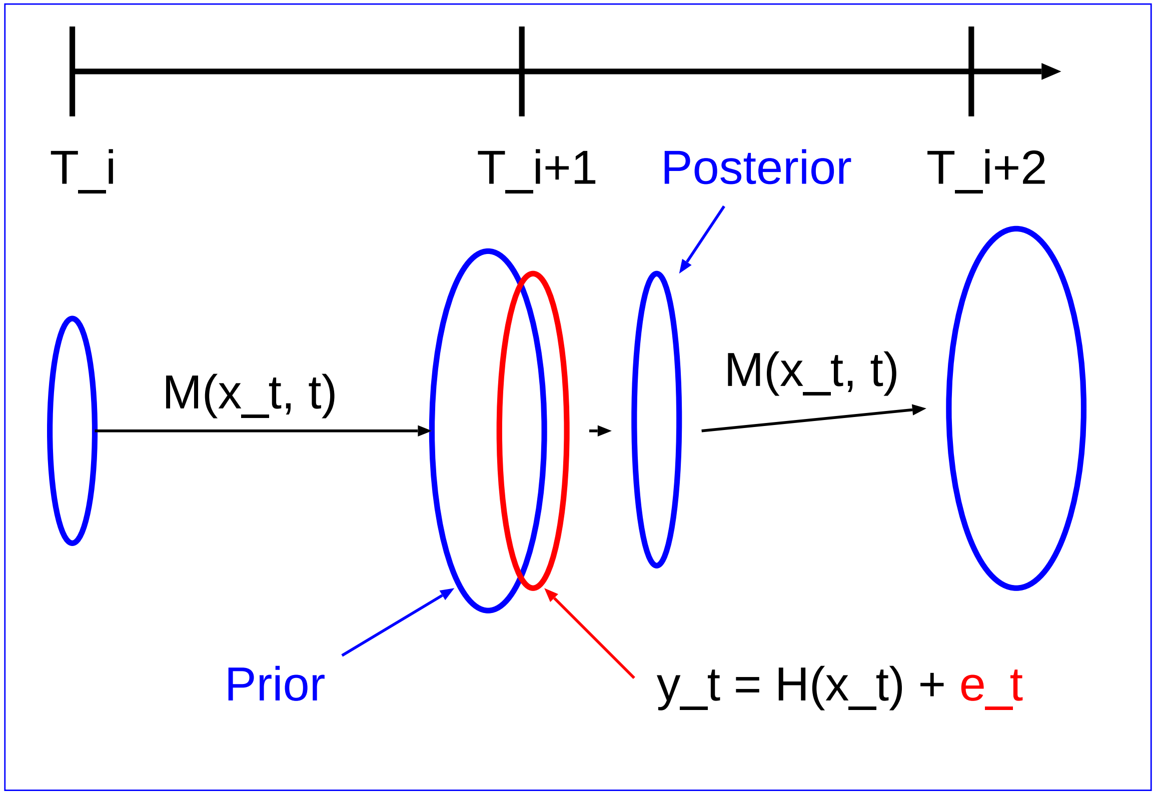
- Time evolution - Liouville equation

A (biased) view of sequential data assimilation



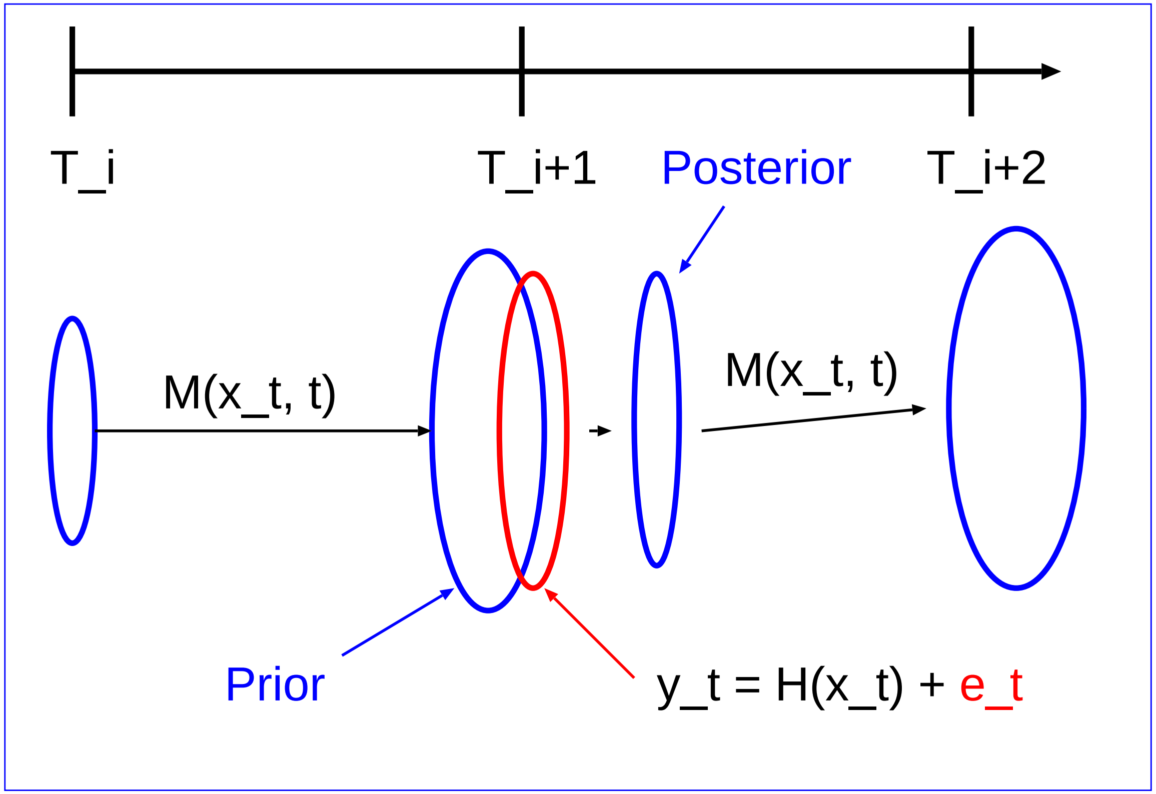
- Updating - at t_{i+1} update system pdf using Bayes rule

A (biased) view of sequential data assimilation



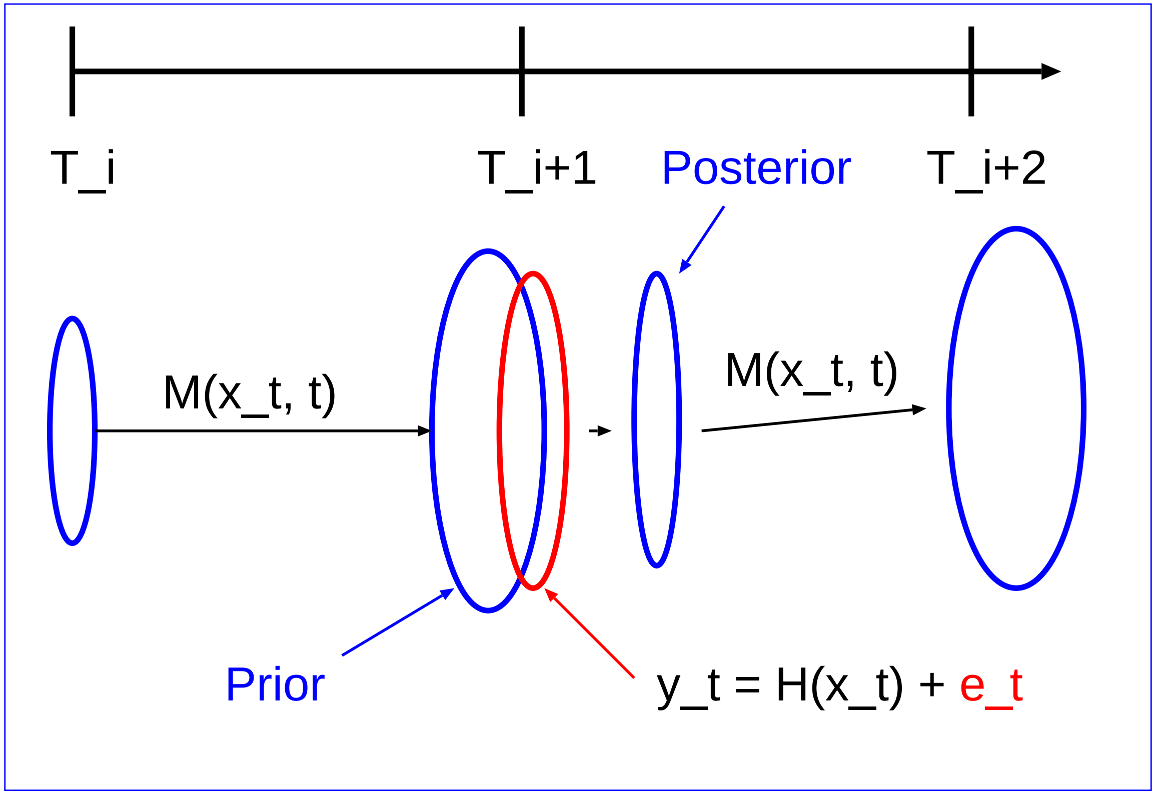
- Problem! $\mathbf{x}$ is order 10^7

A (biased) view of sequential data assimilation



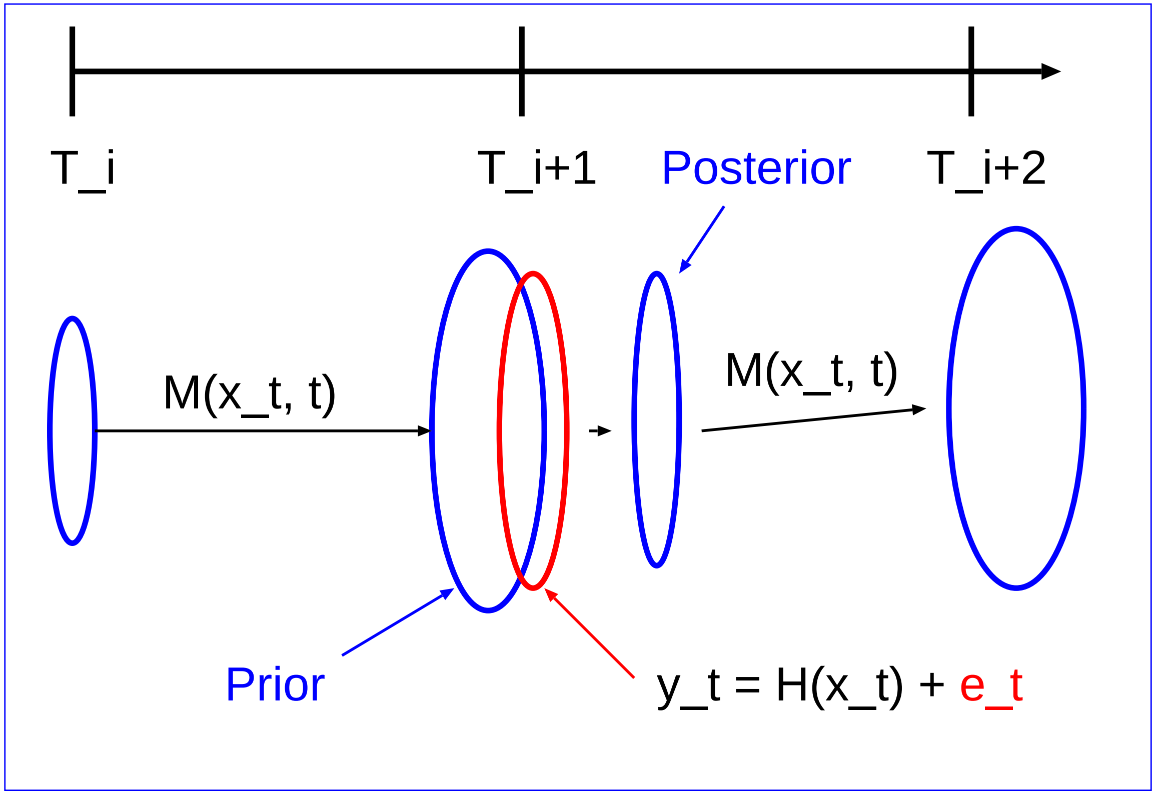
- Time evolution - evolve an ensemble of state estimates - samples of the pdf

A (biased) view of sequential data assimilation



- Practical applications of interest - ensemble size $<<$ degrees of freedom

A (biased) view of sequential data assimilation



- In practice - approximations are introduced in update algorithm - Gaussianity (KF)

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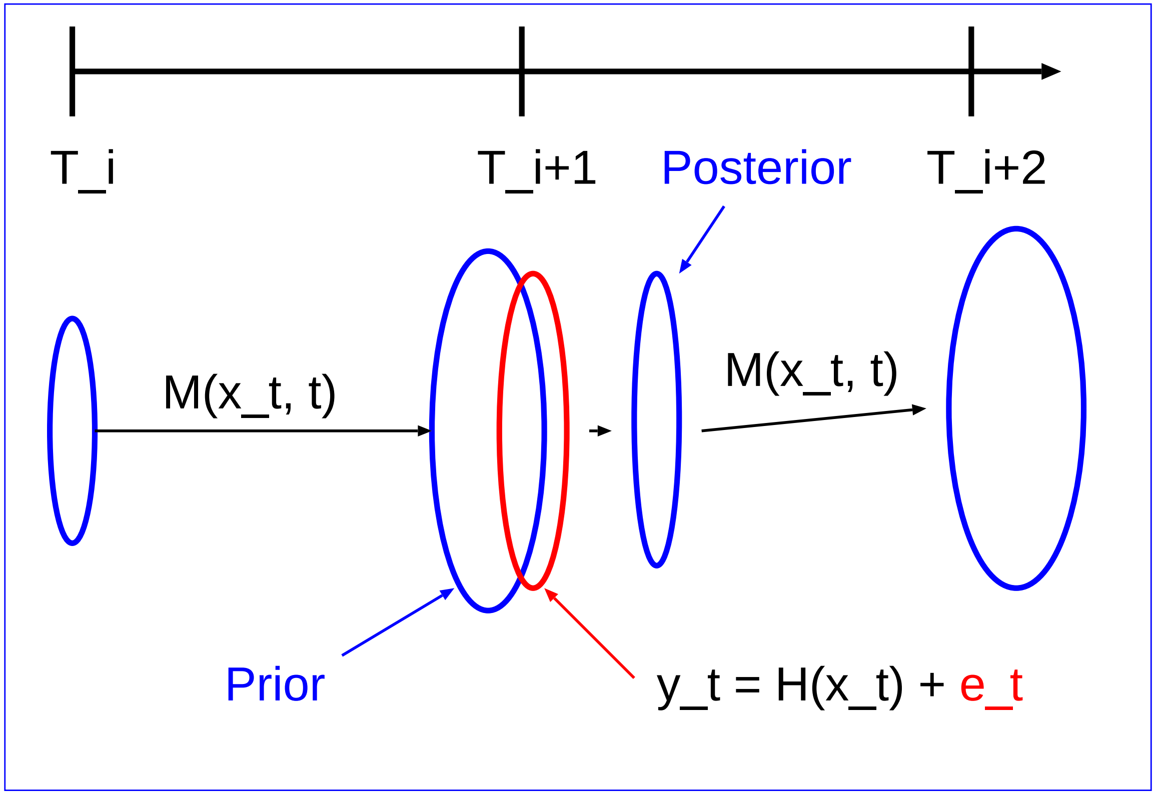

- Within subspace spanned by the ensemble - mean and covariance updated consistent with Kalman Filter update equations (EnKF)

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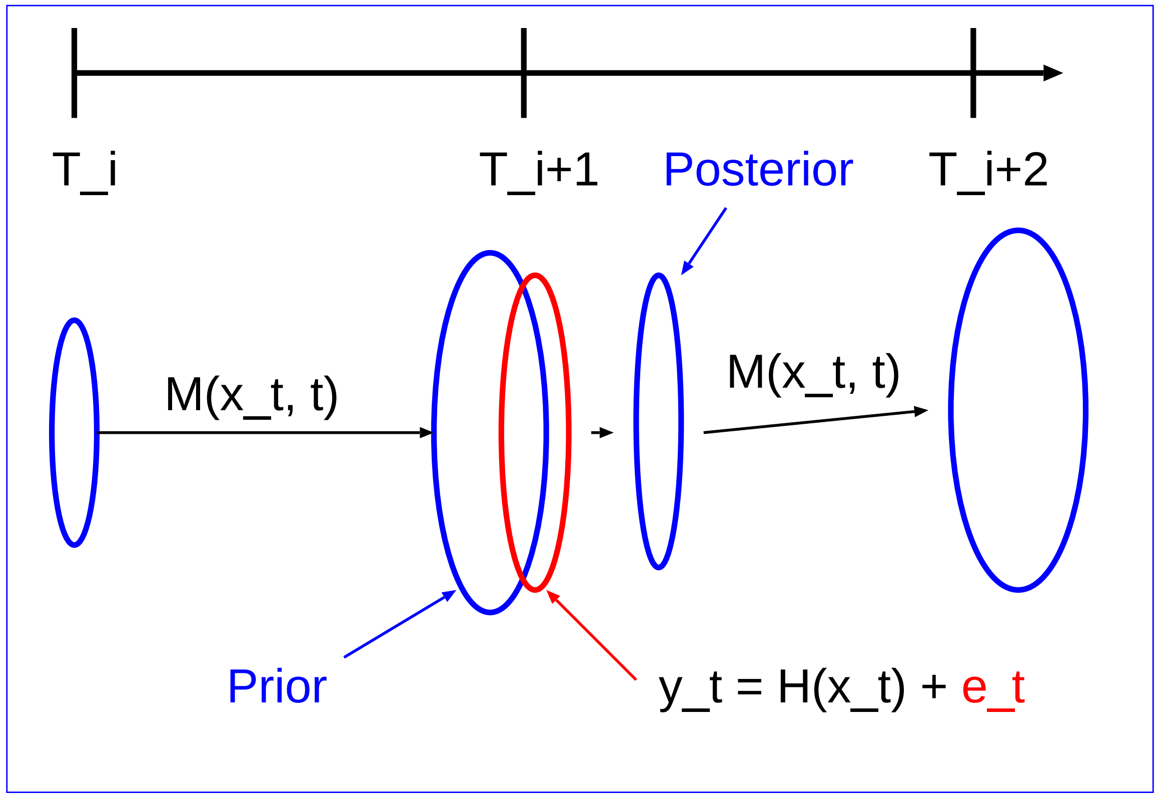
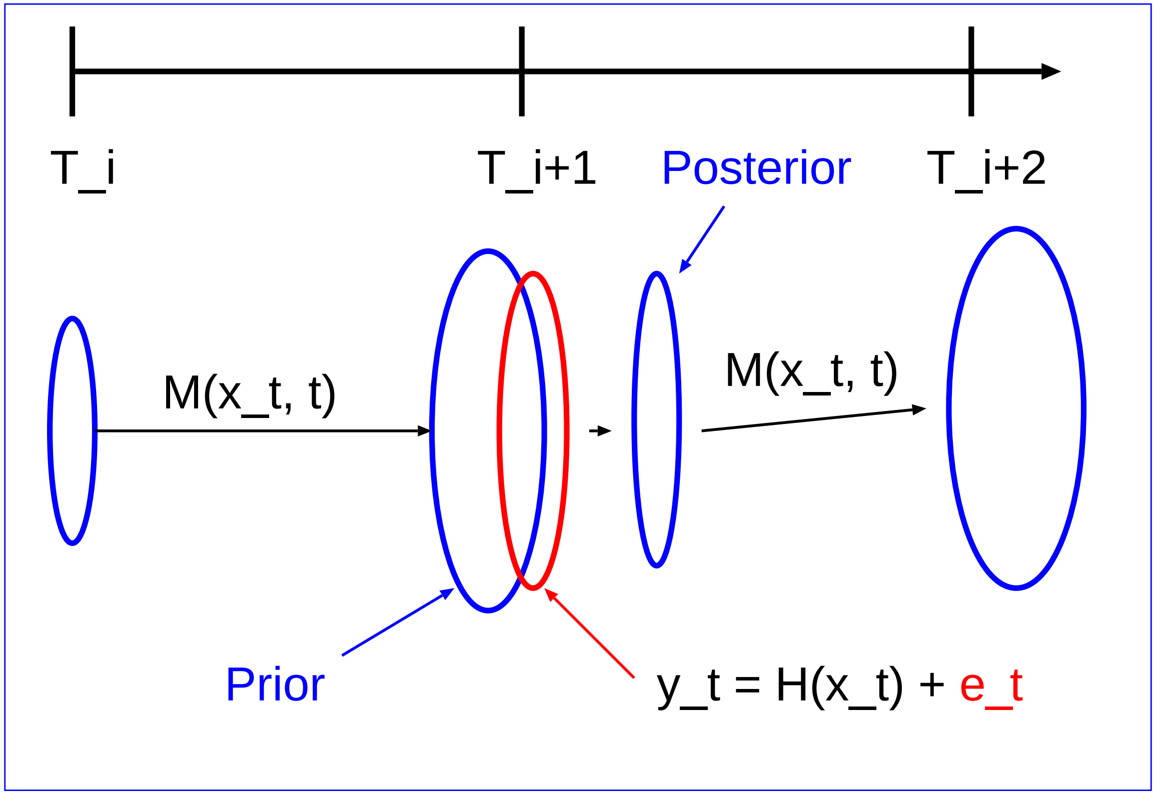


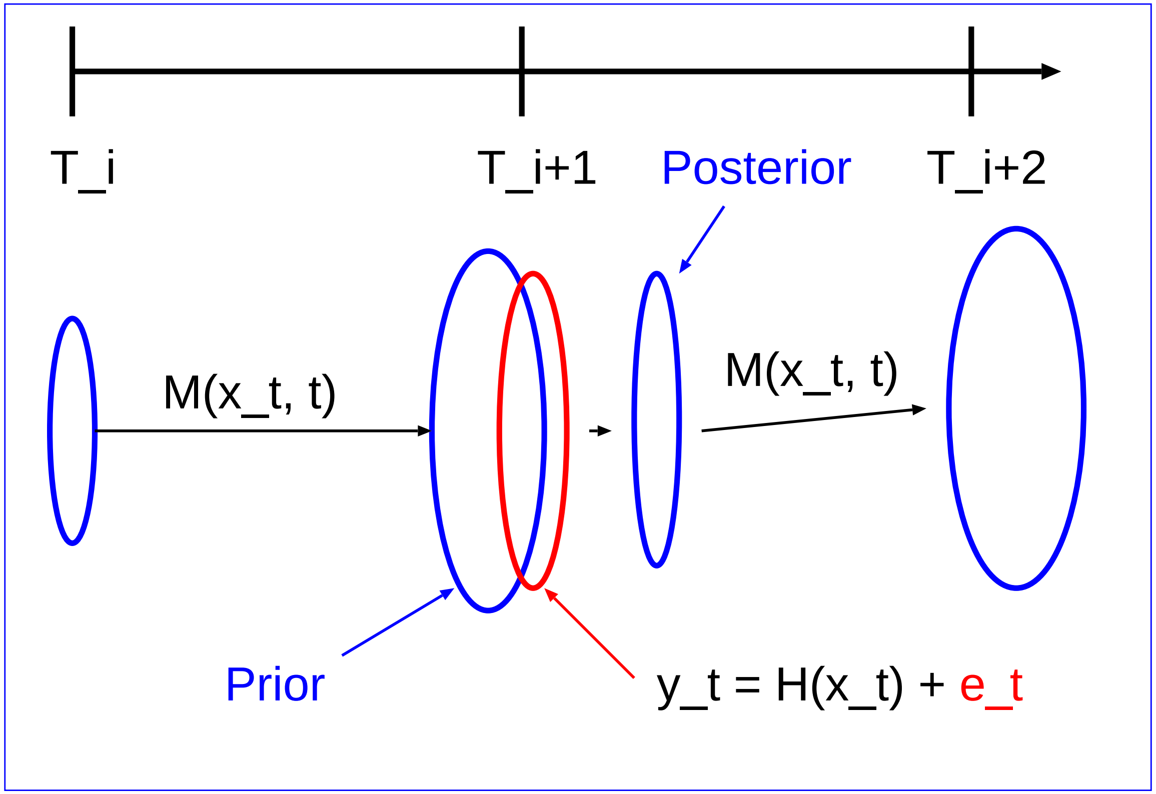
- Deterministic Ensemble Kalman Filters - we use EAKF (Anderson 2001, 2003)

A (biased) view of sequential data assimilation



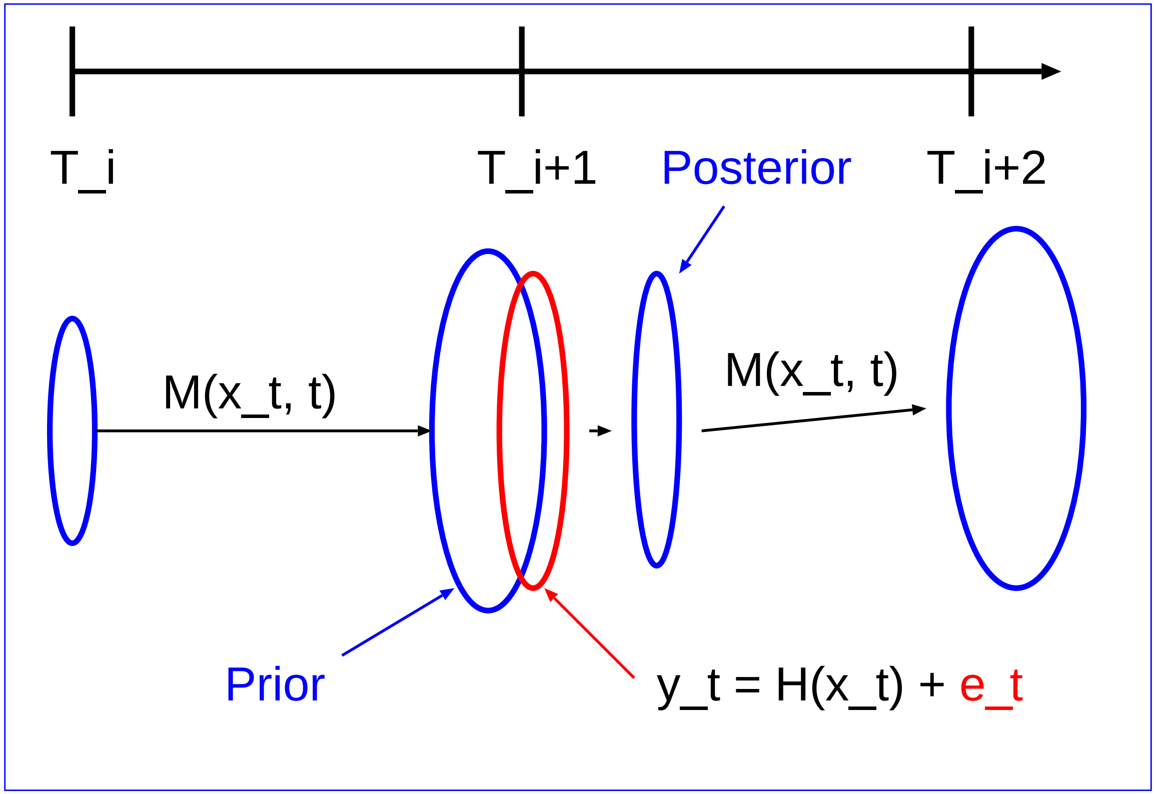
- Damping sampling error is critical - can handle sampling error systematically with EnKF algorithms

A (biased) view of sequential data assimilation



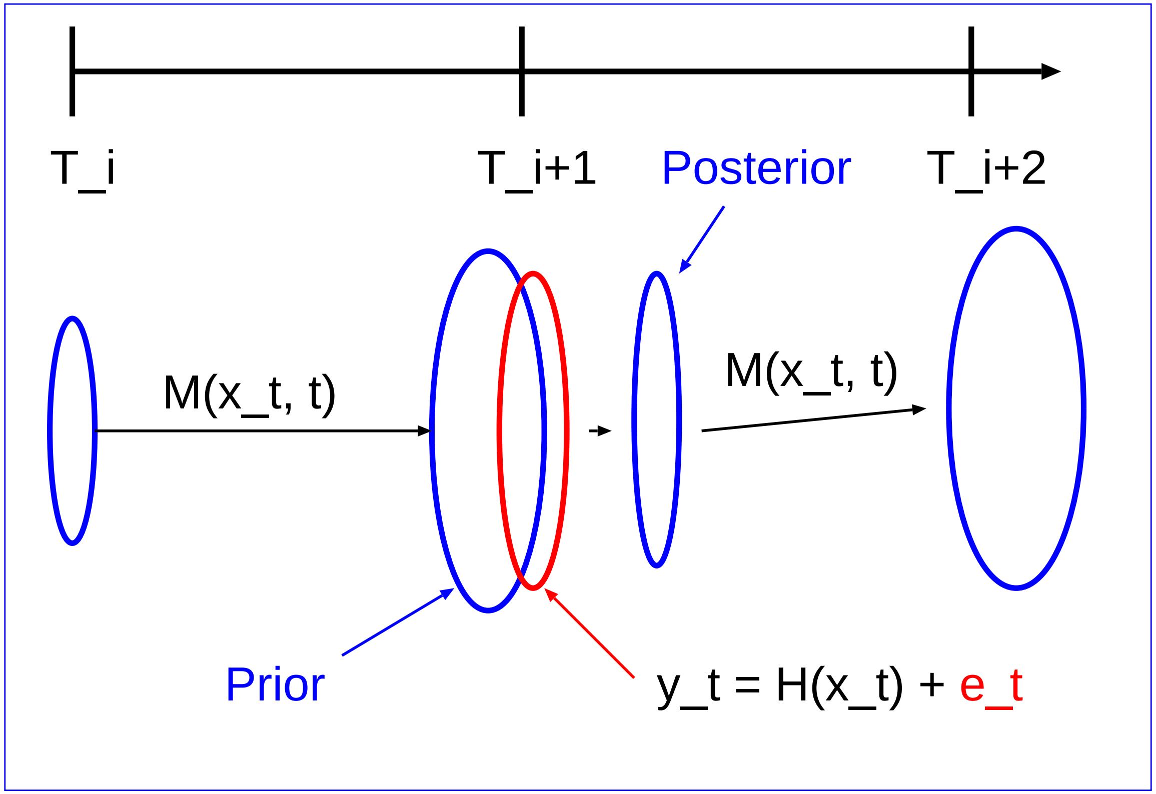
- The ability to handle sampling error $\longrightarrow$ algorithms can be applied to systems with many d.o.f.

A (biased) view of sequential data assimilation



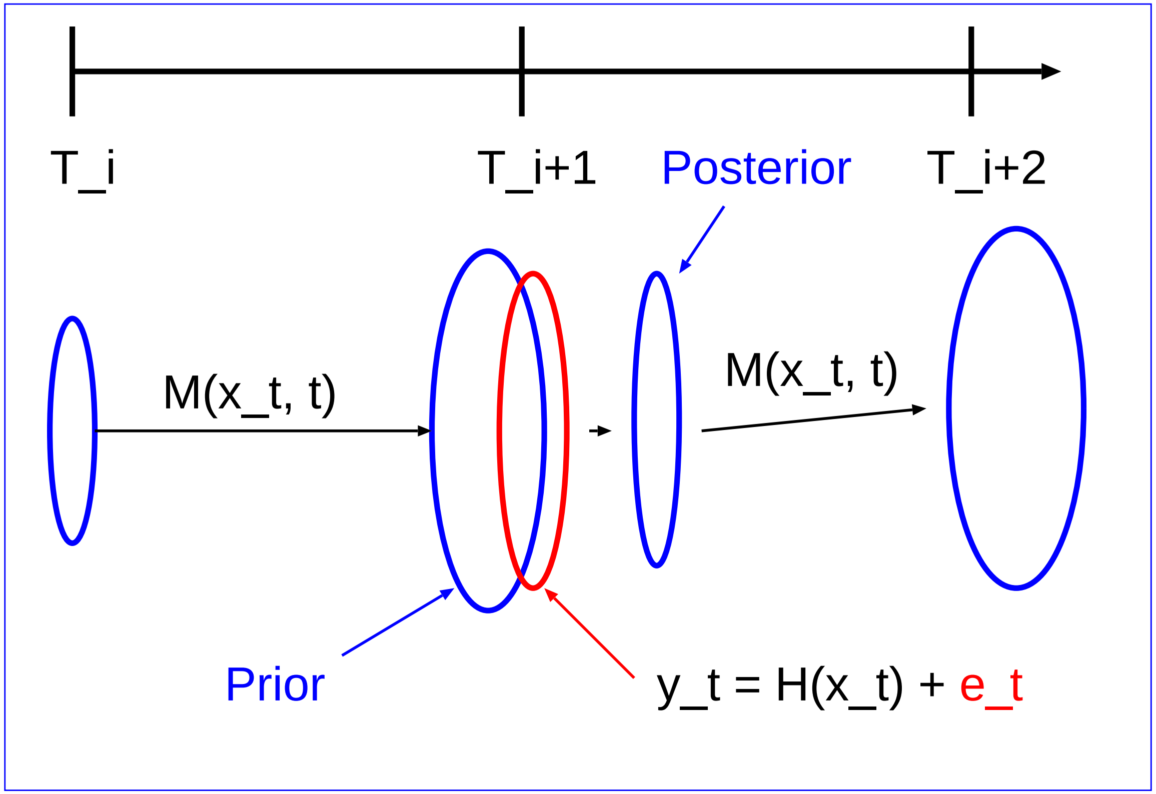
- Fully nonlinear sequential filters have achieved marginal success in systems with many d.o.f.

A (biased) view of sequential data assimilation



- Can we interpret ensembles generated via approximate data assimilation schemes probabilistically?

A (biased) view of sequential data assimilation



- Imperfect models - difficult to implement - even harder to interpret probabilistically

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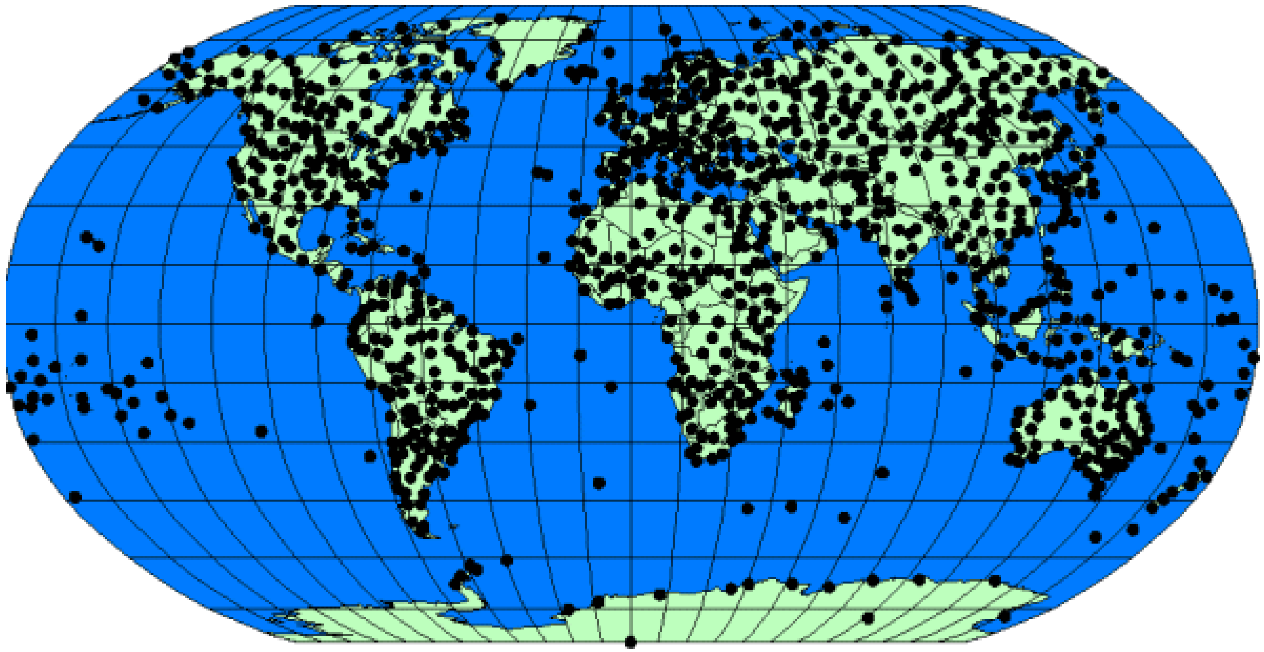
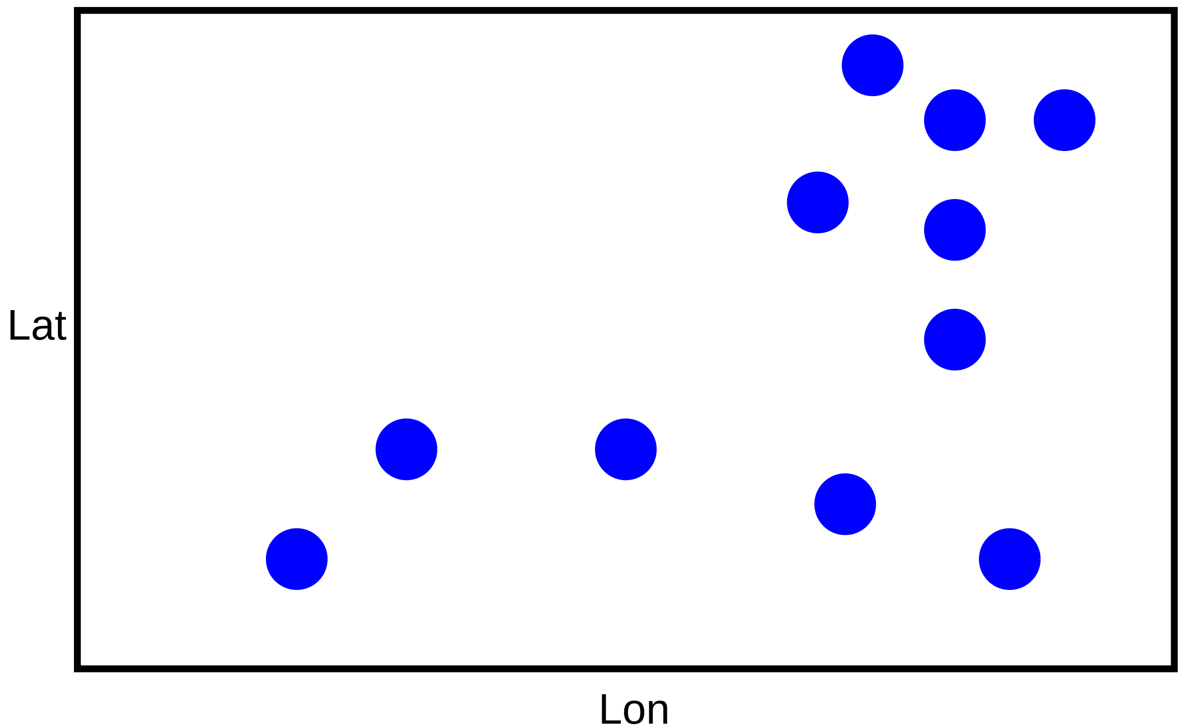
The adaptive observations problem



- Routine observational network
- $\mathbf{y}_t^{routine} = H_t^{routine}(\mathbf{x}_t) + \epsilon_t^{routine}$
- Adaptive observations - supplement to the routine network

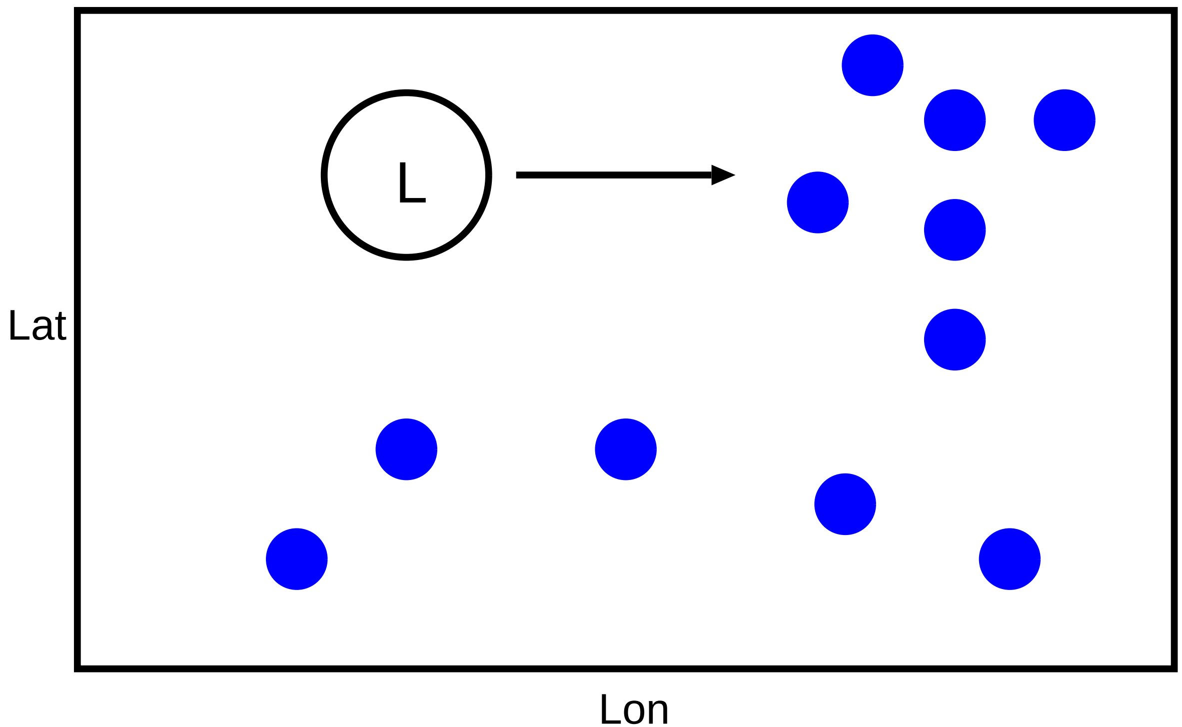
The adaptive observations problem: an example



Today's global observing network

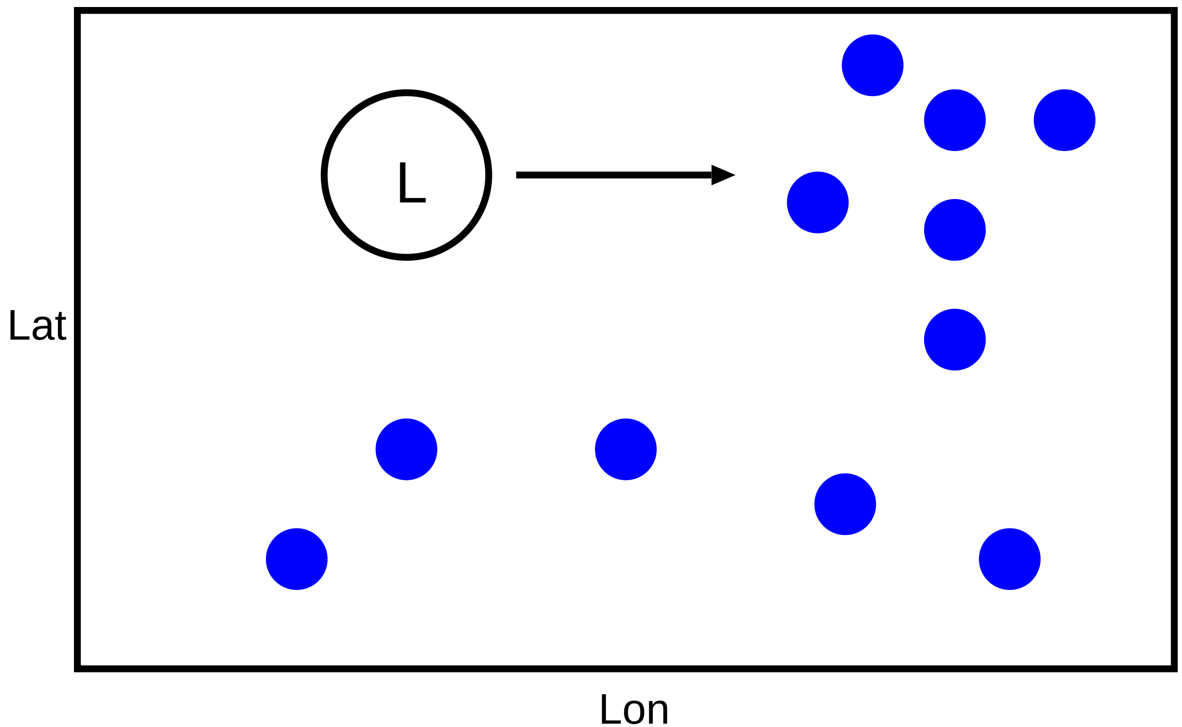
Operator: y_t = H(x_t) + e_t

The adaptive observations problem: an example



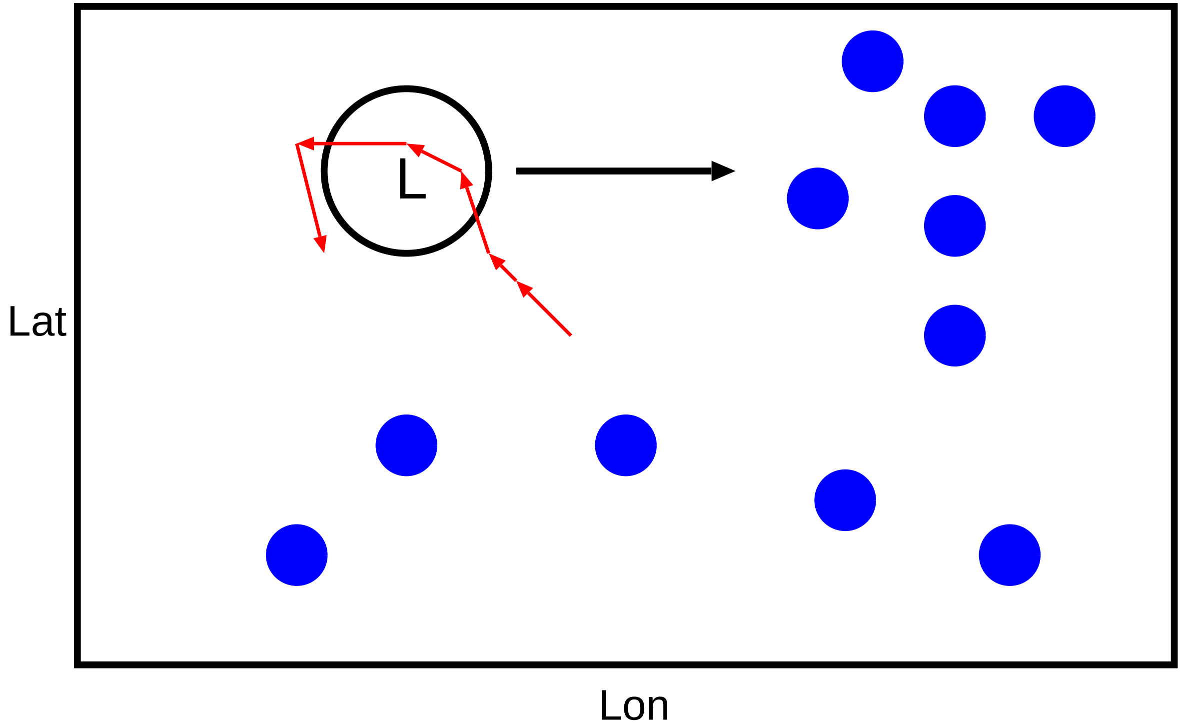
Intense LOW developing over Pacific and is expected to make landfall 4 days from now

The adaptive observations problem: an example



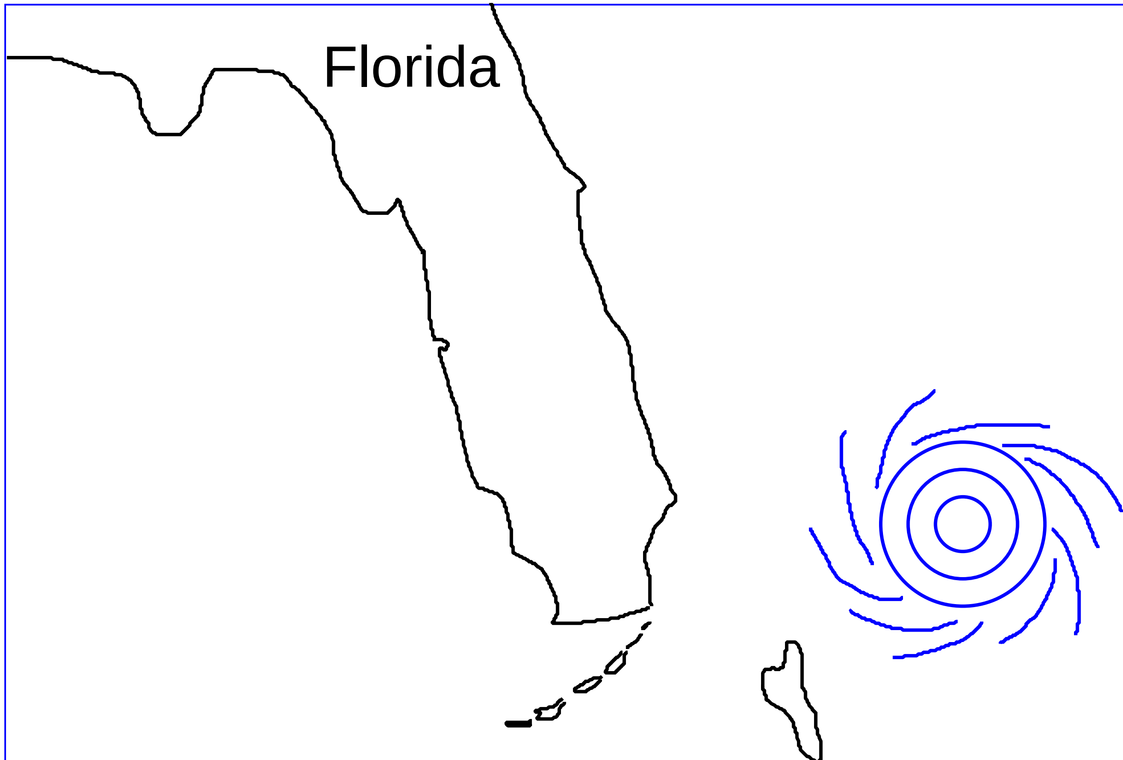
LOW is poorly observed ...
Supplement with Adaptive Observations
to improve prediction

The adaptive observations problem: an example



What is the optimal FLIGHT PATH of an airplane equipped to drop dropsondes?
Operator: $y_t = [\,H_t;\ H_t\,]x_t + e_t$

The adaptive observations problem: an example



Adaptive Observations: important source

of information for Hurricane prediction

The adaptive observations problem: importance

- Provide supplemental observations in data sparse regions (hurricanes, winter storms)

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- Data denial
- Recent field study in Atlantic (THORPEX) shows *positive benefits*
- Winter Storm Reconnaissance Program (WSRP) operational at NCEP since 1999 - uses ensemble based methods

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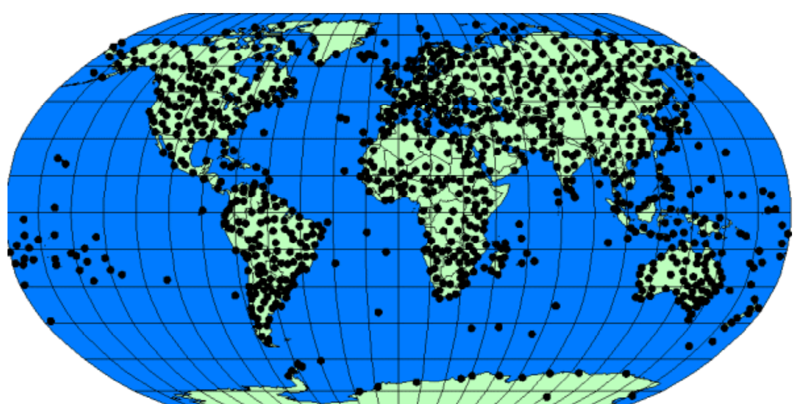
A general Bayesian solution: specific problem

t_initial (t_i) | t_adaptive (t_a) | t_verification (t_v)

H-routine,t_a
+
H-adaptive,t_a

F(t_v|H-adaptive,t_a)



A general Bayesian solution: analytically

- Perfect, deterministic dynamics $d\mathbf{x}/dt = M(\mathbf{x}, t)$

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- Trial observing network: $H_{t_a}^{routine}$, $H_{t_a}^{adaptive}$

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- Entire calculation takes place at t_i

A general Bayesian solution: analytically

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- Integrate Liouville equation to obtain $\mathbf{p}(\mathbf{x}_{t_a})$
- Let $\mathbf{y}_{t_a}$ be some specified set of observations consistent w/ trial observing network
- $\mathbf{p}(\mathbf{x}_{t_a}|\mathbf{y}_{t_a}) = \frac{\mathbf{p}(\mathbf{x}_{t_a})\mathbf{p}(\mathbf{y}_{t_a}|\mathbf{x}_{t_a})}{\mathbf{p}(\mathbf{y}_{t_a})}$

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- Integrate Liouville equation to obtain $\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a})$
- Apply some norm to $\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a})$ - to measure uncertainty

A general Bayesian solution: analytically

- Given $\mathbf{p}(\mathbf{x}_{t_i})$
- Integrate Liouville equation to obtain $\mathbf{p}(\mathbf{x}_{t_a})$
- Let $\mathbf{y}_{t_a}$ be some specified set of observations consistent w/ trial observing network
- $\mathbf{p}(\mathbf{x}_{t_a}|\mathbf{y}_{t_a}) = \frac{\mathbf{p}(\mathbf{x}_{t_a})\mathbf{p}(\mathbf{y}_{t_a}|\mathbf{x}_{t_a})}{\mathbf{p}(\mathbf{y}_{t_a})}$
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- Denote $L(\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a}))$

A general Bayesian solution: analytically

- Example: $L(\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a})) = \int (\mathbf{x}_{t_v} - \overline{\mathbf{x}}_{t_v})^2 \mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a}) d\mathbf{x}_{t_v}$

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- Compute expectation of $L(\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a}))$ over all possible observations consistent with trial network

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- Generally not possible analytically - revert to MC approximations for all above calculations

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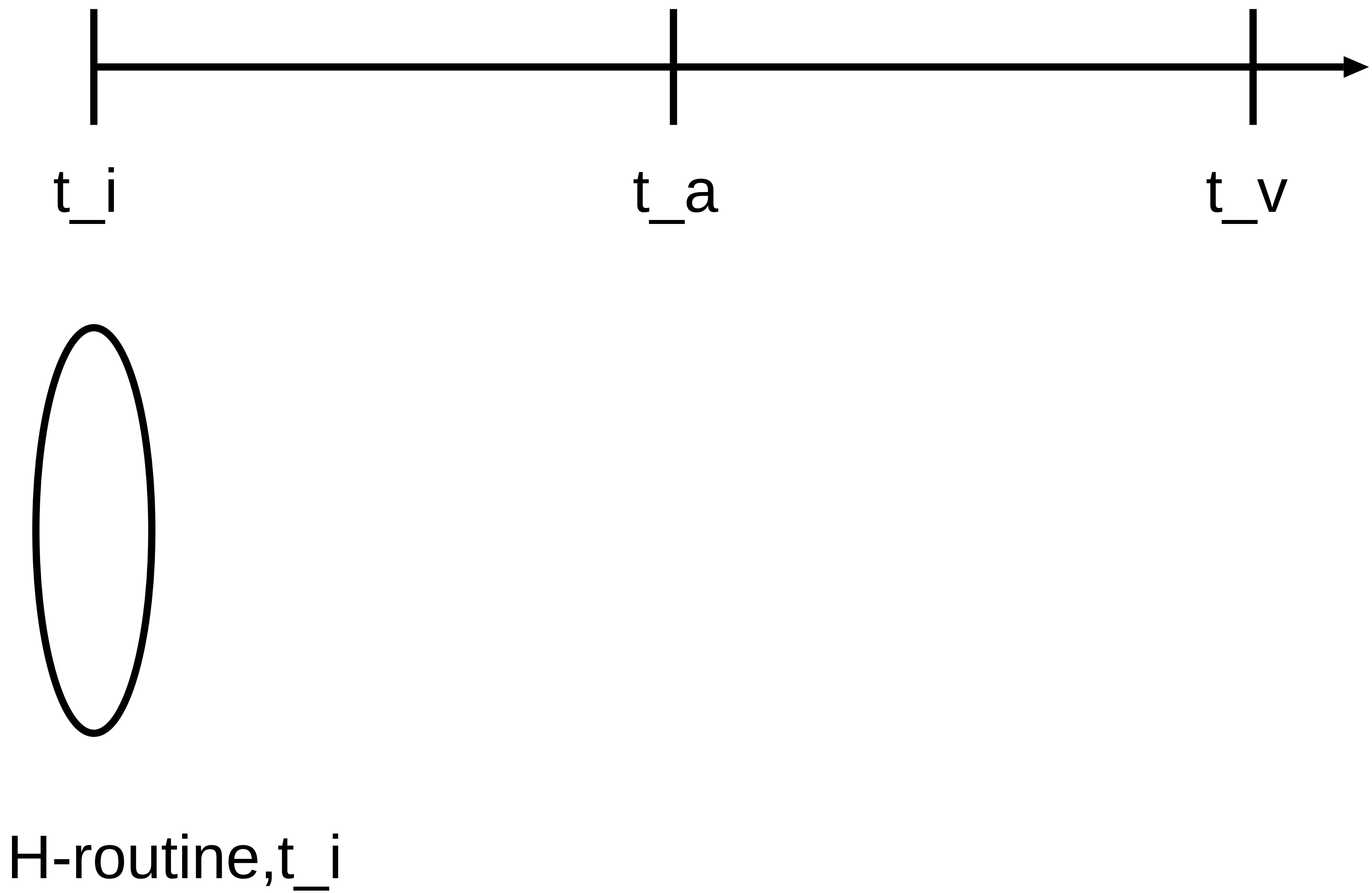
General solution: particle filter approach



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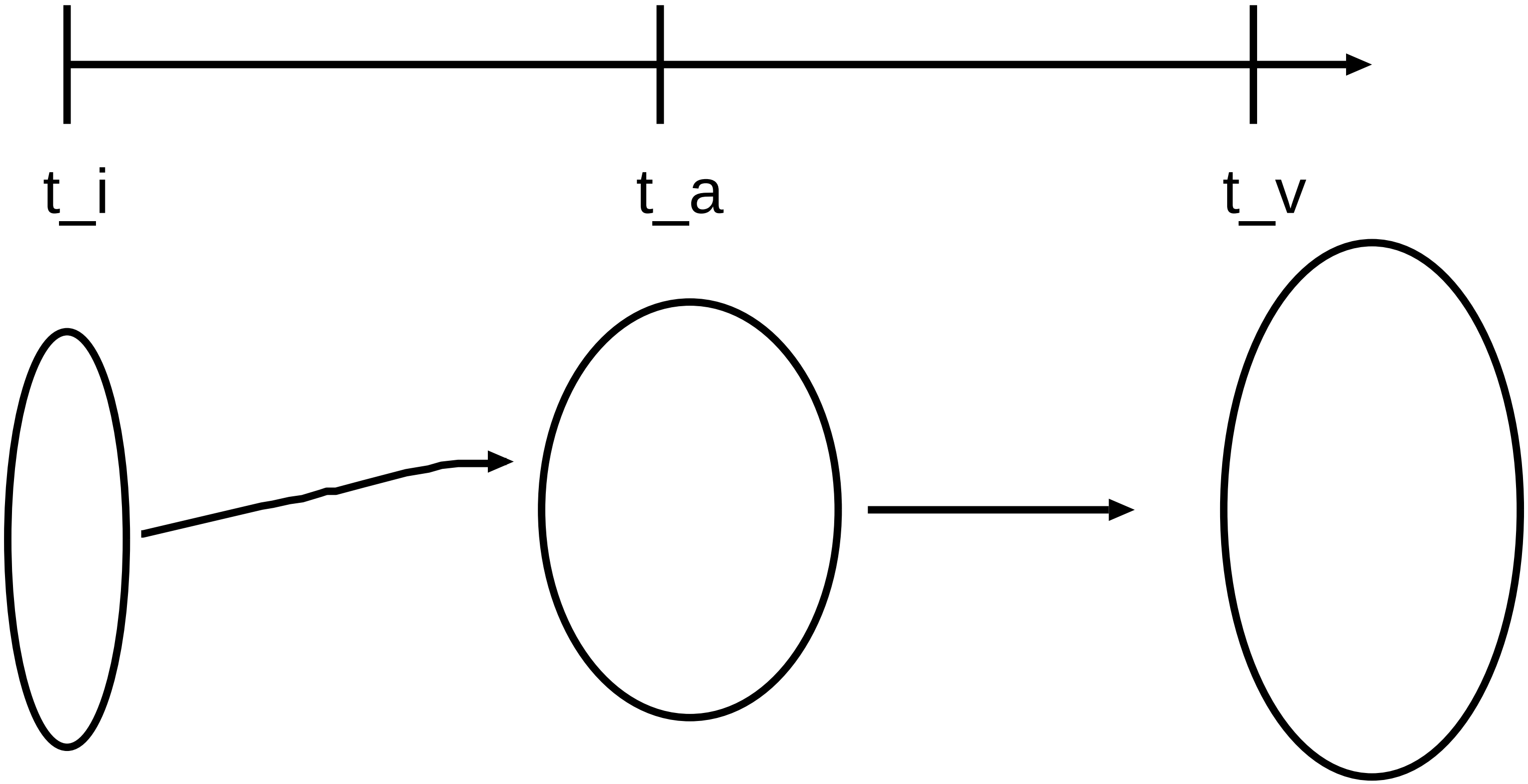
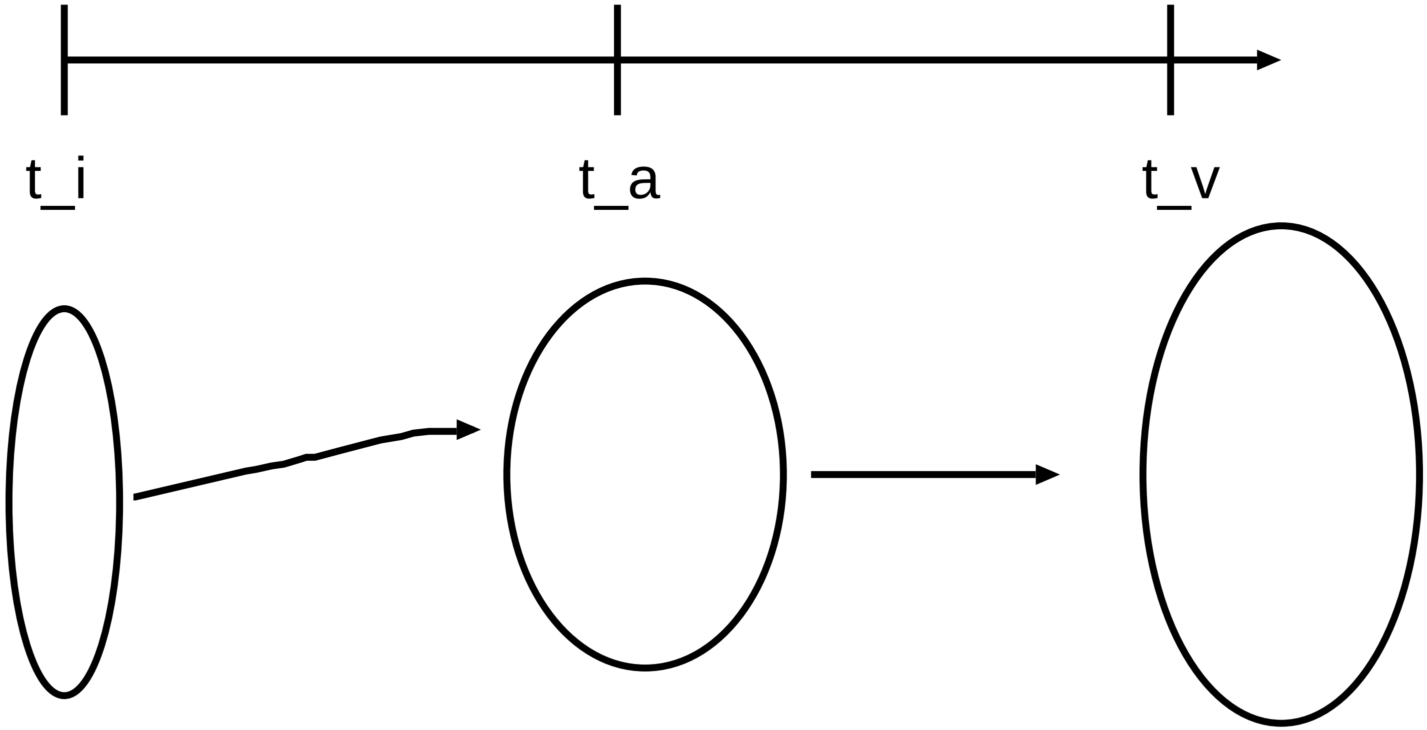


H-routine, t_i

Generate ensemble forecast out to - t_v

General solution: particle filter approach



H-routine,t_i

Trial nework at t_a -

H-routine,t_a ‘+’ H-adaptive,t_a

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- Forecast for t_a - $\mathbf{x}^f_{i,t_a}$ with $i = 1, ..., N$ and weights w^f_{i,t_a}

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- For j^{th} sample, re-weight: $w^u_{i,t_a,j} \sim e^{(-[\mathbf{y}_{t_a,j} - H^{adaptive}_{t_a}(\mathbf{x}^f_{i,t_a})]^T \mathbf{R}^{-1}_{t_a}[\mathbf{y}_{t_a,j} - H^{adaptive}_{t_a}(\mathbf{x}^f_{i,t_a})]/2)} w^f_{i,t_a}$

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- Updated weights apply for t_v - compute j^{th} value for norm using $\mathbf{x}^f_{i,t_v}$ and $w^u_{i,t_a,j}$

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- Updated weights apply for t_v - compute j^{th} value for norm using $\mathbf{x}^f_{i,t_v}$ and $w^u_{i,t_a,j}$
- Take expectation over J samples approximates $F(t_v|H^{adaptive}_{t_a}) = \int L(\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a}))\mathbf{p}(\mathbf{y}_{t_a})d\mathbf{y}_{t_a}$

General solution: particle filter approach

- Forecast for t_a - $\mathbf{x}_{i,t_a}^f$ with $i = 1, ..., N$ and weights w_{i,t_a}^f
- Generate $j = 1, ..., J$ random samples of $\mathbf{p}(\mathbf{y}_{t_a}) = \int \mathbf{p}(\mathbf{x}_{t_a})\mathbf{p}(\mathbf{y}_{t_a}|\mathbf{x}_{t_a})d\mathbf{x}_{t_a}$
- For j^{th} sample, re-weight: $w_{i,t_a,j}^u \sim e^{(-[\mathbf{y}_{t_a,j} - H_{t_a}^{adaptive}(\mathbf{x}_{i,t_a}^f)]^T \mathbf{R}_{t_a}^{-1}[\mathbf{y}_{t_a,j} - H_{t_a}^{adaptive}(\mathbf{x}_{i,t_a}^f)]/2)} w_{i,t_a}^f$
- Updated weights apply for t_v - compute j^{th} value for norm using $\mathbf{x}_{i,t_v}^f$ and $w_{i,t_a,j}^u$
- Take expectation over J samples approximates $F(t_v|H_{t_a}^{adaptive}) = \int L(\mathbf{p}(\mathbf{x}_{t_v}|\mathbf{y}_{t_a}))\mathbf{p}(\mathbf{y}_{t_a})d\mathbf{y}_{t_a}$
- Problems with sampling errors for small ensembles?

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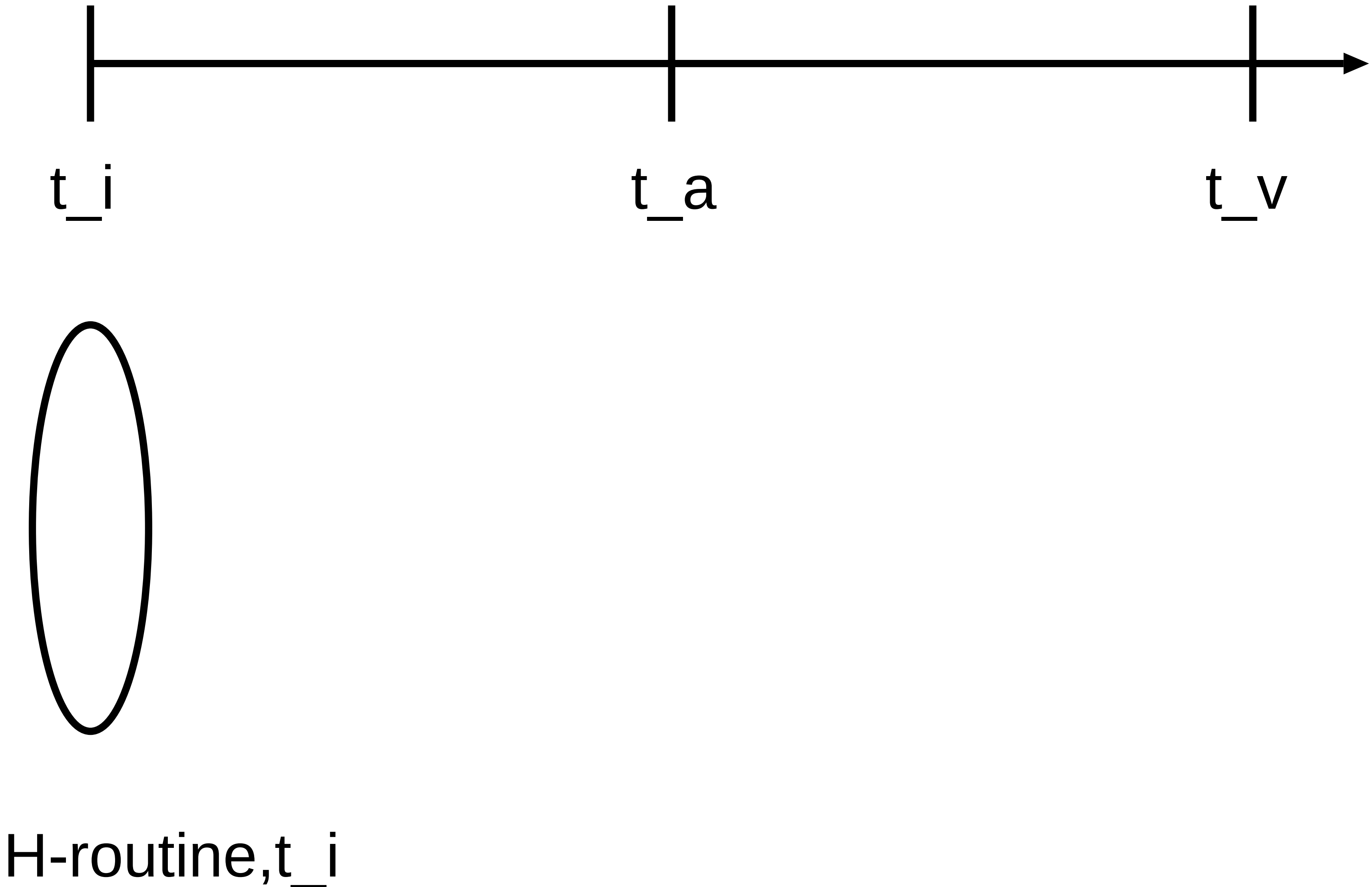
Approximate solution: EnKF approach



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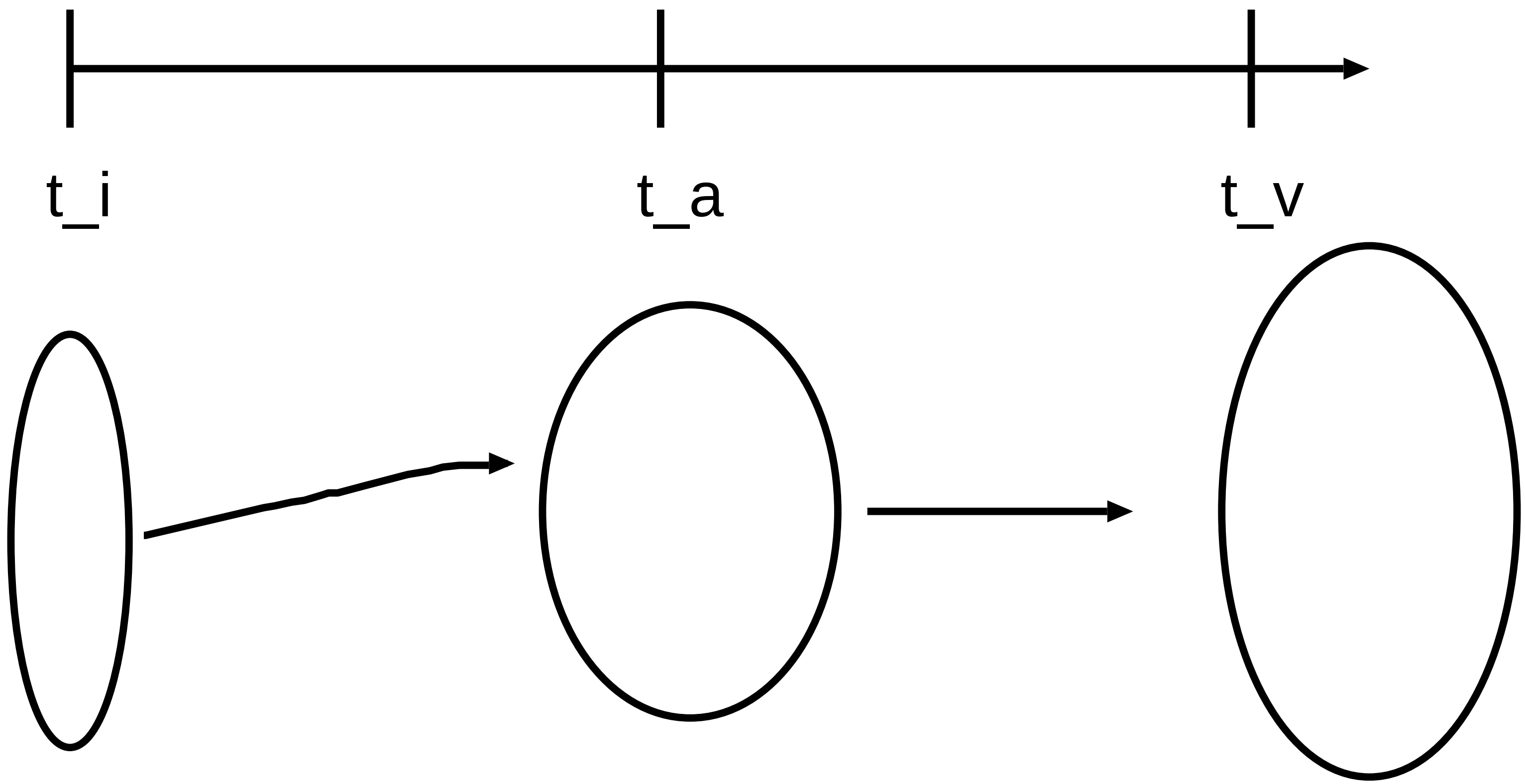

H-routine, t_i

Generate ensemble forecast out to - t_v

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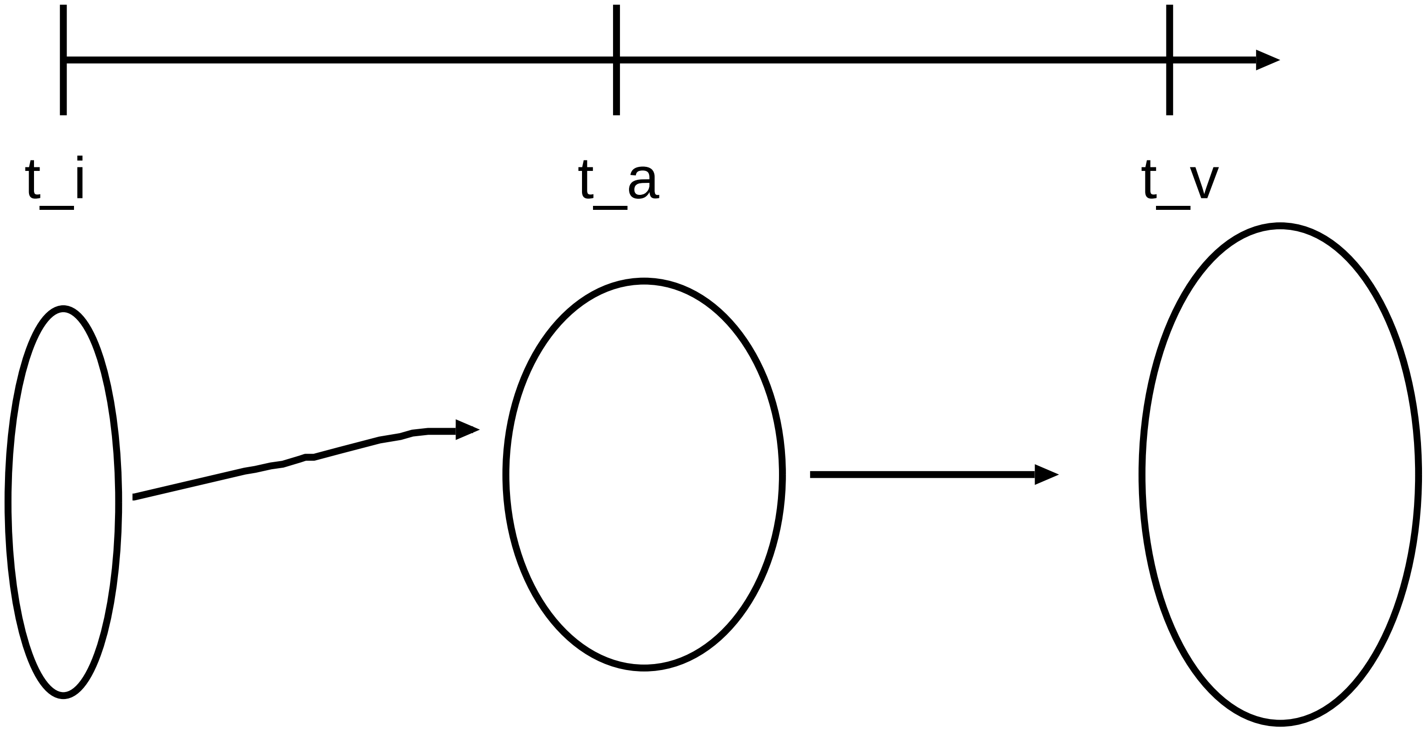

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Trial nework at t_a -

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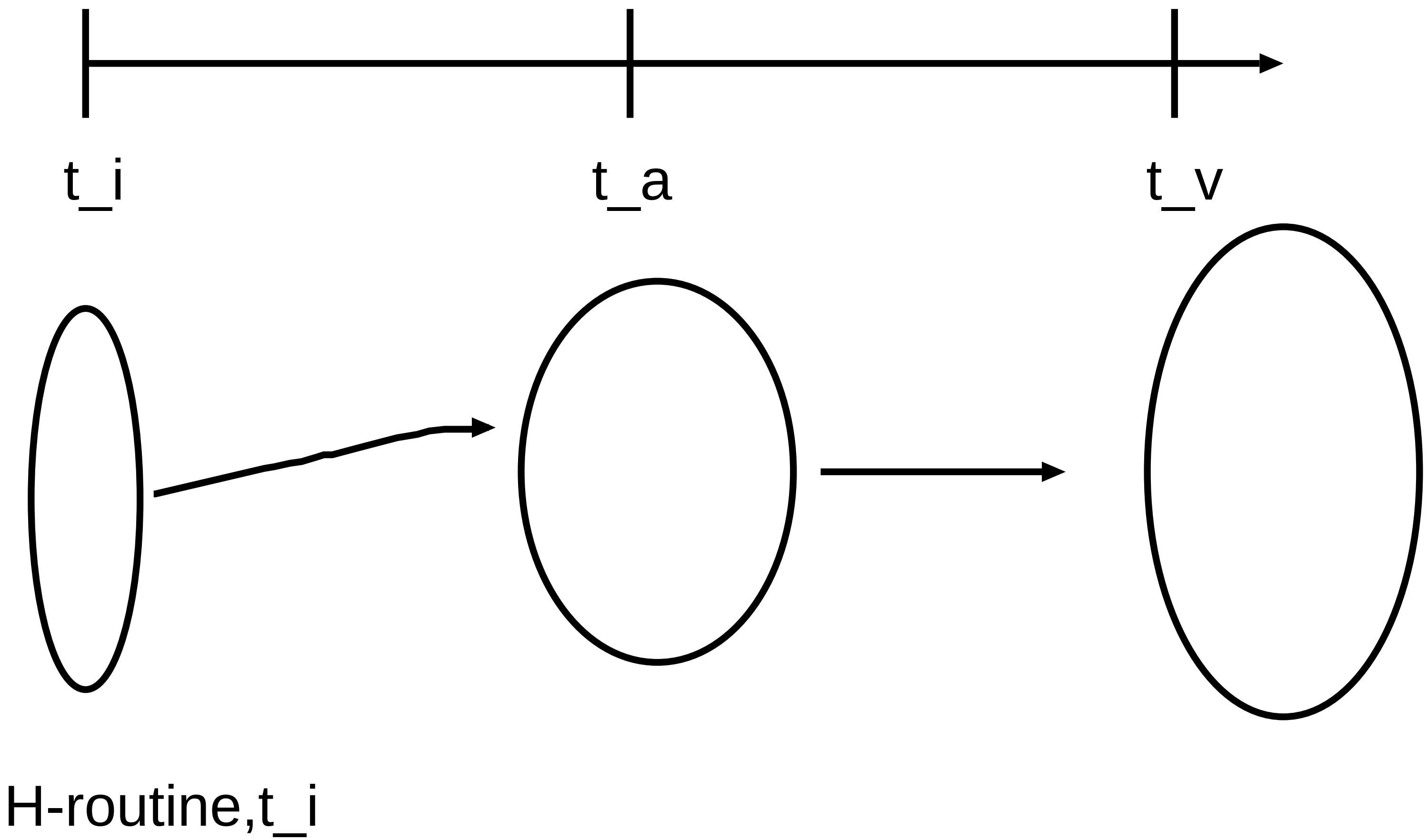
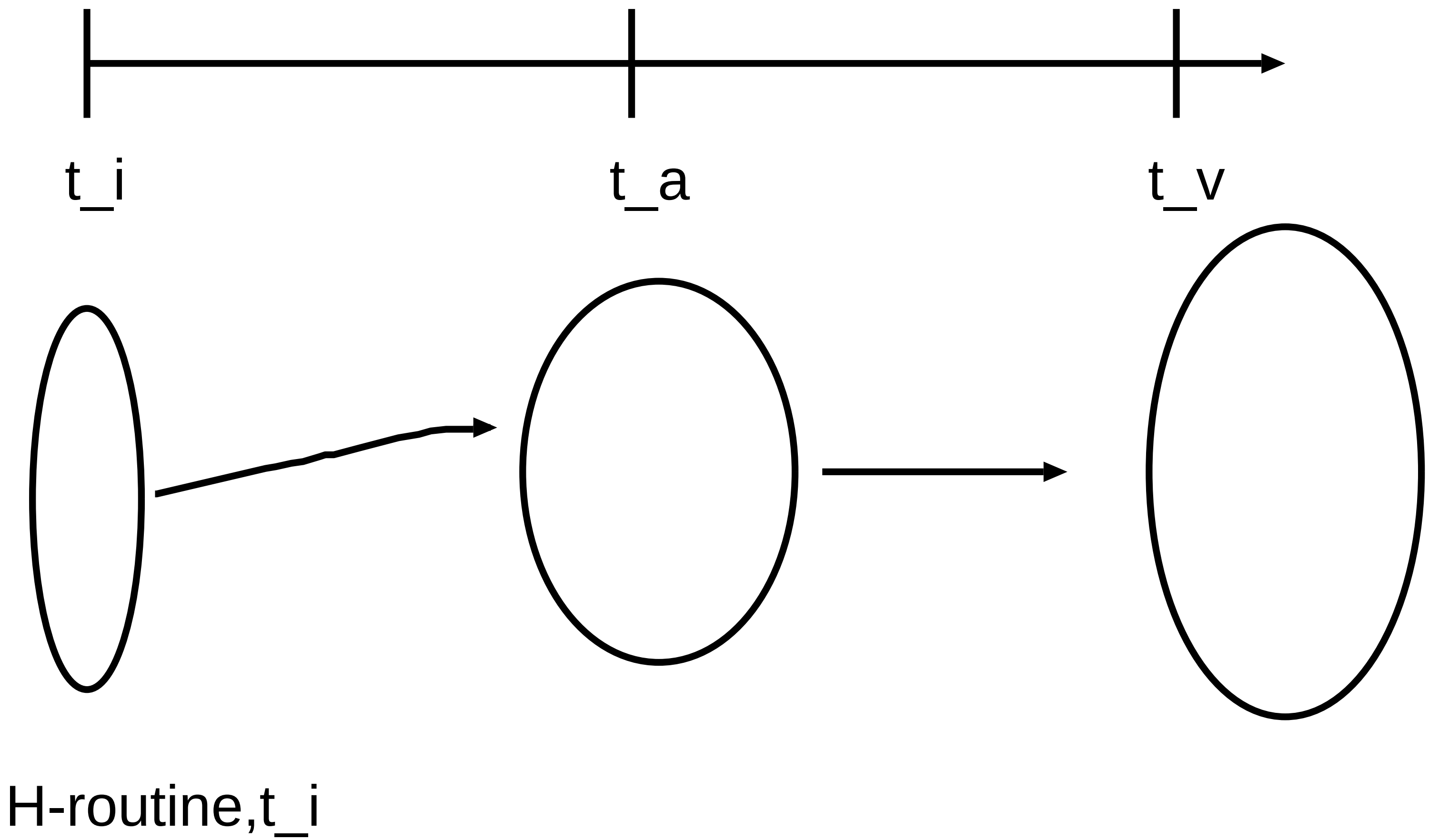


Idea: Use an EnKF based algorithm to update

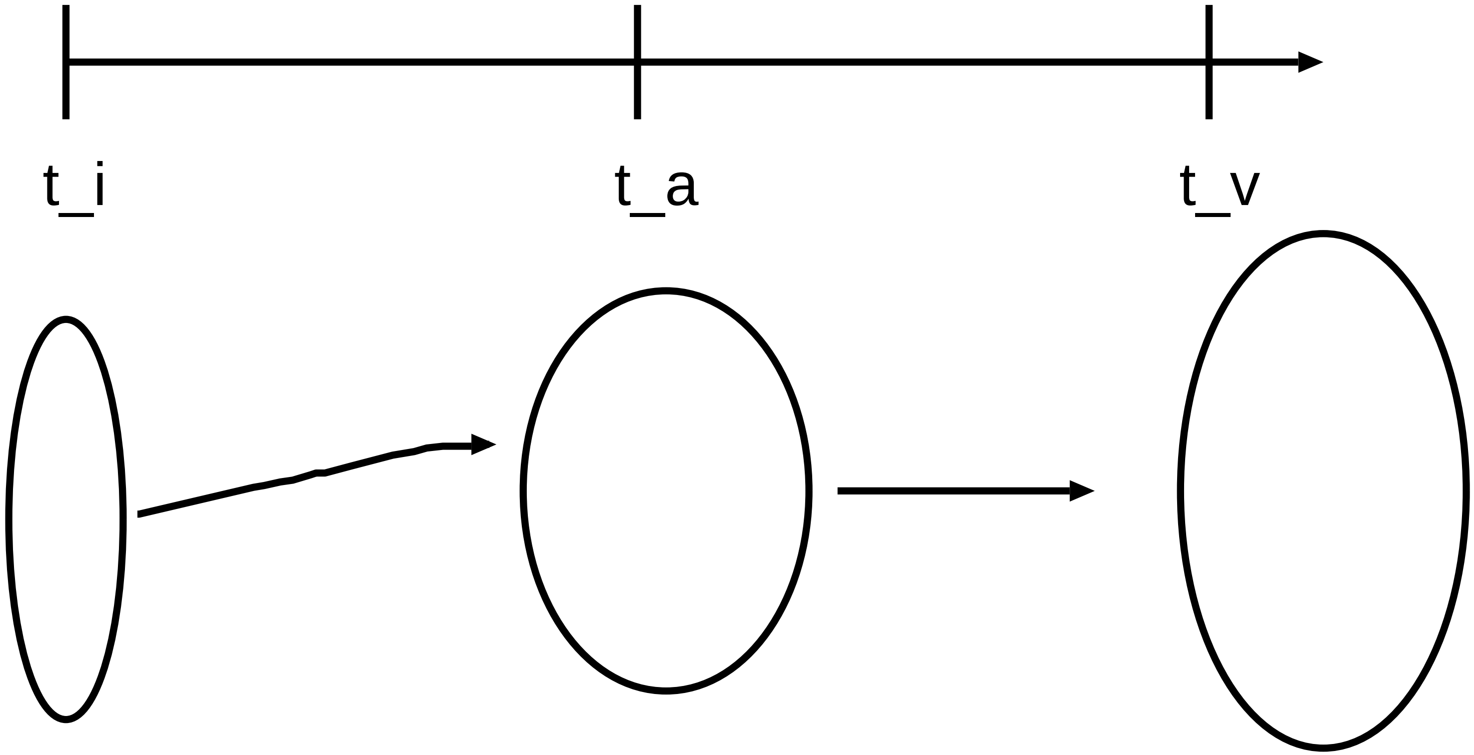
the ensemble forecast

Approximate solution: EnKF approach



This generates an estimate of F(t_v | H-adaptive,t_a)

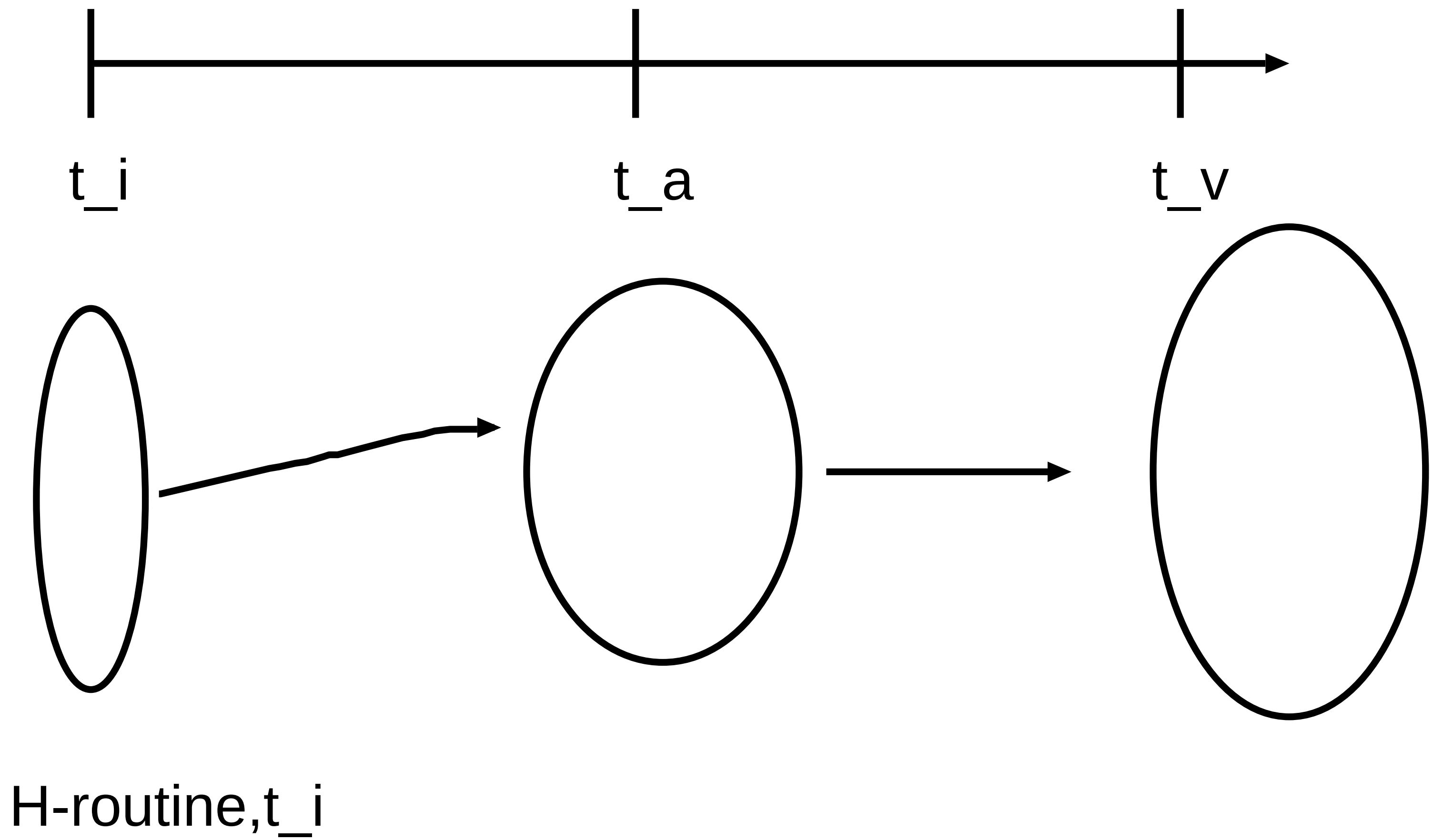
Approximate solution: EnKF approach



H-routine,t_i

*** The KEY - the ensemble forecast need only be generated once ***

Approximate solution: EnKF approach



Can evaluate many H-adaptive,t_a without repeatedly

integrating the forecast model - Comp. Efficient

Approximate solution: EnKF approach

- Interpret ensemble forecast from t_i as a sample of a PRIOR $\mathbf{p}(\mathbf{x}_{t_a}, \mathbf{x}_{t_v})$
- Let the observation values at t_a be: $\mathbf{y}_{t_a} = [H_{t_a}^{routine}; H_{t_a}^{adaptive}]\overline{\mathbf{x}}_{t_a}^f$
- The updated joint state distribution, given the hypothetical network at t_a:

$$\mathbf{p}(\mathbf{x}_{t_a}, \mathbf{x}_{t_v}|\mathbf{y}_{t_a}) = \frac{\mathbf{p}(\mathbf{x}_{t_a}, \mathbf{x}_{t_v})\mathbf{p}(\mathbf{y}_{t_a})}{Normalization} \qquad (0)$$

- Update PRIOR using and EnKF (Gaussian specifications)

A computationally efficient scheme: theory

- Use updated ensemble to compute covariance for $\mathbf{x}_{t_v}$ - can then quantify $F(H_{t_a}^{adaptive})$
- **Theory for Deterministic EnSRF implementations with no covariance localization**
- **Without covariance localization - DEnSRF implementations are equivalent NCEP's operational scheme** (Bishop et al. 2001)
- No repeated integrations of forecast model in evaluating many $H_{t_a}^{adaptive}$ - **operationally feasible**

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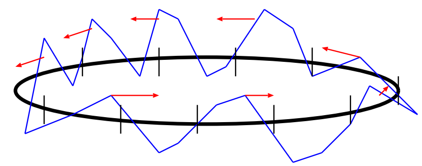
Experiments in an atmospheric ‘toy model’

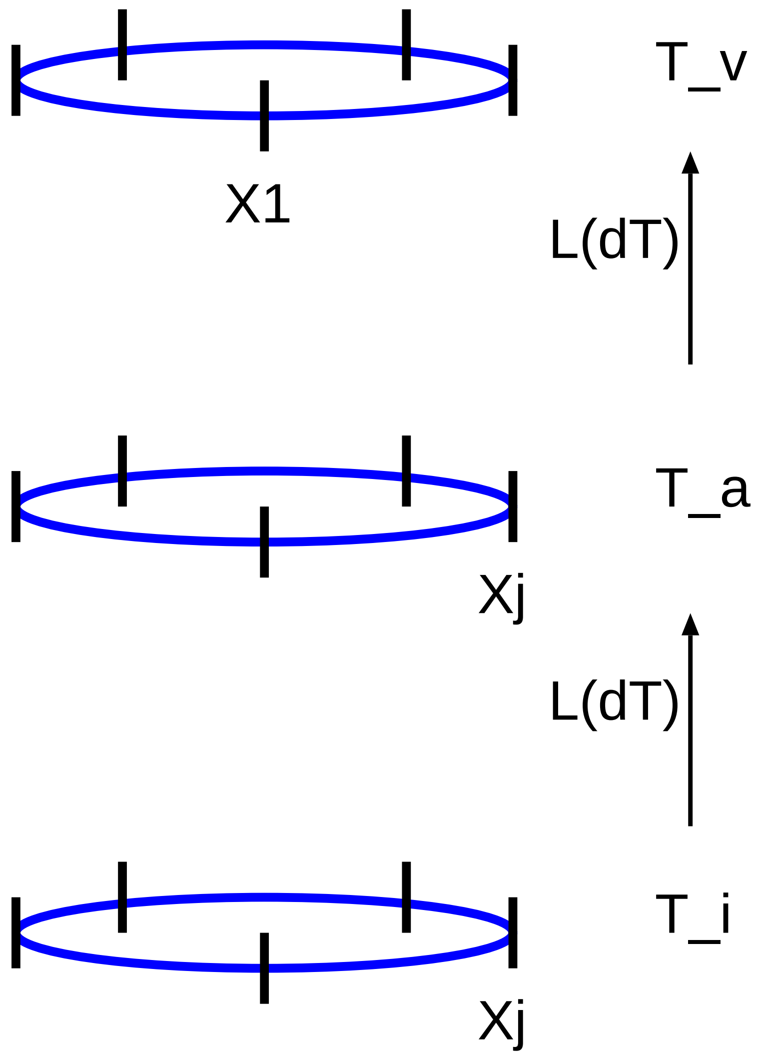


$$\frac{dx_j}{dt} = -x_{j-2}x_{j-1} + x_{j-1}x_{j+1} - x_j + F$$

- Non-linear advection, forcing, linear dissipation, energy conservation $E = (x_1^2 + ... + x_{40}^2)$
- $F = 8$ sensitive dependence on IC’s, $j = 1, ..., 40$, cyclic
- Climatology $\sqrt{\sigma_{climate}} = 3.6$ and $\overline{x} = 2.3$
- Look at small perturbations to steady state solution, $x_j = F$, $\delta x_j = \sum_k exp(-i\omega t) exp(ikj)$

$$\omega(k) = -[sin(k) + sin(2k)]F + i[(cos(k) - cos(2k))F - 1]$$

Experiments in an atmospheric ‘toy model’



Lorenz 1996 (F = 8, j = 1, ..., 40)

dT = 0.05 units = 6 ‘hours’

Look at results from 10000 consecutive experiments

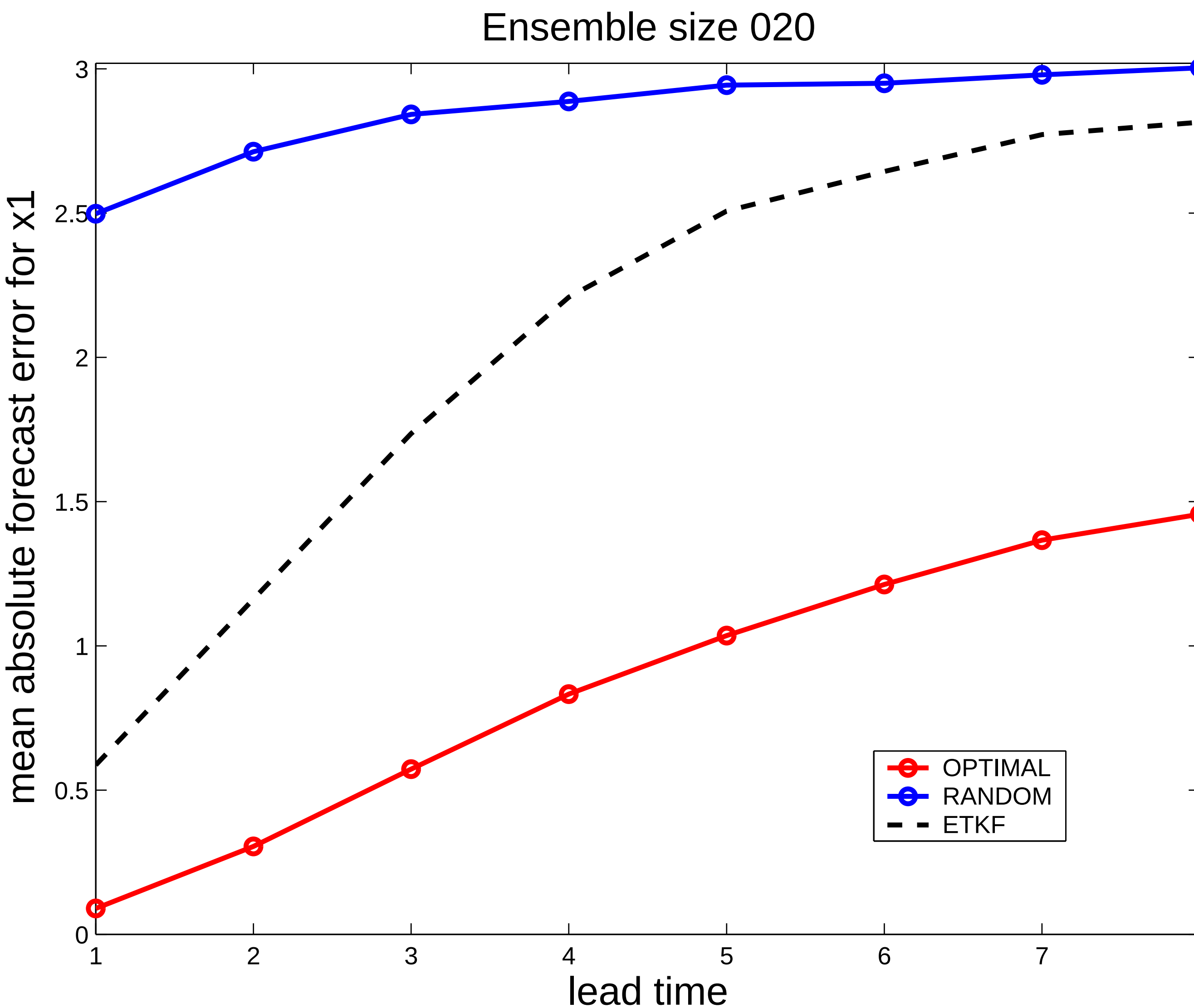
Find optimal location of one accurate observation at each t_a

Verification region - x1

N = 20, EAKF assimilation with inflation and GC localization

T_v - T_a = T_a - T_i = L(dT)

Experiments in an atmospheric ‘toy model’



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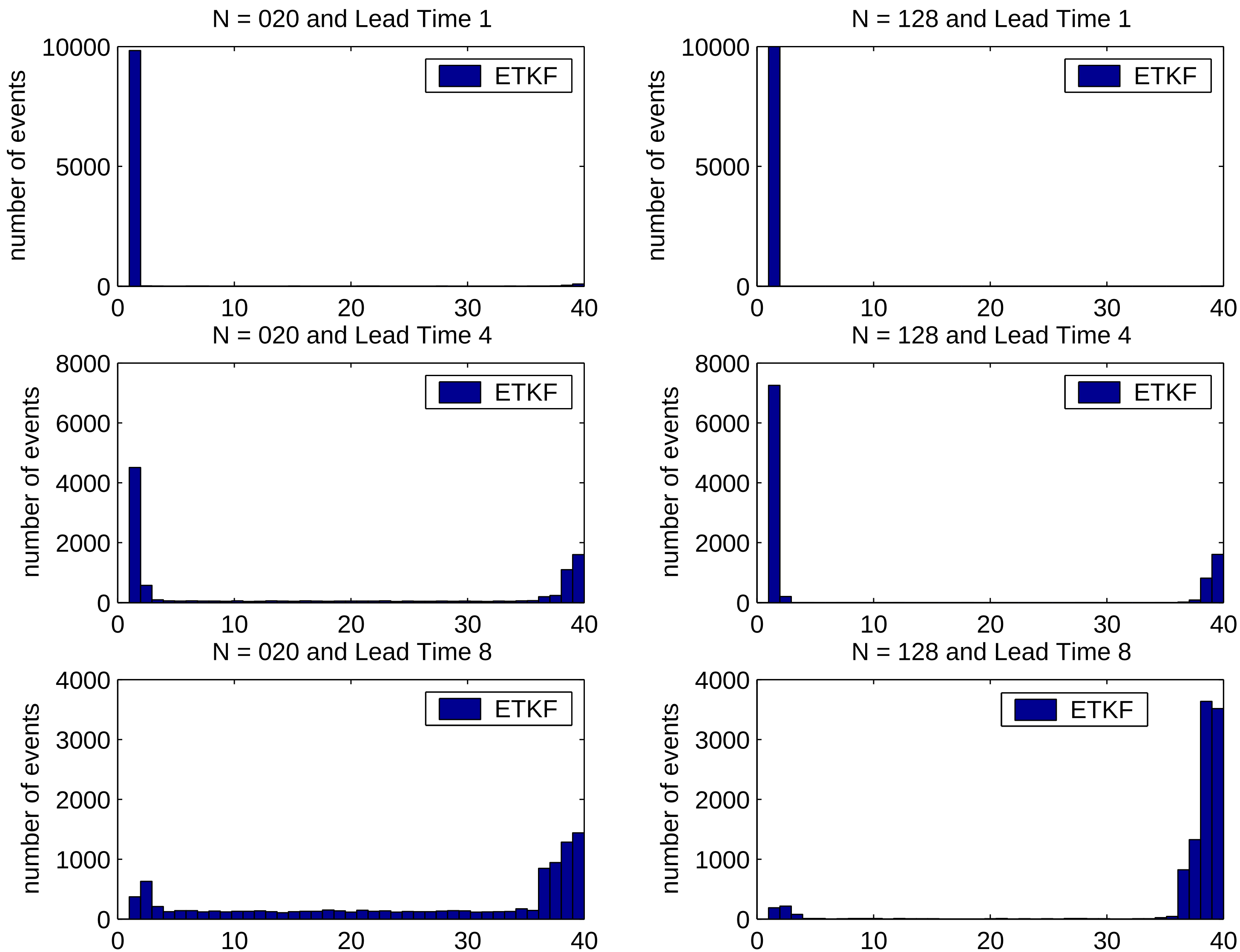

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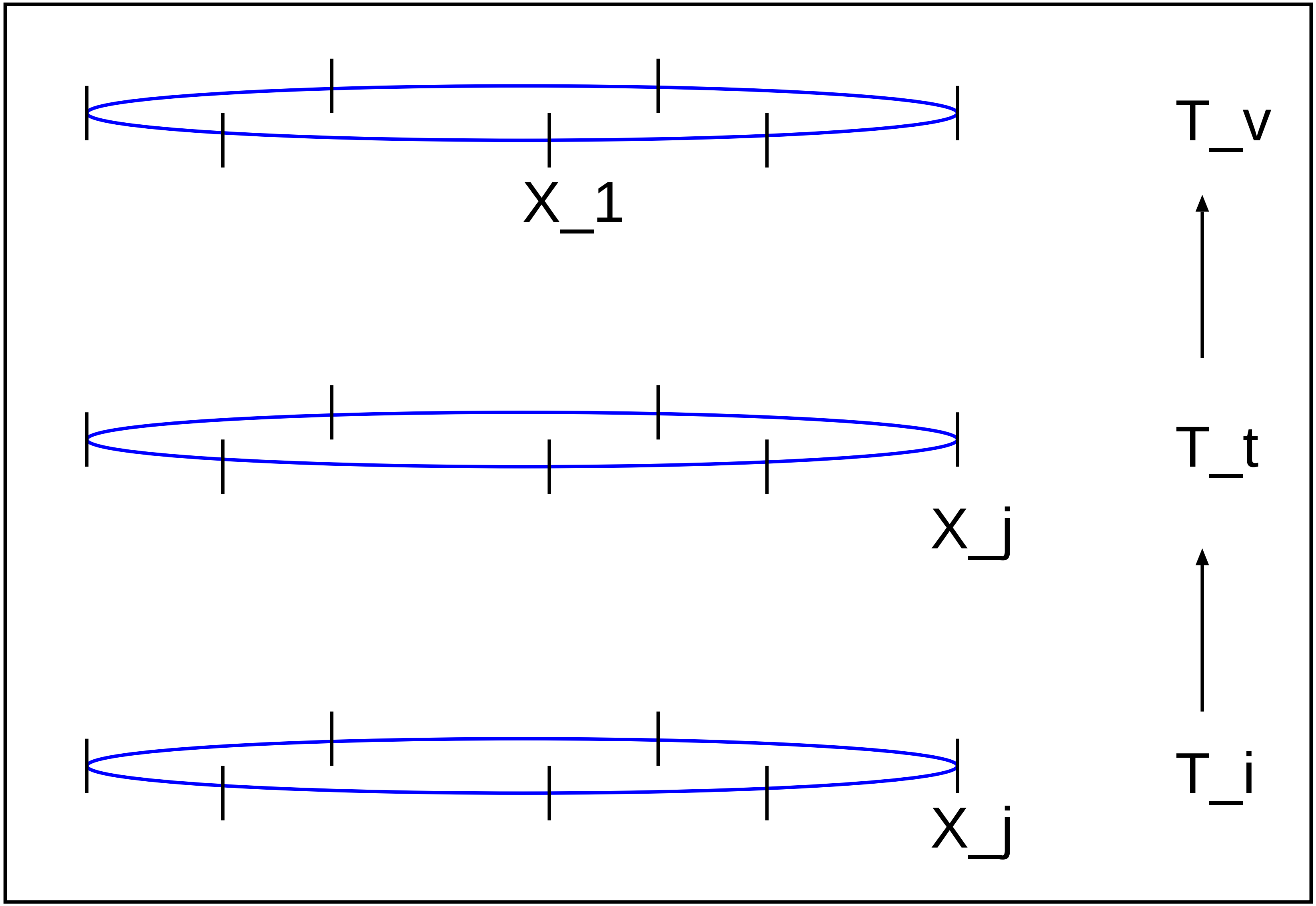

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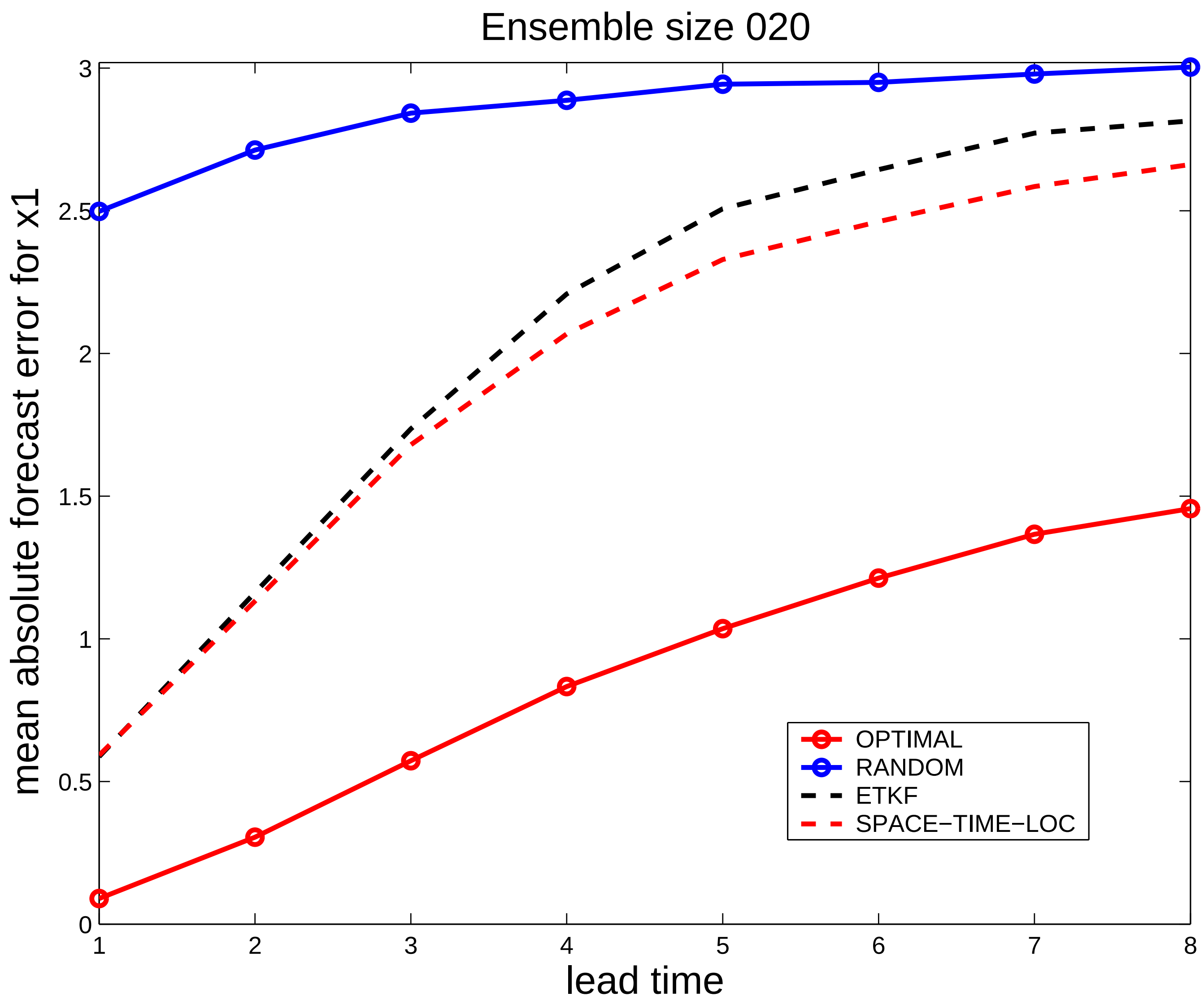

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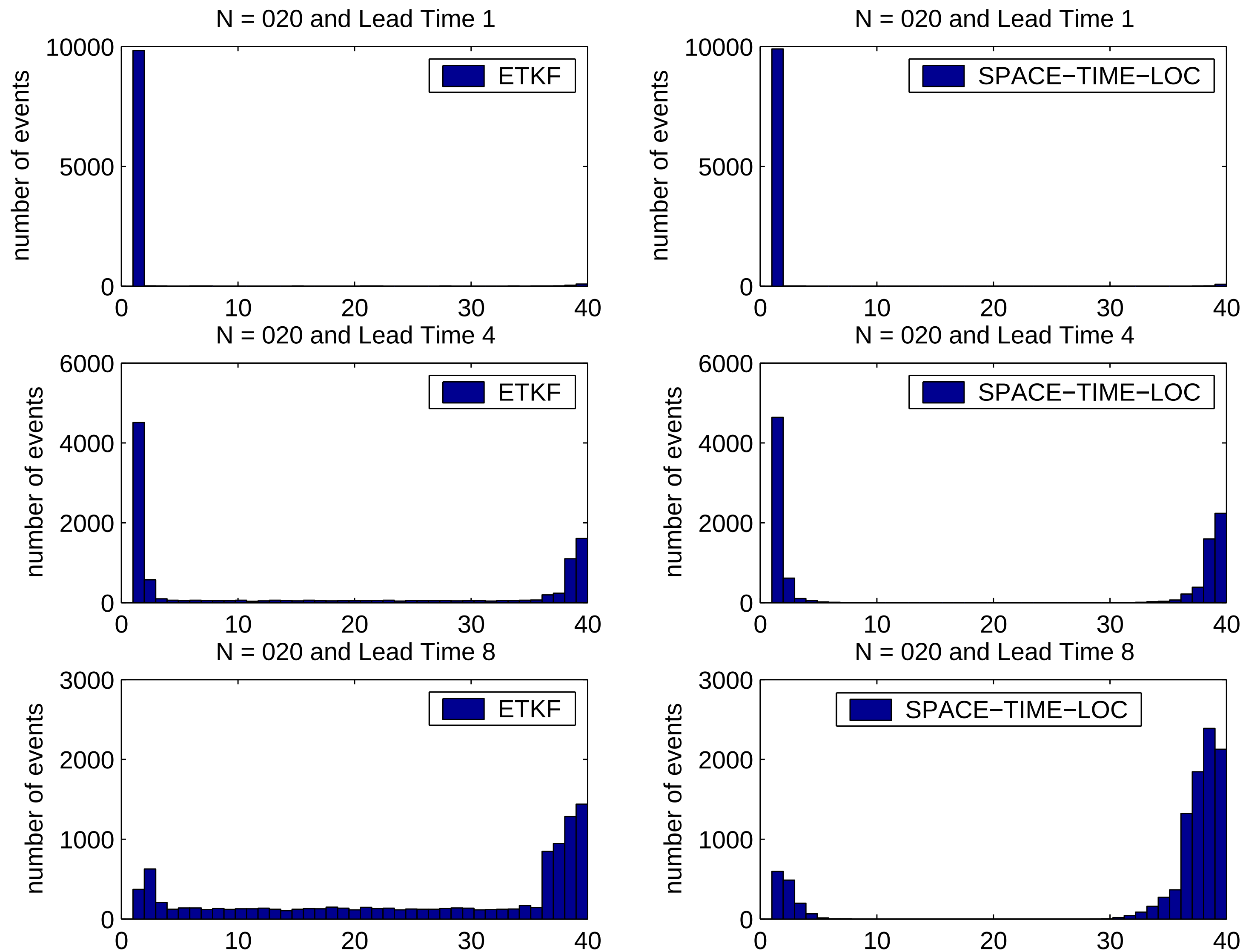

Summary and conclusions

- **Outlined general theory in perfect model setting - particle filter implementation**

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Summary and conclusions

- Outlined general theory in perfect model setting - particle filter implementation
- Computationally efficient approximation - ETKF scheme used operationally at NCEP
- Key result - handling sampling errors crucial to improving operational ETKF scheme

Key future areas in adaptive observations

- **Using optimization - non-trivial - flight paths, air traffic control, physical restrictions ...**

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Key future areas in adaptive observations

- **Using optimization - non-trivial - flight paths, air traffic control, physical restrictions ...**
- **Data denial (THORPEX)**
- **How do we quantify uncertainty under influence of $H_{t_a}^{adaptive}$ in the imperfect model case?**