Judgment, Communication and Decisions Under Uncertainty: A Psychological Perspective

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IMAGE TOY: Uncertainty in Climate Change Research: An Integrated Approach
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Overview

• Why is the psychological perspective relevant to the CC community?
• Some background and terminology
• Judgment of uncertainty (probability)
• Combining probabilities
• Judge – DM interaction and communication
• Individual decision making under uncertainty and with imprecise information
• Group decision making (Social dilemmas) under uncertainty
Why is the psychological perspective relevant?

• To turn science into action, the models’ results (forecasts, estimates and the corresponding uncertainties) need to be communicated to the public, stakeholders and DMs who do not necessarily understand the science, the model and the process.

• Sometimes the models depend on subjective probabilities (parameters, outcomes, etc.) that must be elicited from experts or stakeholders.

• Introspection and insight—learn about one’s own tendencies, biases, etc.
Background and terminology
Behavioral Work in J/DM

- Empirical (mostly experimental) research on the way DMs (individuals, small interacting groups) make decisions
- Normative (axiomatic) – models of the decision problem
- Descriptive (behavioral) – models of the decision maker
- Phenomenon (data) driven
Probability in Psychological Studies

• The three interpretations of probability
  – Classical (Laplace)
  – Relative Frequency (Von Mises, Reichenbach): *The probability of E is...*  
  – Subjective/Personal (Ramsey, De Finetti, Savage): *My probability of E is...*

• The three approaches are not always differentiated and are often used interchangeably

• This has been an area of (sometimes heated) theoretical disagreements
Variants of Uncertainty (K&T, 1982)

• External to the judge (Aleatory)
  – Distributional (“Outside” view)
  – Singular (“Inside” view)

• Internal to the judge (Epistemic)
  – Reasoned (Arguments)
  – Direct (Experience)

• These are useful as special cases but the classification is subjective and, often, ambiguous
Sources of Imprecision in Communication of Uncertainty (Budescu & Wallsten 1995)

- **Nature of the event**: An event is *unambiguous* (*ambiguous*) if its definition allows one to determine unequivocally if any given outcome is (is not) an exemplar of the target event.

- **Type of uncertainty underlying the event**: Uncertainty is *precise* if it can be expressed by means of a proper probability function, and it is *vague* if such a function cannot be specified.

- **Language of communication**: Numerical probabilities are the language of the *precise communication*, whereas intervals (*e.g.* 0.2 – 0.4), qualified values (*e.g.* around 0.6), and linguistic probabilities (*e.g.* highly likely) are examples of *imprecise ones*. 
Judgment of probability
Types of Probabilities Judged
(Lichtenstein, Fischhoff, Phillips, 1982)

• Discrete events
  – Single target event & full range
  – C (C ≥ 2) alternatives & 1/C range

• Continuous events (full distributions)
  – Fixed Value
  – Fixed Probability

• Direct and indirect methods

• Most of the work on discrete events (full or half range) and direct estimation
Started in July 2011 and will continue for several years.
Collects forecasts from voluntary judges
Items from business, economy, military, policy, politics, social, sports, health, science and technology, etc.
FP and FV

![Graph of FP and FV](image-url)
Indirect Method (Abbas, Budescu et al. 2007)

You win $20 if

we spin the wheel and it lands on orange.

<25 degrees C

the high temperature (in Celsius) in Palo Alto on Tuesday December 12th 2006 is less than 25.

Choose  Indifferent  Choose
Examples of Distributions (Abbas, Budescu et al. 2007)
Assessing Joint Probabilities (Abbas, Budescu, Gu, 2010)
Assessing Joint Probabilities (Abbas, Budescu, Gu, 2010)

You win $20 if

<6'1"  
and
<148 lbs

the height of a randomly selected UIUC male undergraduate student is less than 6'1" and his weight is less than 148 lbs

choose

indifferent

<6'2"
and
<112 lbs

the height of a randomly selected UIUC male undergraduate student is less than 6'2" and his weight is less than 112 lbs

choose
Assessing Quality of Judgements
(Wallsten & Budescu, 1983)

• Reliability / stability
• Coherence / internal consistency (static and dynamic)
• External validity
• Converging validity (procedural invariance)
• Calibration
Calibration Curve
Measuring Calibration

- Murphy (1972) decomposition of $S(P)$
  - For designated form: $S(P) = V - R' + C'$
    - Variance of outcome proportion
    - Resolution
    - Calibration
    - All vary with designation (as does bias)
  - Brier Score: $S(P) = \frac{1}{n} \sum_{i=1}^{n} (p_i - x_i)^2$

\[
V = \bar{x}(1 - \bar{x})
\]
\[
R' = \frac{1}{N} \sum_{j=1}^{J} N_j (x_j - \bar{x})^2
\]
\[
C' = \frac{1}{N} \sum_{j=1}^{J} N_j (p_j - x_j)^2
\]
Alternative View (Yates, 1990)
Patterns of Confidence

- Curve A: over-prediction
- Curve B: under-estimation
- Curve C: over-extremity
- Curve D: under-extremity
- Diagonal line E: perfect calibration
Empirical results – Over-extremity
Empirical Results: Hard Easy Effect
Possible Explanations

• Optimistic overconfidence
• Confirmation
• Ecological models
• Random errors with asymmetric distributions
Factors Affecting Judgments

• Cognitive (Heuristics)
  – Representativeness
  – Availability & Simulation
  – Anchoring & Adjustment

• Motivational
  – Desirability
  – Confirmation / Motivated Reasoning
  – Affect

• The structuring of the problem / task
  – Framing
  – Packing / Unpacking
  – Format / labeling / colors
  – Representation (probabilities vs frequencies)
Representativeness

- Judge likelihood of event, E, according to the degree that it resembles (it is perceived as representative of) a model or process, M
  - Base-rate frequencies/prior probabilities
  - Insensitivity to sample size
  - Misconception of chance
  - Conjunction fallacy
  - Insensitivity to predictability
  - Misconception of regression (toward the mean)
Availability & Simulation

• Judge likelihood of event, E, according to the ease with which it can be recalled, retrieved or imagined
  – Retrievability of instances
  – Effectiveness of a search set
  – Imaginability

Both Representativeness and Availability can be explained by “attribute substitution” (Kahneman & Frederick, 2002)

Link to two systems
Affect

• The quality of “goodness/badness” that is experienced as a feeling state and marks the stimuli as positive/negative
  – Affect is attached to images that influence judgments
  – The evaluability of an attribute/dimension drives the precision of its affective feelings and increases its weight
  – Quantities evaluated may convey little meaning in the absence of appropriate context to induce the appropriate affective perspective

• Examples:
  – Gambles become more attractive when a small loss is added!
  – Common ratio effect (Imagining the numerator)
Labels (Global Warming / Climate Change)
Wisdom of the Crowds: Averaging Probability Judgments
Within-Item Crowd Size Analysis

• We analyze responses to 90 distinct items with at least 64 respondents

• For each item we construct “pseudo-crowds” with
  \( n = 1, 2, 4, 8, 16, 32, 64 \) respondents

• We average judges while keeping total amount of information constant:
  \( n \) individuals, \( n/2 \) dyads, \( n/4 \) tetrads, \( n/8 \) octads, etc…

• We replicate the process 500 times

• Our score is a transformation of the Brier Score:

\[
Score = 100 - 50 \left[ \sum_{\text{Items}} \sum_{\text{outcomes}} (\text{group forecast} - \text{eventual outcome})^2 \right]
\]

  - 100 = perfect, 0 = worst, 75 = “chance” (answer 50%)
Examples: Crowds Don’t necessarily Improve Performance

Hard Item: Crowd does not improve much
Easy Item: Crowd does not improve much
Examples: Performance Peaks at n=8

Slow Improvement

More Pronounced Improvement
More on the Effects of the Crowd

- A more conservative approach focuses on other values of the distribution of forecasts (not the mean) in the distribution’s lower tail
  - For example, one could model the values associated with the 25\textsuperscript{th} percentile (Q1) or the 5\textsuperscript{th} percentile of the distribution of forecasts
- Since the starting point of these statistics is much lower, the rate of growth as a function on $n$ is steeper and more impressive

- The effects of aggregation peak later and require larger groups (For Q1, $n \approx 41$)
Examples Revisited
Judge – DM interaction and communication
Preferences

• In general, judges prefer to communicate uncertainty in vague terms but DMs prefer to receive precise information (Budescu & Wallsten, 1995; Brun & Teigen)

• This is also the modal individual pattern
The Congruence Hypothesis

• DMs are best served if uncertainty information is communicated in a way that matches the nature of the events and the underlying uncertainty.

• It makes no sense to use precise language to communicate vague uncertainty about ambiguous events (The chances of an *abrupt* drop in the market in the *near future* is 0.2348)

• DMs are best served when the nature and language of communication match the type of uncertainty and context (Erev et al. 1991)

• DMs prefer communication modes that match the type of uncertainty (Olson & Budescu, 1997)
When (and Why) Do DMs Prefer Vague Information? (Du & Budescu, 2010)

- Earnings per share (EPS) forecasts issued by management of publically traded companies (33,625 quarterly forecasts by 4,744 companies issued between 1996 and 2006)

  Range only – 36%; Point only – 22%
  Mixed – 42% (Range / Points = 3 / 1)

_Hypothesis_ – Management anticipates the investors’ expectations and seeks to communicate to them information that is congruent with the level and nature of uncertainty underlying the forecasts
Experimental results

- DMs (MBA students) indicate preference to purchase companies that issue range forecasts
- This preference is more pronounced when the information underlying the forecasts is vague
- Non-monotonic pattern of attitude to imprecision – preference for moderate levels of vagueness (Yaniv & Foster)
- DMs (MBA students) judged range forecasts to be more informative, accurate and credible
- DMs *correctly* expect range forecasts to be more accurate (not because they are wider!)
Judged Informativeness In Various Domains

Estimated Marginal Means of informative
IPCC communication
<table>
<thead>
<tr>
<th>Phrase</th>
<th>Likelihood of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtually certain</td>
<td>&gt; 99%</td>
</tr>
<tr>
<td>Very likely</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>Likely</td>
<td>&gt; 66%</td>
</tr>
<tr>
<td>About as likely as not</td>
<td>33% to 66%</td>
</tr>
<tr>
<td>Unlikely</td>
<td>&lt; 33%</td>
</tr>
<tr>
<td>Very unlikely</td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>Exceptionally Unlikely</td>
<td>&lt; 1%</td>
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</tbody>
</table>
Linguistic Probabilities

• Verbal lexicons of probabilistic terms vary widely across individuals
• Interpretations of probabilistic words are personal, subjective, susceptible to self-serving interpretations, and vary as a function of context
• It is difficult to “standardize” or “legislate” the meaning of language
• All these facts set the stage for a potential ”Illusion of Communication”
The Goals of the Studies

We investigate

• To what degree is the meaning of the IPCC probabilistic statements understood, as intended, by the public?
• Are there systematically biased misinterpretations?
• Are there simple ways of improving the efficiency of these communications?
• Do individual differences (gender, age, education, ideological, attitudes, etc.) affect systematically people’s interpretations of these communications?
Budescu, Por & Broomell (2011)

• Extension of Budescu, Broomell & Por (2009)
• National (US) Sample:
  – Random sample representative of US population (n = 556, 41.% men; Age: mean = 48 SD=17).
• Task:
  – Read statements extracted from the IPCC summary reports containing 4 terms: very likely, likely, unlikely, and very unlikely (2 statements for each term).
  – Provide best estimate for each term.
• Attitudes toward Global Climate Change (Belief in GCC, Personal Experience with GCC, Causes of GCC, Consequences of GCC)
• Numeracy scores

• 3 Groups: Control, Translation, Verbal-Numerical (VN)
Reconstructions of climate data for the past 1,000 years also indicate that this warming was unusual and is *unlikely* to be entirely natural in origin.

On a scale from 0 to 100%, please indicate what is your best estimate of the probability conveyed by this statement.

Type in the number for the answer

30
It is very unlikely that climate changes of at least the seven centuries prior to 1950 were due to variability generated within the climate system alone.

On a scale from 0 to 100%, please indicate what is your best estimate of the probability conveyed by this statement.

Type in the number for the answer:

<table>
<thead>
<tr>
<th>Phrase</th>
<th>Likelihood of Occurrence/Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtually certain</td>
<td>&gt;99%</td>
</tr>
<tr>
<td>Very likely</td>
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<td>&lt;10%</td>
</tr>
<tr>
<td>Exceptionally Unlikely</td>
<td>&lt;1%</td>
</tr>
</tbody>
</table>
Temperatures of the most extreme hot nights, cold nights and cold days are **likely (greater than 66%)** to have increased due to anthropogenic forcing. [Note: Anthropogenic forcing refers to the influences on the environment by human, rather than natural, factors.].

On a scale from 0 to 100%, please indicate what is your best estimate of the probability conveyed by this statement.

Type in the number for the answer:

75
Consistency Rates

Cumulative % of Respondents

Consistency Rates

Control (n=191)
Mean Consistency = 20.8%

Translation (n=170)
Mean Consistency = 18.8%

Verbal-Numerical (n=185)
Mean Consistency = 30.1%
Adjusted Mean Values (ANCOVA)

Adjusted Mean Probability Estimates

- Very Unlikely (<10%)
- Unlikely (<33%)
- Likely (>66%)
- Very Likely (>90%)

Control (n=162)
Translation (n=167)
Verbal-Numerical (n=171)
## Correlation with Covariates

<table>
<thead>
<tr>
<th>Probability Estimates</th>
<th>Consistency Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Likely</td>
<td>Likely</td>
</tr>
<tr>
<td>BGCC</td>
<td>0.56</td>
</tr>
<tr>
<td>PE</td>
<td><strong>0.49</strong></td>
</tr>
<tr>
<td>PCa</td>
<td><strong>0.48</strong></td>
</tr>
<tr>
<td>PCo</td>
<td><strong>0.47</strong></td>
</tr>
<tr>
<td>Numeracy Score</td>
<td><strong>0.17</strong></td>
</tr>
</tbody>
</table>

Note:
- **BGCC** - Belief in Global Climate Change
- **PE** - Personal Experience
- **PCA** - Perception it is Caused by Human Activities
- **PCO** - Perception of Consequences

Correlations in italicized bold are significant (p < 0.05).
Numeracy Items

1. Imagine that we roll a fair, six-sided die 1,000 times. Out of 1,000 rolls, how many times do you think the die would come up as an even number? Answer: _______

2. In the BIG BUCKS LOTTERY, the chances of winning a $10.00 prize are 1%. What is your best guess about how many people would win a $10.00 prize if 1,000 people each buy a single ticket from BIG BUCKS? Answer: _______ people

3. A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost? Answer: ______ cents

4. If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets? _____ minutes

5. In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? Answer: ______ days
Relation to Numeracy

Consistency Rate vs. Numeracy Scores

- Control (n=193)
- Translation (n=175)
- VN (n=188)
Relation to Political Affiliation

Probability Estimates (Mean)

Very Unlikely (<10%)  Unlikely (<33%)  Likely (>66%)  Very Likely (>90%)

- Republican (n=156)
- Lean Republican/Independent/Lean Democrat (n=110)
- Democrat (n=181)

(With a graph showing the probability estimates for each category based on political affiliation.)
Summary And Conclusions

– The public consistently misinterprets the probabilistic statements in the IPCC report
– Less extreme than intended;
– High inter-individual variability;
– This variance is associated (to a large degree) with individual differences (mostly ideological)
– The dual (verbal – numerical) scale is superior to the current mode of communication as it
  • increases the level of differentiation between the various terms;
  • increases the level of consistency with the IPCC guidelines.
– Most importantly, these positive effects are independent of the respondents’ ideological and environmental views.
Recommendations

• Avoid probabilistic pronouncements about ambiguous events and scenarios
• Continue use of several probability terms, but
• To improve quality of communication these words should be accompanied by appropriate numerical ranges
• Use different ranges to match the uncertainty of specific target events
Modeling Words with Membership Fs.

![Graph showing membership functions for likely, improbable, and certain categories.]
Decision making with imprecise numerical information (ranges)
The type of decision

Luce & Raiffa (1958) distinguish between decisions making under:

- **Certainty**
- **Risk**
- **Uncertainty (Ignorance)**

In reality these are just (easy to axiomatize and analyze) points along a continuum
Decisions under risk

The worth of a binary prospect $V(x,p;y,q)$ is modeled as a bilinear combination of functions of the values, and the beliefs:

$$V(X,p;Y,q) = f(p)v(X) + f(q)v(Y)$$

<table>
<thead>
<tr>
<th>Beliefs</th>
<th>Values</th>
<th>Bernoulli Utility</th>
<th>VN-M Utility</th>
<th>Value Function</th>
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<tbody>
<tr>
<td>Probabilities</td>
<td>EV</td>
<td></td>
<td>EU</td>
<td></td>
</tr>
<tr>
<td>Subjective Probabilities</td>
<td>SEV</td>
<td></td>
<td>SEU</td>
<td></td>
</tr>
<tr>
<td>Decision weights</td>
<td></td>
<td></td>
<td></td>
<td>PT</td>
</tr>
</tbody>
</table>
Modeling *Precise* Prospects

- **Prospect Theory** (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) distinguishes between the editing and evaluation of prospects.
- The worth of a binary prospect $V(x,p;y,q)$ is modeled as a bilinear combination of its value function, $v(X)$, and the decision weights, $f(p)$:
  
  $$V(X,p;Y,q) = f(p)v(X) + f(q)v(Y)$$

- The two functions have several distinctive properties.
Modeling *Precise* Prospects

Figure 2.3. A hypothetical value function.

Figure 5.1. Weighting functions for gains ($w^+$) and losses ($w^-$).
Typical Instantiations of the Functions

\[ v(x) = x^\alpha \text{ if } x \geq 0 \]

\[ v(x) = -\lambda (-x)^\beta \text{ if } x \leq 0 \]

\[ f(p) = \frac{p^\gamma}{\left[ p^\gamma + (1-p)^\gamma \right]^{1/\gamma}} \]
The type of decision

Of particular interest is the “gray area” between risk and ignorance that includes many realistic cases, such as:

- Second order probability distributions
- Multiple priors (e.g., Ranking of states by their respective probabilities)
- Unreliable (imprecise) probabilities
- Upper Bound > Prob(Si) > Lower Bound
- Prob(Si) belongs to a fuzzy category (e.g. “likely”, “around, or in the, 80s”)
Decisions with Vague Probabilities

• Urn A: 50 Red and 50 Blue balls
• Urn B: ?? Red and (100-??) Blue balls
• Q1: If a Red ball is drawn you win $X. Do you prefer to draw from Urn A or B?
• Q2: If a Blue ball is drawn you win $X. Do you prefer to draw from Urn A or B?

• Ellsberg’s paradox: In choices among a precise and an unspecified distribution people tend to prefer the precise option
Ellsberg’s Paradox

Explanations of the prototypical pattern

• Weighting by an “inferred/assumed” probability distribution (Ellsberg, 1961)

• Avoidance of vagueness (ambiguity) and preference for precision

• Competence (Heath & Tversky, 1991)
Ellsberg’s Paradox

Explanations of the prototypical pattern

• Weighting by an “inferred/assumed” probability distribution (Ellsberg, 1961)

• Avoidance of vagueness (ambiguity) and preference for precision

• Competence (Heath & Tversky, 1991)
Ellsberg’s Paradox (Kramer & Budescu 2004)

Precise-Vague cases

Vague-Vague cases

midpoint =20
midpoint =50
midpoint =80

classic paradox (n=562)
consistency (n=417)
weak indifference (n=287)
indifference (n=190)
reverse paradox (n=149)

classic paradox (n=988)
consistency (n=834)
weak indifference (n=608)
indifference (n=405)
reverse paradox (n=375)
A common metric for vagueness

The prospects ($p=0.05$ to win *between $45$ and $105$*), and ($p$ *between 0.03 and 0.07* to win $75$) are equally vague since they span the same range of EVs (and identical midpoints).

<table>
<thead>
<tr>
<th>Probs</th>
<th>Outcomes</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$45$</td>
<td>$75$</td>
<td>$105$</td>
<td>$45-105$</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>0.05</td>
<td></td>
<td>$3.75$</td>
<td></td>
<td>$2.25-5.25$</td>
<td></td>
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<tr>
<td>0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03-0.07</td>
<td></td>
<td>$2.25-5.25$</td>
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</tr>
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</table>
Certainty Equivalents of Prospects with Vague Parameters (Du & Budescu, 2005)

Expected value (EV) in gain or loss domain

Certainty Equivalent
Summary of Attitudes to Various Sources of Vagueness (Du & Budecu 2005)

<table>
<thead>
<tr>
<th>Task Used</th>
<th>Type of Comparison</th>
<th>Domain</th>
<th>Gain</th>
<th>Loss</th>
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</thead>
<tbody>
<tr>
<td>CE</td>
<td>PP vs. PV</td>
<td>V. Seeking</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>CE</td>
<td>PP vs. VP</td>
<td>No effect</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>PC</td>
<td>PP vs. PV</td>
<td>V. Avoidance</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>PC</td>
<td>PP vs. VP</td>
<td>V. Avoidance</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>CE</td>
<td>PV vs. VP</td>
<td>Outcome V. more salient</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>PC</td>
<td>PV vs. VP</td>
<td>No effect</td>
<td>No effect</td>
<td>No effect</td>
</tr>
</tbody>
</table>
Modeling Vague Prospects

• In the editing phase, the DM "resolves the vagueness" of the range of probabilities or outcomes, so the decision problem is converted to one with "equivalent" precise parameters.
• This editing operation is performed by relying on the focal end-points of the interval
• The relevant range is replaced by a weighted average of its end points

\[ v(Range X_l - X_u) = v(w_x X_l + (1 - w_x) X_u) \]

\[ f(Range p_l - p_u) = f(w_p p_l + (1 - w_p) p_u) \]
Modeling *Vague* Prospects

- The coefficients $w_x$ and $w_p$ represent measures of attitude towards vagueness. For both:
  
  $w = 0.5$ indifference to vagueness, 
  
  $w > 0.5$ vagueness aversion, and 
  
  $w < 0.5$ vagueness preference.  

- The quantity $|w - 0.5|$ measures *intensity of attitude towards vagueness* 

- Comparison of $|w_p - 0.5|$ and $|w_x - 0.5|$ contrasts the *relative sensitivity to the sources of vagueness*
Decisions with Vague Probabilities with Decision Aids (Budescu et al. 2009)

• Instead of focusing on the contrast between vagueness and precision we examine how choice between various actions is affected by the level of imprecision
• The decision problem is designed to mimic some climate change problems
• We also study the effect of two classes of decision aids
The Decision Problem

- DM is endowed with $e$, and faces an urn that contains 100 Red and White balls

\[ P_l \leq P(\text{Red}) \leq P_h. \]
The Decision Problem

Choose one of 3 actions:

- **Risky**: Two balls are sampled. If both are Red, DM loses the endowment

- **Riskless**: Surrender a portion of the endowment, $f$

- **Reduced Risk**: Pay fee ($y$); draw 2 balls; if both are Red, draw a 3rd. If it is also Red pay another $z$. 
The Decision Problem

<table>
<thead>
<tr>
<th>Risky</th>
<th>1 – $p^2$</th>
<th>$e$</th>
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</thead>
<tbody>
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<td>$p^2$</td>
<td>$0$</td>
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</table>

<table>
<thead>
<tr>
<th>Riskless</th>
<th>$(e – f)$</th>
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</thead>
</table>

<table>
<thead>
<tr>
<th>Reduced Risk</th>
<th>1 – $p^2$</th>
<th>$(e – y)$</th>
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<tbody>
<tr>
<td></td>
<td>$p^2$</td>
<td>$1 – p$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$(e – y – z)$</td>
</tr>
</tbody>
</table>
Relating the Decision Problem to Abrupt Climate Change

- **Risky** Option represents continued emissions (BAU).
- **Riskless** Option represents capping of greenhouse gas emissions at very low levels immediately.
- **Reduced Risk** Option represents less costly options that reduce some (but not all) the risks.
Example of Decision Problem:

\[0.2 \leq p \leq 0.8, \ e=\$10, \ f=\$4, \ y=\$2, \ z=\$5\]
Experimental Details: Display

>> Ndisplays;
Welcome to the program!
What is your ID?  2
What is the lower probability of a red ball?  0
What is the upper probability of a red ball?  1
What is the endowment amount?  10
What is the amount that you will return if you choose option A?  4
What is the amount you will return if you choose option C?  3
What is the amount that you lose if the 3 balls are red (in option C)?  4
Is the information you entered correct?  y

Current plot held
You have seen the expected outcomes for the various options under all possible compositions of the urn.
Please rate the attractiveness of options A, B, C by using a 7-point scale.
1 = very unattractive and 7 = very attractive.
Option A:

![Graph showing the attractiveness of options A, B, C over a probability range.](image)
Experimental Details: Calculator

```
>> summary;
Welcome to the program!
What is your ID?  2
What is the lower probability of a red ball?  0
What is the upper probability of a red ball?  .6
What is the endowment amount?  10
What is the amount that you will return if you choose option B?  4
What is the amount you will return if you choose option C?  3
What is the amount that you lose if the 3 balls are red (in option C)?  4
Is the information you entered correct?  y

uncertaintyfig

<table>
<thead>
<tr>
<th>Interval</th>
<th># of balls</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4-0.5</td>
<td>Decrease</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Increase</td>
<td>3</td>
</tr>
</tbody>
</table>

Please close this window when finished
```
Attractiveness of the Actions as a Function on Vagueness

As vagueness increases, the Riskless (most conservative) action becomes more attractive at the expense of the Risky action.

<table>
<thead>
<tr>
<th>Vagueness</th>
<th>Risky</th>
<th>Riskless</th>
<th>Reduced Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>41</td>
<td>26</td>
<td>33</td>
</tr>
<tr>
<td>0.60</td>
<td>37</td>
<td>29</td>
<td>34</td>
</tr>
<tr>
<td>0.80</td>
<td>39</td>
<td>36</td>
<td>34</td>
</tr>
<tr>
<td>1.00</td>
<td>30</td>
<td>38</td>
<td>33</td>
</tr>
</tbody>
</table>
Attractiveness of Actions as a Function of the Decision Aid

The availability of decision aids (especially the EV calculator) make the *Risky* Option more attractive, at the expense of the *Riskless* one.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Risky</th>
<th>Riskless</th>
<th>Reduced Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>37%</td>
<td>34%</td>
<td>29%</td>
</tr>
<tr>
<td>Display</td>
<td>43%</td>
<td>27%</td>
<td>30%</td>
</tr>
<tr>
<td>Summary</td>
<td>50%</td>
<td>22%</td>
<td>28%</td>
</tr>
</tbody>
</table>
Modal Choice as a Function of Range and Midrange

High and vague probabilities cause subjects to prefer the riskless action, while low and less vague probabilities cause subjects to prefer the risky action.

<table>
<thead>
<tr>
<th>Midrange</th>
<th>Range</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>Risky</td>
<td>Risky</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>Risky</td>
<td>Risky</td>
<td>Risky</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>Riskless</td>
<td>Riskless</td>
<td>Riskless</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td>Riskless</td>
<td></td>
</tr>
</tbody>
</table>
## Decision Models Being Compared

<table>
<thead>
<tr>
<th>Class</th>
<th>Model</th>
<th>Model</th>
<th>Key Index</th>
<th>Choice Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Maxi-min</td>
<td>Lowest outcome for each action</td>
<td>Maximal</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Maxi-max</td>
<td>Highest outcome for each action</td>
<td>Maximal</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Mini-max Regret</td>
<td>Highest regret for each action</td>
<td>Minimal</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Mini-min Regret</td>
<td>Lowest regret for each action</td>
<td>Minimal</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>$P(U) = P_U/(1 + P_U - P_L)$</td>
<td>Proportional to the upper bound</td>
<td>Maximize EV</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$P(L) = P_L / (1 + P_L - P_U)$</td>
<td>Proportion to the lower bound</td>
<td>Maximize EV</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>$P(M) = (P_U + P_L)/2$</td>
<td>Mid-range</td>
<td>Maximize EV</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>Laplace</td>
<td>Expected value for each action</td>
<td>Highest EV</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Starr</td>
<td>Expected value for each action</td>
<td>Highest EV for most probabilities</td>
</tr>
</tbody>
</table>
Models That Describe Best the DMs’ Choices

As the vagueness increases DM’s choices are better predicted by a conservative (ignorance based) MaxiMin model
Social Dilemmas
With Uncertain Resources
The setup

- Social dilemma in group with N participants which can cooperate (C) or defect (D):
  - $V(D) > V(C)$ each person prefers D
  - $All(C) > All(D)$ the collective benefits from C

- Examples:
  - Common Pool Resource
  - Provision of Public Goods

- Uncertainty about the size of the pool or the size of the PG

- Experiments on CPR with random resources (Rapoport, Suleiman & Budescu)
## Mean Individual Requests
(N=5, E(Pool size) = 500)

<table>
<thead>
<tr>
<th>Range</th>
<th>Sample size</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>167</td>
<td>106</td>
<td>55</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>113</td>
<td>75</td>
</tr>
<tr>
<td>70</td>
<td>60</td>
<td>115</td>
<td>78</td>
</tr>
<tr>
<td>200</td>
<td>120</td>
<td>122</td>
<td>85</td>
</tr>
<tr>
<td>250</td>
<td>55</td>
<td>125</td>
<td>96</td>
</tr>
<tr>
<td>380</td>
<td>60</td>
<td>110</td>
<td>72</td>
</tr>
<tr>
<td>500</td>
<td>80</td>
<td>122</td>
<td>103</td>
</tr>
<tr>
<td>560</td>
<td>120</td>
<td>137</td>
<td>109</td>
</tr>
<tr>
<td>750</td>
<td>55</td>
<td>135</td>
<td>112</td>
</tr>
<tr>
<td>1000</td>
<td>90</td>
<td>162</td>
<td>130</td>
</tr>
</tbody>
</table>
# Provision of Requests

<table>
<thead>
<tr>
<th>Range</th>
<th>Group Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>96</td>
</tr>
<tr>
<td>100</td>
<td>29</td>
</tr>
</tbody>
</table>
Summary

• Behavioral research on judgment and decision making under conditions of vagueness / ambiguity can provide numerous important insights and sometimes subtle and surprising insights.

• Some of the accepted “common wisdom” about the impacts of imprecision hat permeate the literature may be over-generalizations.
Summary

How to communicate uncertainty?

• Scientific evidence should be presented at the highest level of precision warranted by the available data (but not more precisely)

• DMs are best served if uncertainty is communicated in a way that matches the nature of the events and the underlying uncertainty.

• Uncertainty communication should be associated with unambiguous events / statements

• Relatively small changes in presentation format may have considerable effects of the quality of the message
Caveats

The results of these experiments are

• Based primarily on samples of students
• Involve small monetary amounts
• Do not involve extreme probabilities
• Detached from organizational / ideological / political contexts