

The Data Assimilation Research Testbed: Powerful Adaptive Fast Assimilation for Geophysical Applications



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IMAGE Data Assimilation Research Section (DAReS)

Thanks to Nancy Collins, Tim Hoar, Hui Liu, Kevin Raeder

The Geophysical Data Assimilation Problem:

Given: 1. A physical system (atmosphere, ocean, climate system...)

2. Observations of the physical system

Often sparse and irregular in time and space.

Instruments have error of which we have a (poor) estimate.

Observations may be of quantities not found in model.

Many observations may have very low information content.

3. A model of the physical system

Usually approximates time evolution.

Truncated representation of 'continuous' physical system.

Often quasi-regular discretization in space and/or time.

Generally characterized by 'large' systematic errors.

Often ergodic with some sort of 'attractor'.

Assimilation increases information about all three pieces:

1. Get an improved estimate of state of physical system.

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Includes time evolution and ‘balances’.

High quality analyses (re-analyses).

2. Get better estimates of observing system error characteristics.

Estimate value of existing or planned observations.

Design observing systems that provide increased information.

3. Improve model of physical system.

Evaluate model systematic errors.

Forward and backward sensitivity analysis (adjoint and linear tangent replacement).

Select appropriate values for model parameters.

DART/CAM NWP Assimilation: January, 2003

Model: CAM 3.1 T85L26.

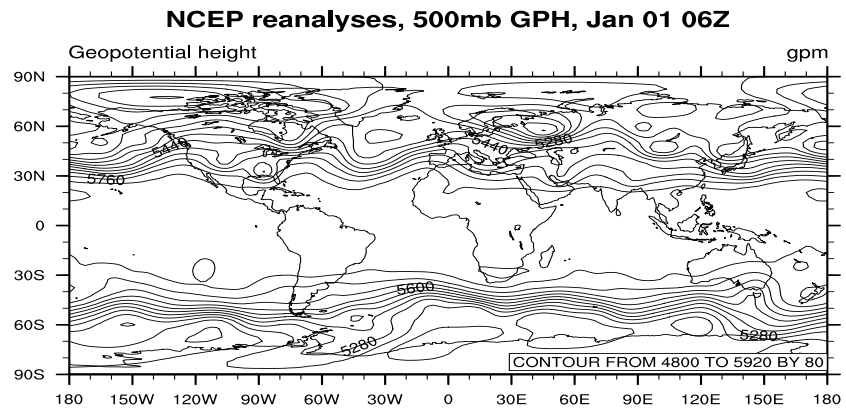
Initialized from a climatological distribution (huge spread).

Observations: Radiosondes, ACARS, Satellite Winds.

Subset of observations used in NCAR/NCEP reanalysis.

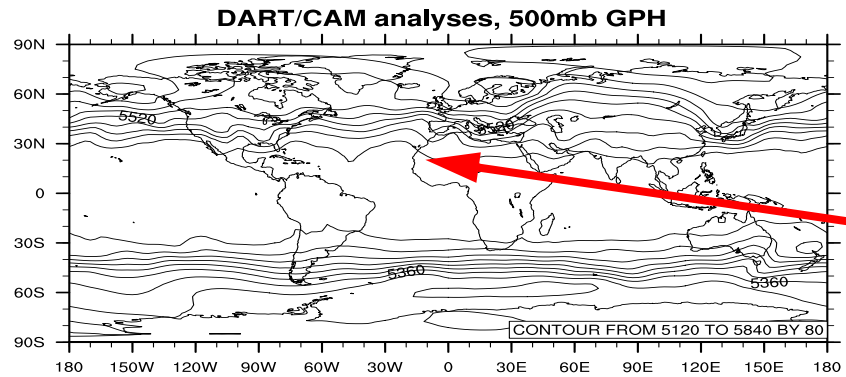
Compare to NCEP operational, T254L64, uses radiances.

NCEP



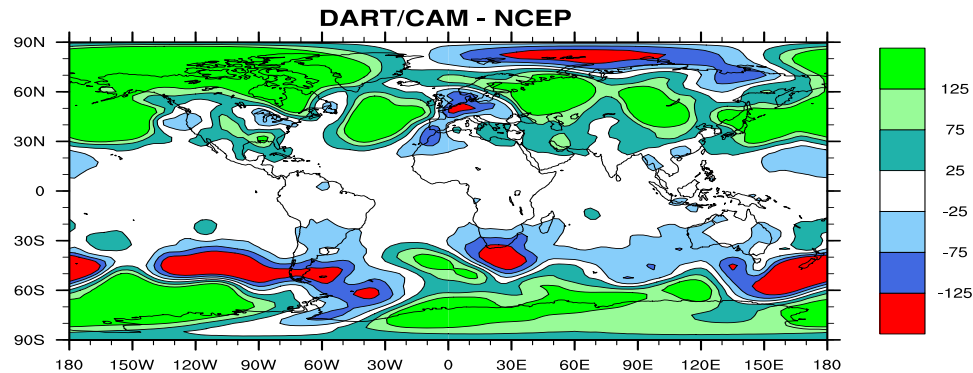
After 6 hours.

DART/CAM



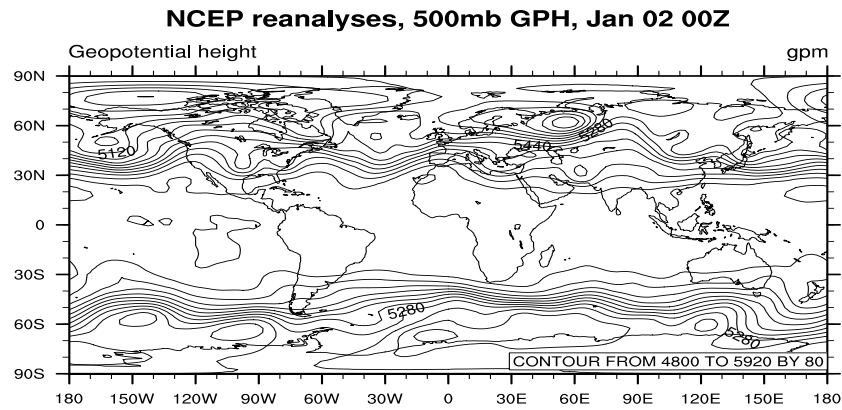
CAM starts with climatology! Nearly zonal.

Difference.

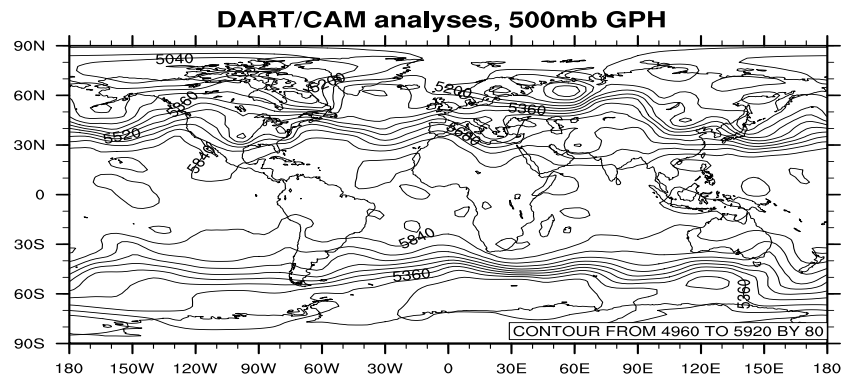


After 1 day.

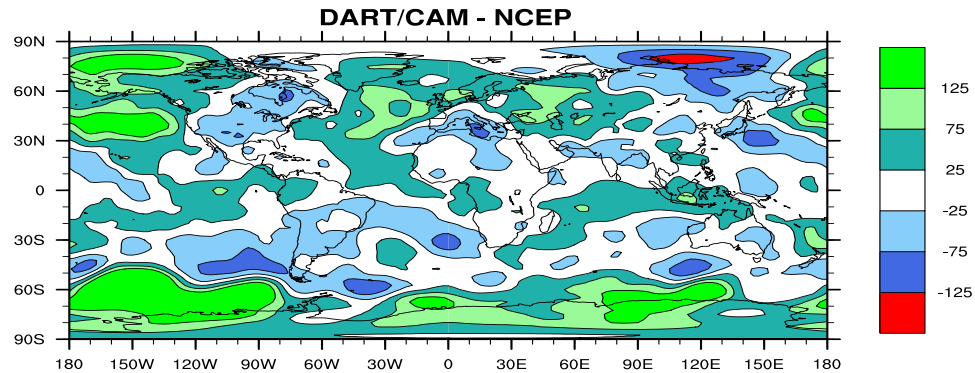
NCEP



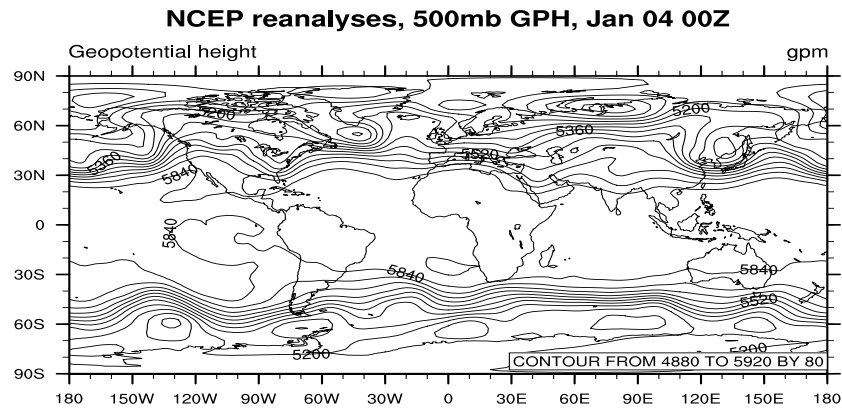
DART/CAM



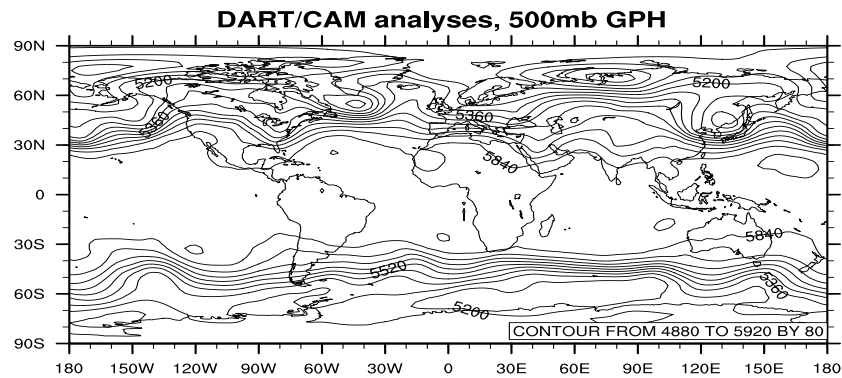
Difference.



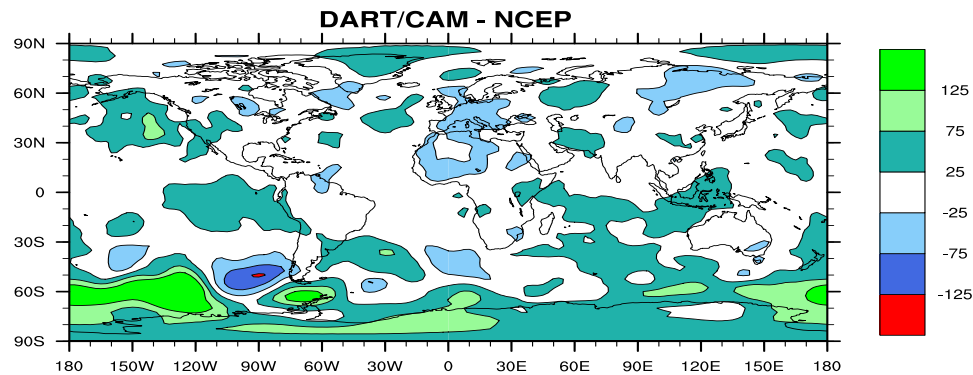
NCEP



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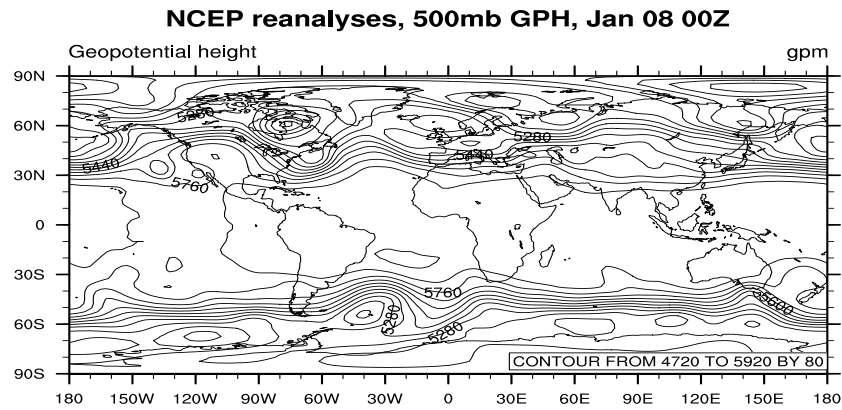
Difference.



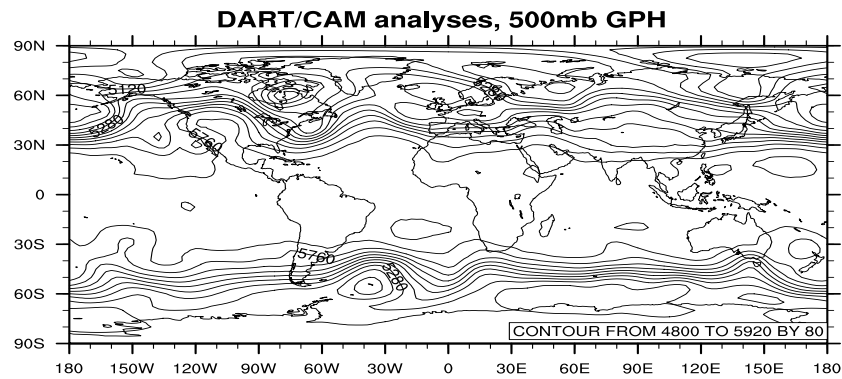
After 3 days.

CAM gains zonal structure.

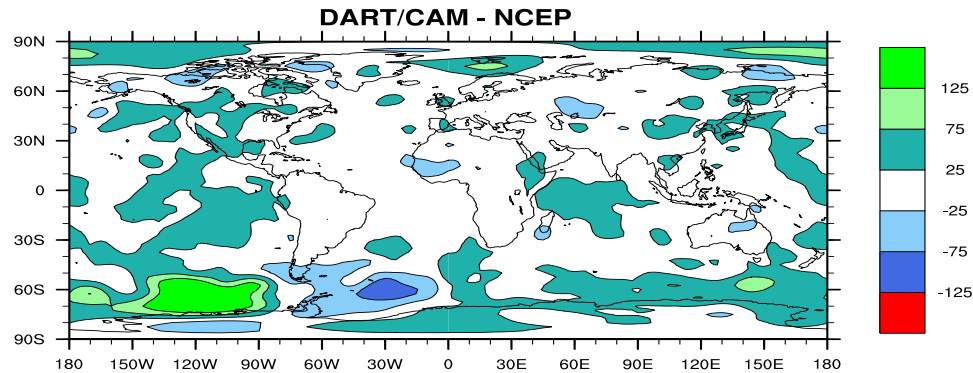
NCEP



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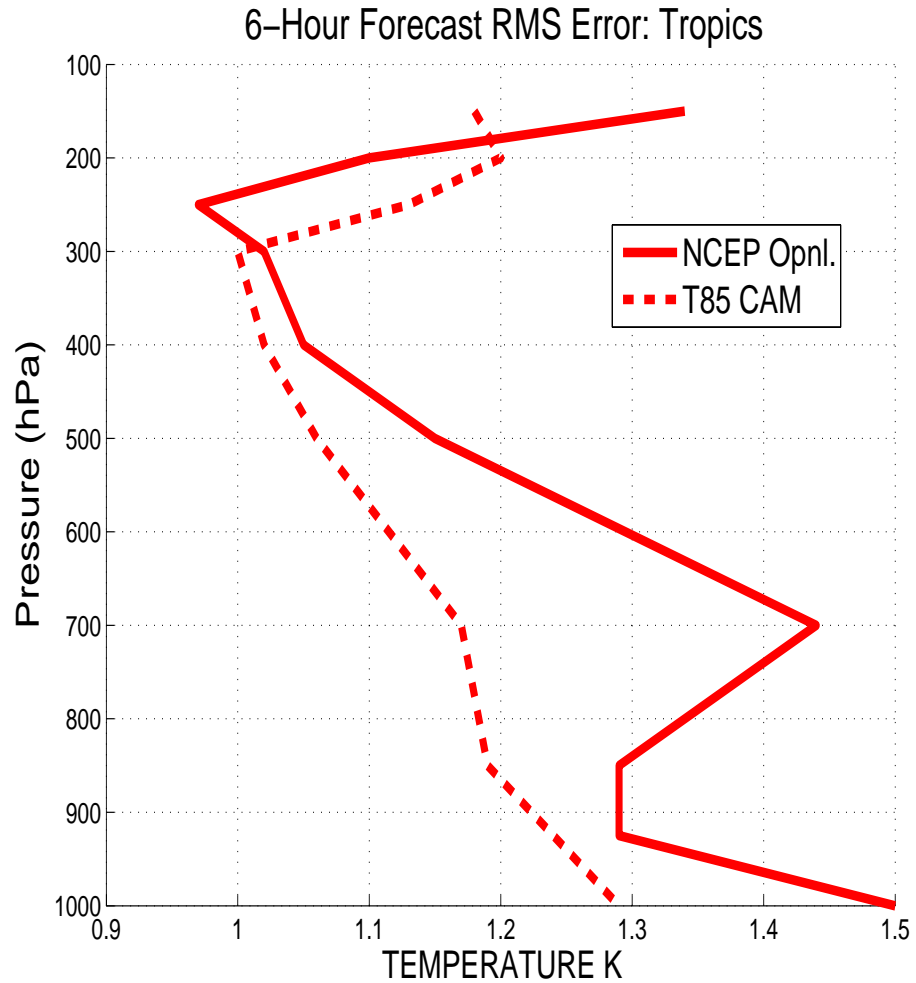
Difference.



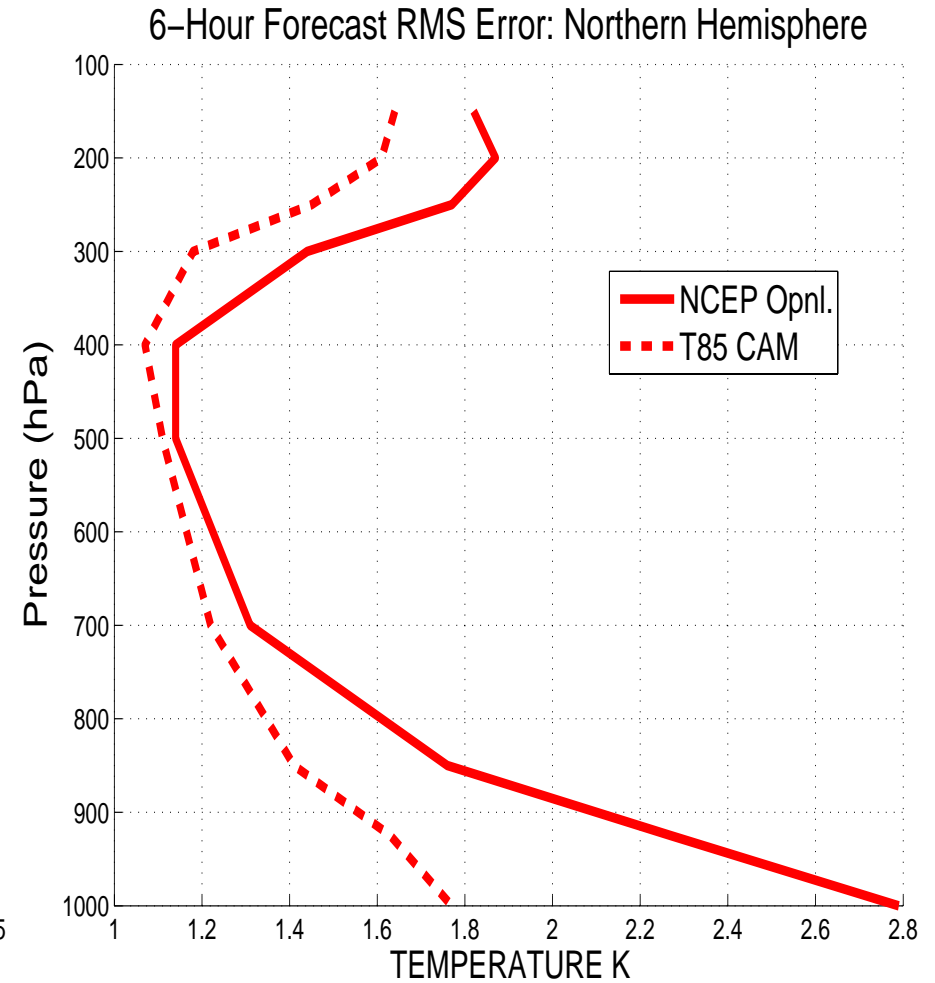
After 7 days.

NH converged.
SH poorly observed.

6-Hour Forecast Observation Space Temperature RMS



Tropics



Northern Hemisphere

DART/CAM competitive with operational NWP system.

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Forward and backward sensitivity analysis (adjoint and linear tangent replacement).

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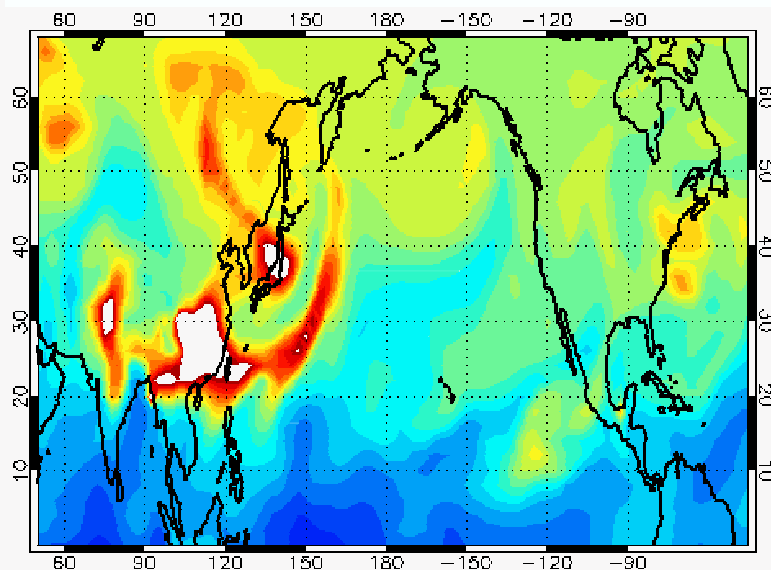
High-quality analysis of CO in Finite Volume CAM-CHEM model.

Assimilate standard observations plus MOPITT CO observations.

Work by Ave Arellano and Peter Hess supported by Kevin Raeder.

Impact of Assimilation in Modeled CO

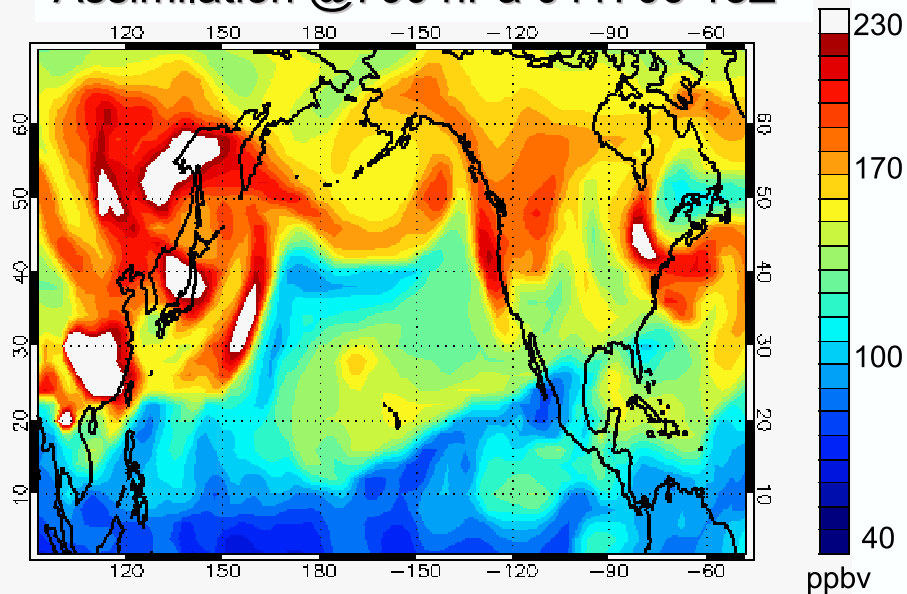
No Assimilation @700 hPa 041706 18Z



Assimilating MOPITT CO provides important constraints to regional CO distribution in the troposphere.

Suggests the utility of assimilation in providing better initial/boundary conditions to regional CO forecasts.

Assimilation @700 hPa 041706 18Z



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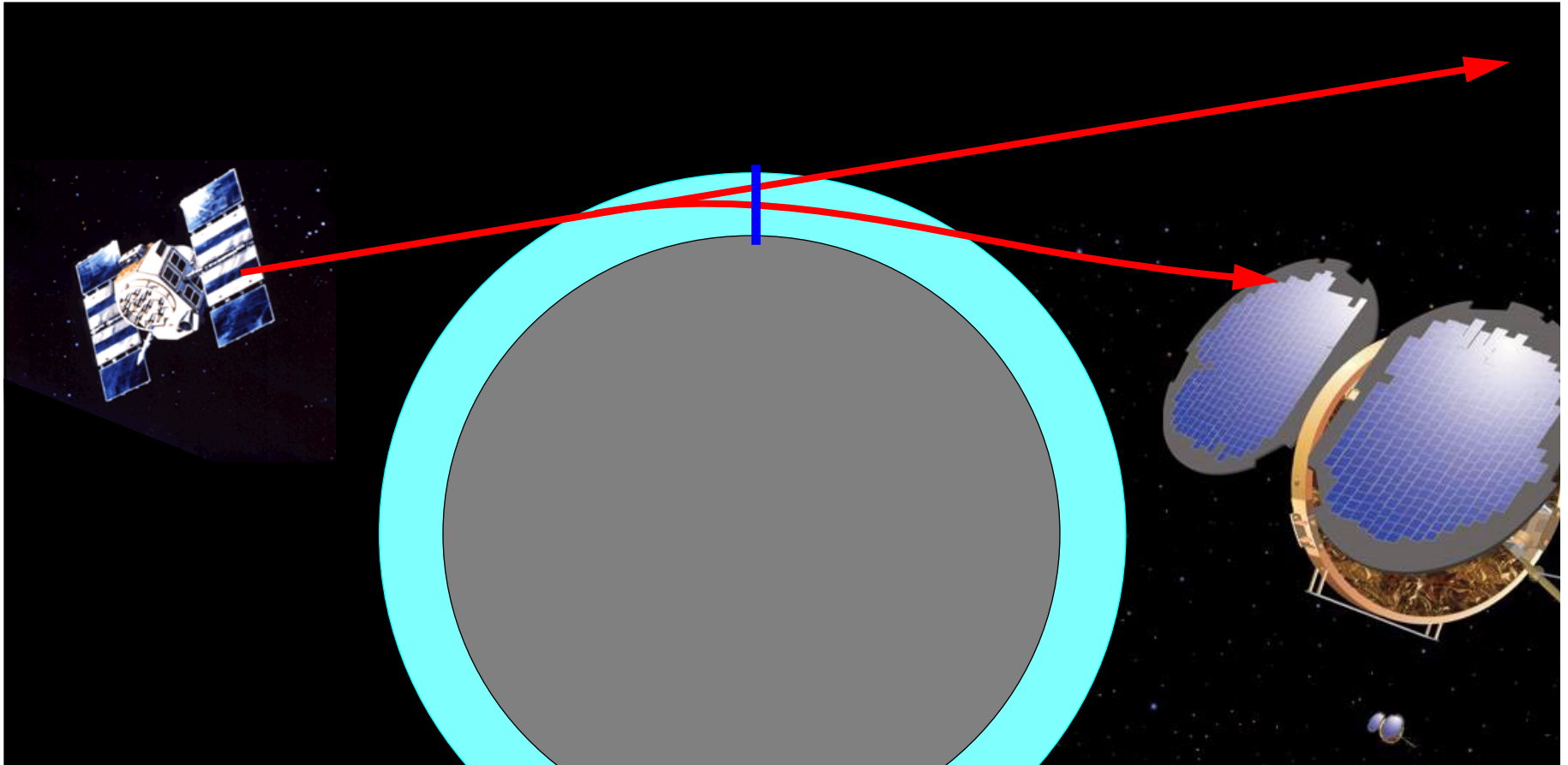
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Assimilating GPS Radio Occultation Observations in WRF

Assimilated as refractivity along beam path.

Complicated function of T, Q, P and ionospheric electric field.

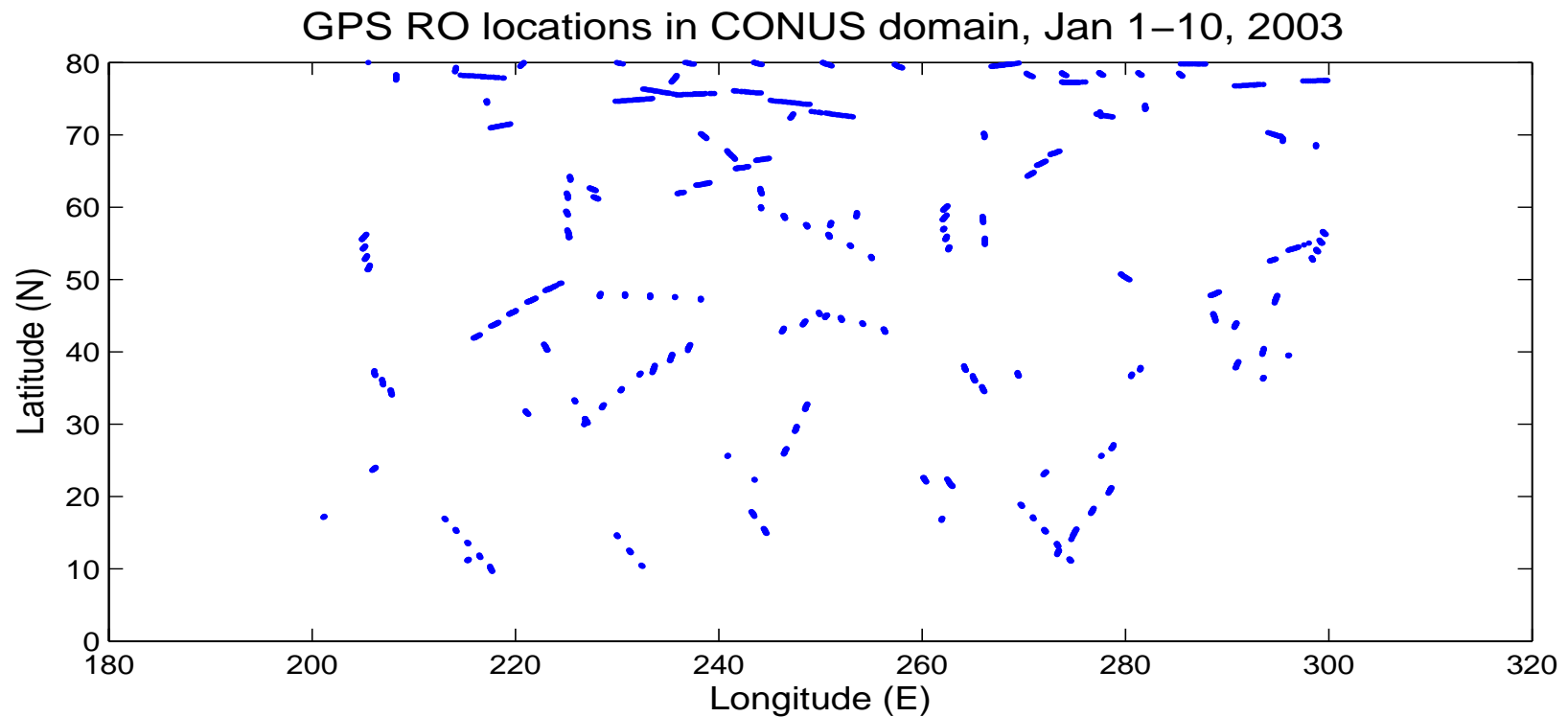


Get a sounding as GPS satellite sets relative to low earth satellite.

Assimilating GPS Radio Occultation Observations in WRF

Weather Research and Forecasting Model.
Regional Weather Prediction model.
Configured for CONUS domain, 50 km grid.

Several hundred profiles available from CHAMP satellite.



Assimilating GPS Radio Occultation Observations in WRF

Evaluating Impact of GPS Observations.

Case 1: Assimilate radiosondes EXCEPT those close to GPS profiles.

Case 2: Also assimilate GPS profiles.

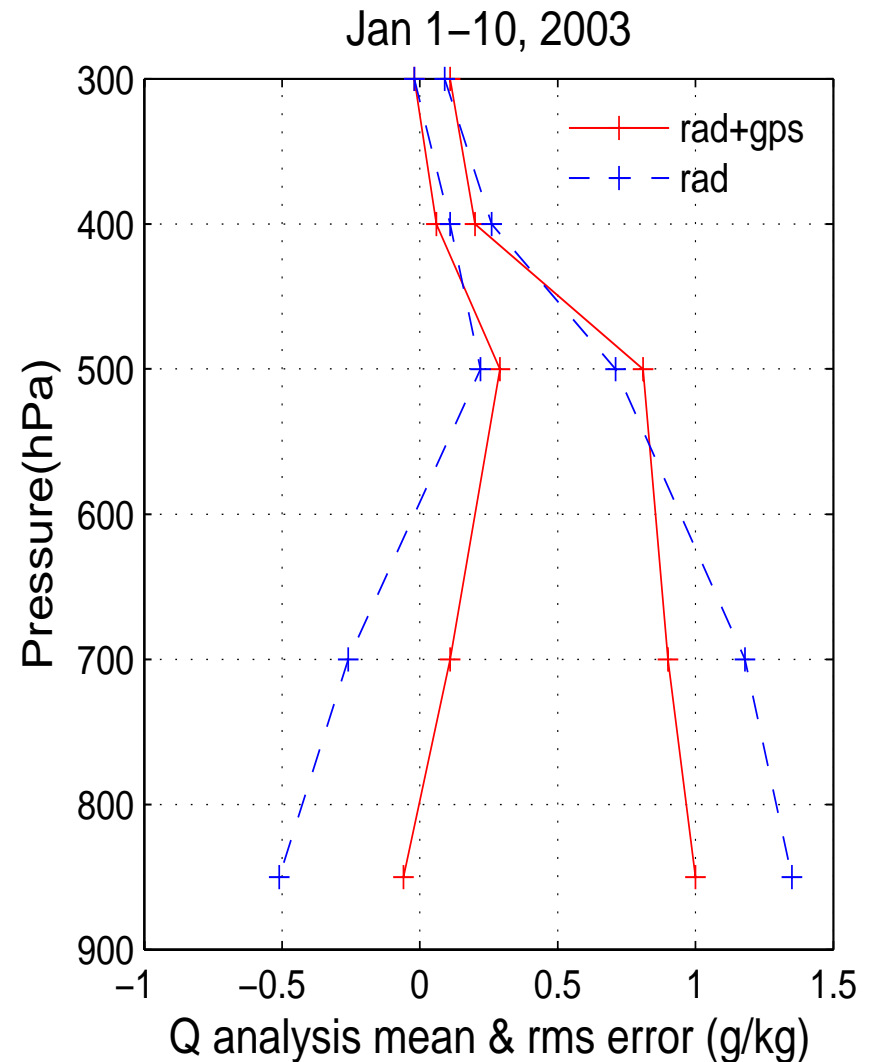
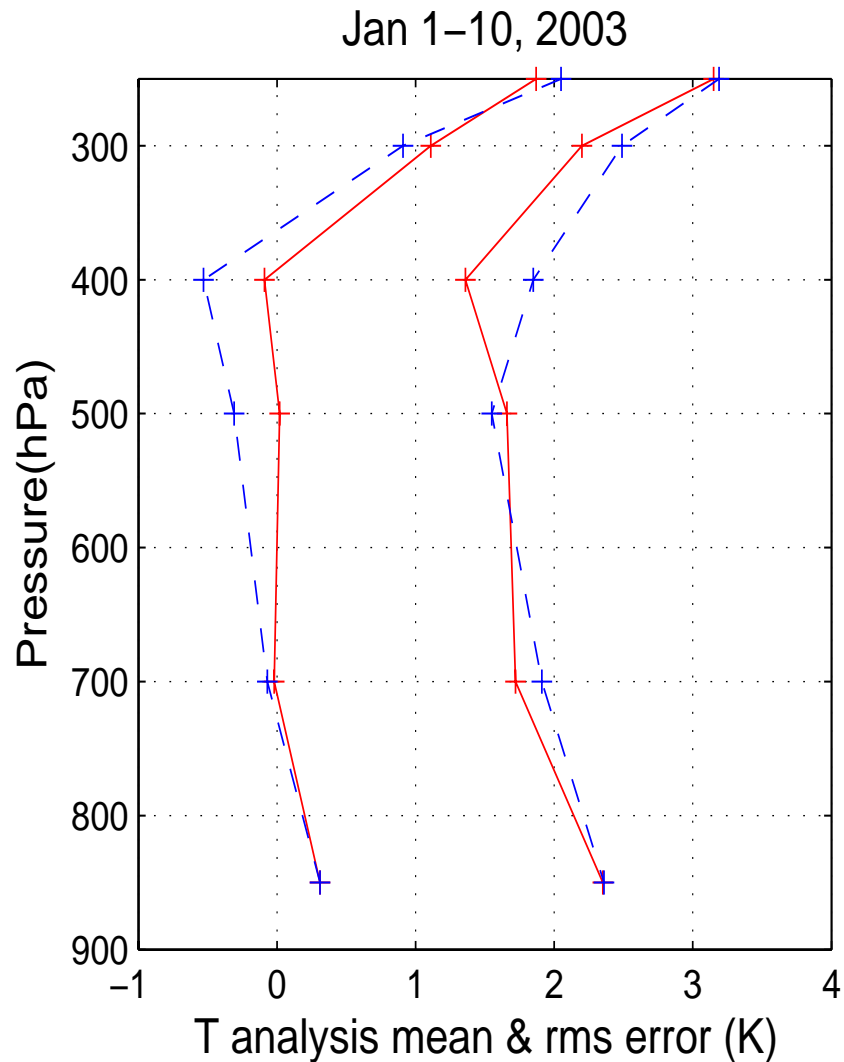
Look at reduction in error from close (unused) radiosonde profiles.

NOTE: Identical code allows assimilation in CAM, GFDL, GFS...

GPS Radio Occultation Impact on T and Q Errors in WRF

Each plot displays bias (left pair) and RMS (right pair).

Red curves include GPS: reduced bias and RMS.



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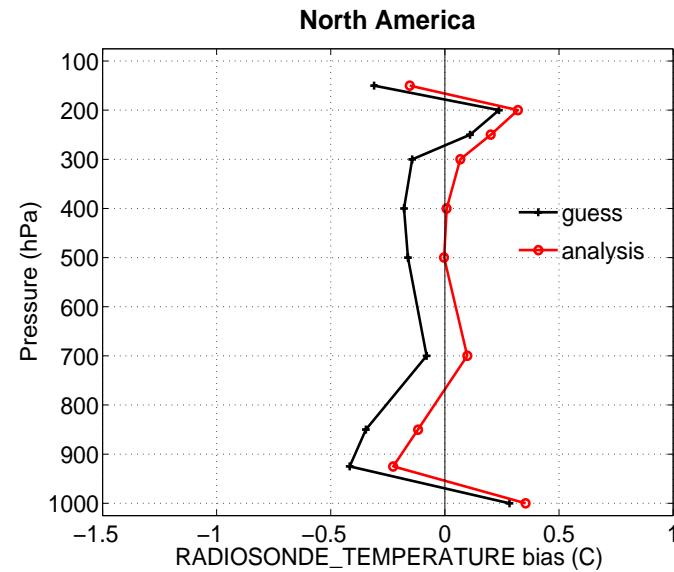
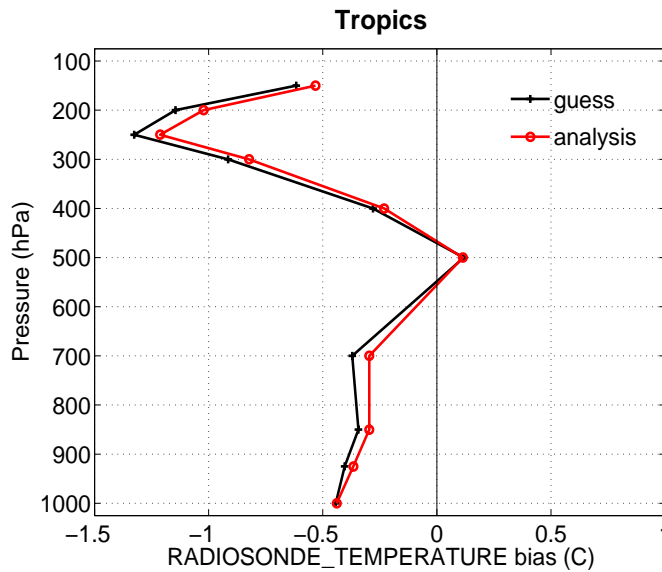
Forward and backward sensitivity analysis (adjoint and linear tangent replacement).

Select appropriate values for model parameters.

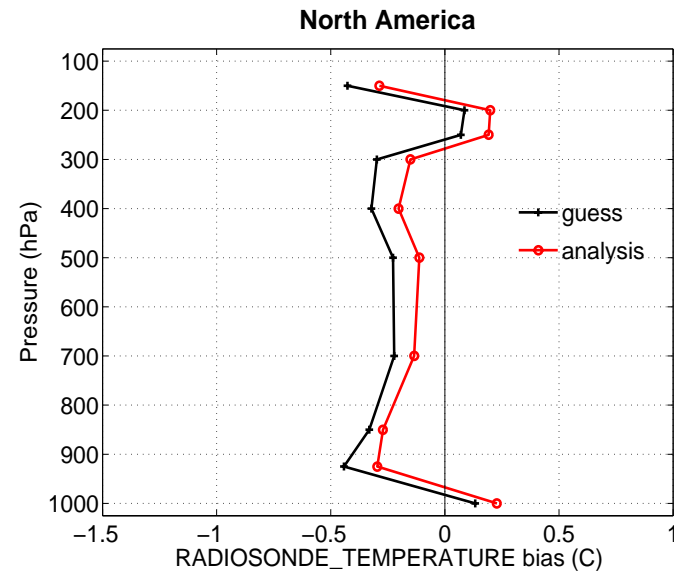
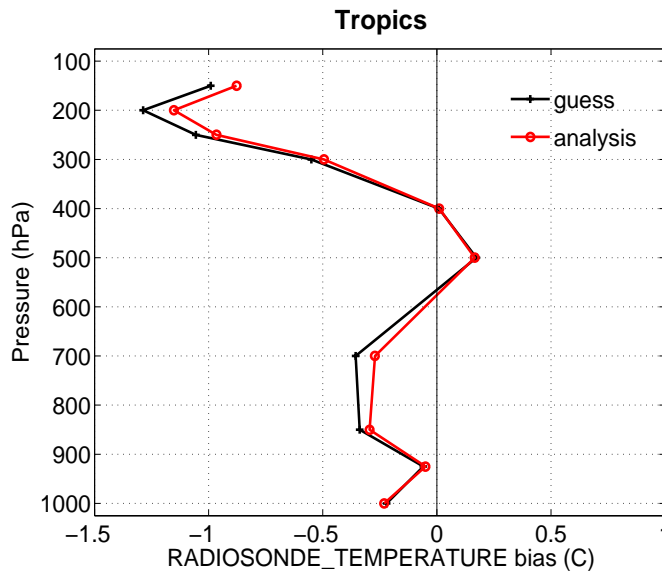
Example of low-resolution assimilation comparisons.

CAM spectral vs. FV for January, 2003: **Temperature Bias**

Spectral
T21



Finite
Volume
2x2.5



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Forward/backward sensitivity analysis (adjoint/linear tangent proxy).

Select appropriate values for model parameters.

Ensemble Sensitivity Analysis

Can compute correlation (covariance) between ANY forecast or analysis quantity and ALL other forecast and analysis quantities or functions thereof at any time lag.

Can get same information as unlimited number of adjoint and linear tangent integrations over arbitrary periods.

Explore relations between variables, observations, or functions thereof.

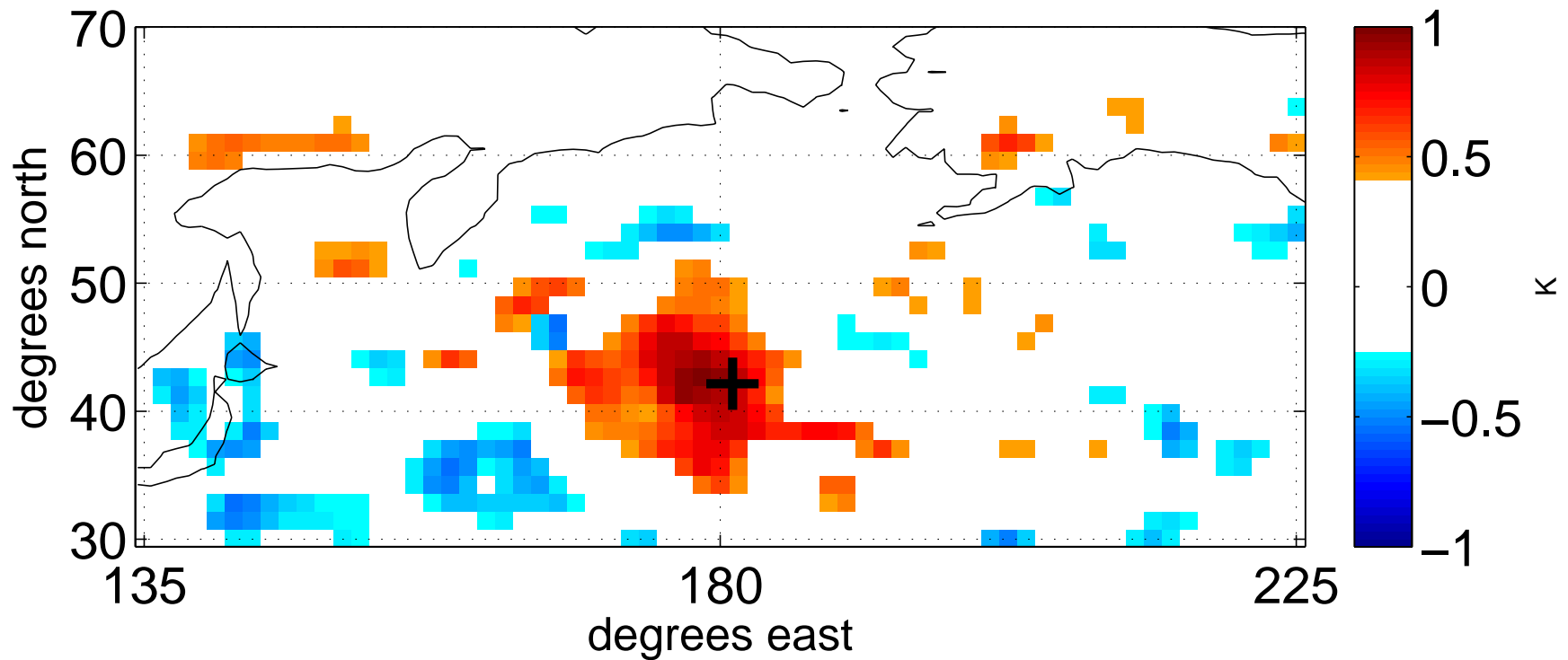
Example 1: Base point is 500 hPa mid-latitude temperature.
Look at impact on evolution of 500hPa temperatures.

Similar to linear tangent integration.

Significant correlations from 20 member T85 ensemble.

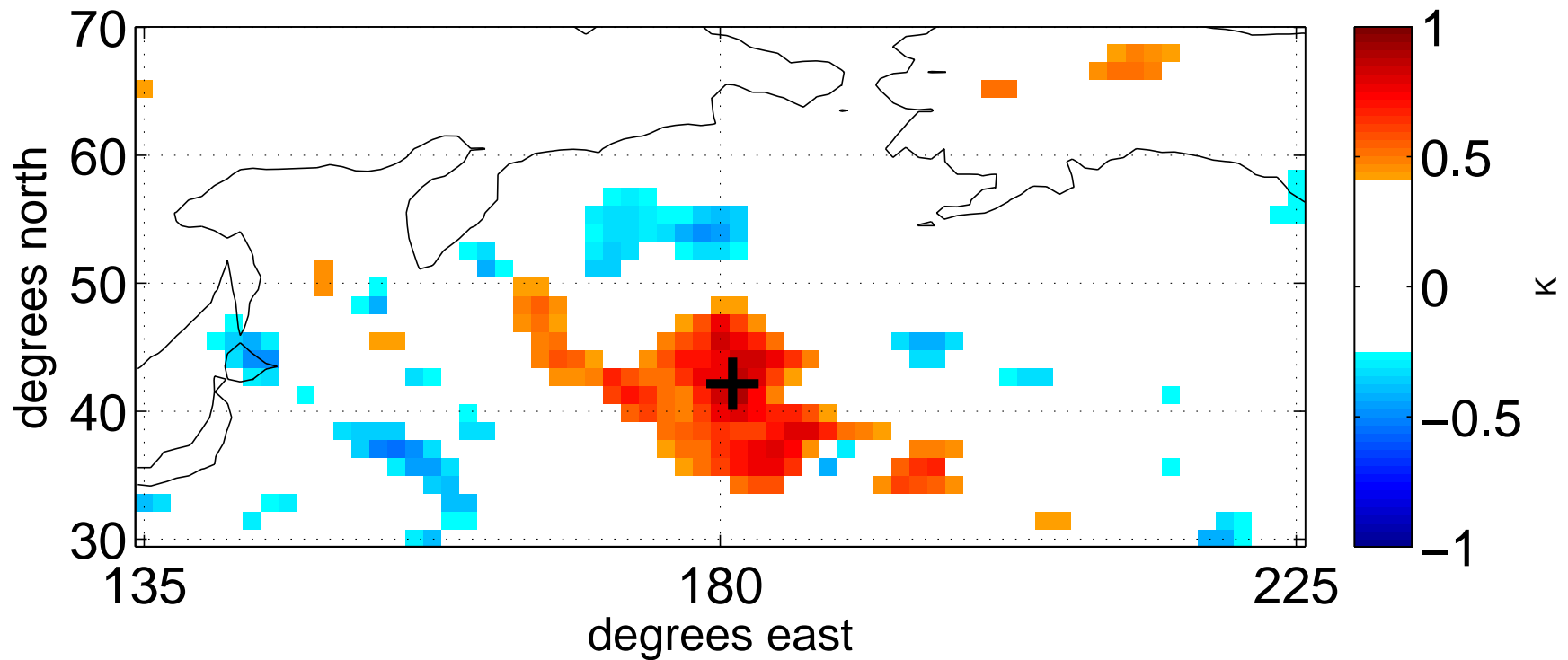
Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 00 hours: 500 hPa Temperature to 500 hPa Temperature



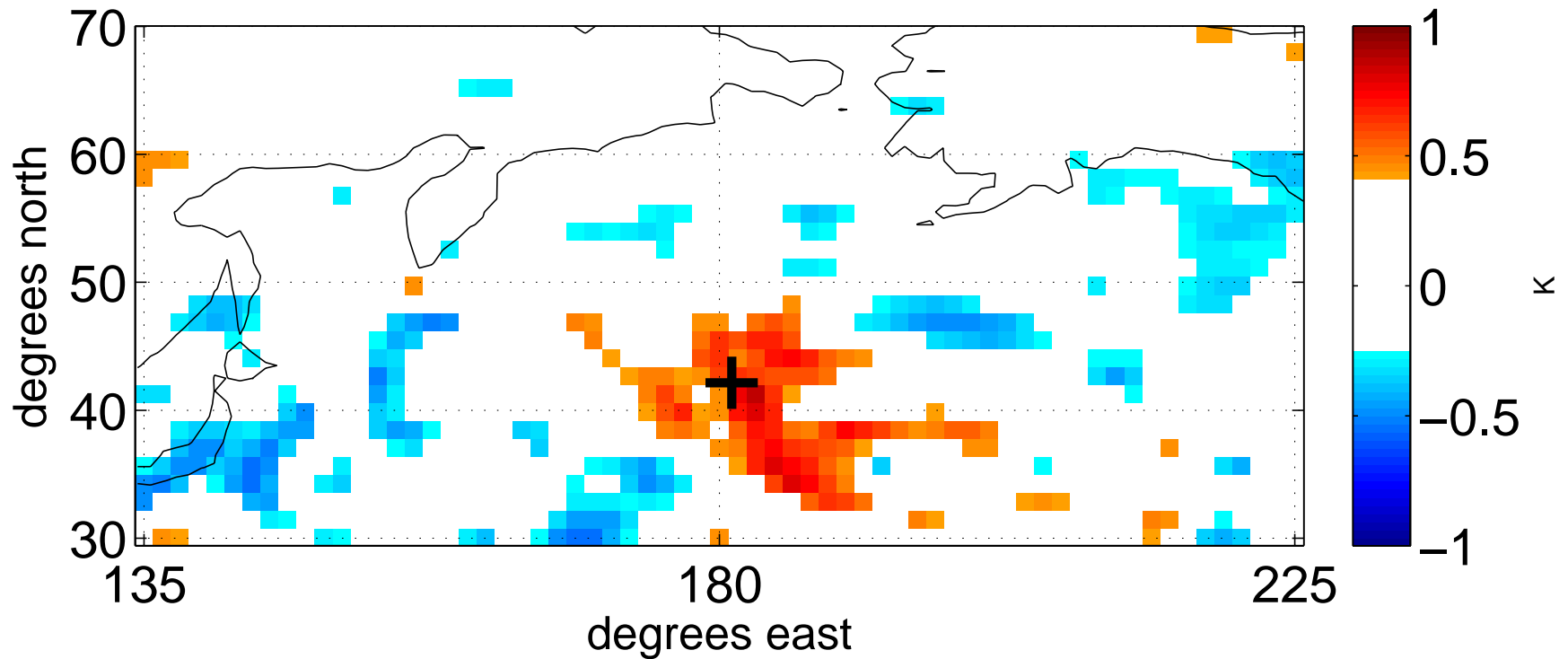
Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 06 hours: 500 hPa Temperature to 500 hPa Temperature



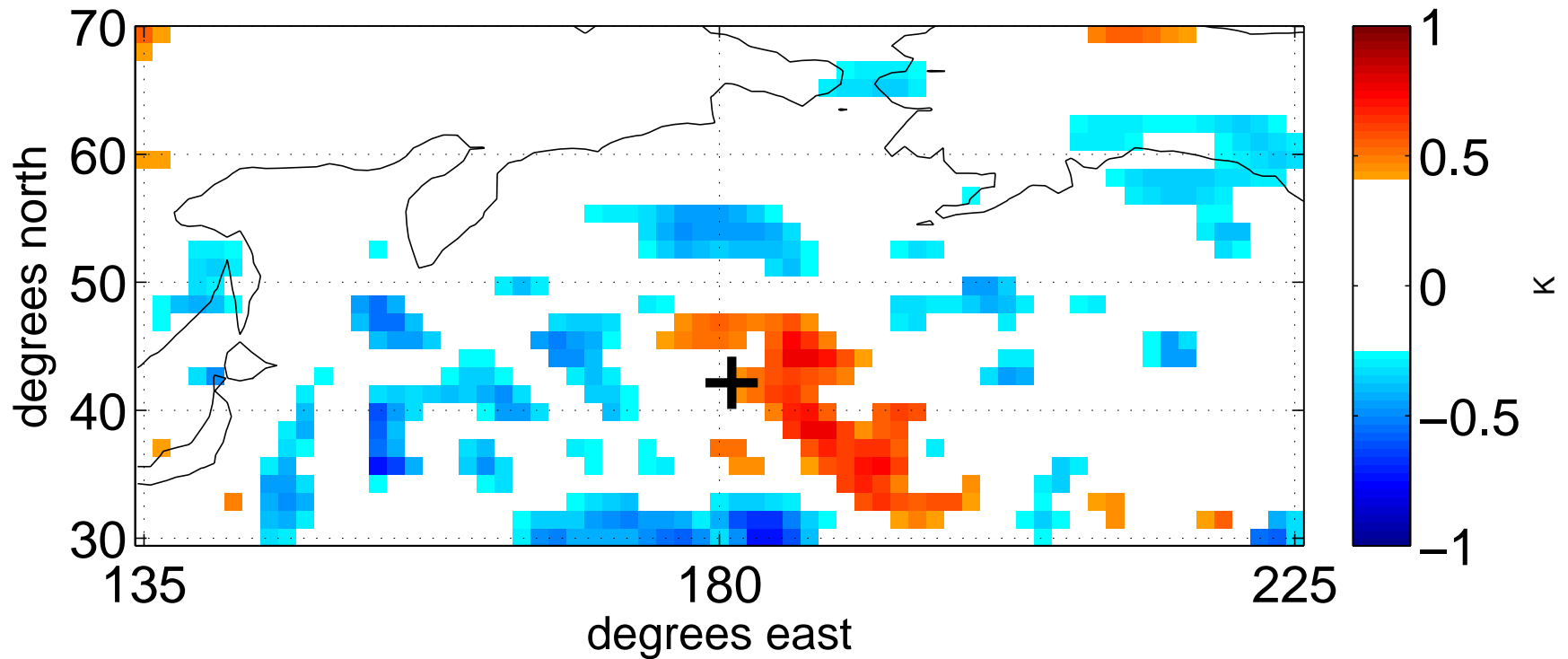
Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 12 hours: 500 hPa Temperature to 500 hPa Temperature



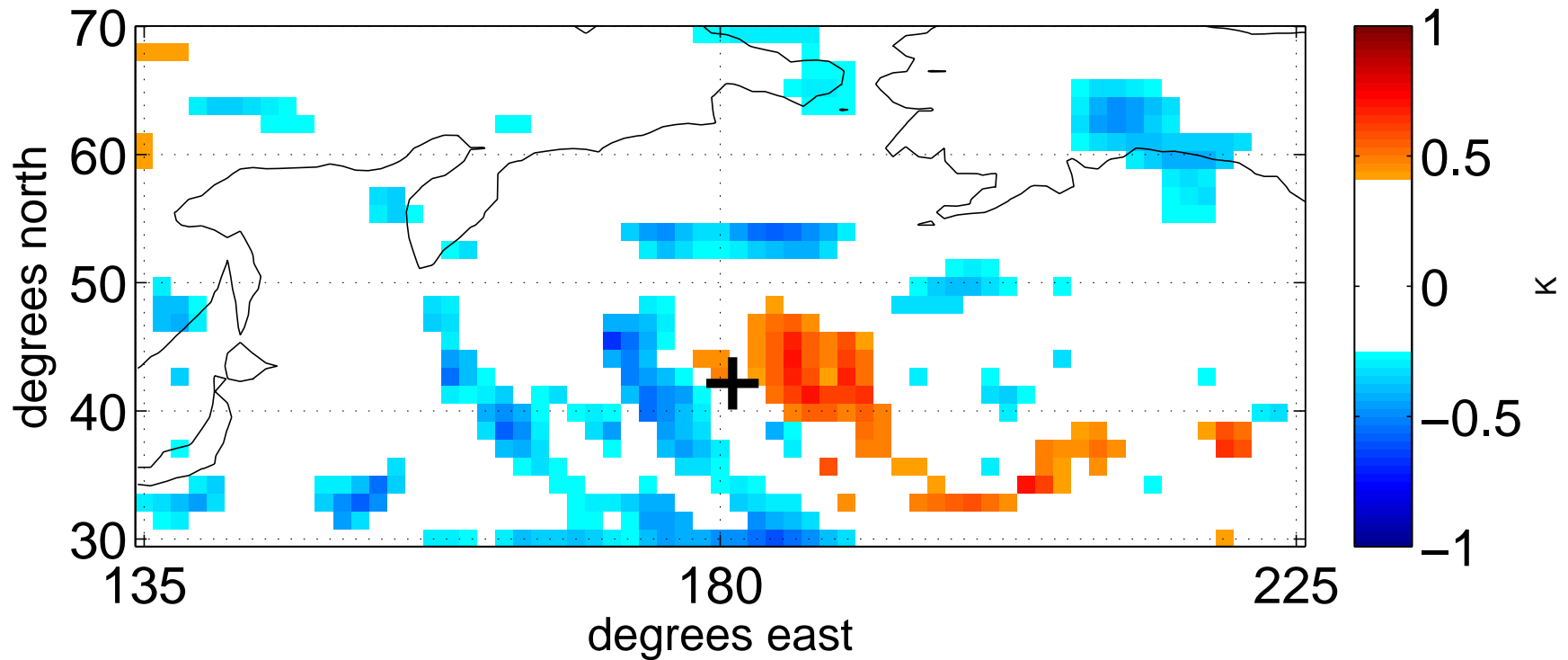
Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 18 hours: 500 hPa Temperature to 500 hPa Temperature



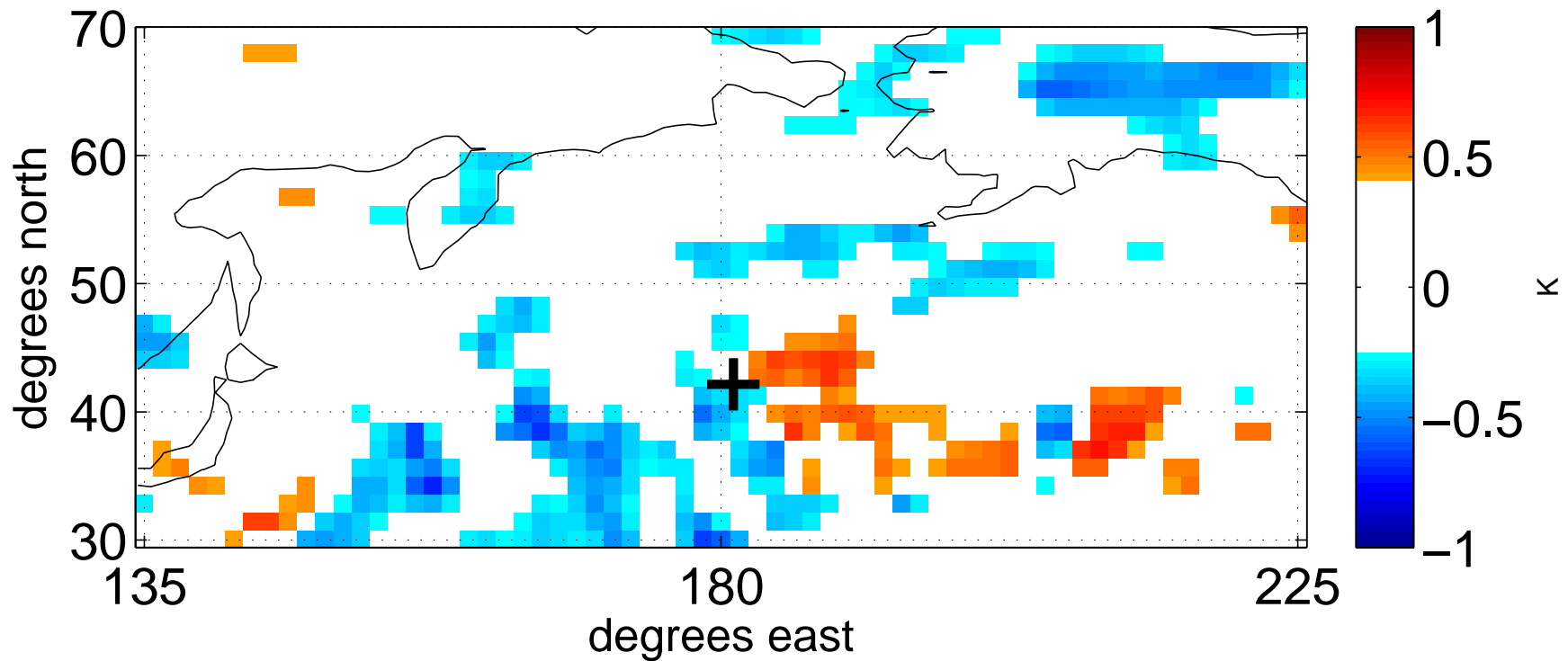
Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 24 hours: 500 hPa Temperature to 500 hPa Temperature



Forward in Time Sensitivity (Linear Tangent equivalent)

Time lag 30 hours: 500 hPa Temperature to 500 hPa Temperature



Ensemble Sensitivity Analysis

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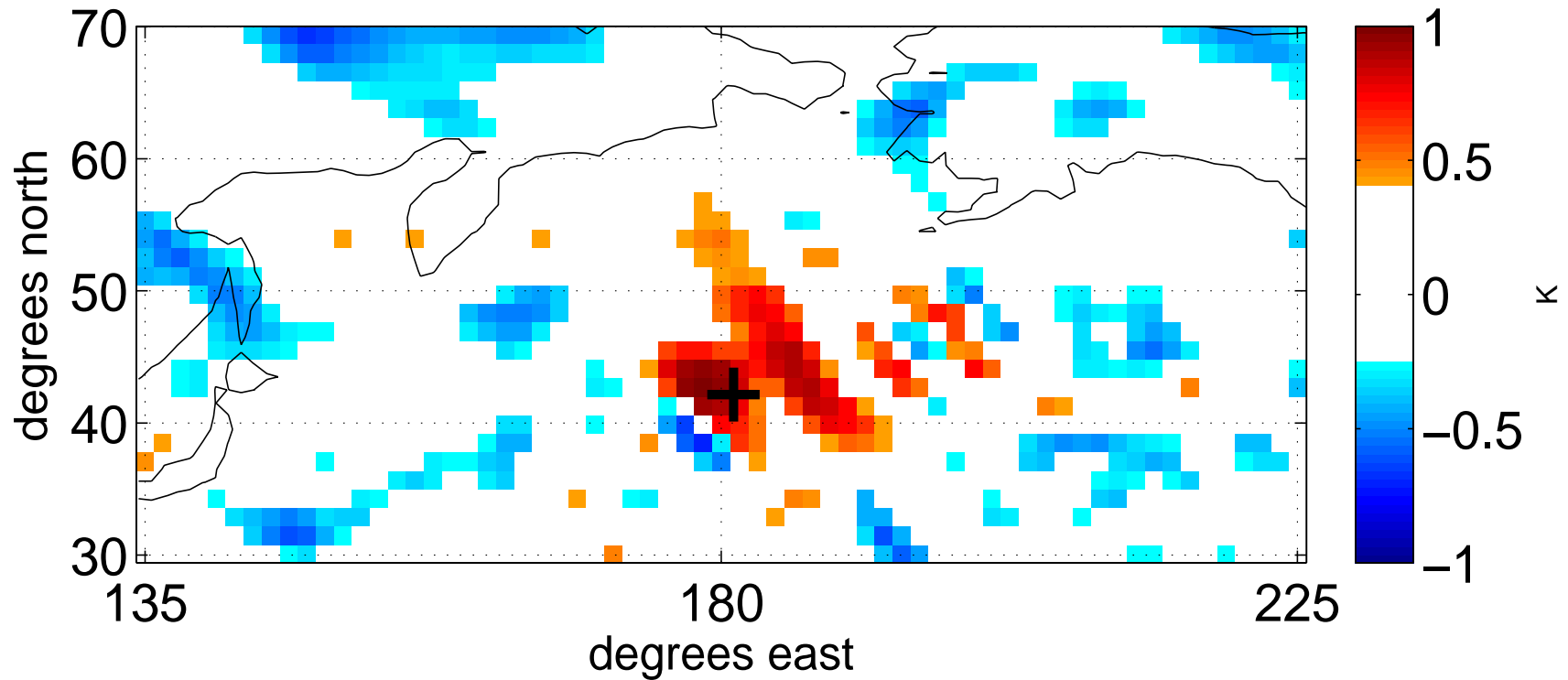
Explore relations between variables, observations, or functions thereof.

Example 2: Base point is 500 hPa mid-latitude zonal velocity.
Look at impact of previous 500 hPa temperature.

Compare to an adjoint integration.

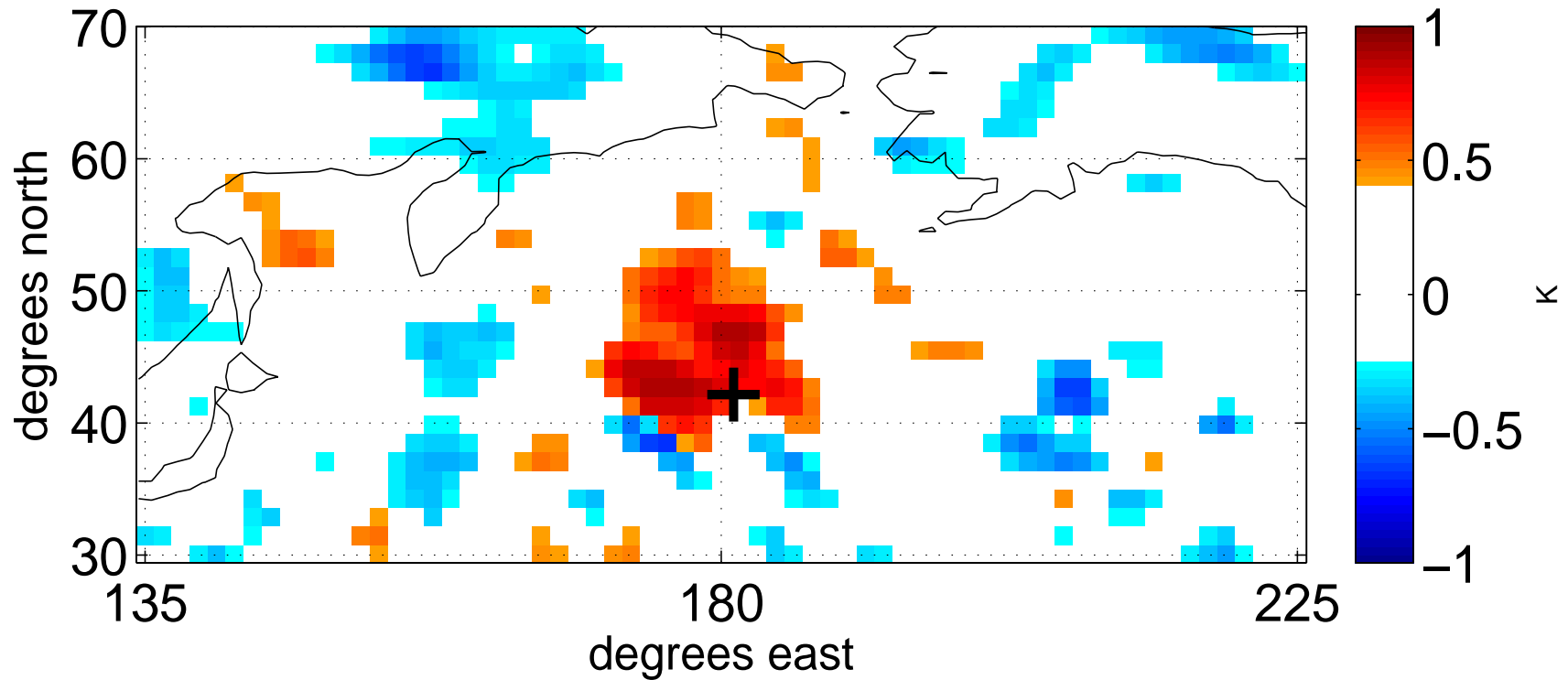
Backward in Time Sensitivity (Adjoint equivalent)

Time lag -00 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



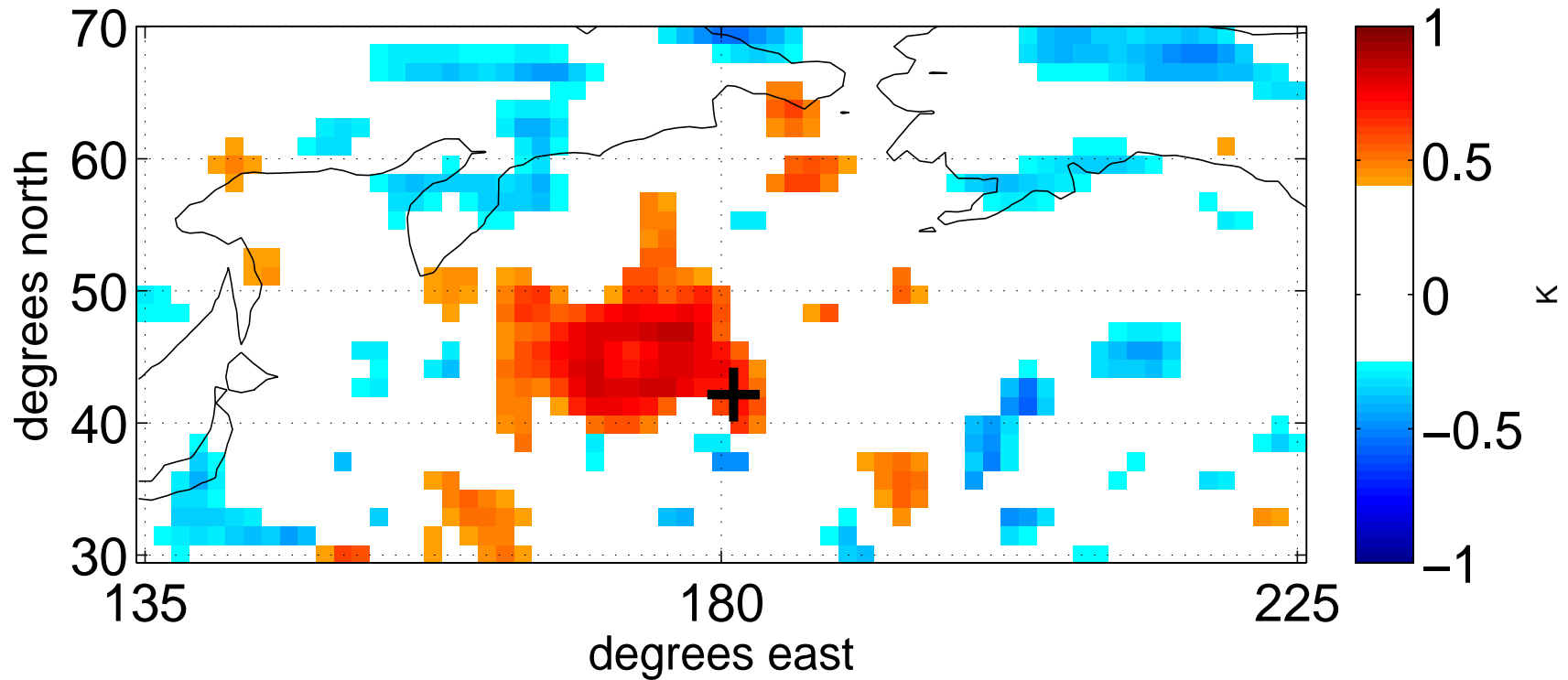
Backward in Time Sensitivity (Adjoint equivalent)

Time lag -06 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



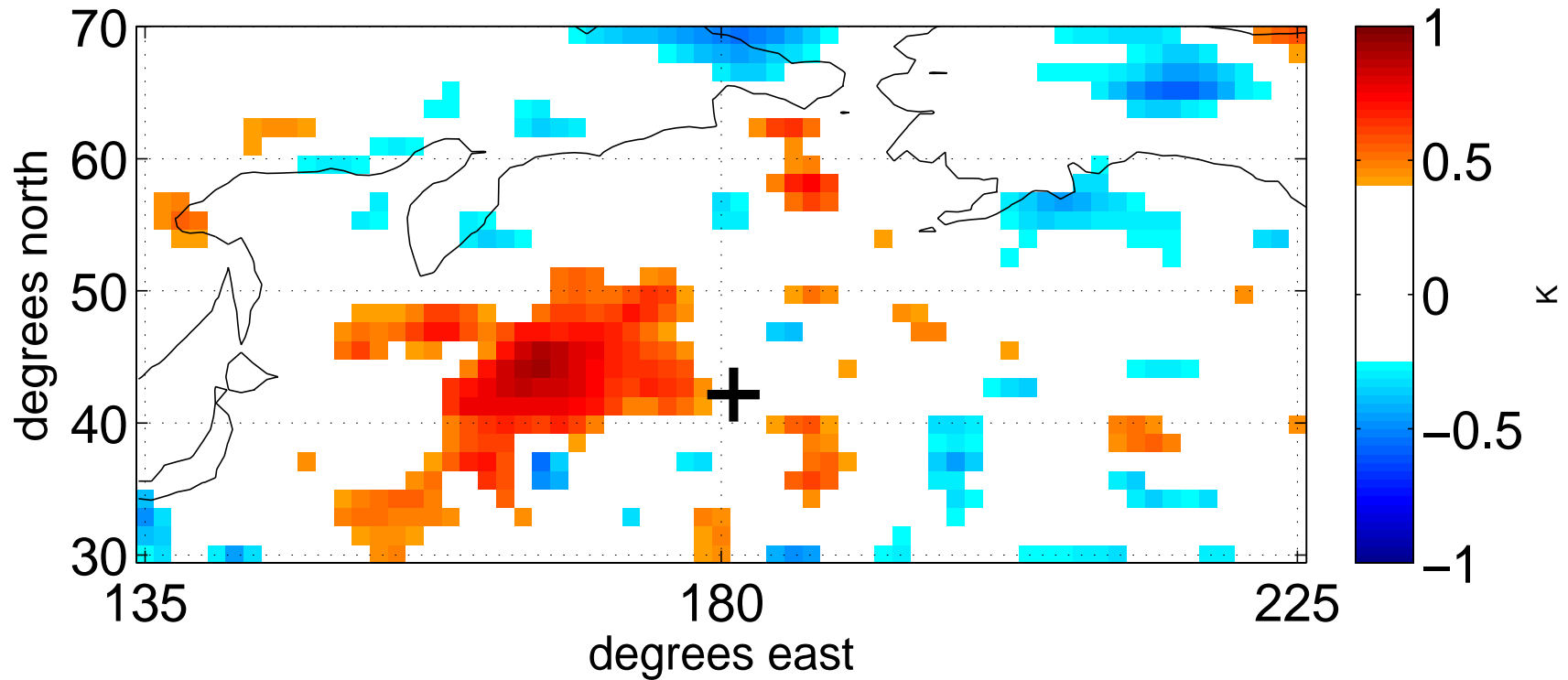
Backward in Time Sensitivity (Adjoint equivalent)

Time lag -12 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



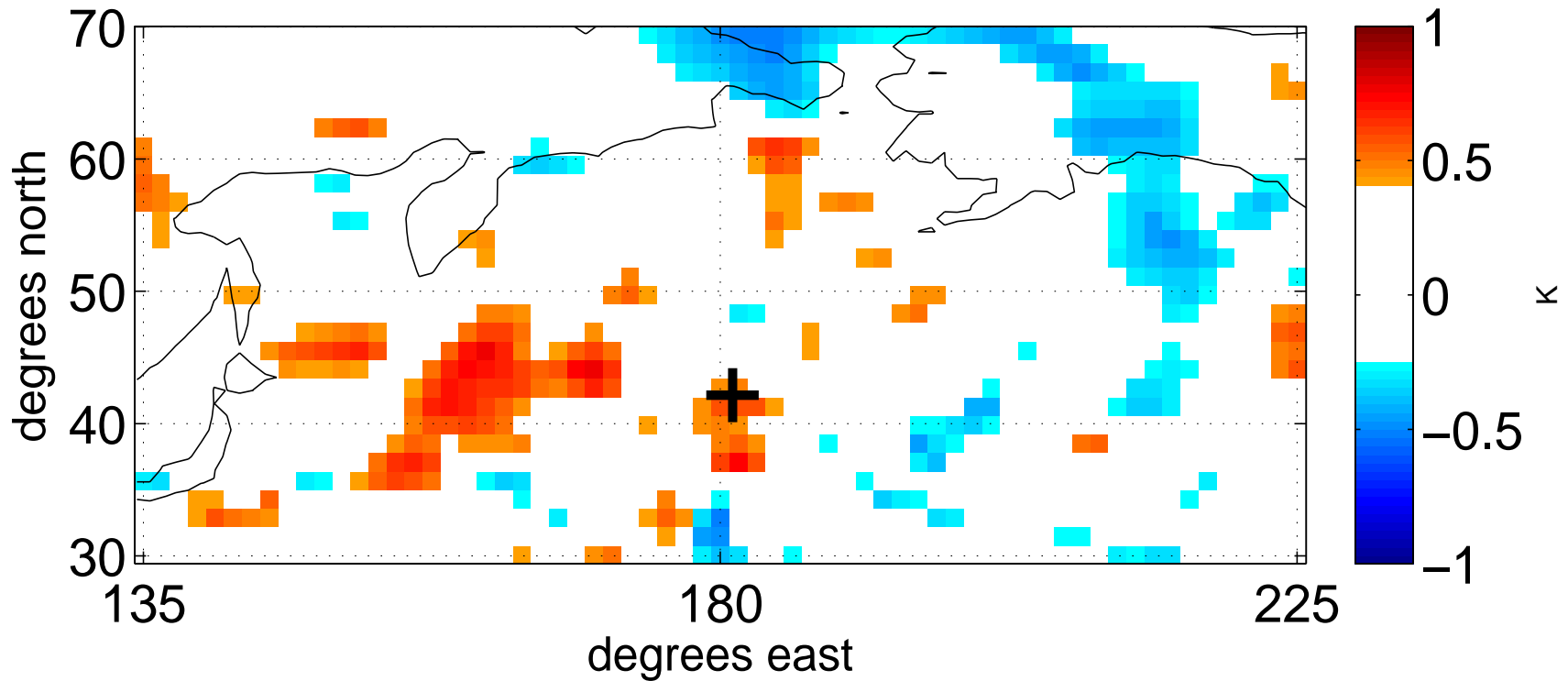
Backward in Time Sensitivity (Adjoint equivalent)

Time lag -18 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



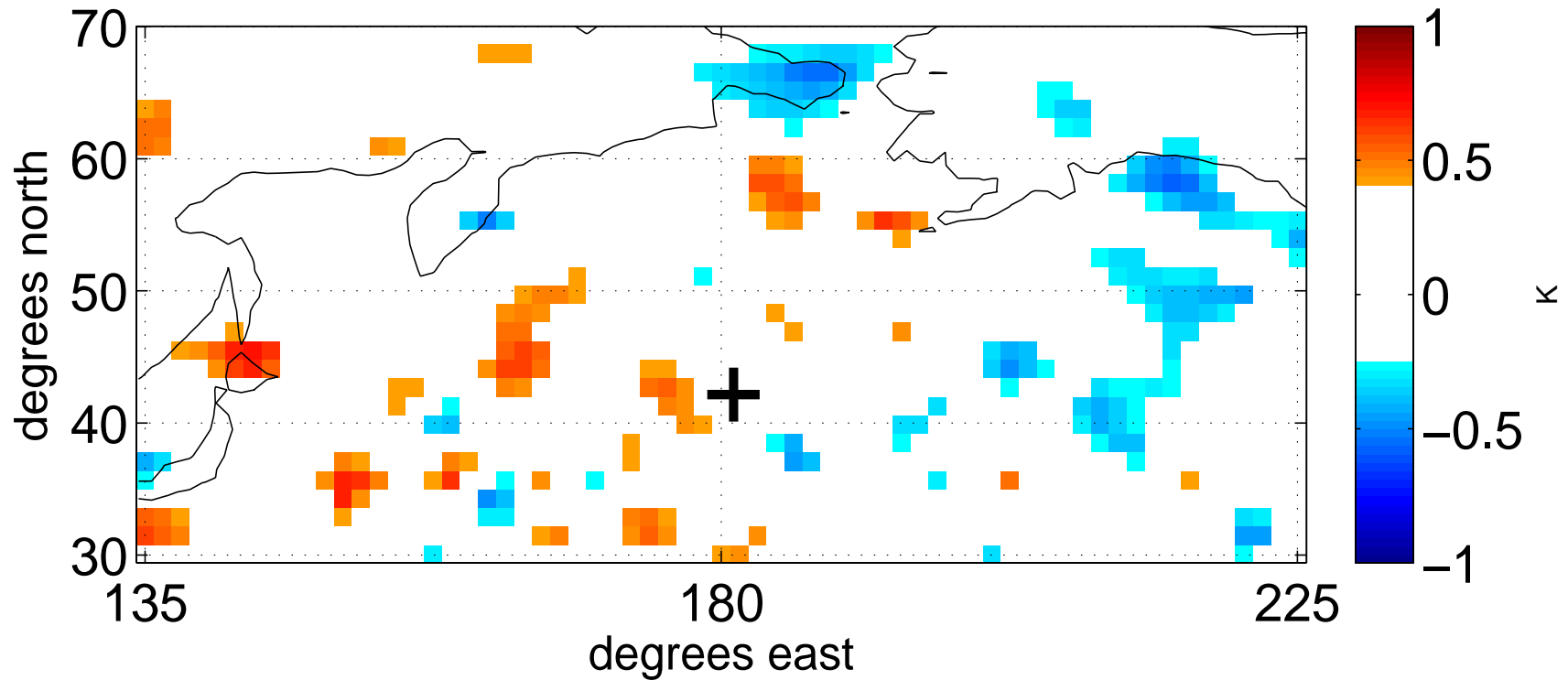
Backward in Time Sensitivity (Adjoint equivalent)

Time lag -24 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



Backward in Time Sensitivity (Adjoint equivalent)

Time lag -30 hours: 500 hPa Zonal Velocity to 500 hPa Temperature



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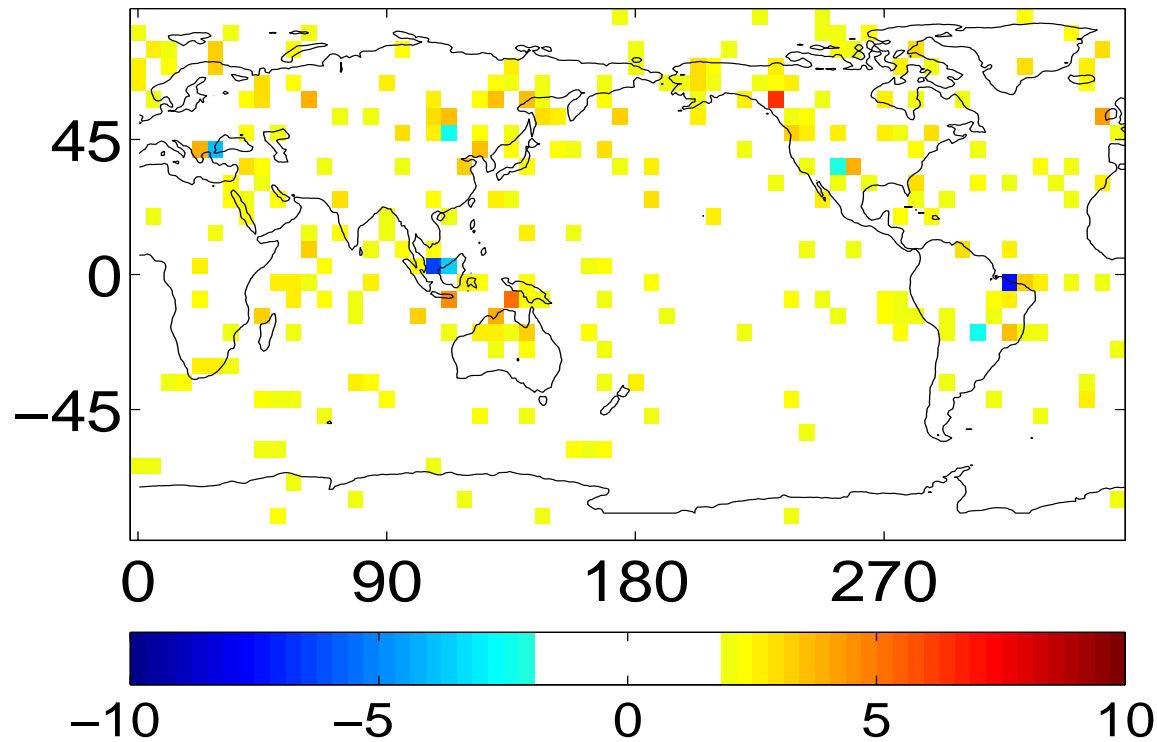
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Climate Model Parameter Estimation via Ensemble Data Assimilation.



T21 CAM assimilation of gravity wave drag efficiency parameter.

Oceanic values are noise (should be 0).

$0 < \text{efficiency} < \sim 4$ suggested by modelers.

Positive values over NH land expected.

Problem: large negative values over tropical land near convection.

May reduce wind bias in tropical troposphere, but for 'Wrong Reason'.

Assimilation tries to use free parameter to fix ALL model problems

Requirements for a Community Assimilation Facility

1. Assimilation that works with variety of models and obs. types.
2. Coding to add model must be easy (weeks max).
3. Coding to add observation type must be easy (weeks max).
4. Assimilations must be nearly as good as best available system.
5. Performance/scalability must be very good.

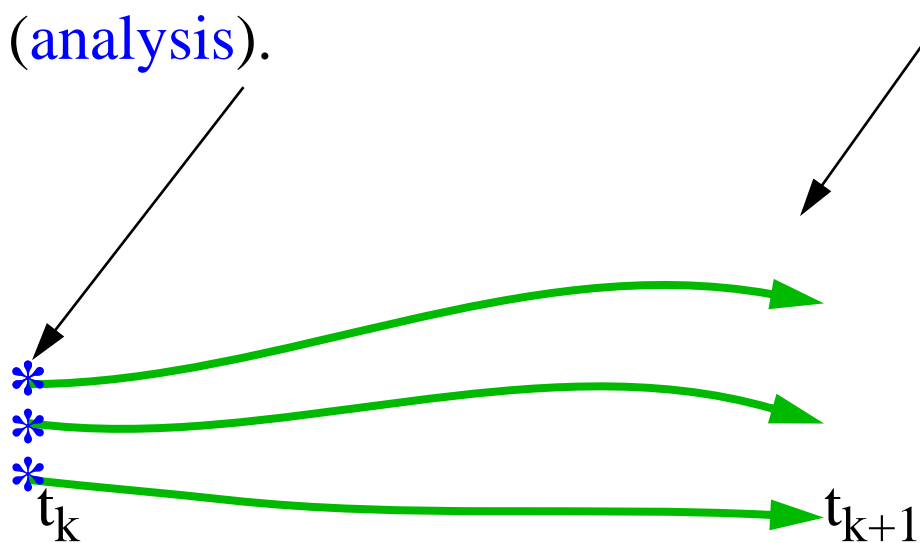
Adaptive ensemble filters with software engineering can do this...

How an Ensemble Filter Works for Geophysical Data Assimilation

1. Use model to advance **ensemble** (3 members here) to time at which next observation becomes available.

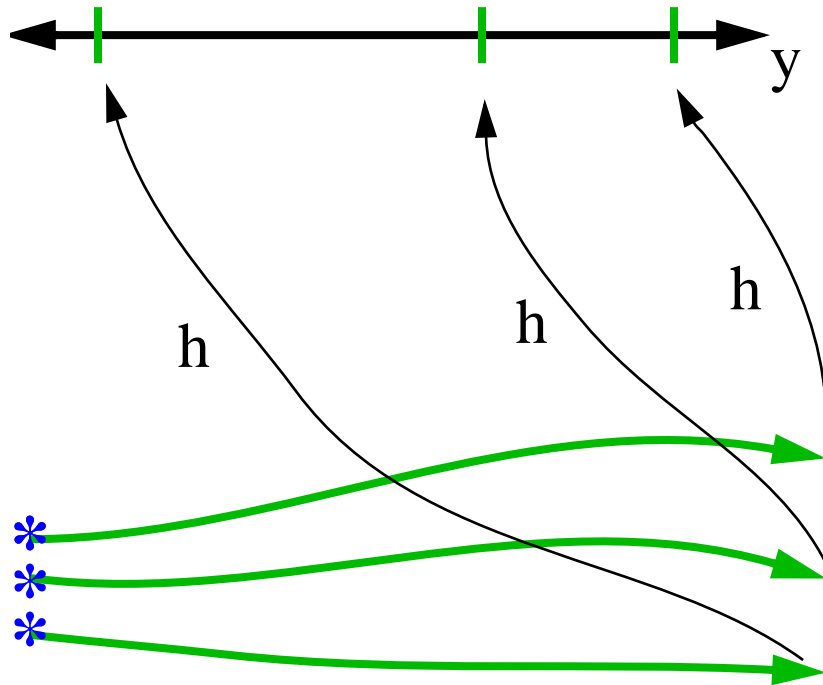
Ensemble state estimate after using previous observation (**analysis**).

Ensemble state at time of next observation (**prior**).



How an Ensemble Filter Works for Geophysical Data Assimilation

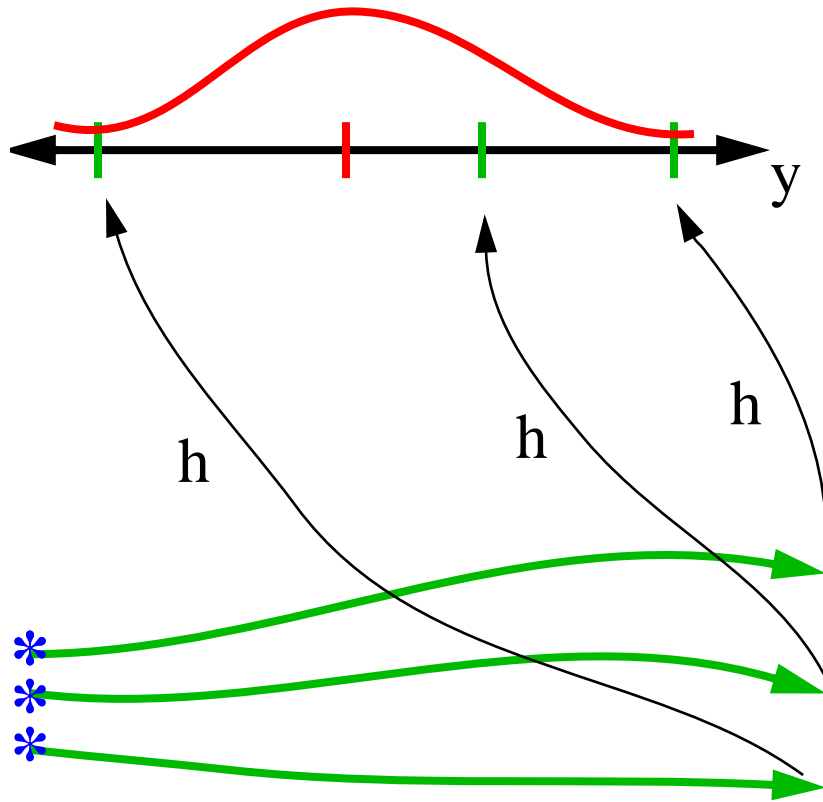
2. Get prior ensemble sample of observation, $y=h(x)$, by applying forward operator h to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially.

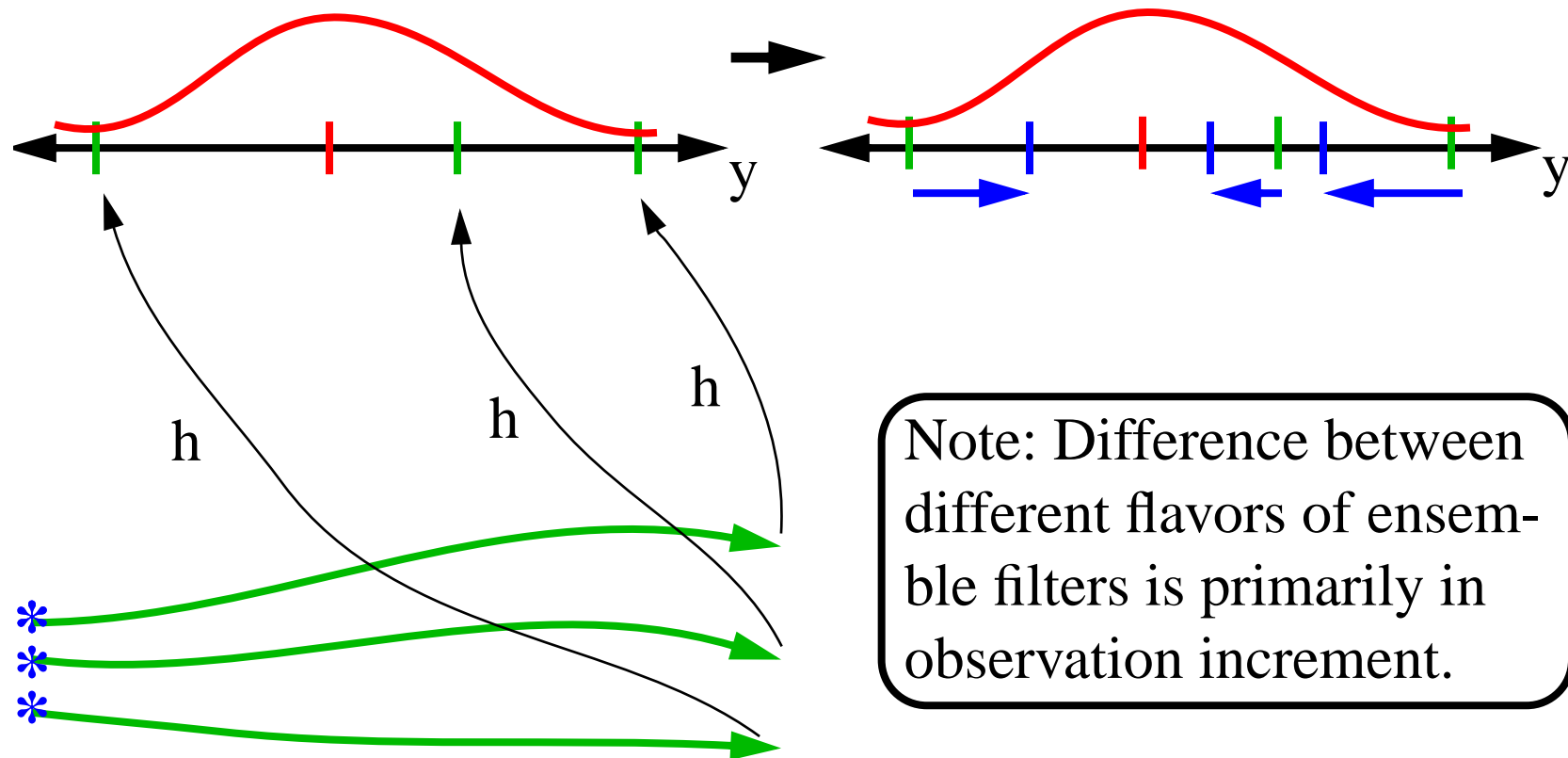
How an Ensemble Filter Works for Geophysical Data Assimilation

3. Get **observed value** and **observational error distribution** from observing system.



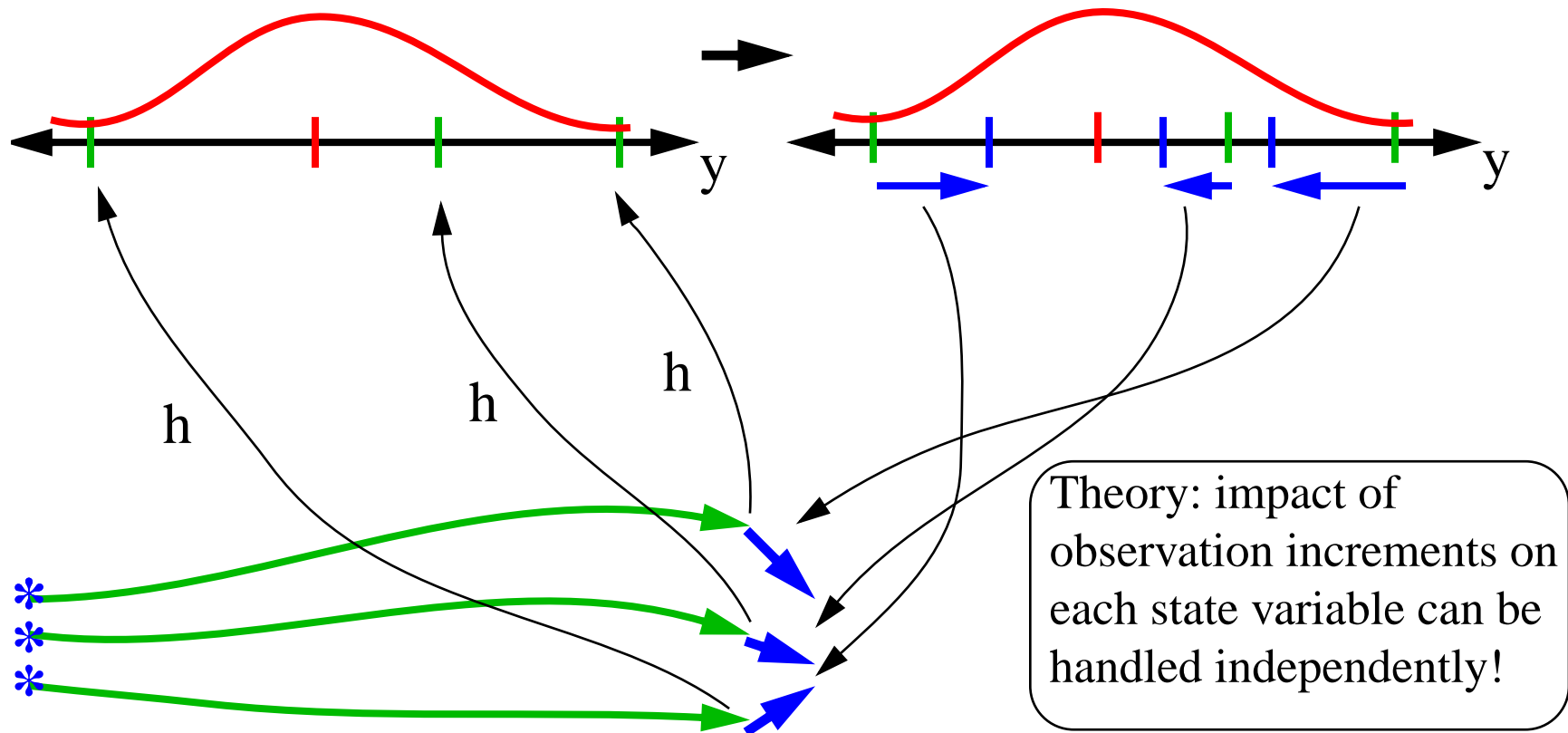
How an Ensemble Filter Works for Geophysical Data Assimilation

4. Find **increment** for each prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



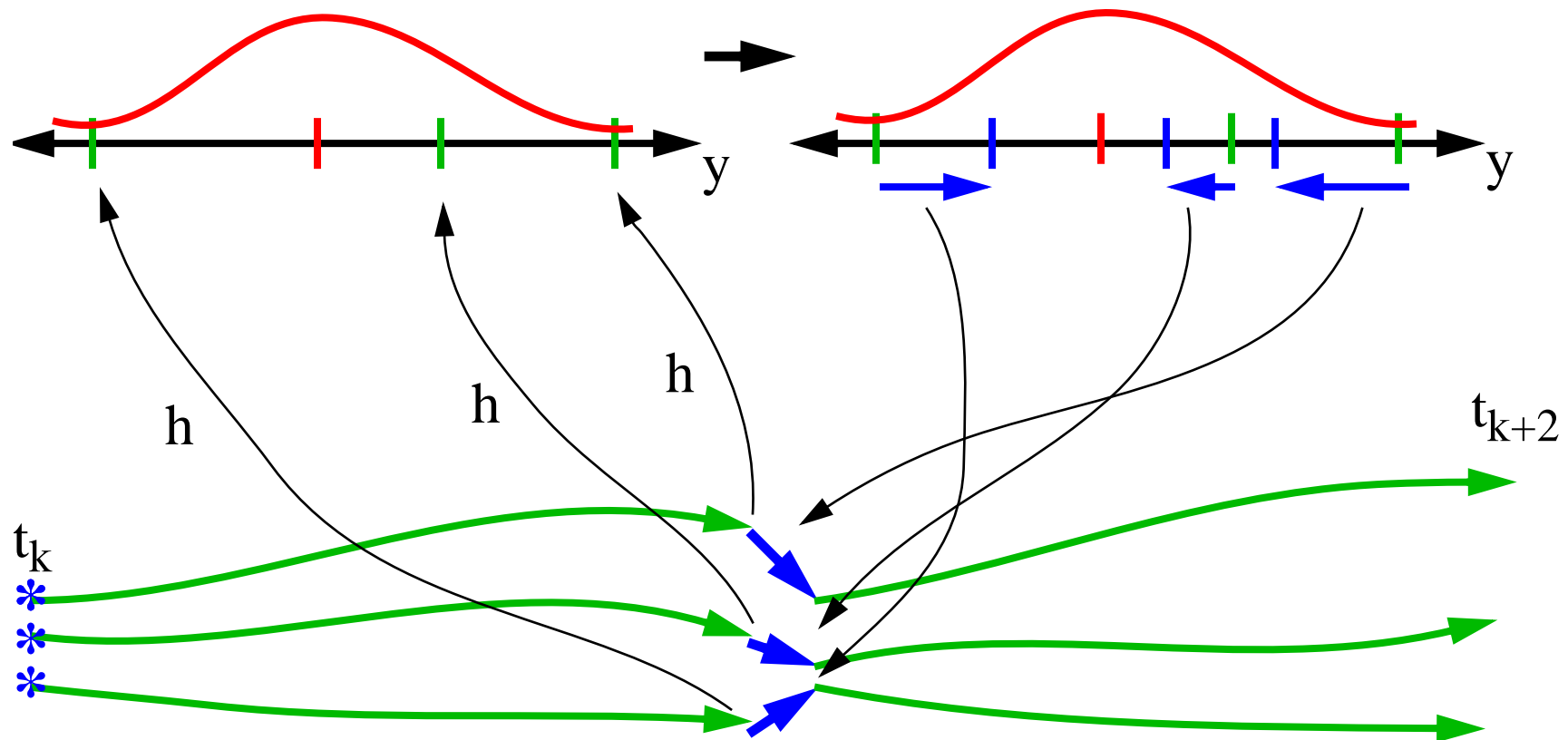
How an Ensemble Filter Works for Geophysical Data Assimilation

5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



How an Ensemble Filter Works for Geophysical Data Assimilation

6. When all ensemble members for each state variable are updated, have a new analysis. Integrate to time of next observation...

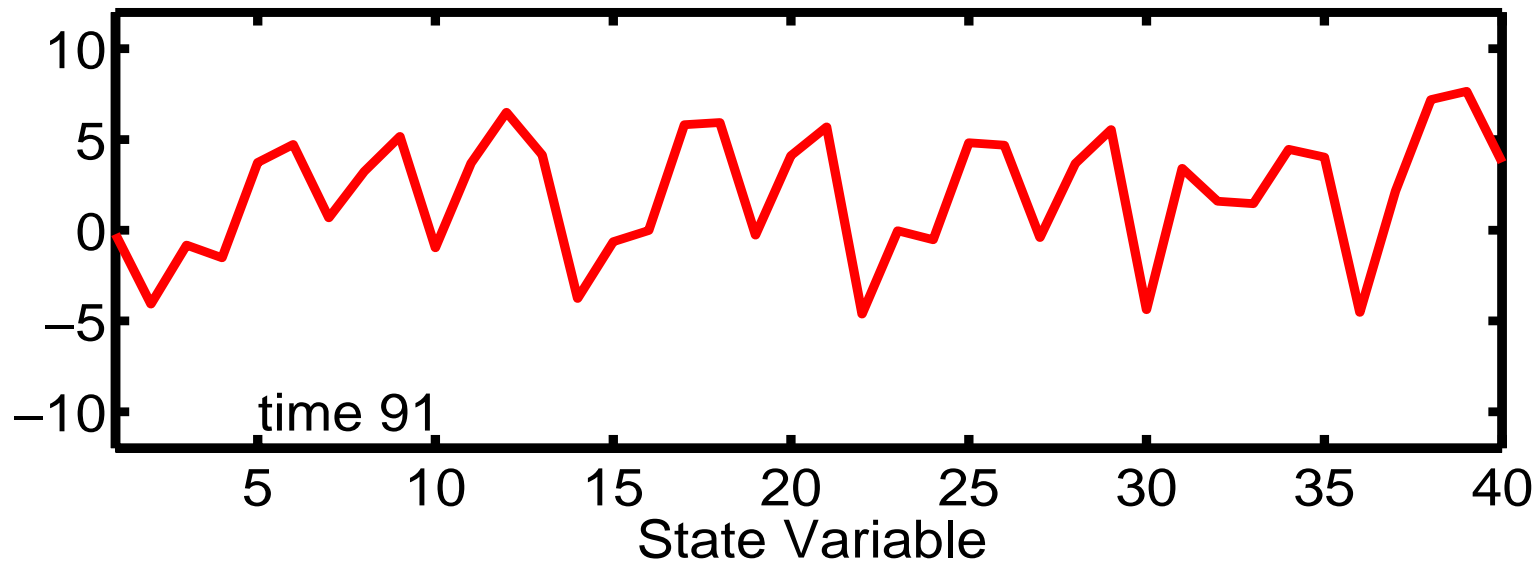


Ensemble Filter for Lorenz-96 40-Variable Model

40 state variables: X_1, X_2, \dots, X_{40} .

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

Acts 'something' like synoptic weather around a latitude band.

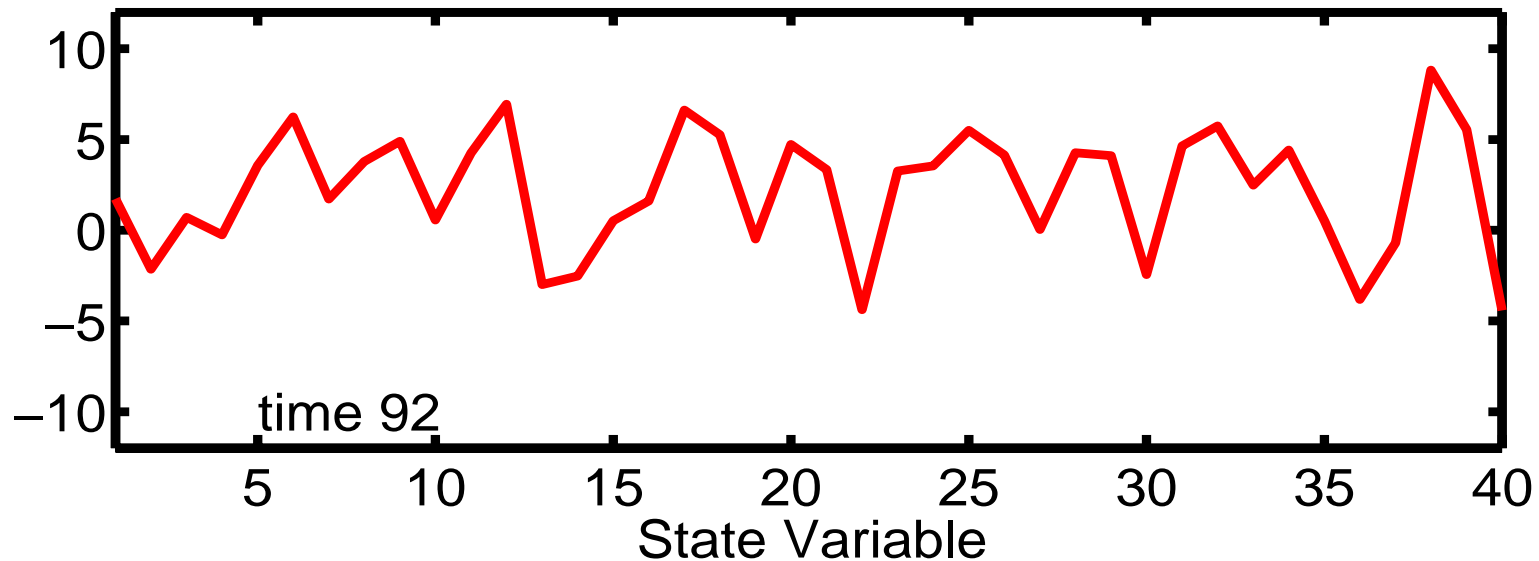


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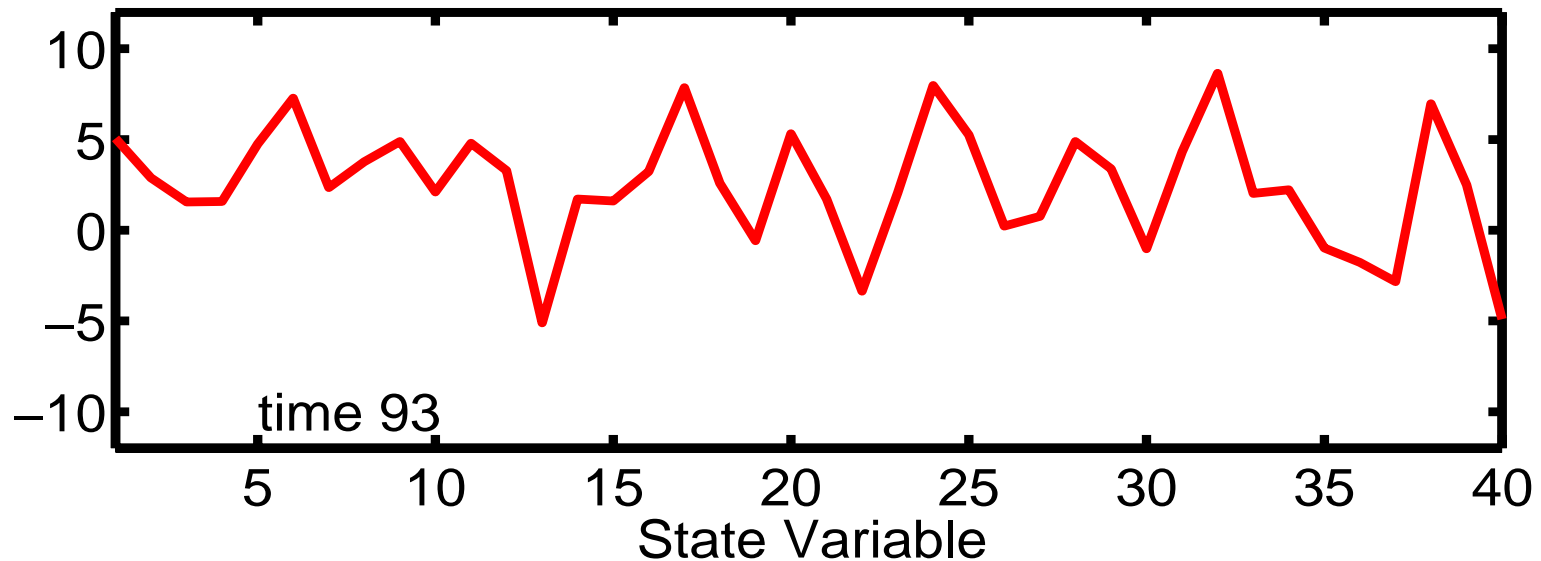


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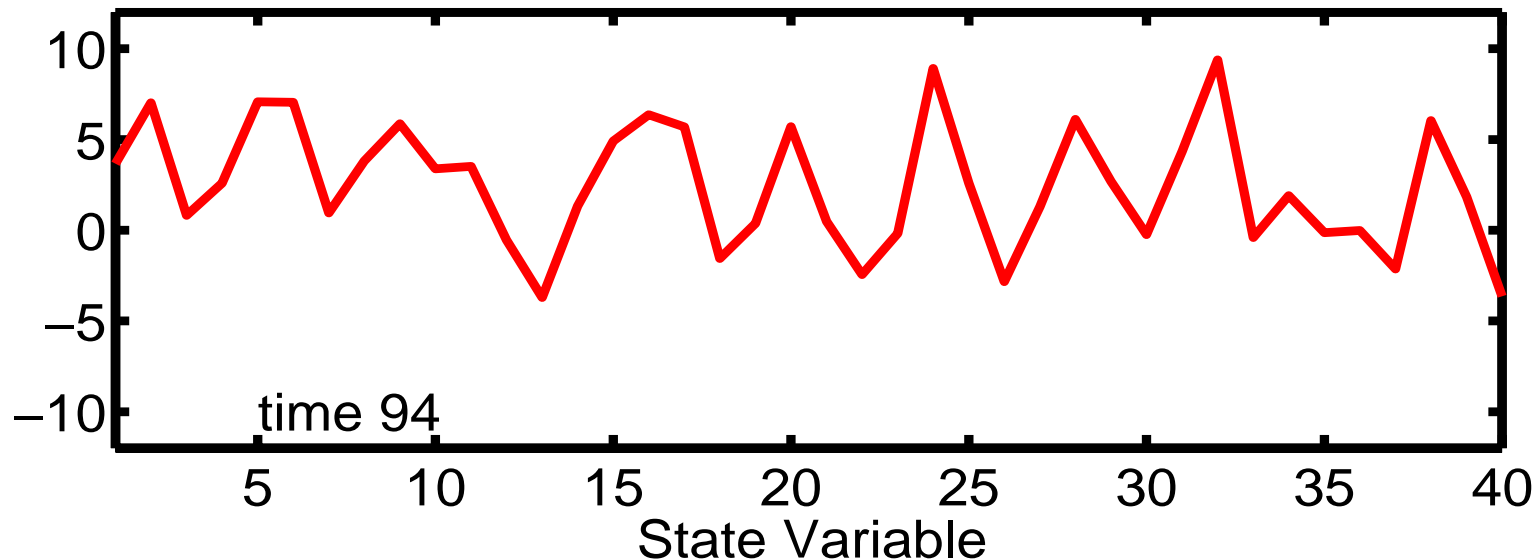


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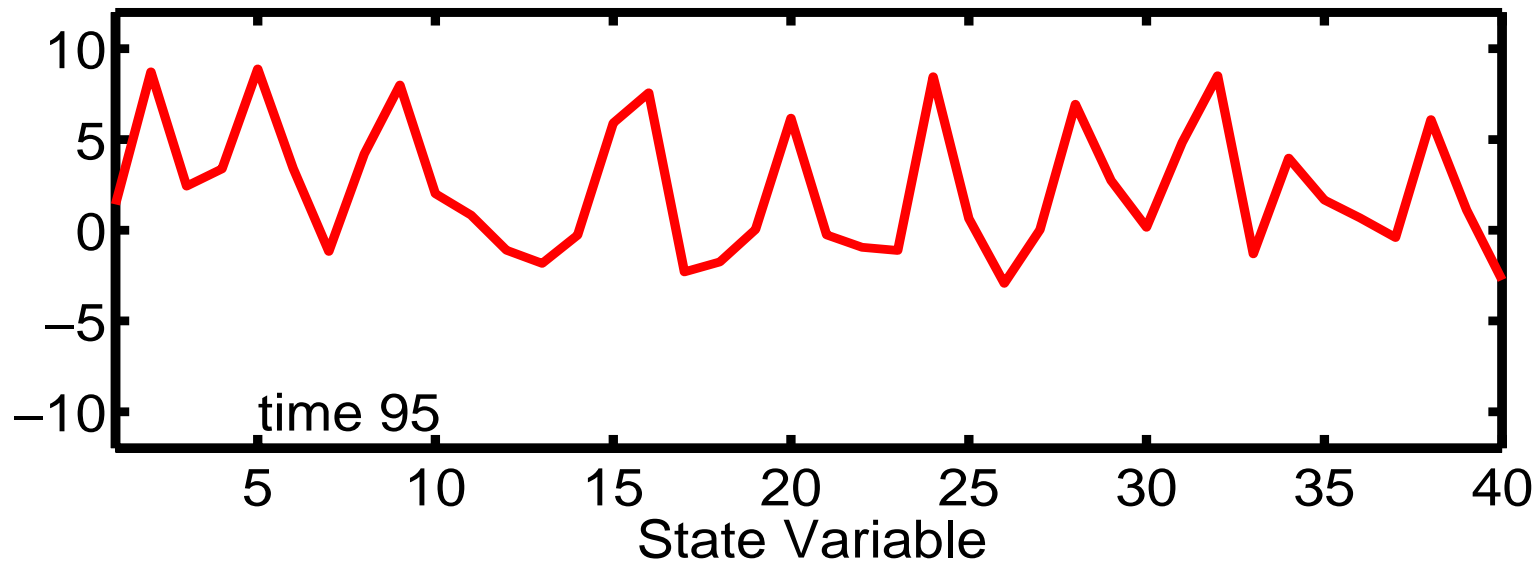


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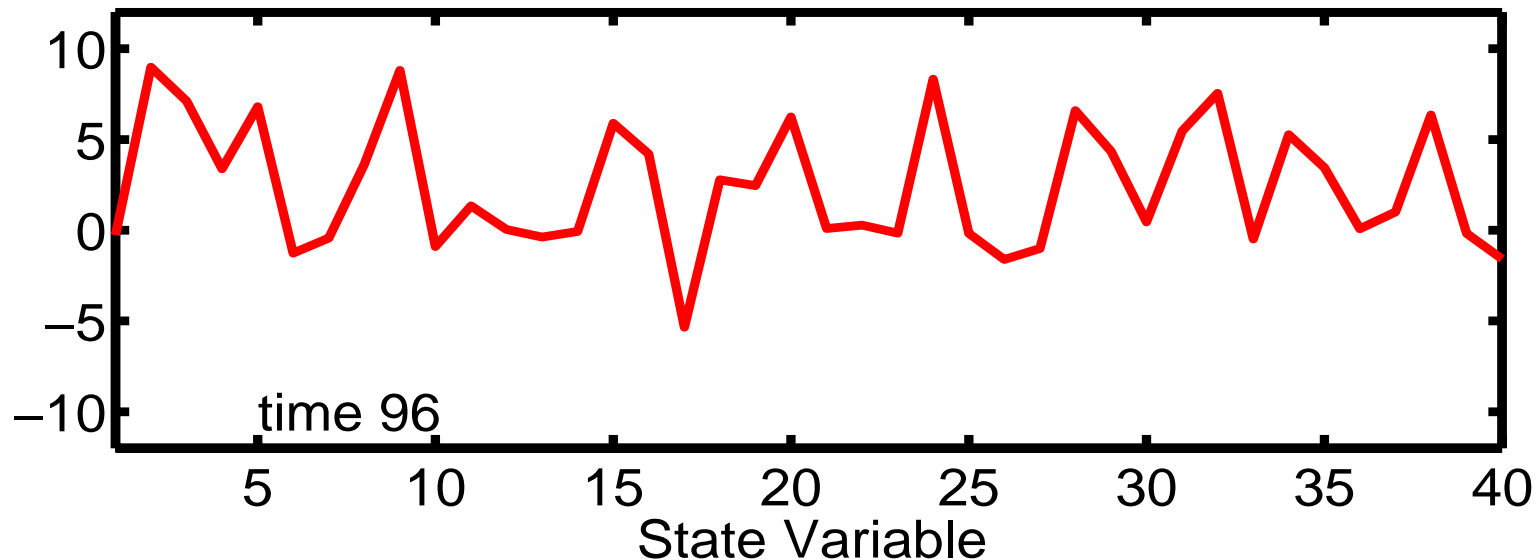


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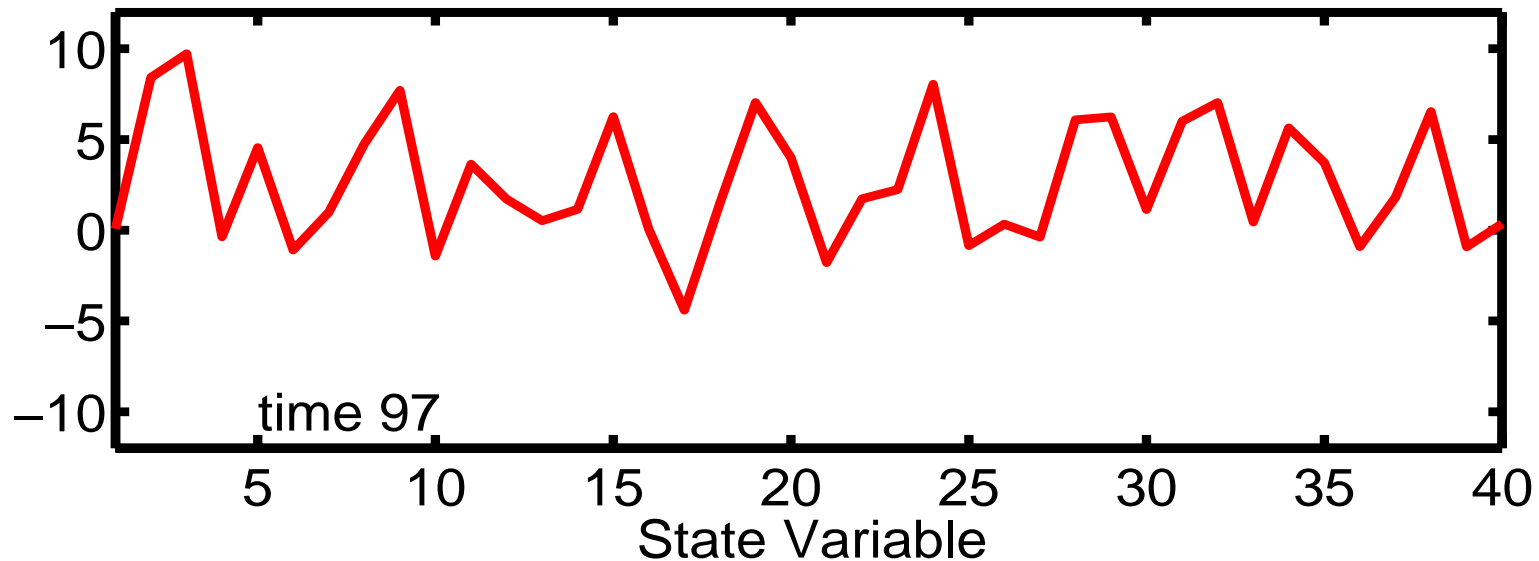


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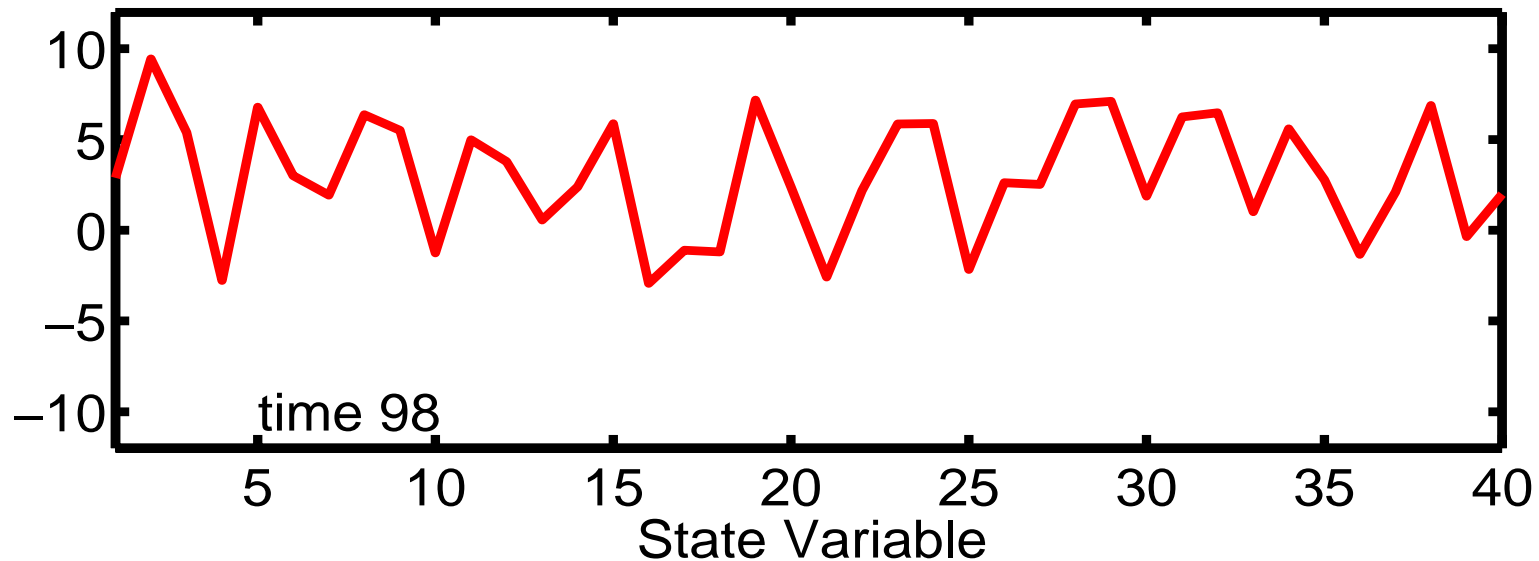


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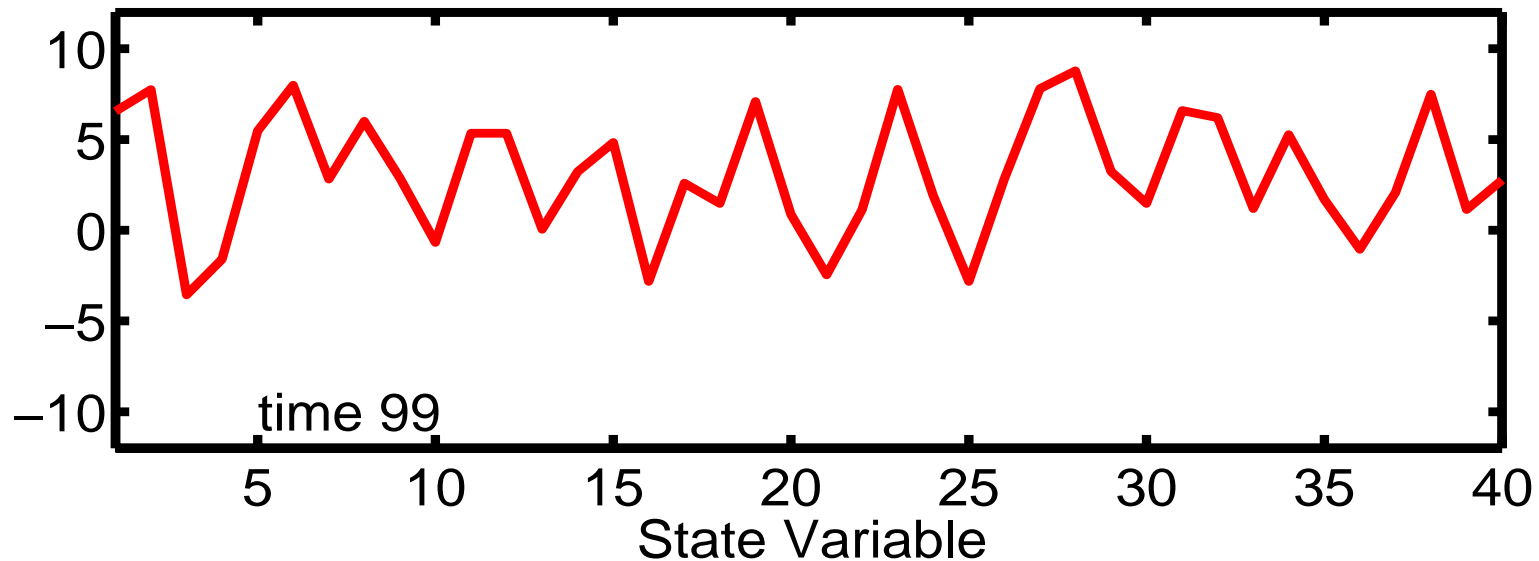


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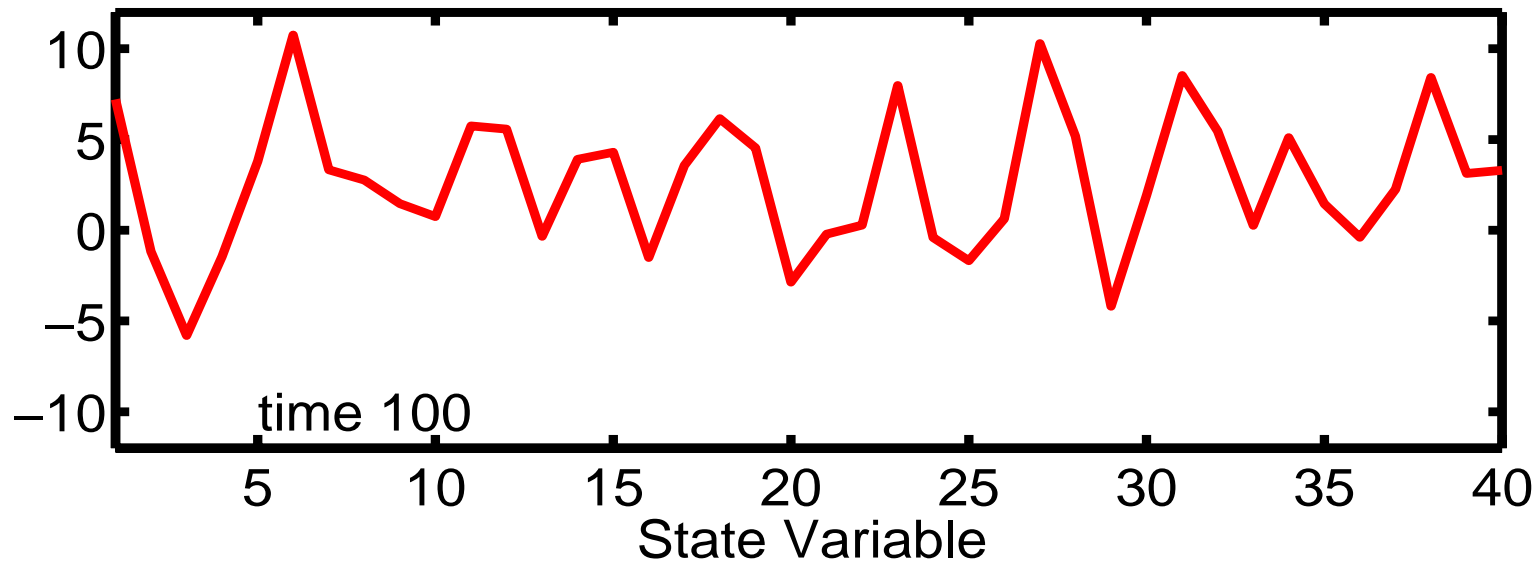


Ensemble Filter for Lorenz-96 40-Variable Model

40 state variables: X_1, X_2, \dots, X_{40} .

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

Acts ‘something’ like synoptic weather around a latitude band.

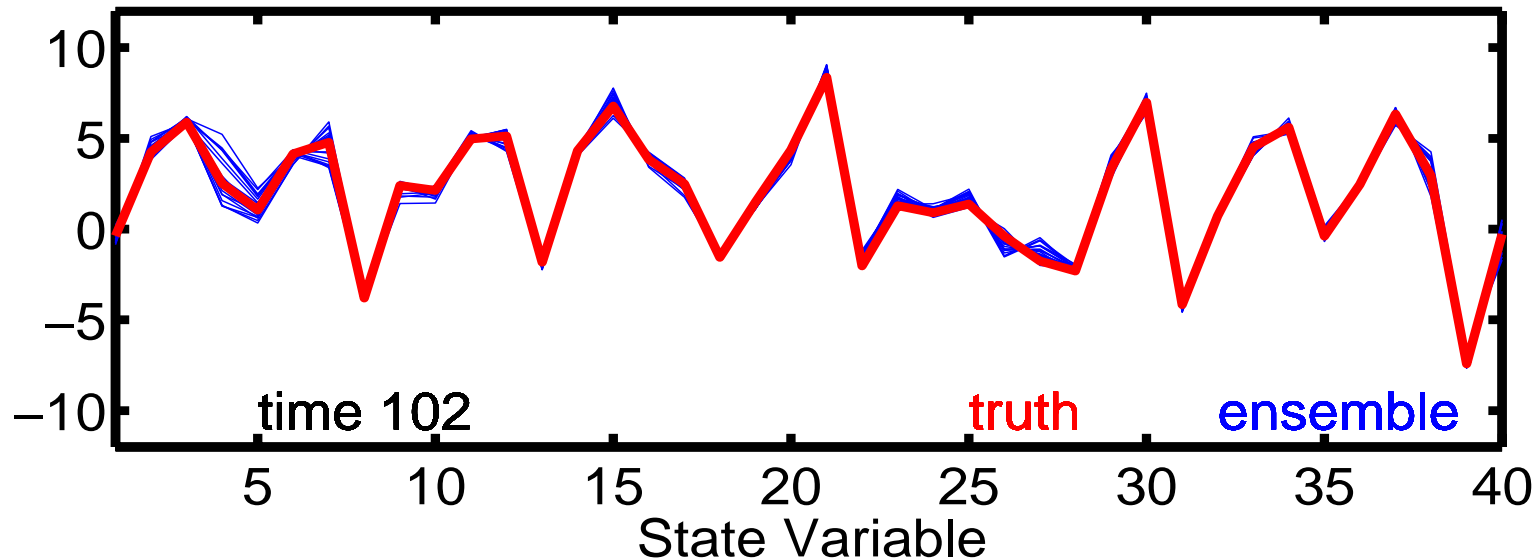


Lorenz-96 is sensitive to small perturbations

Introduce 20 'ensemble' state estimates.

Each is slightly perturbed for each X_i at time 100.

Refer to unperturbed control integration as 'truth'.

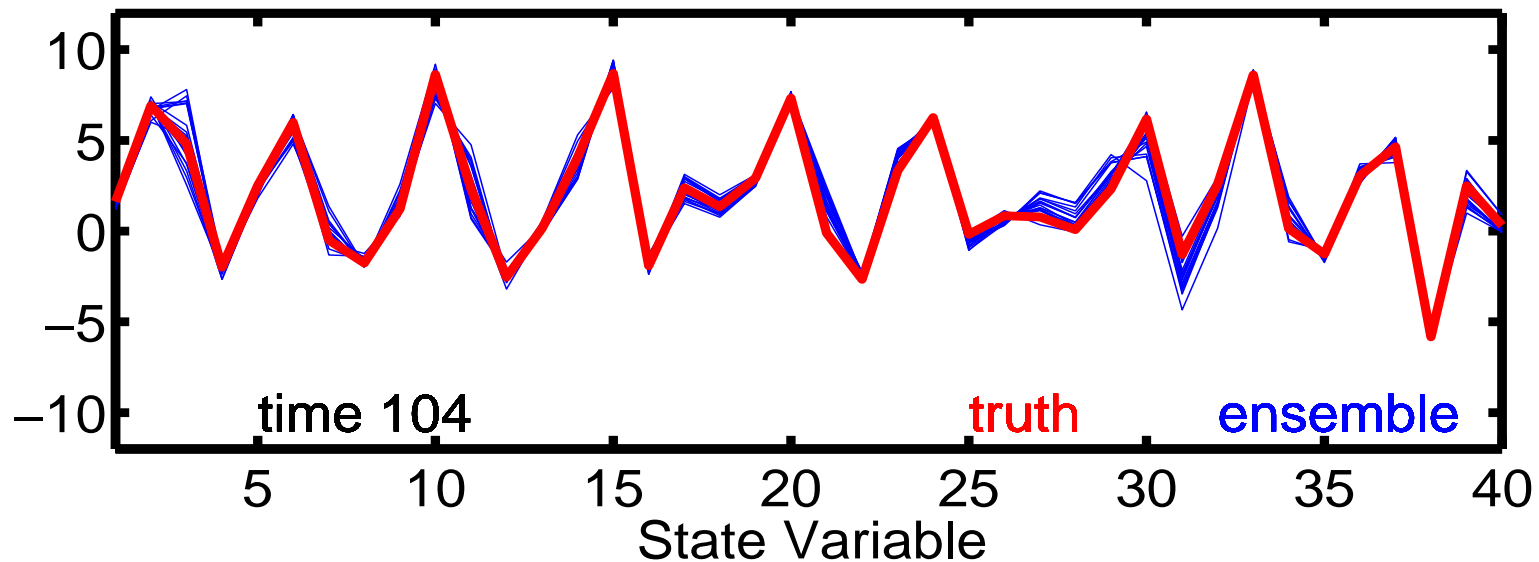


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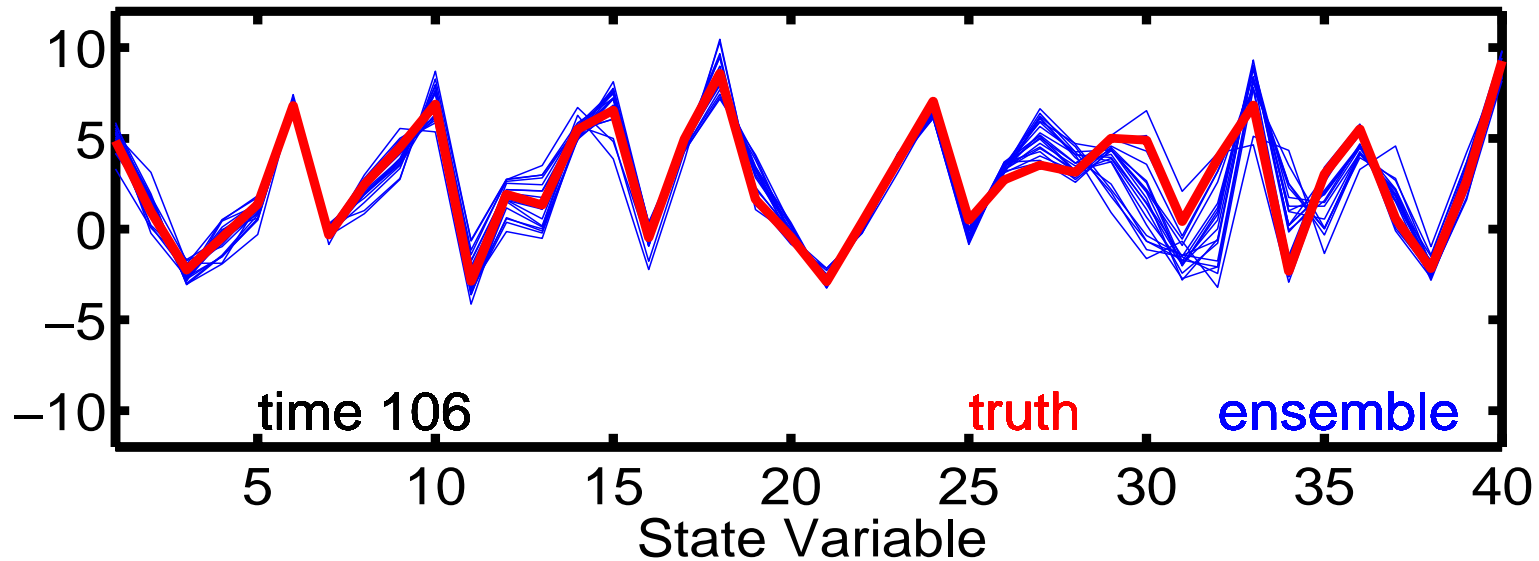


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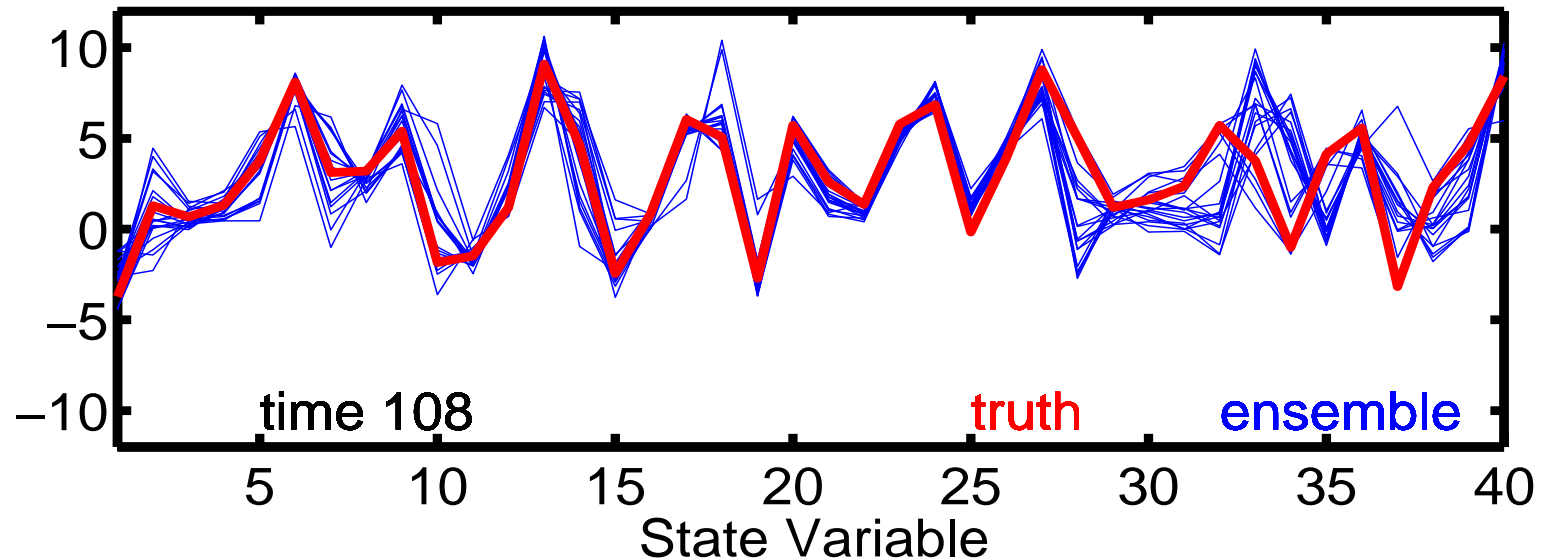


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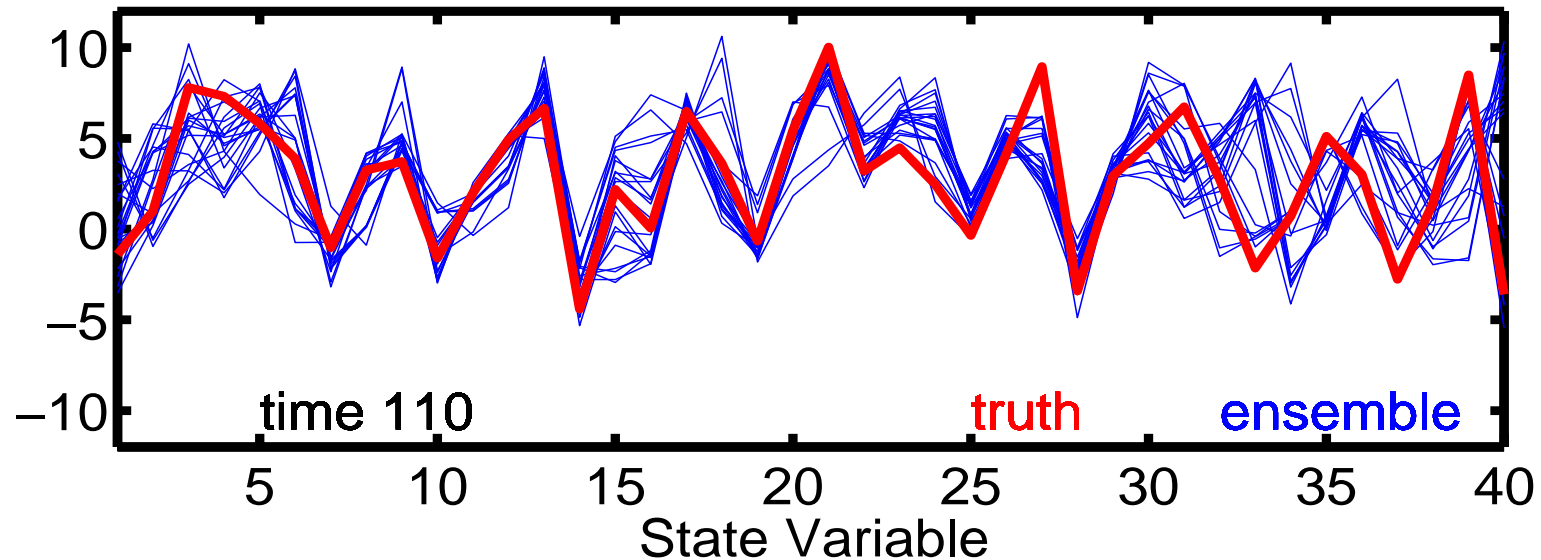


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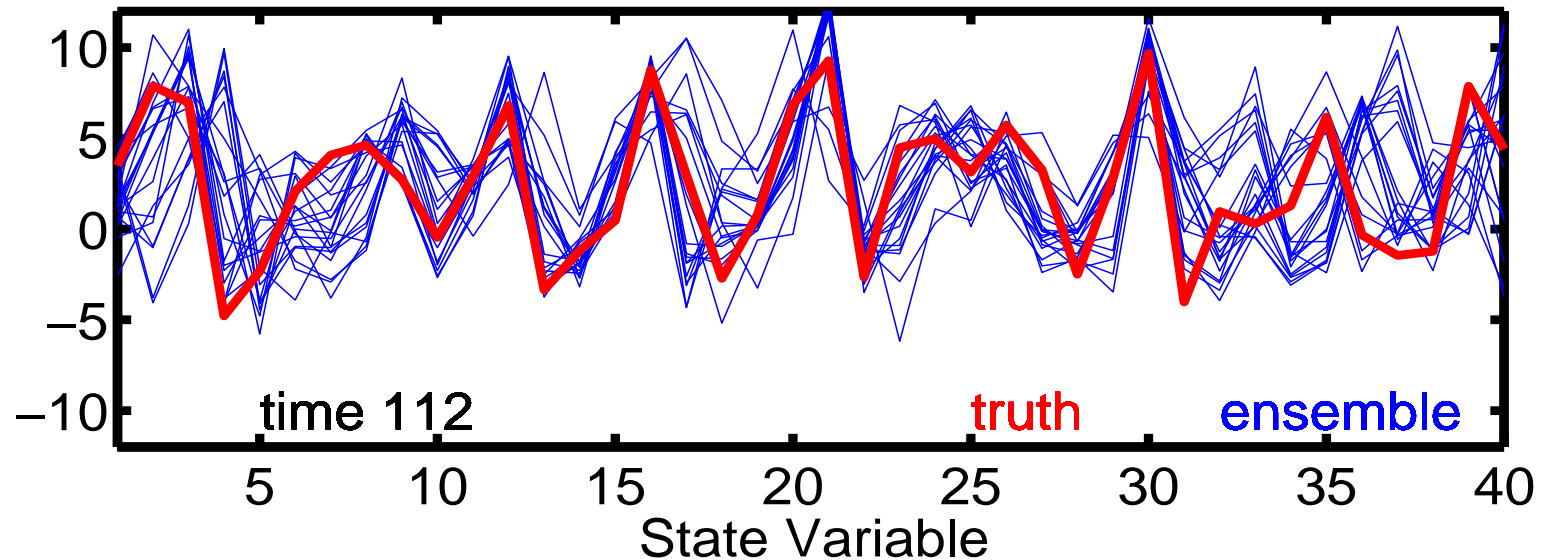


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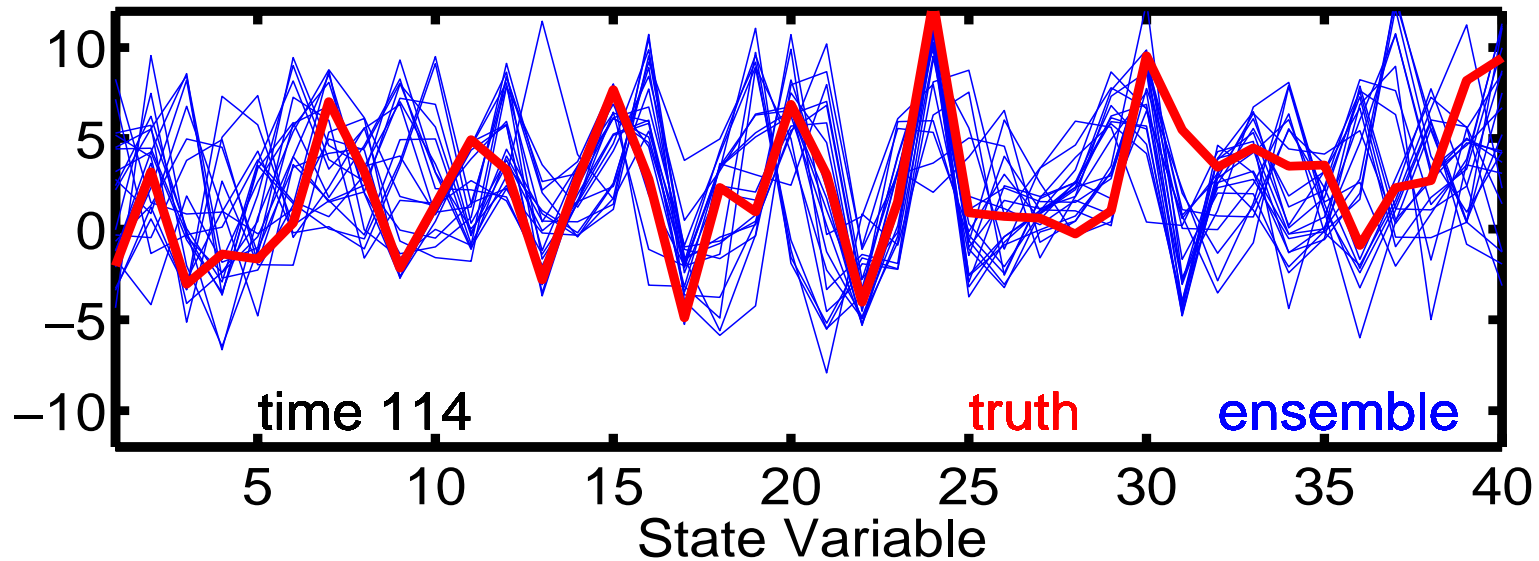


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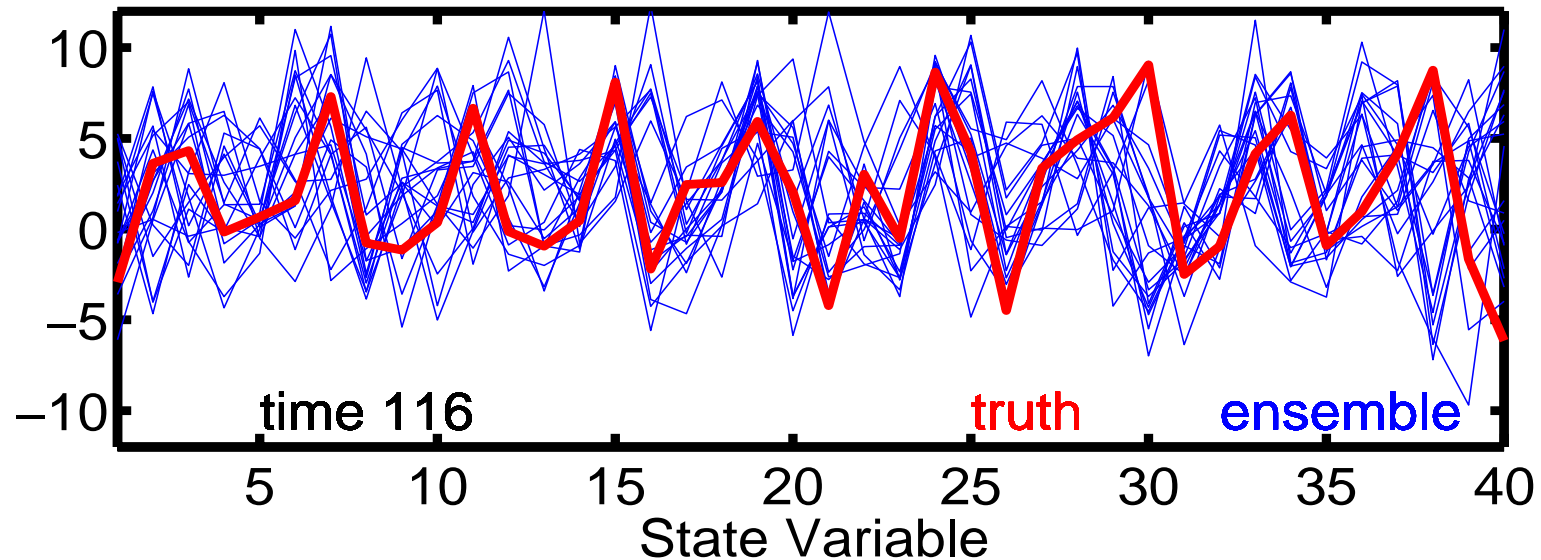


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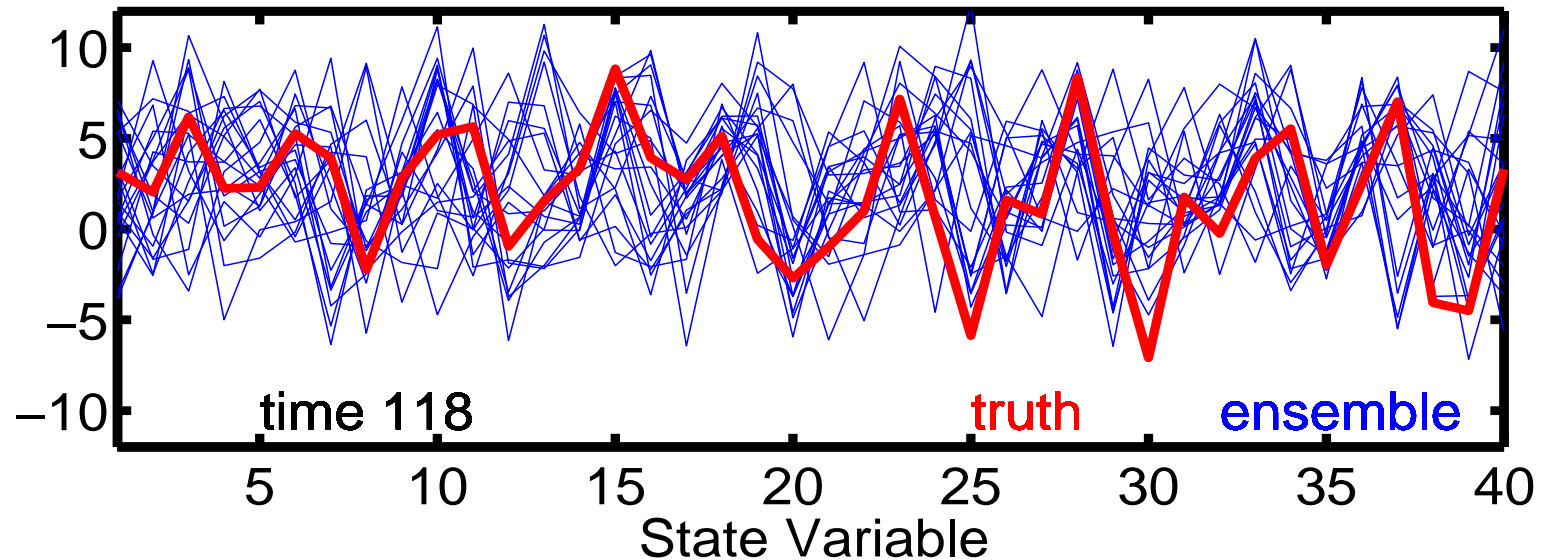


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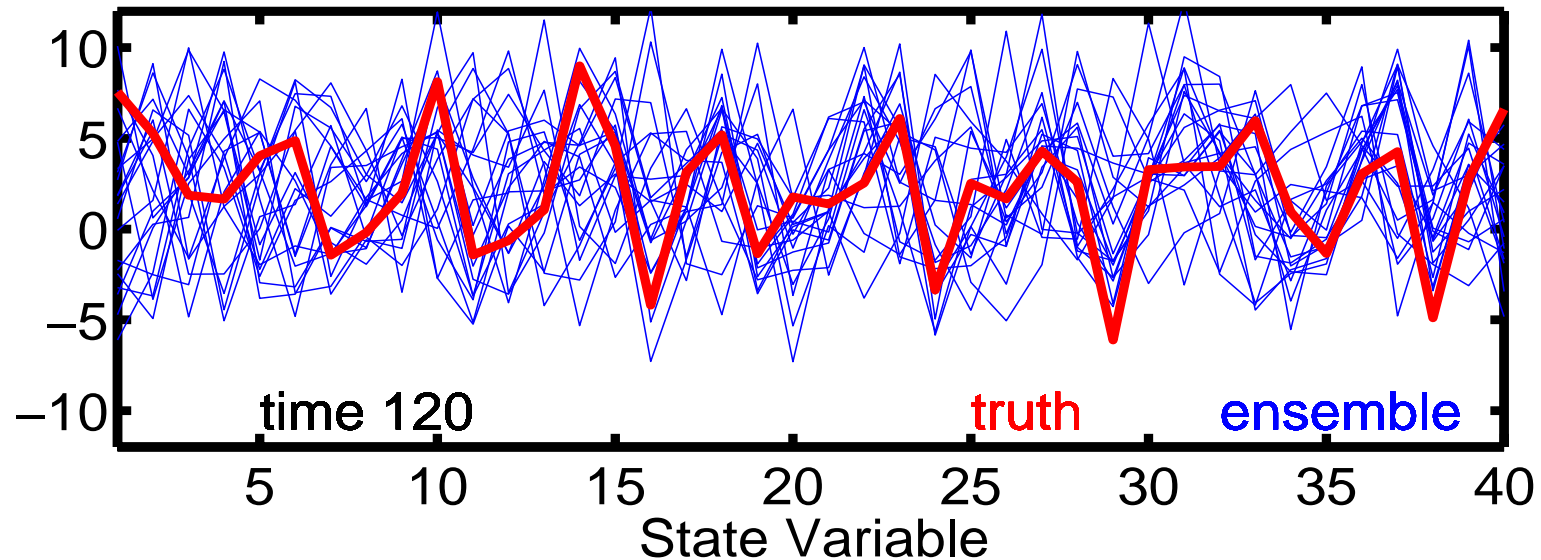


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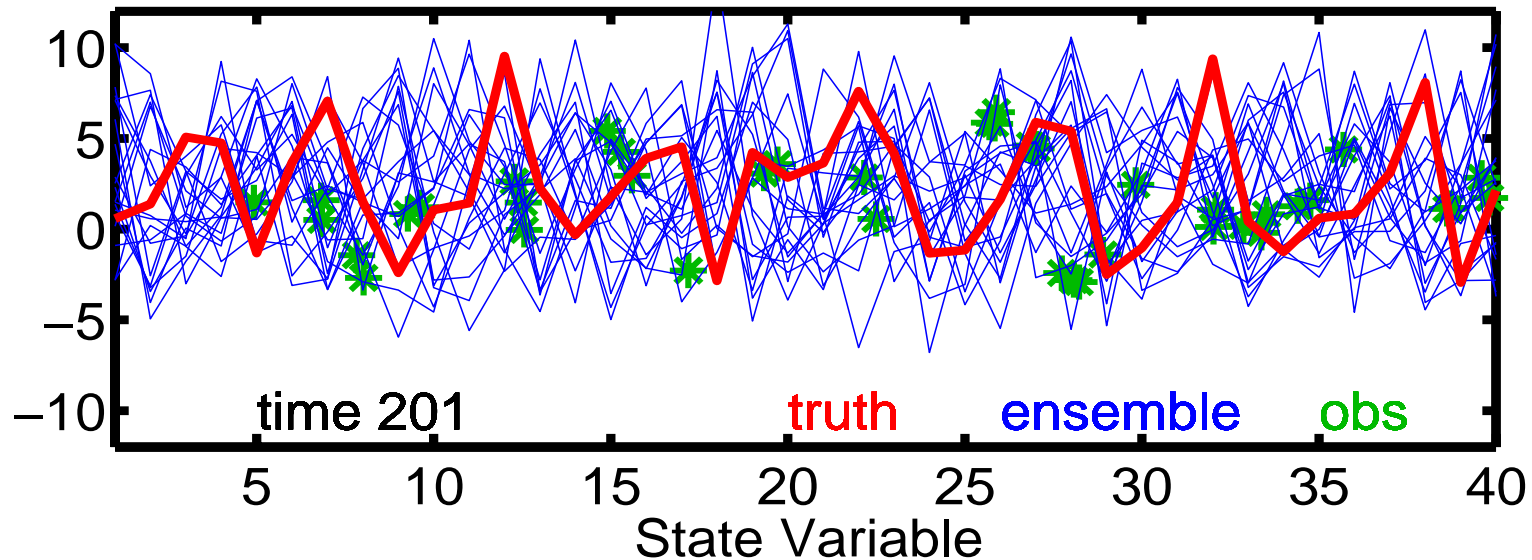


Assimilate ‘observations’ from 40 randomly located stations each step.

Observations generated by interpolating truth to station location.

Simulate observational error: Add random draw from $N(0, 1)$ to each.

Start from ‘climatological’ 20-member ensemble.

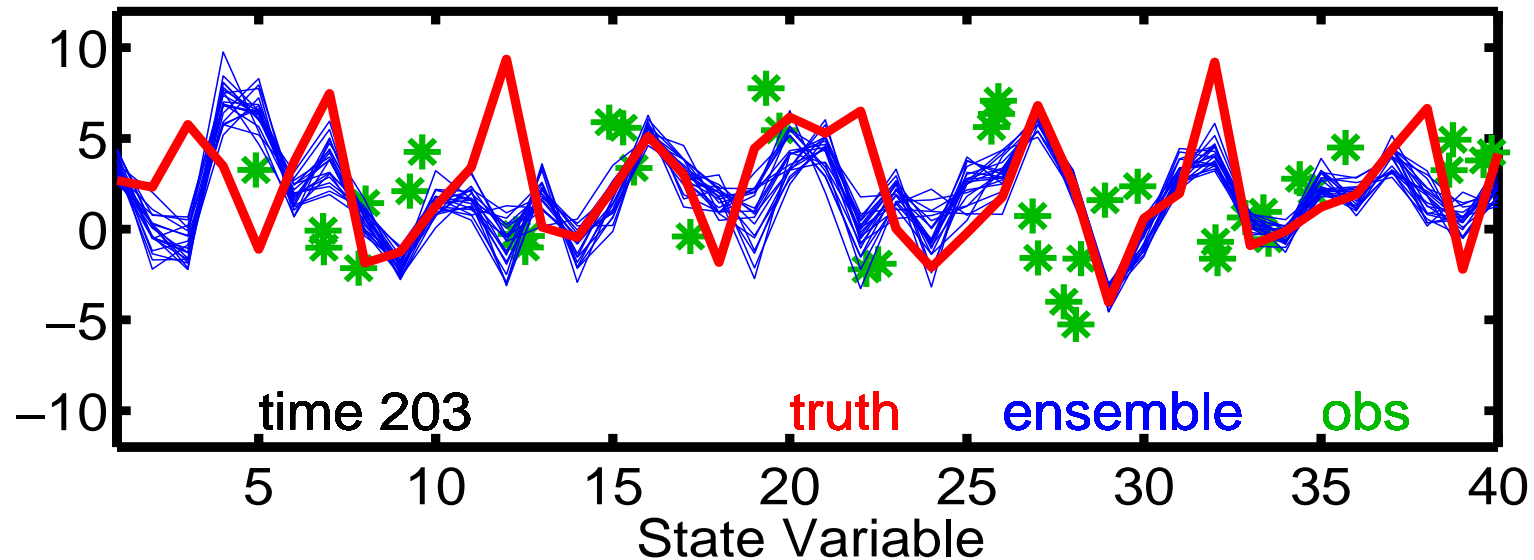


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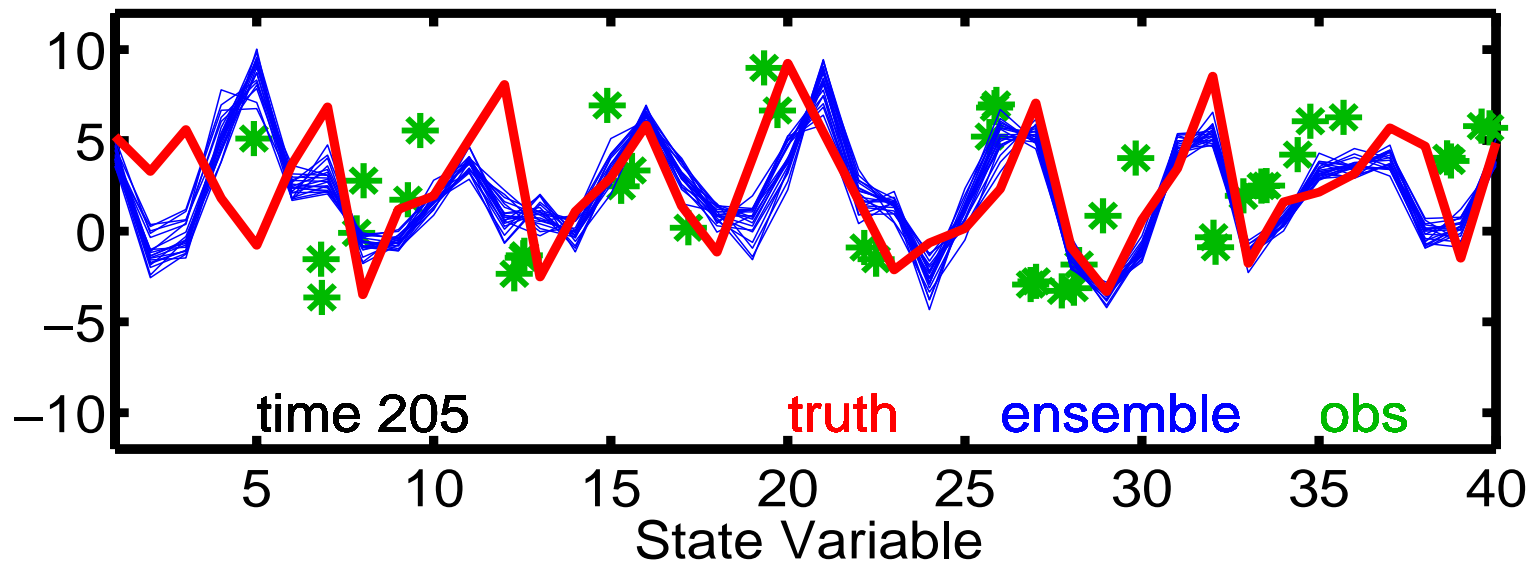


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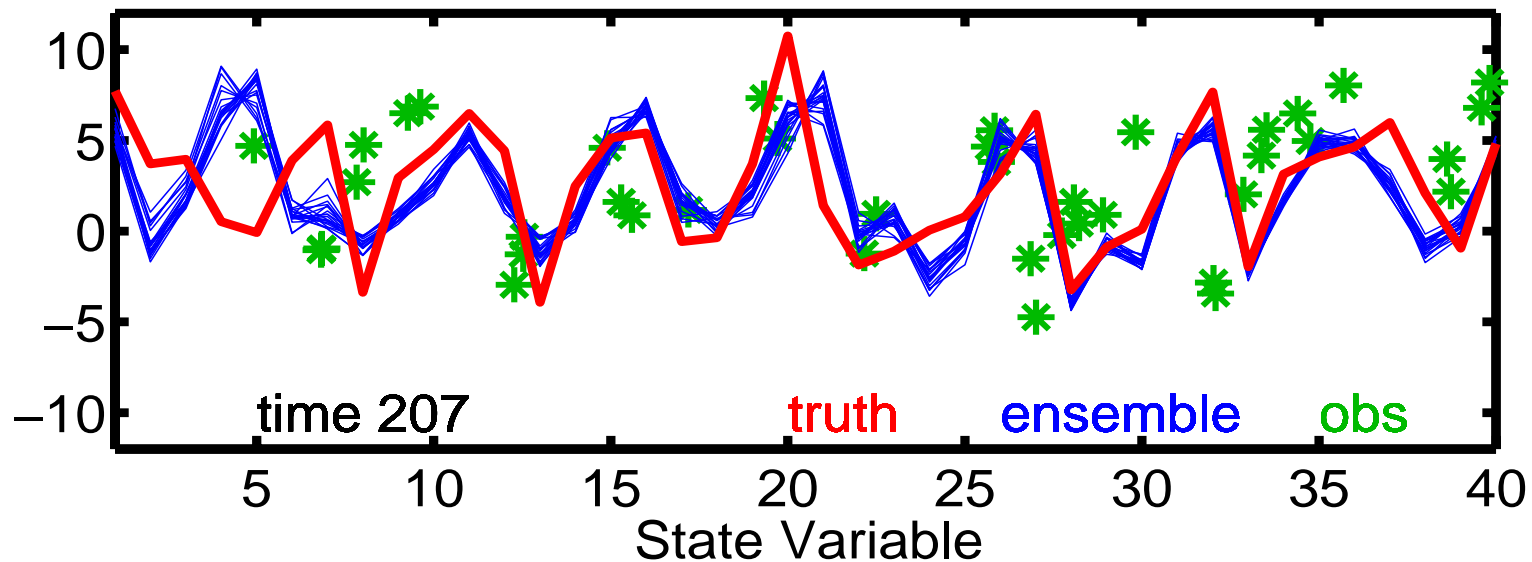


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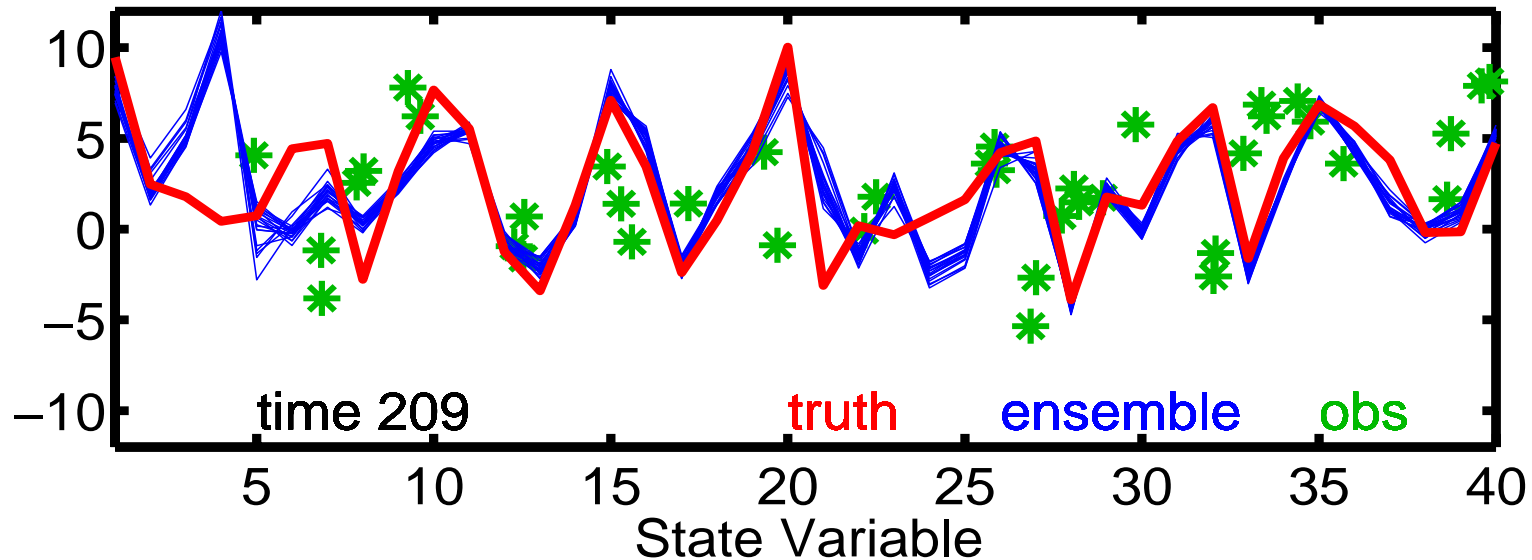


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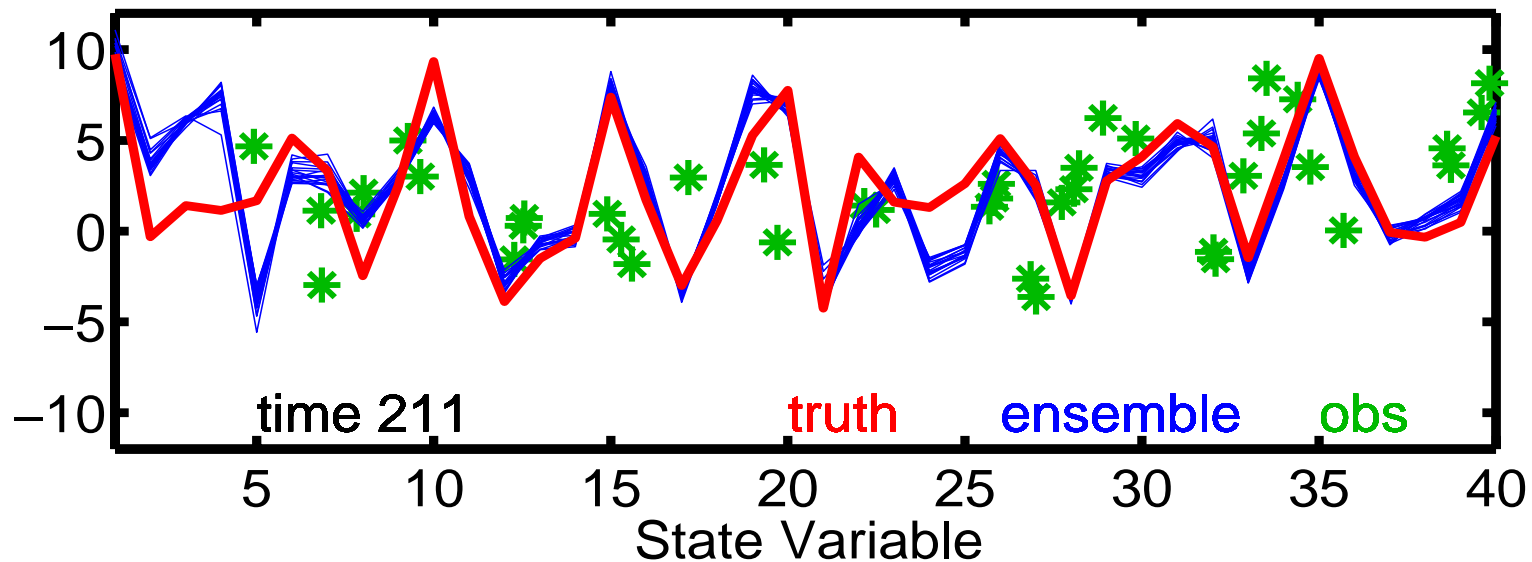


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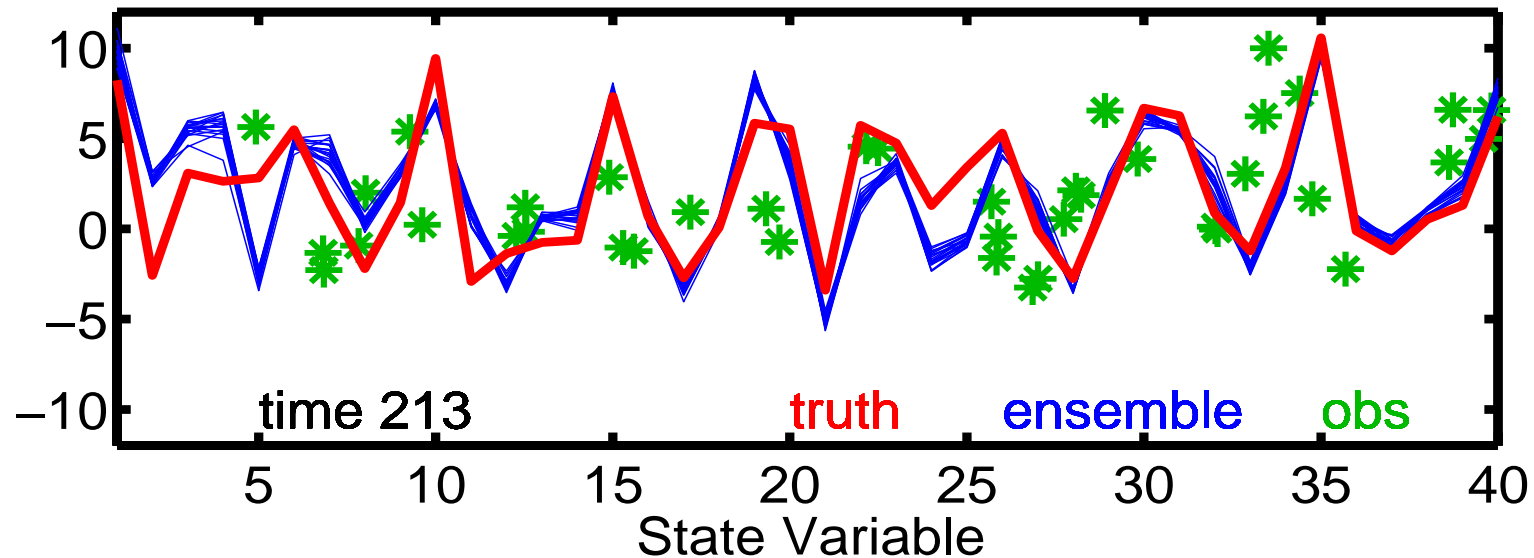


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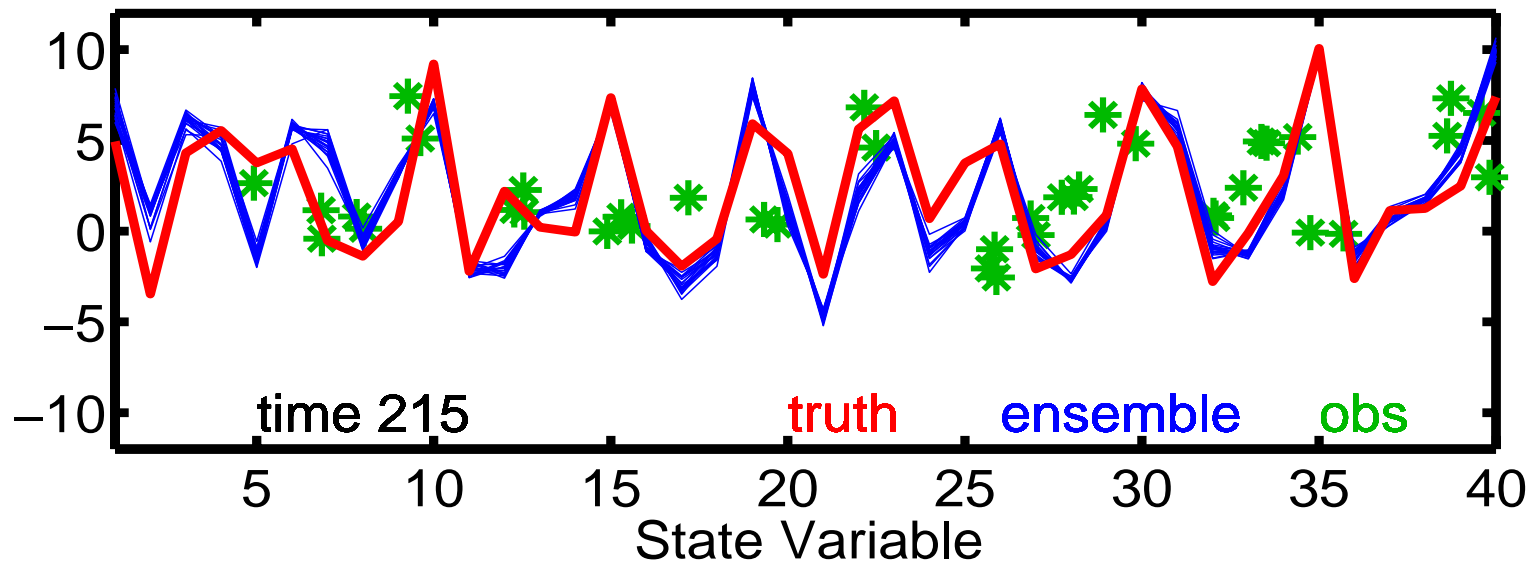


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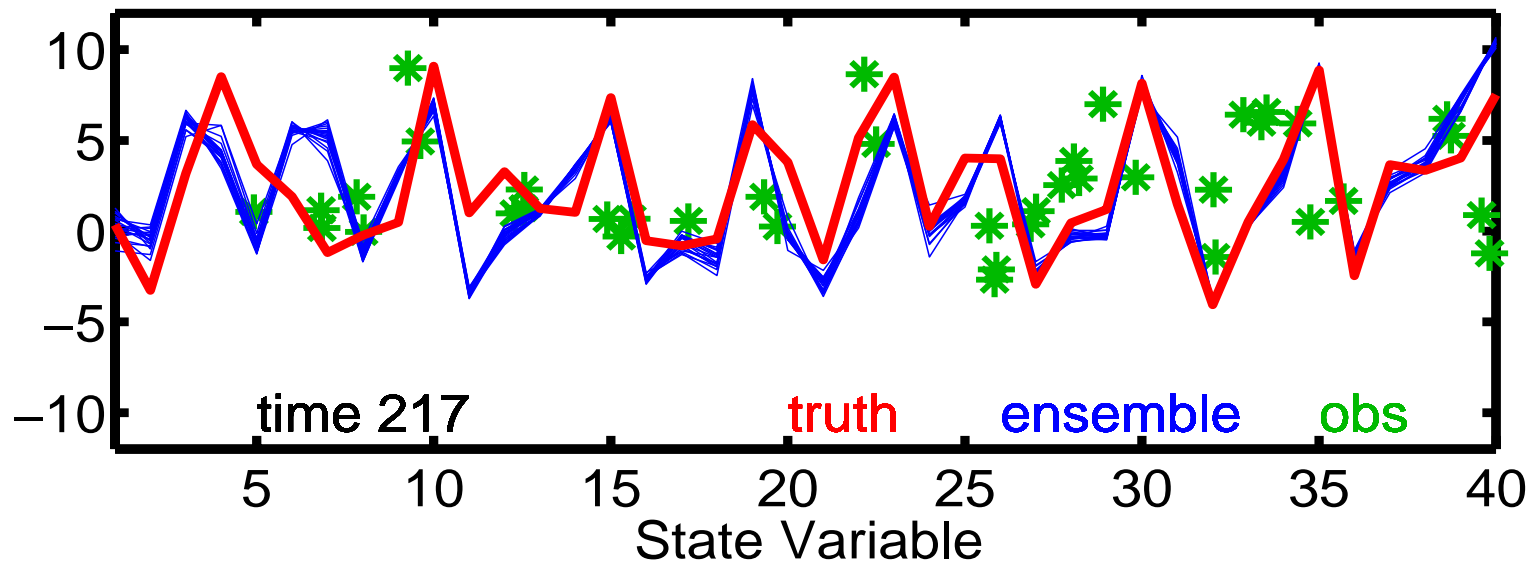


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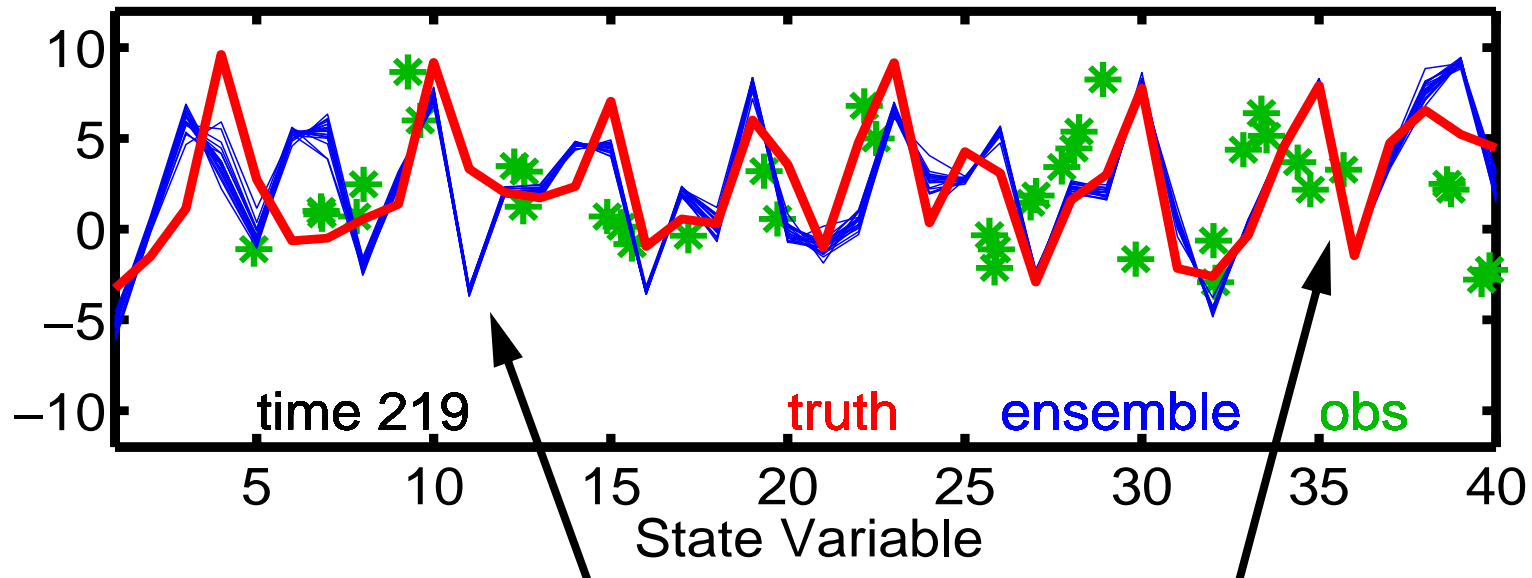


Assimilate 'observations' from 40 randomly located stations each step.

This isn't working very well.

Ensemble spread is reduced, but....

Ensemble is inconsistent with truth most places.



Confident and WRONG.

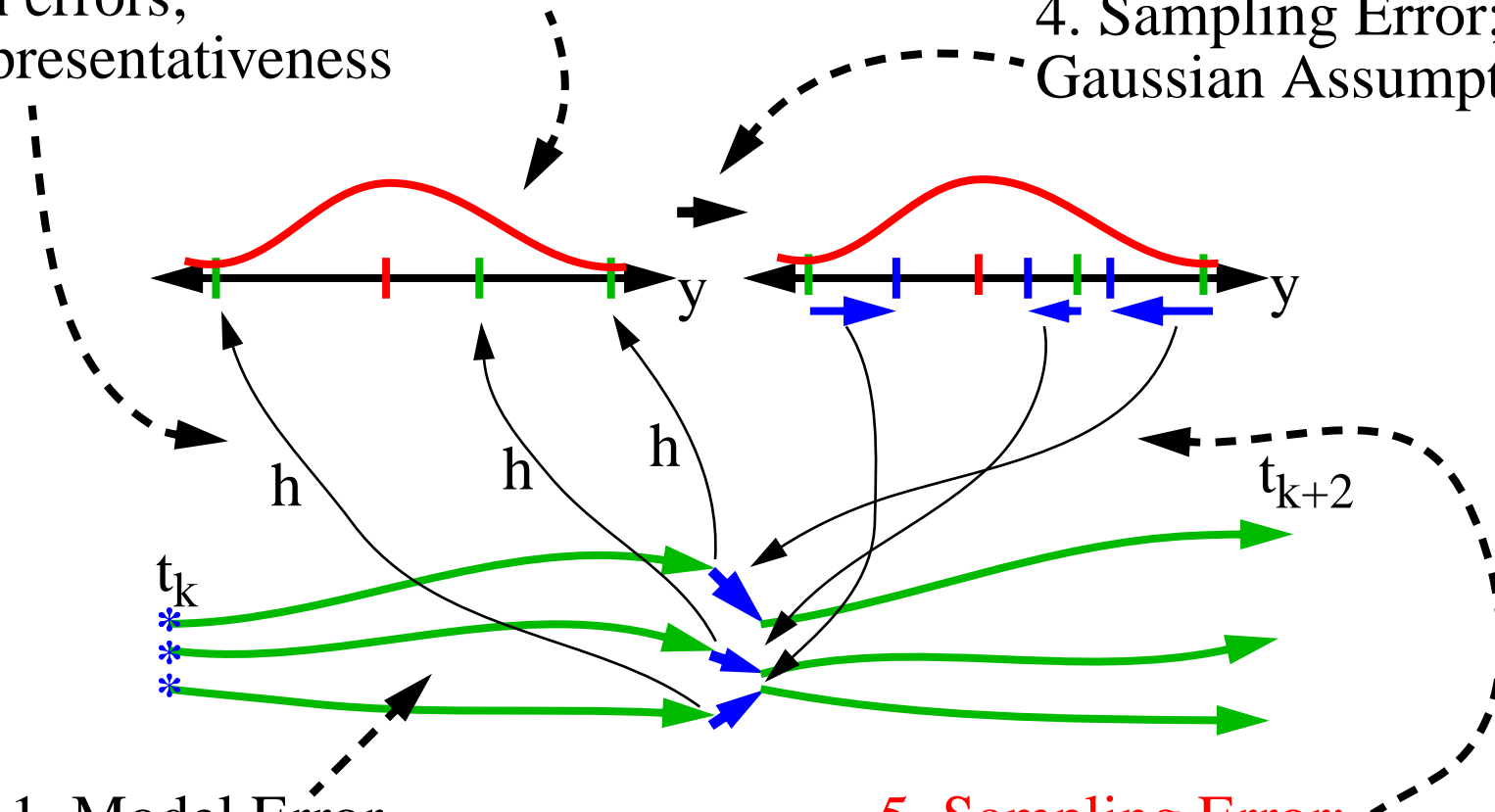
Confident and right!

Some Error Sources in Ensemble Filters

3. 'Gross' Obs. Errors

2. h errors;
Representativeness

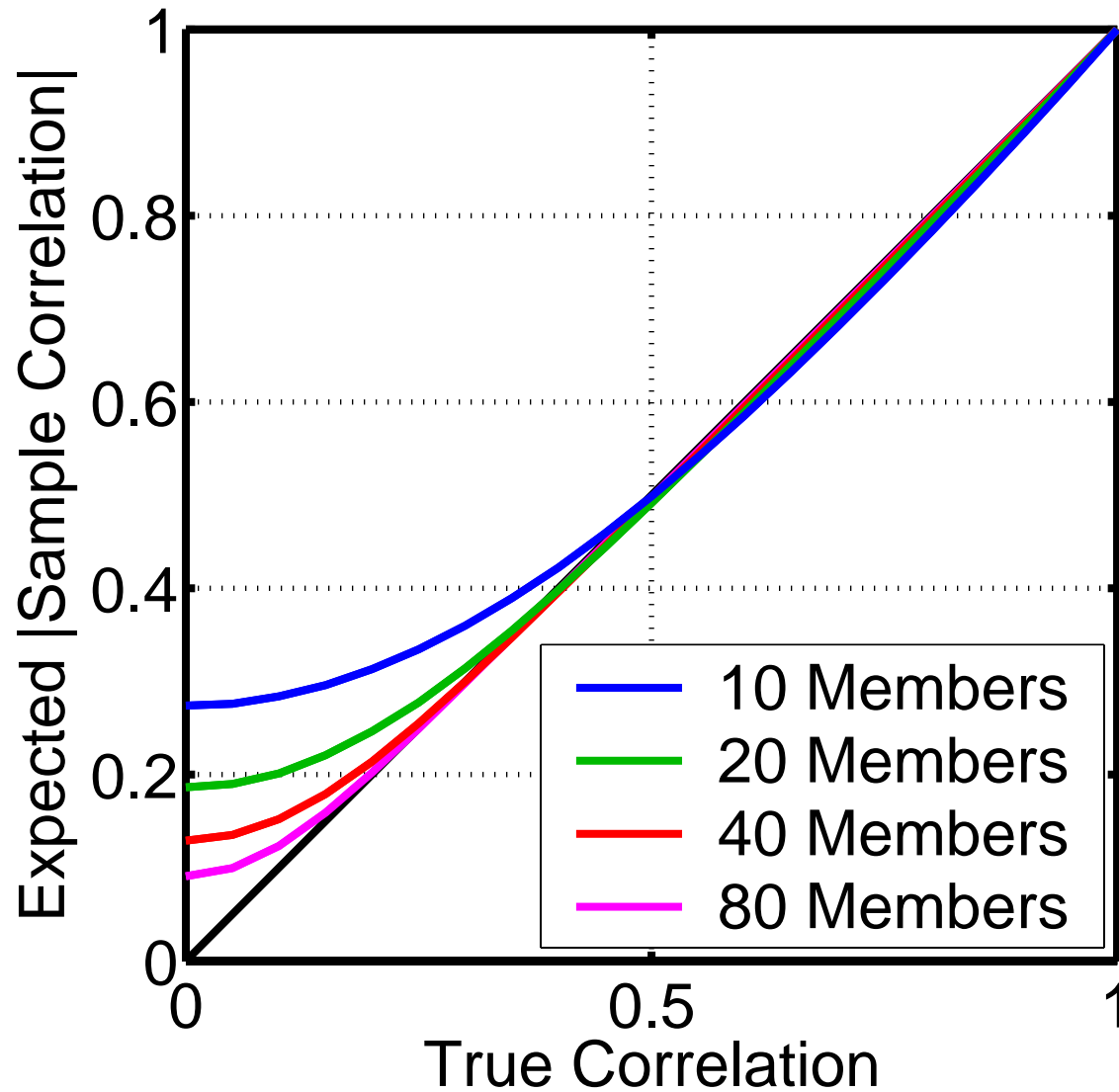
4. Sampling Error;
Gaussian Assumption



1. Model Error

5. Sampling Error;
Assuming Linear
Statistical Relation

Observations impact unrelated state variables through sampling error.



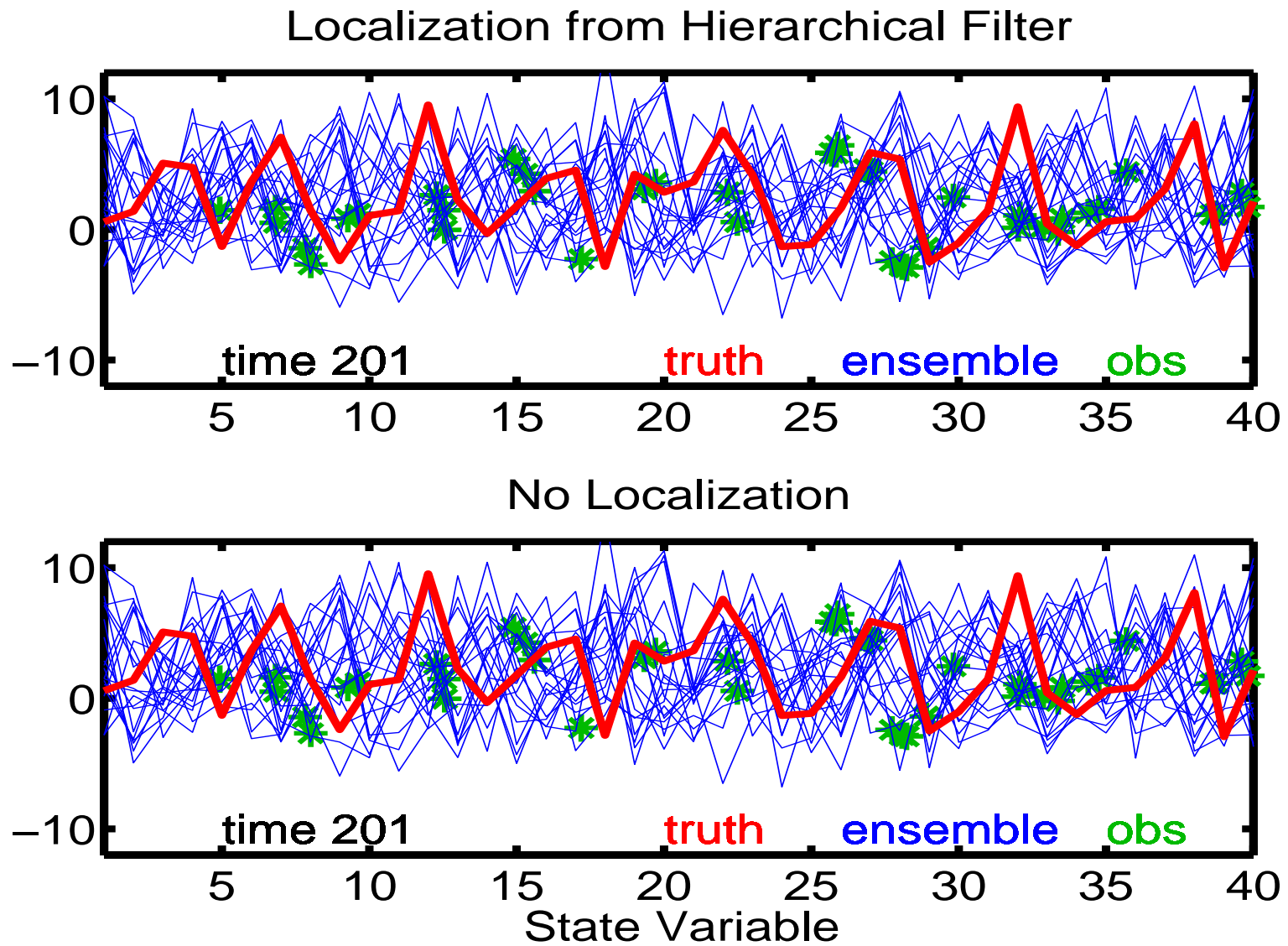
Plot shows expected absolute value of sample correlation vs. true correlation.

Unrelated obs. reduce spread, increase error.

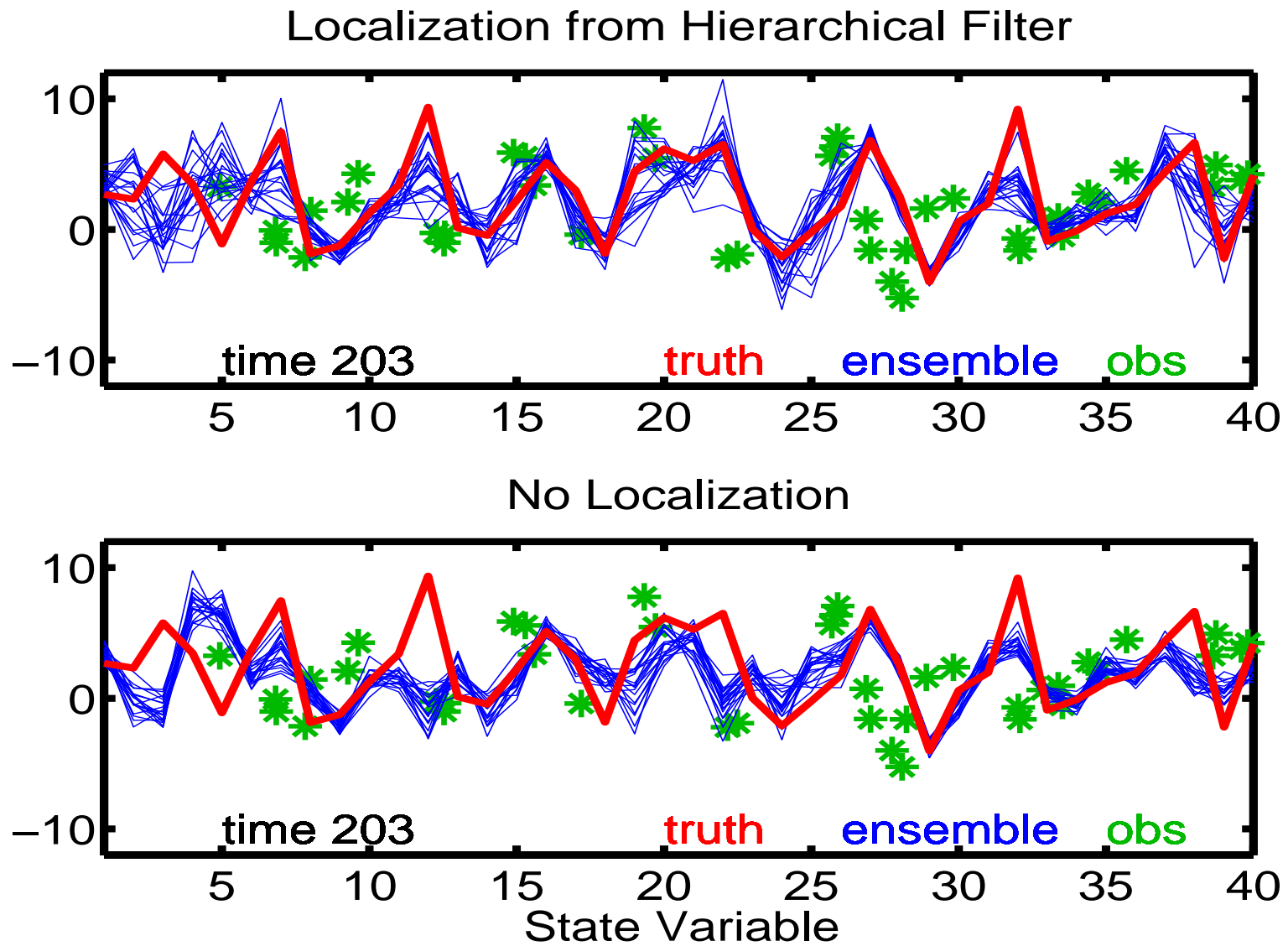
Attack with localization.

Don't let obs. impact unrelated state.

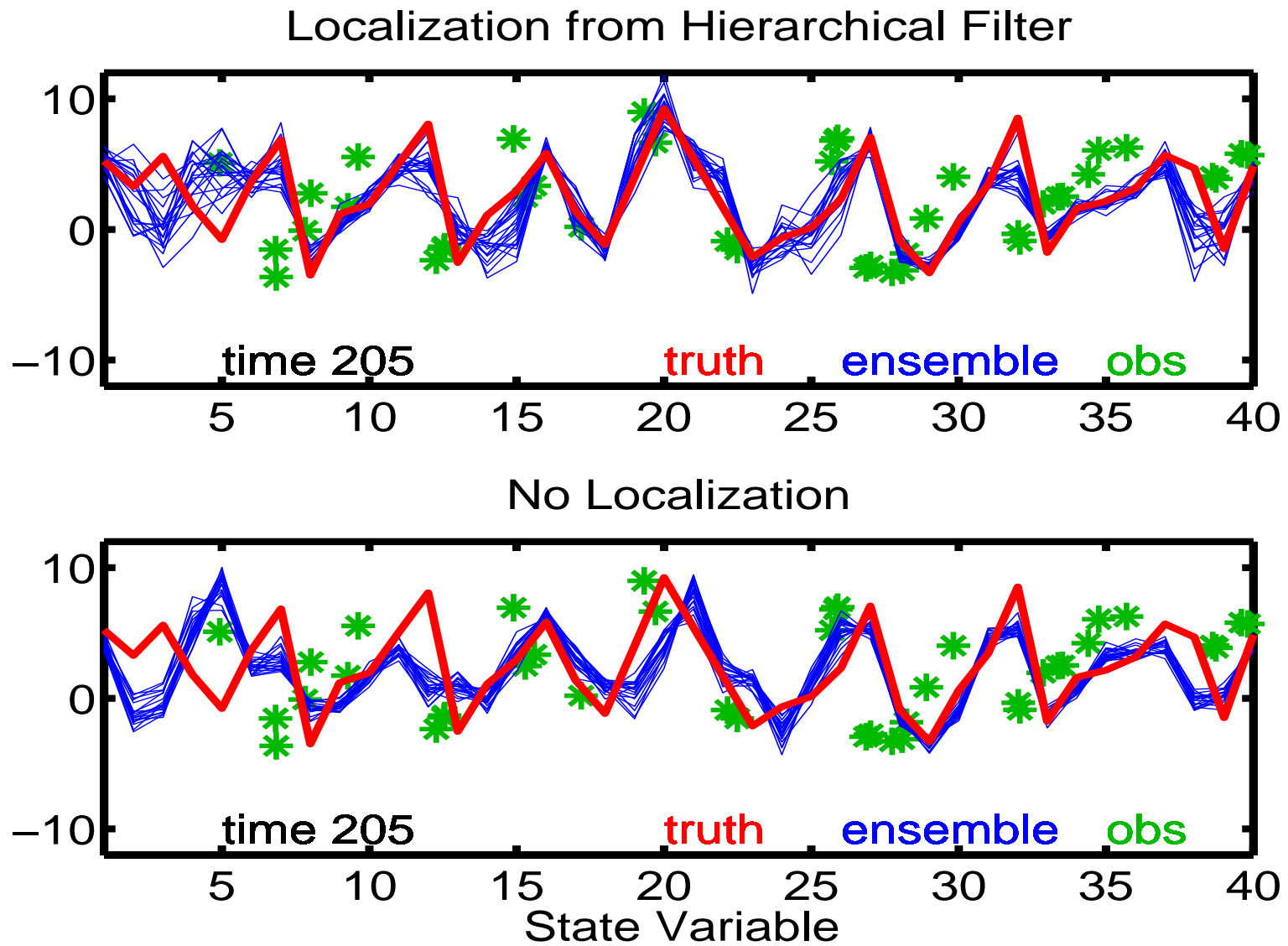
Lorenz-96 Assimilation with localization of observation impact.



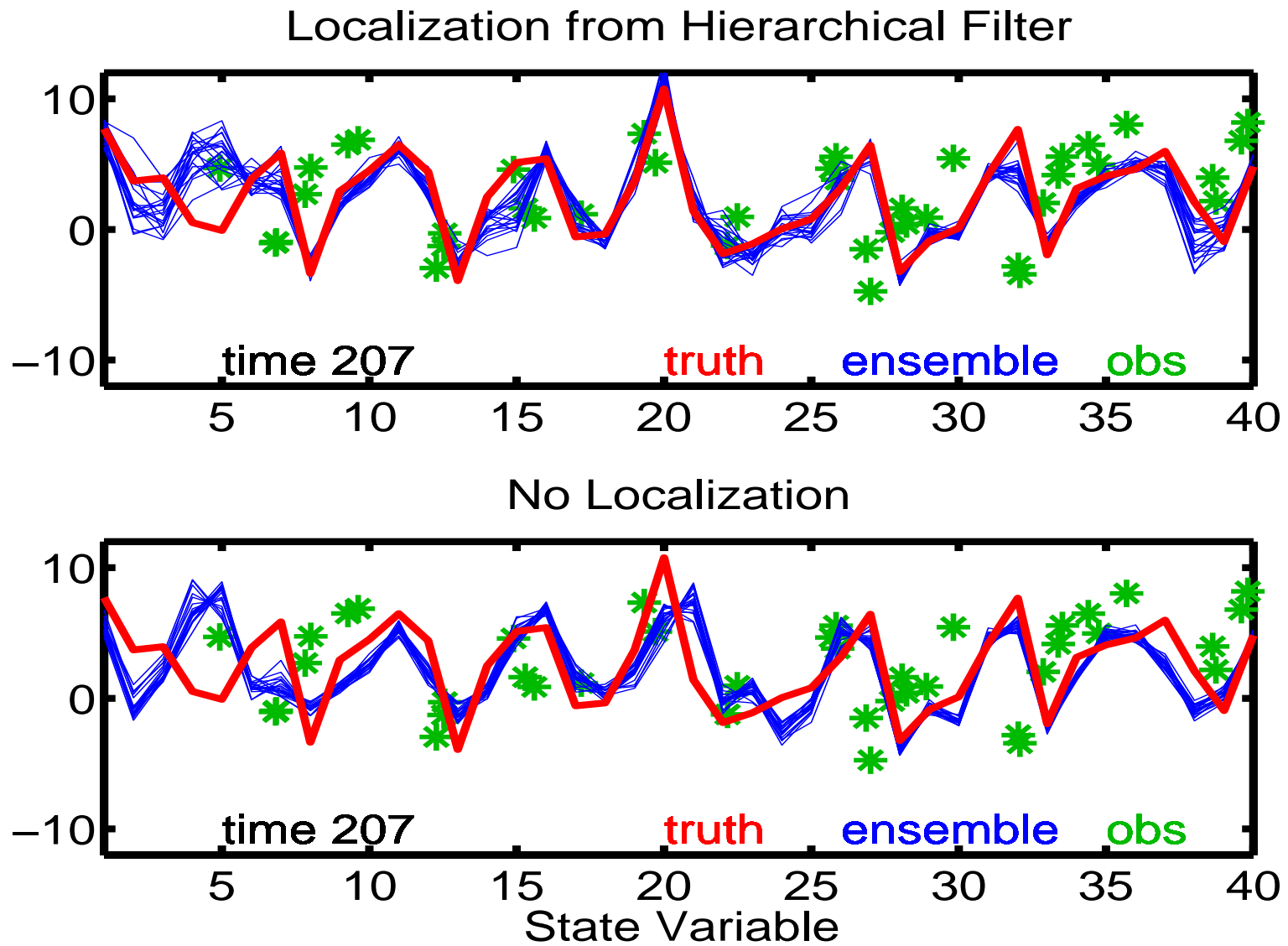
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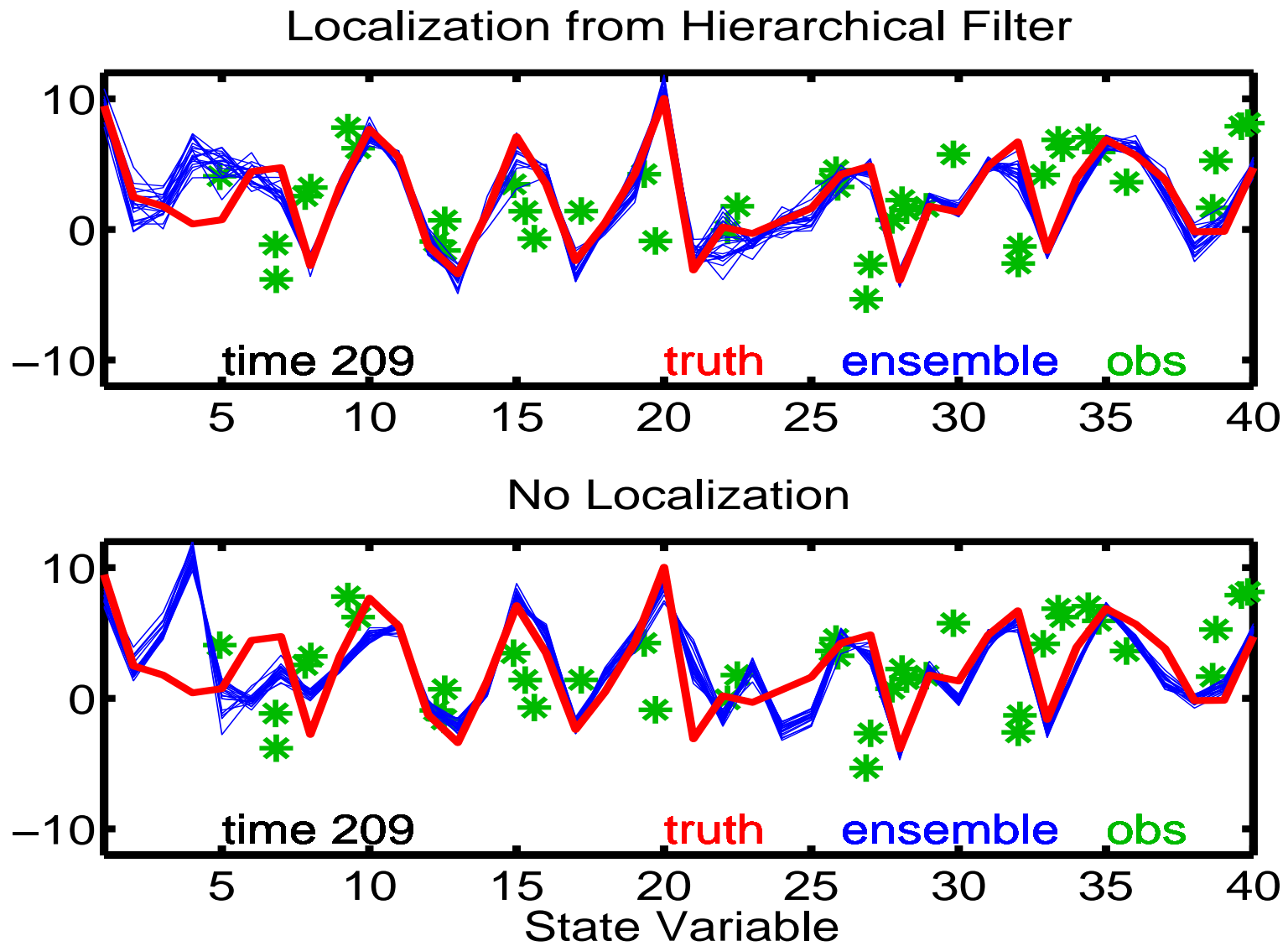
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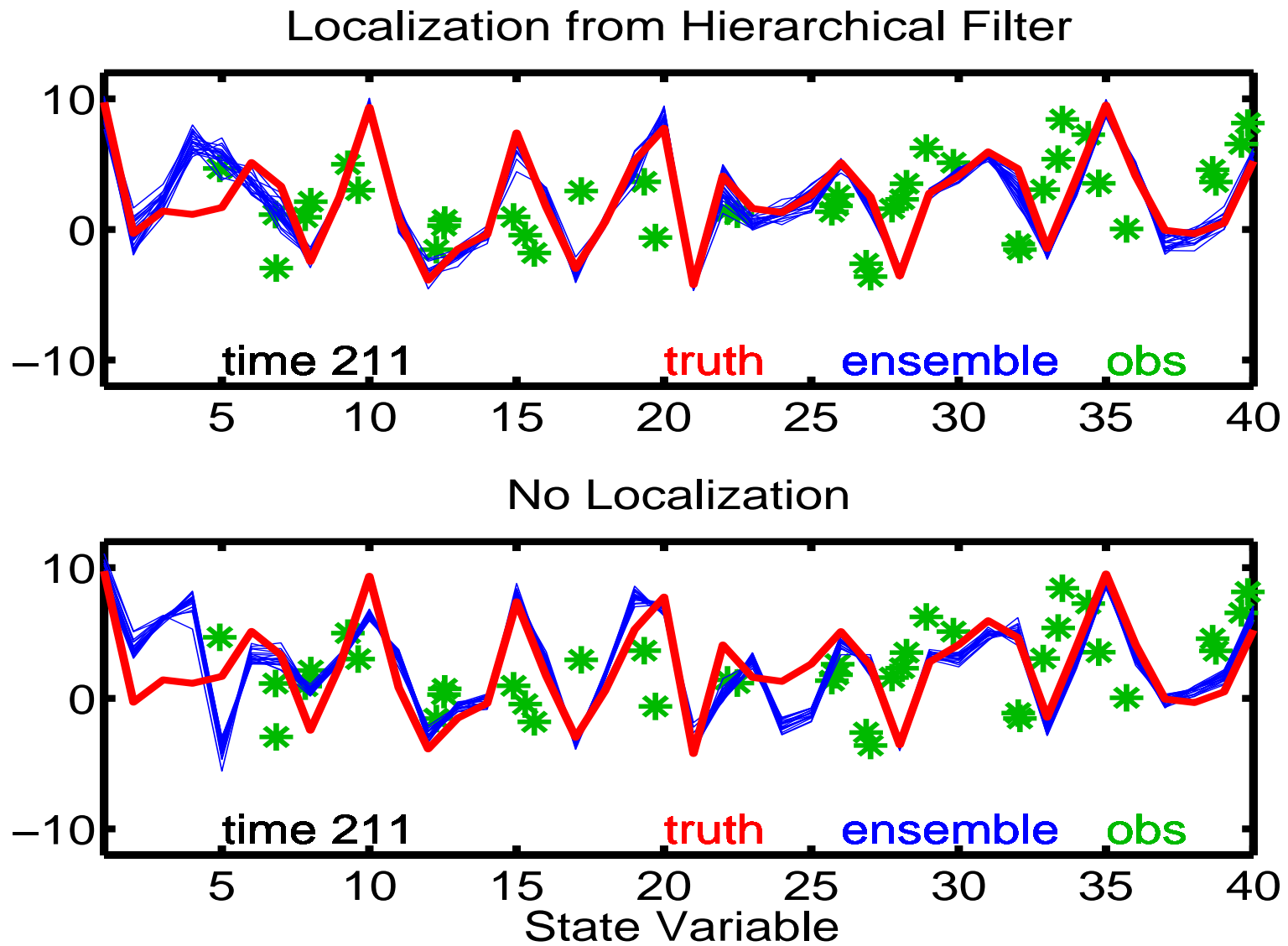
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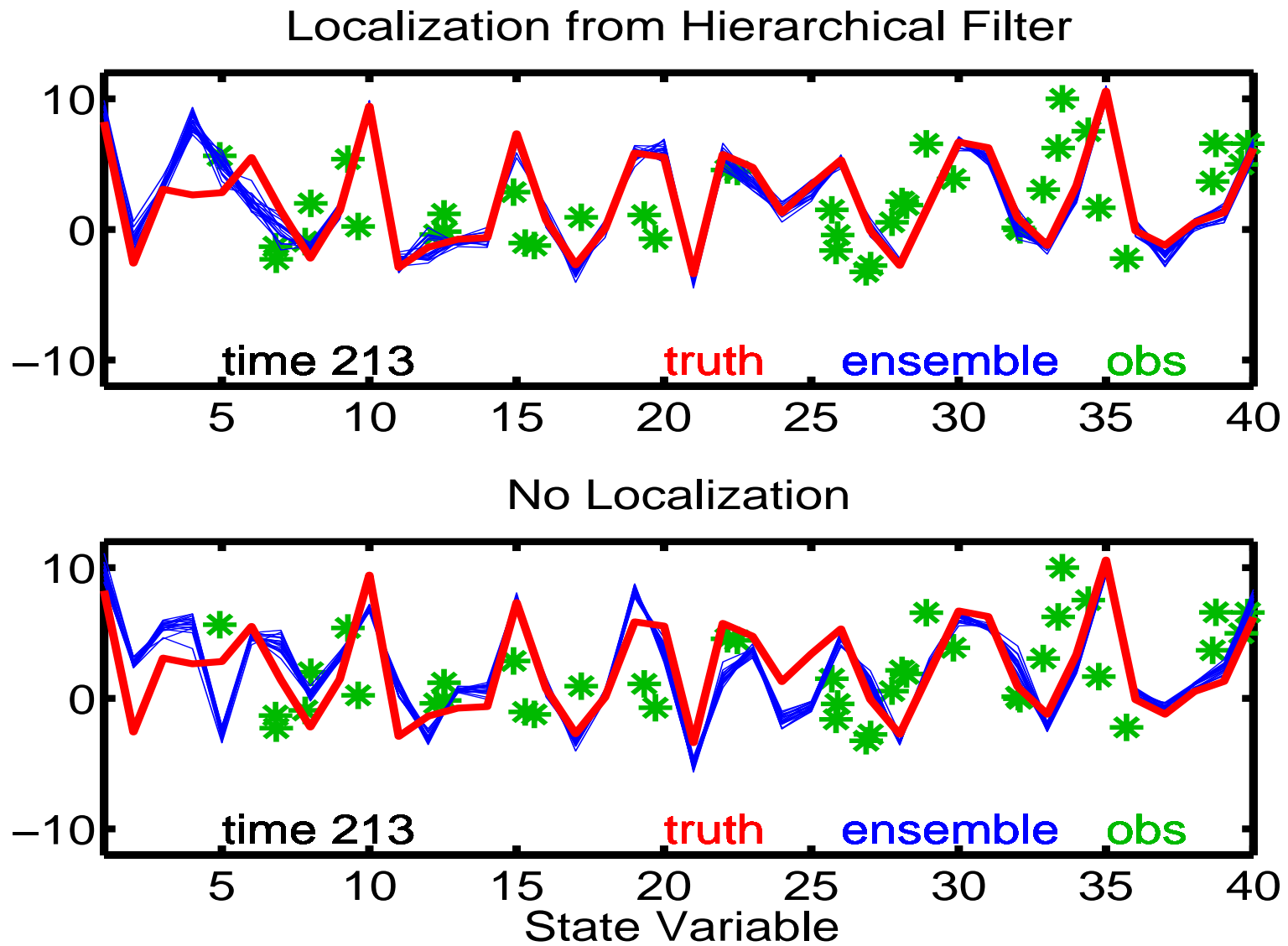
Lorenz-96 Assimilation with localization of observation impact.



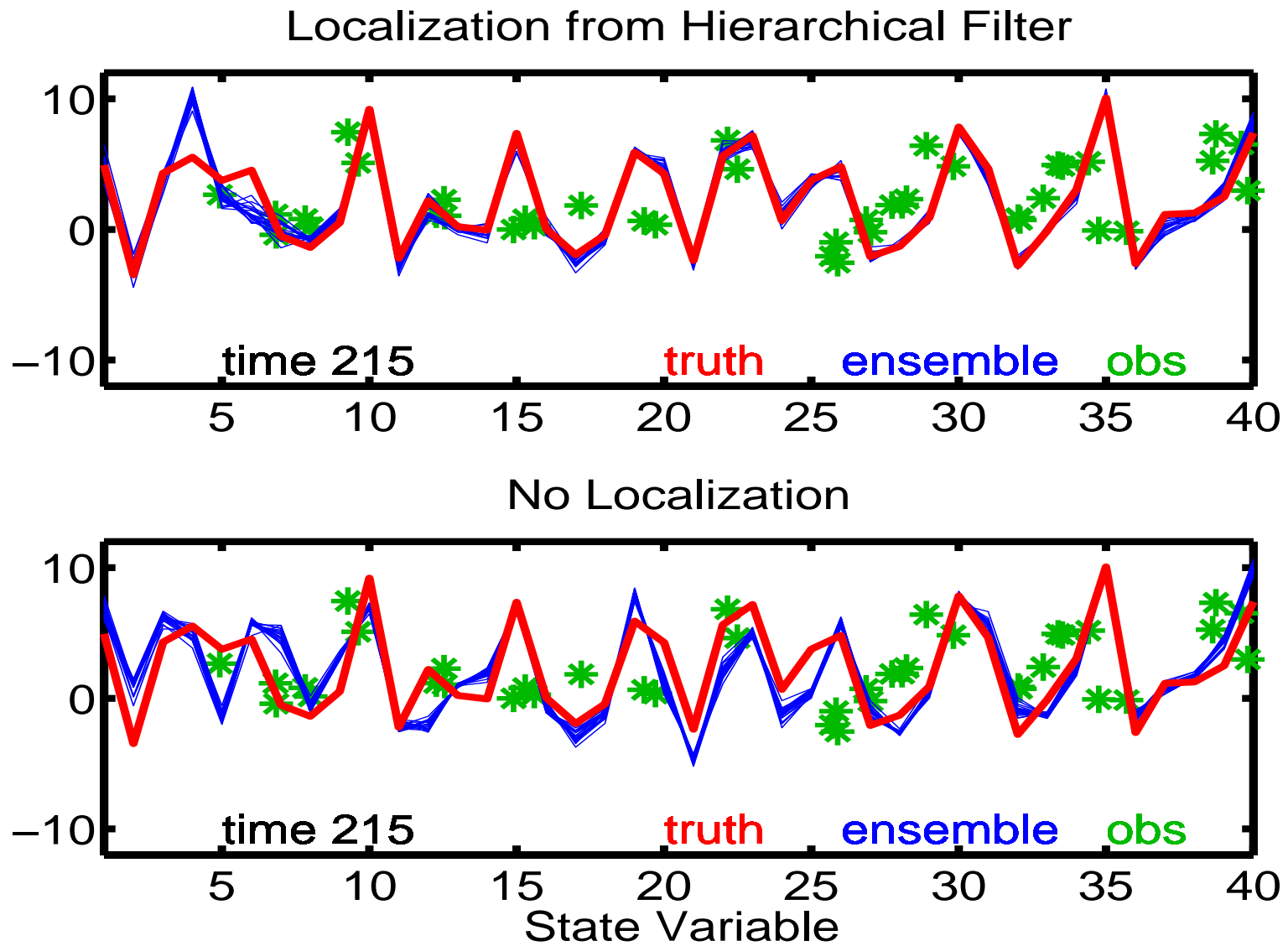
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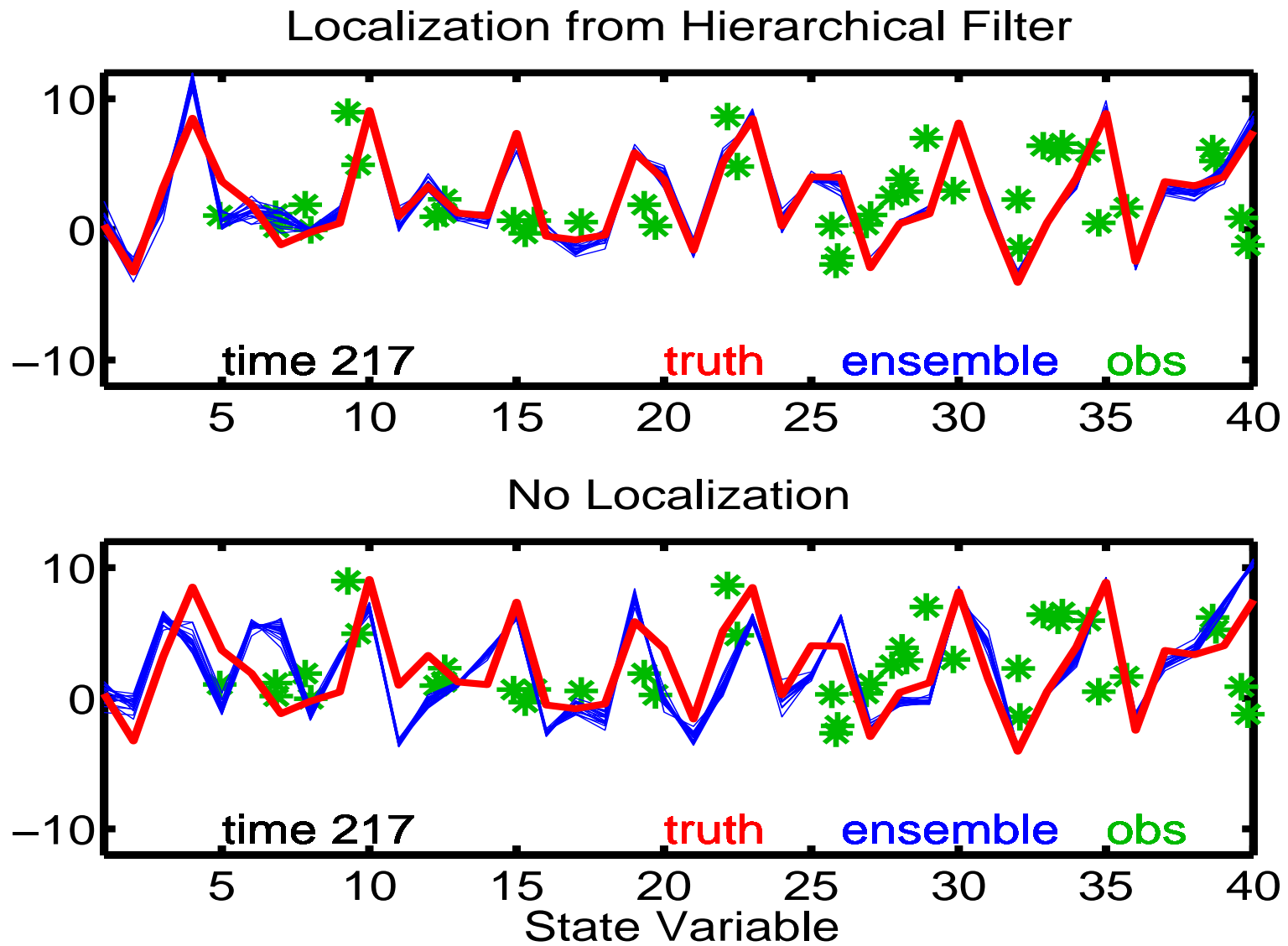
Lorenz-96 Assimilation with localization of observation impact.



Lorenz-96 Assimilation with localization of observation impact.



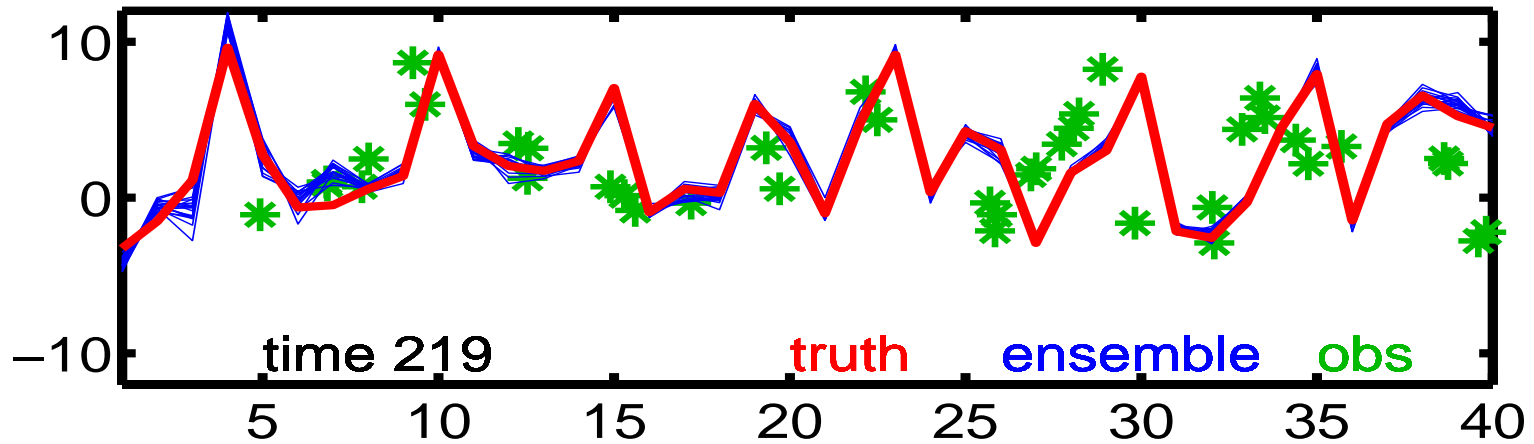
Lorenz-96 Assimilation with localization of observation impact.



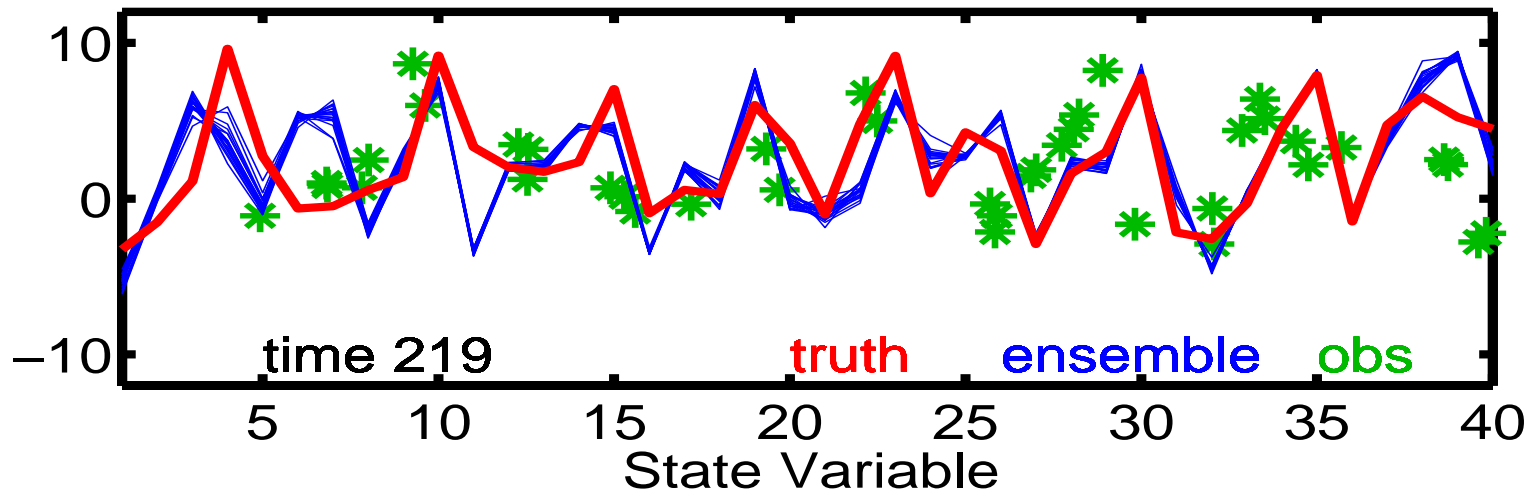
Lorenz-96 Assimilation with localization of observation impact.

Ensemble is much more consistent with truth.

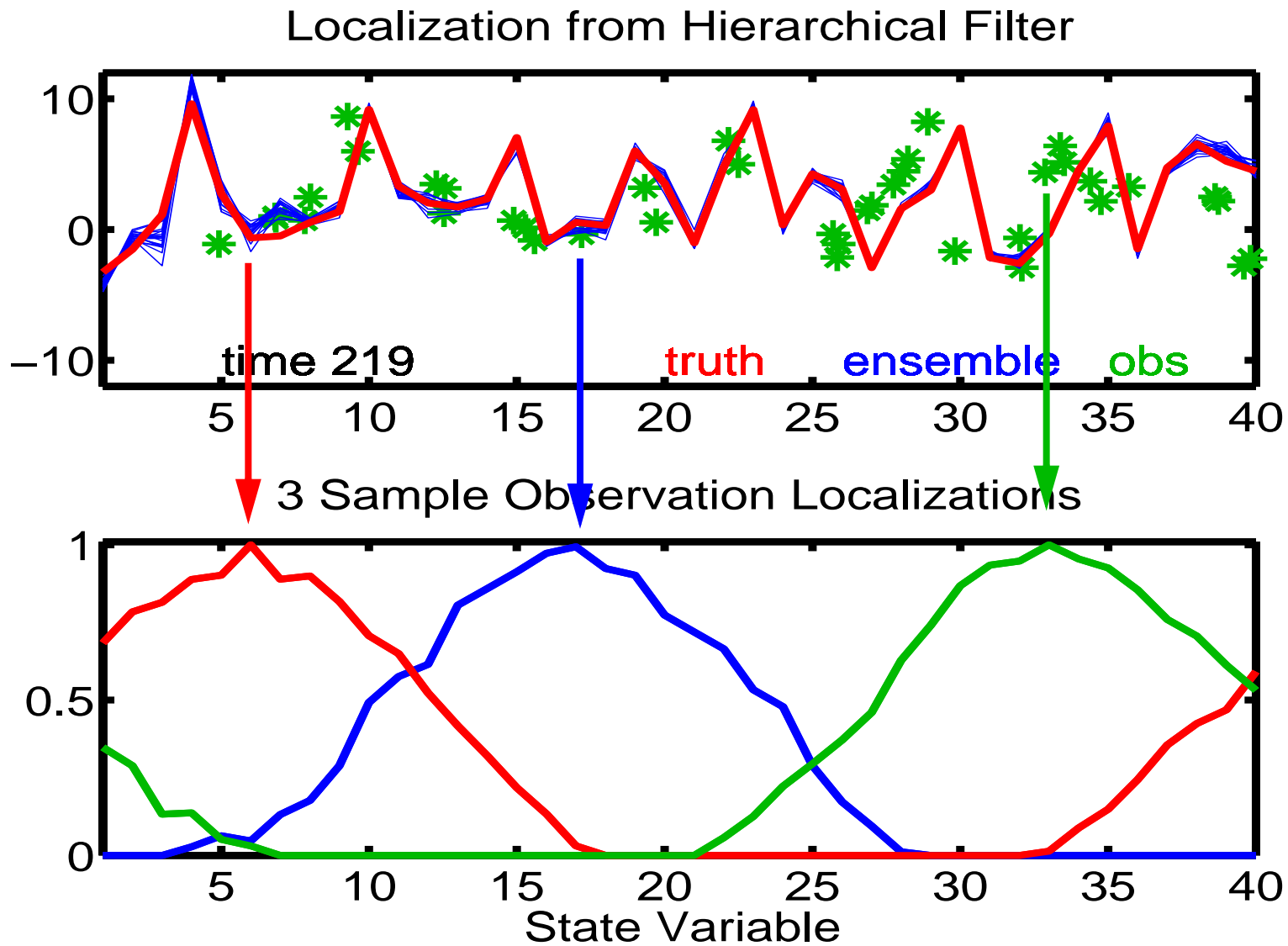
Localization from Hierarchical Filter



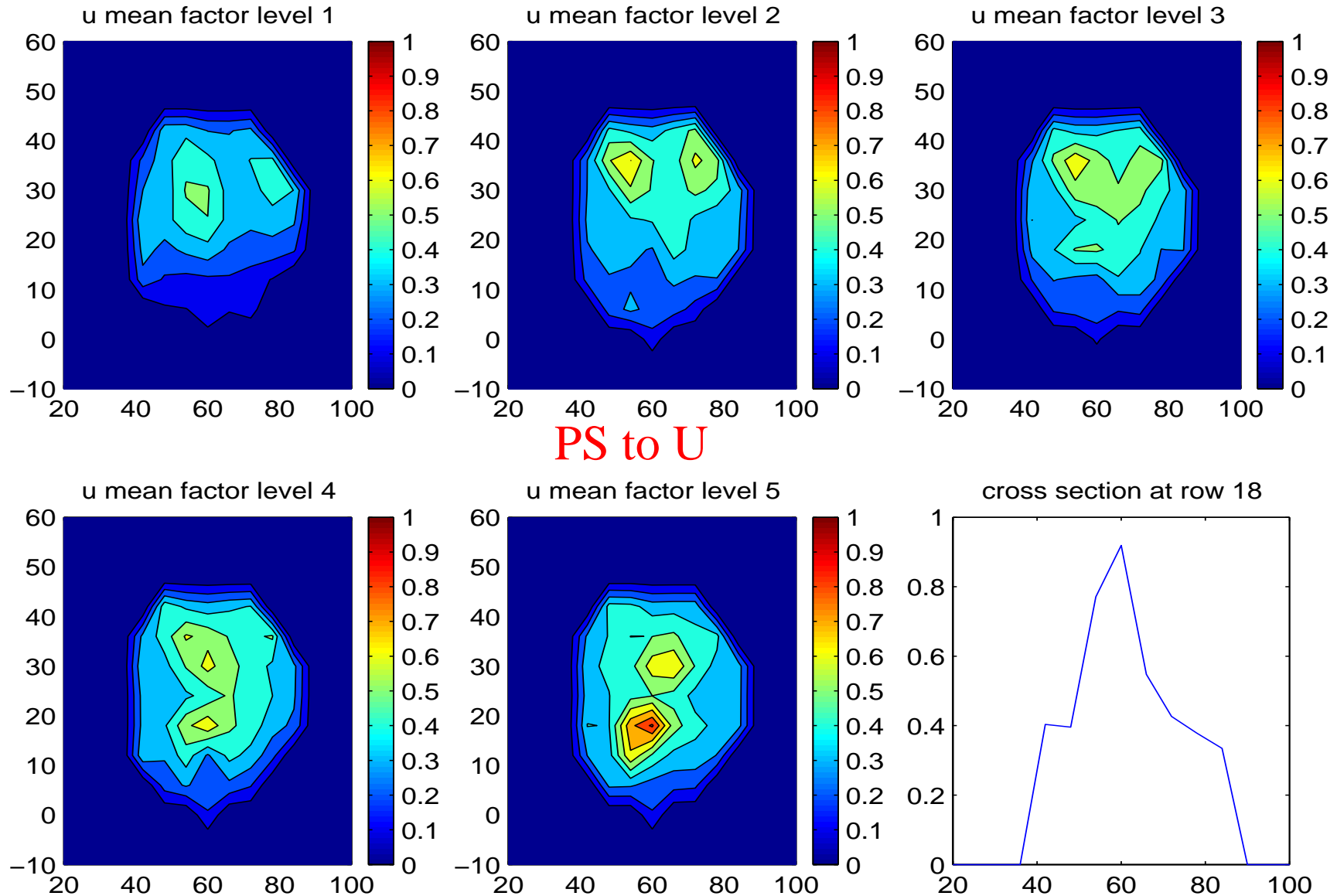
No Localization



Localization computed by adaptive hierarchical filter.
A tuning run of 4, 20-member ensembles maximizes signal.



Localization in GCM can be very complex. Surface Pressure Obs. at 20N, 60E



PS to U

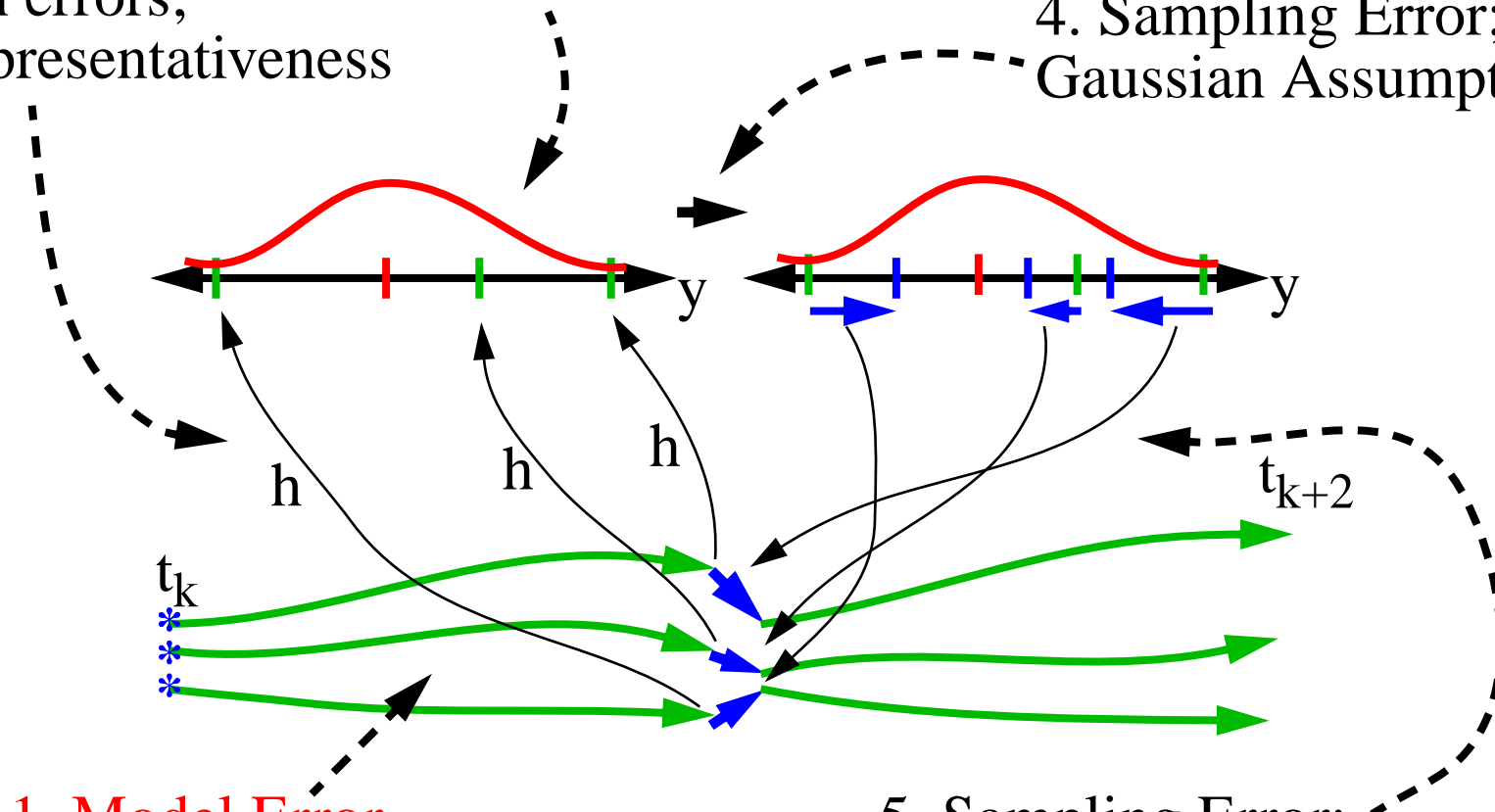
MUST HAVE ADAPTIVE HELP FOR NON-EXPERT USERS.

Some Error Sources in Ensemble Filters

3. 'Gross' Obs. Errors

2. h errors;
Representativeness

4. Sampling Error;
Gaussian Assumption



1. Model Error

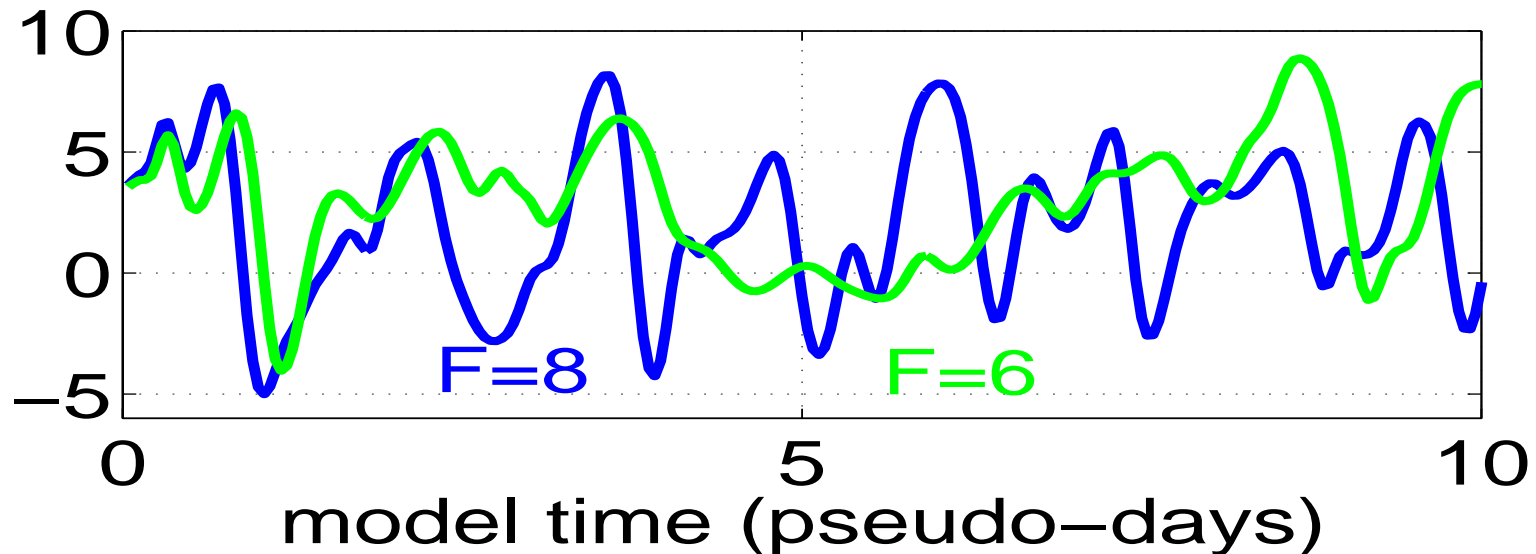
5. Sampling Error;
Assuming Linear
Statistical Relation

Assimilating in the presence of simulated model error.

$$dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$$

For truth, use $F = 8$.

In assimilating model, use $F = 6$.



Time evolution for X_1 shown.

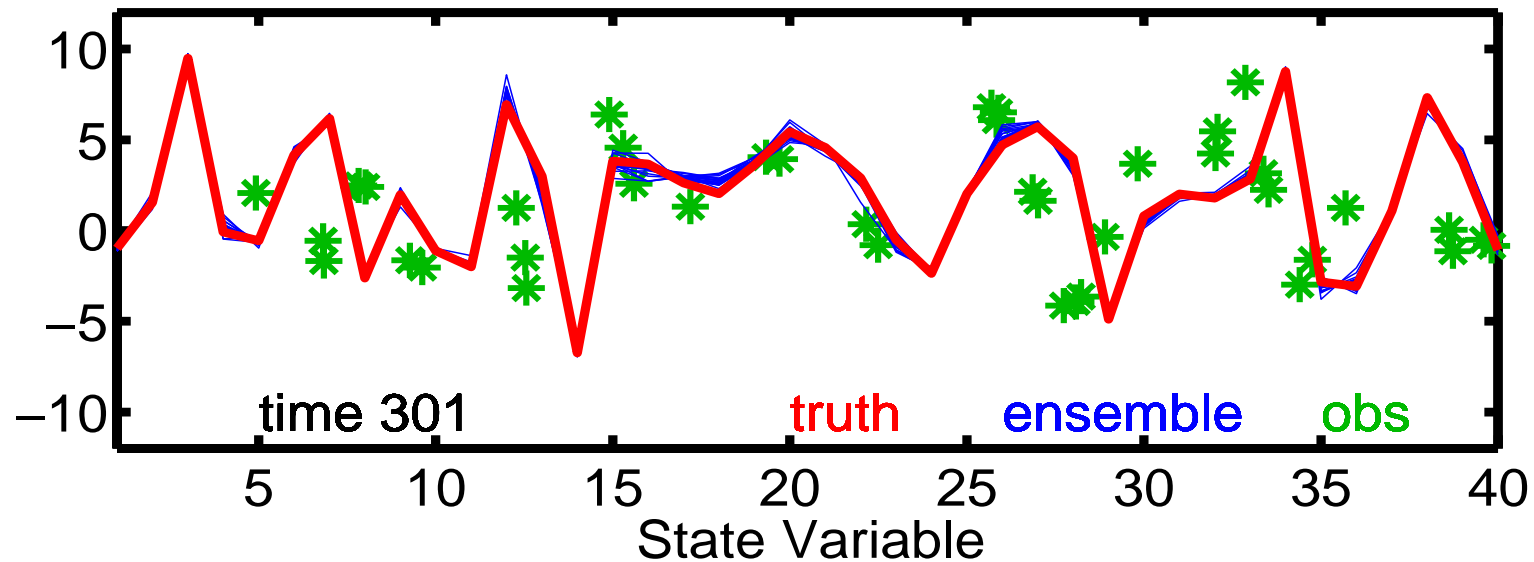
Assimilating model quickly diverges from 'true' model.

Assimilating in the presence of simulated model error

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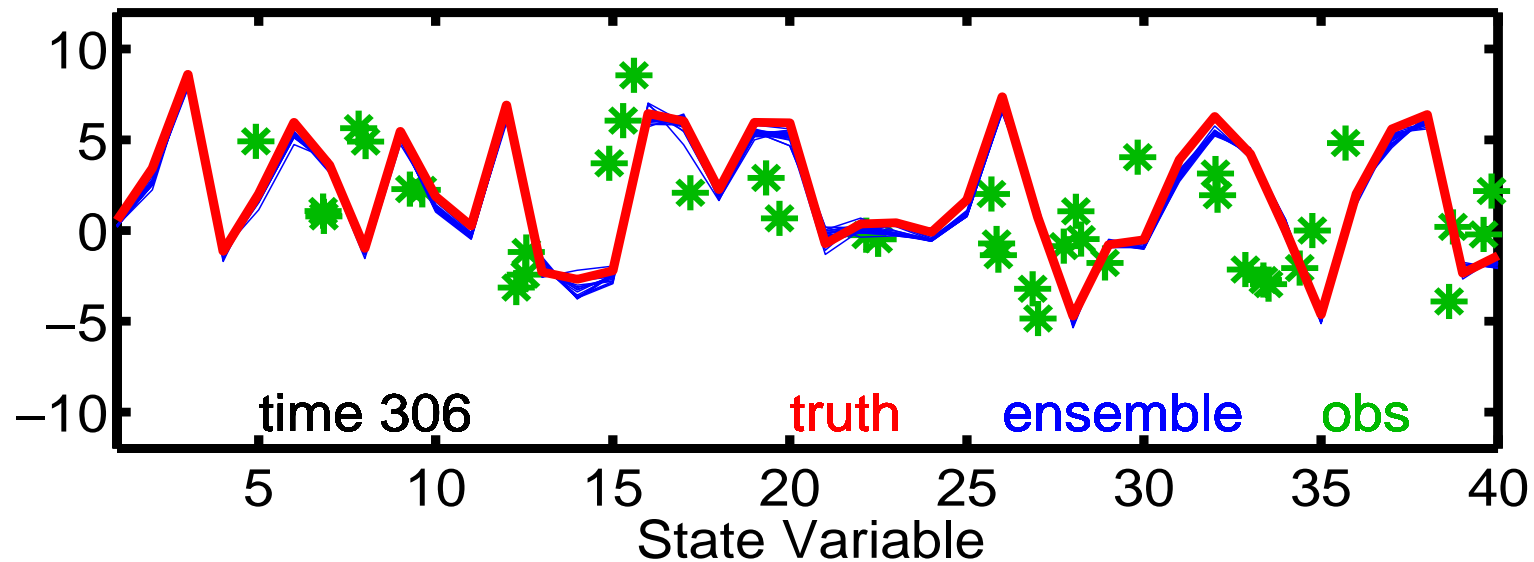


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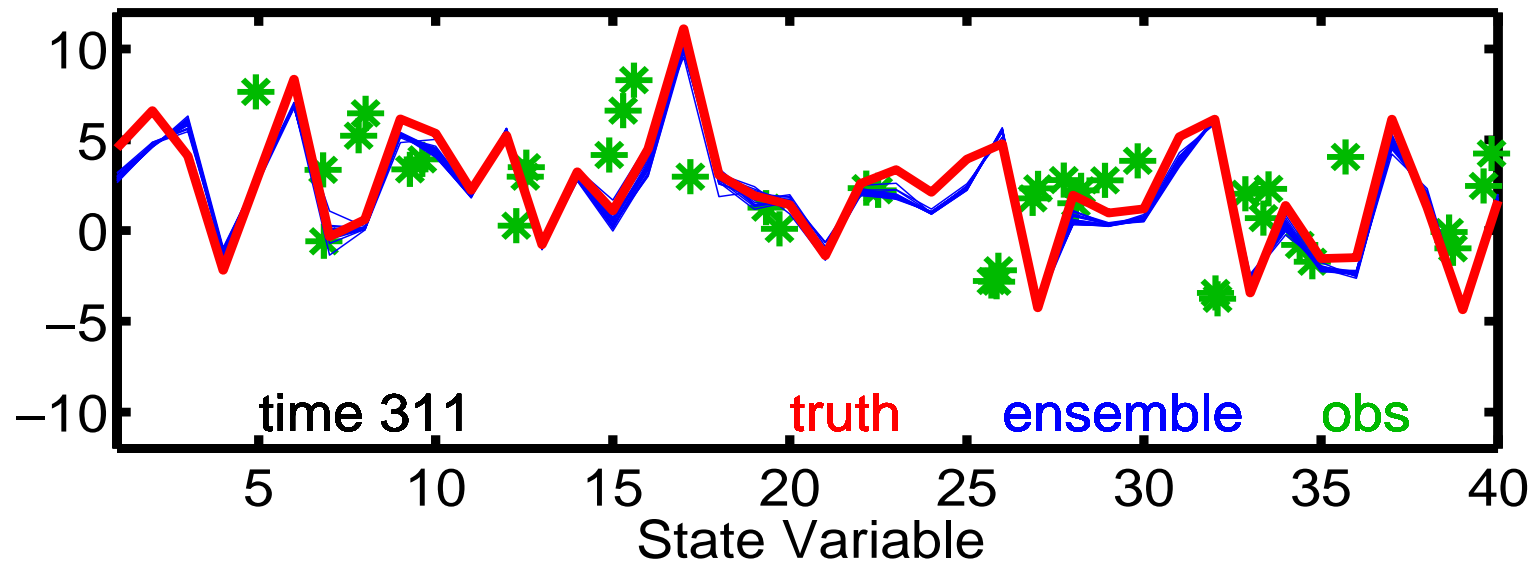


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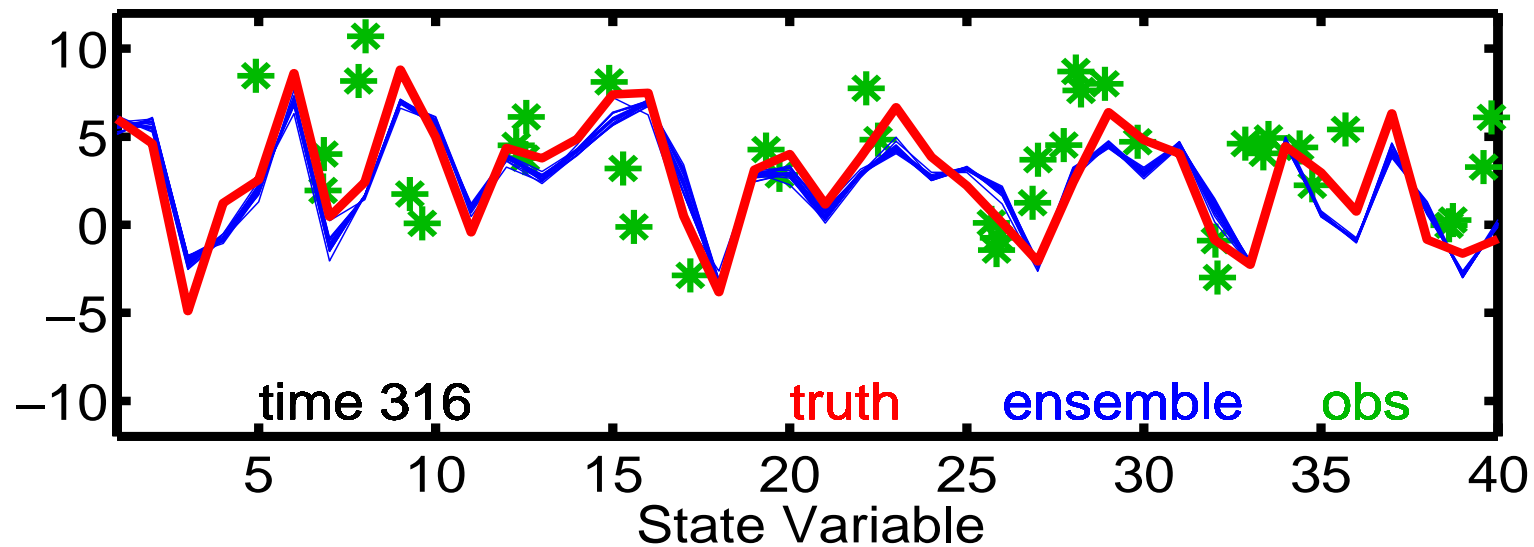


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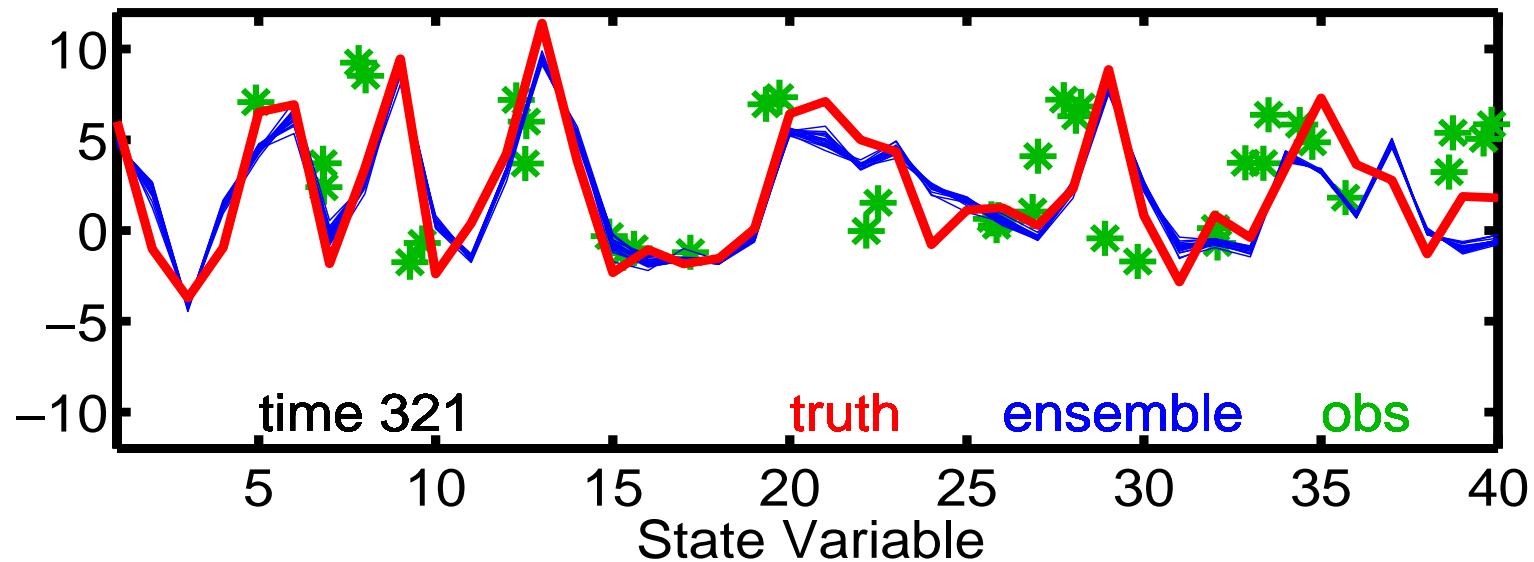


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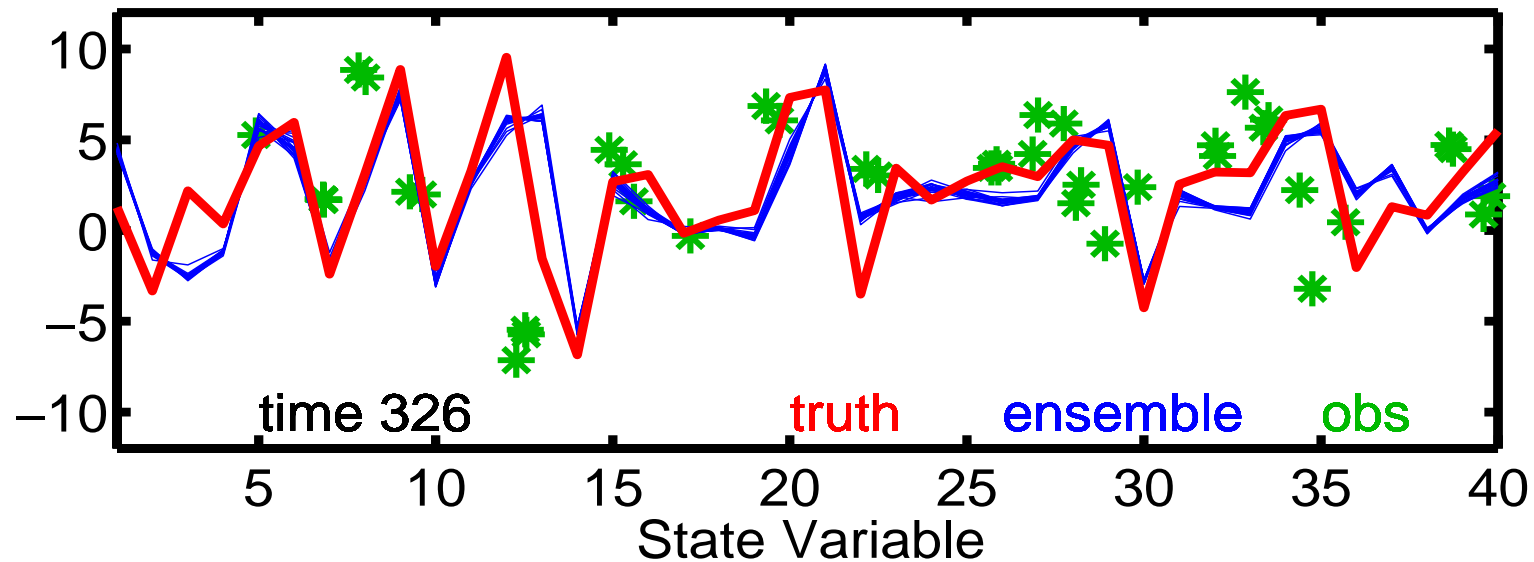


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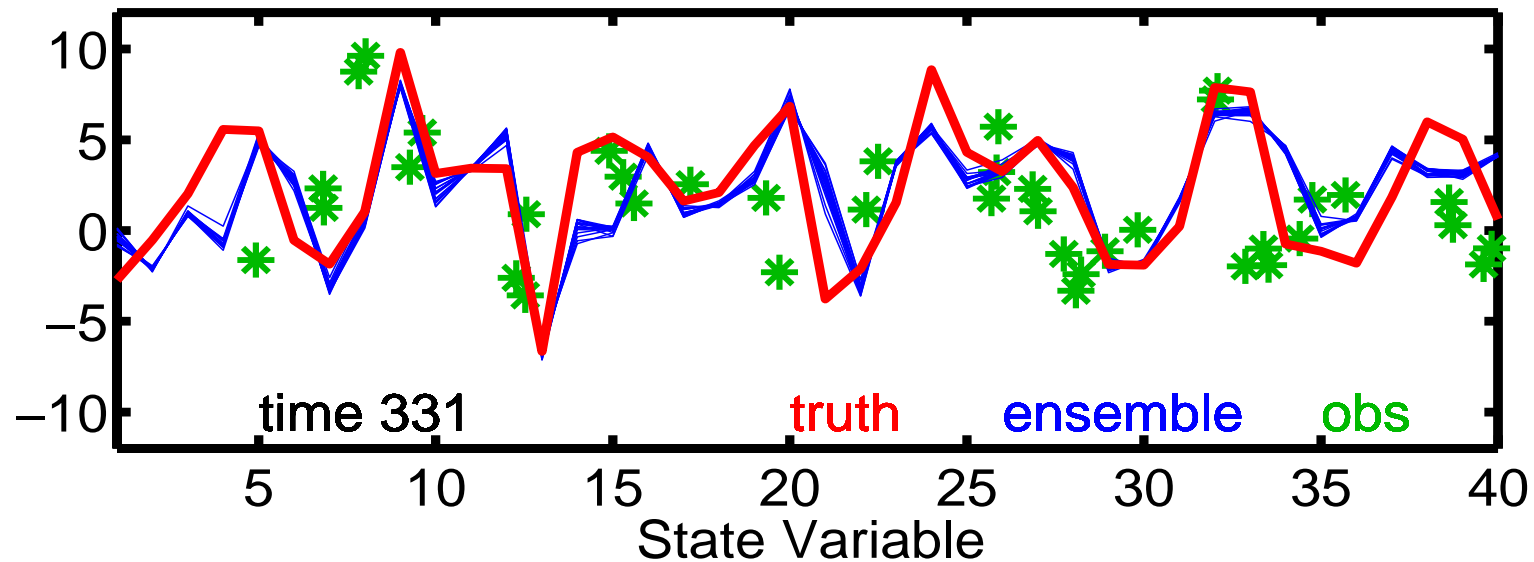


Assimilating in the presence of simulated model error

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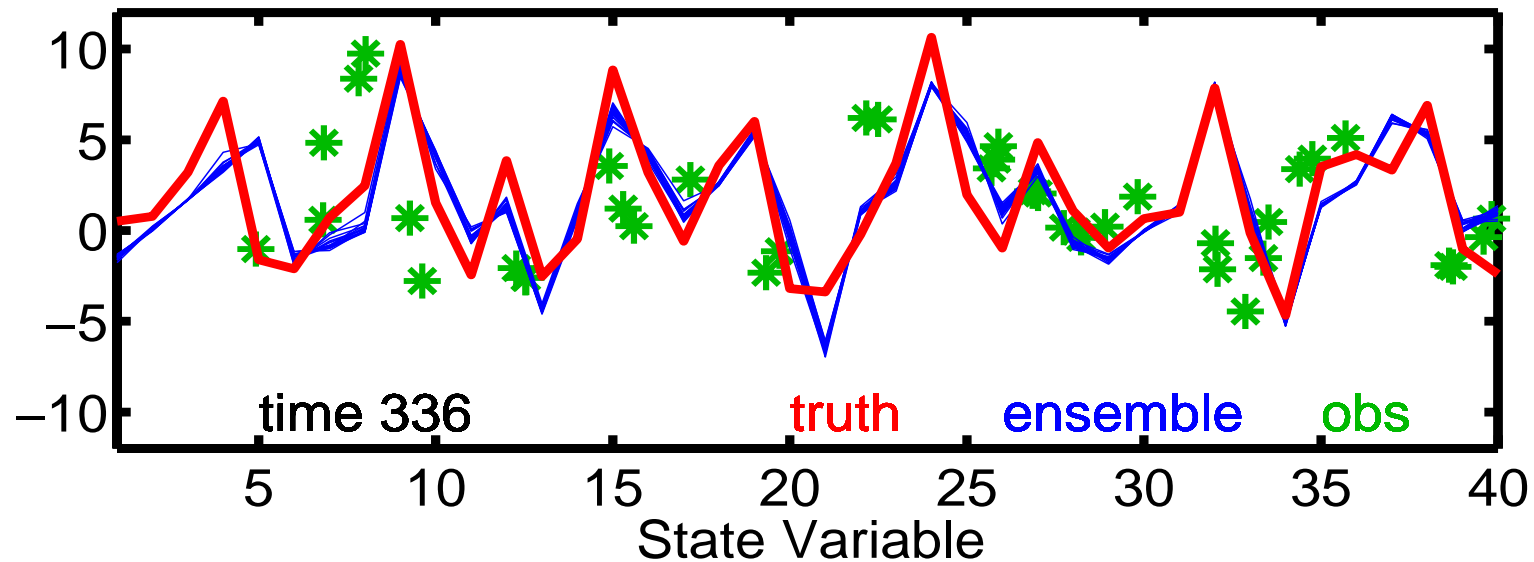


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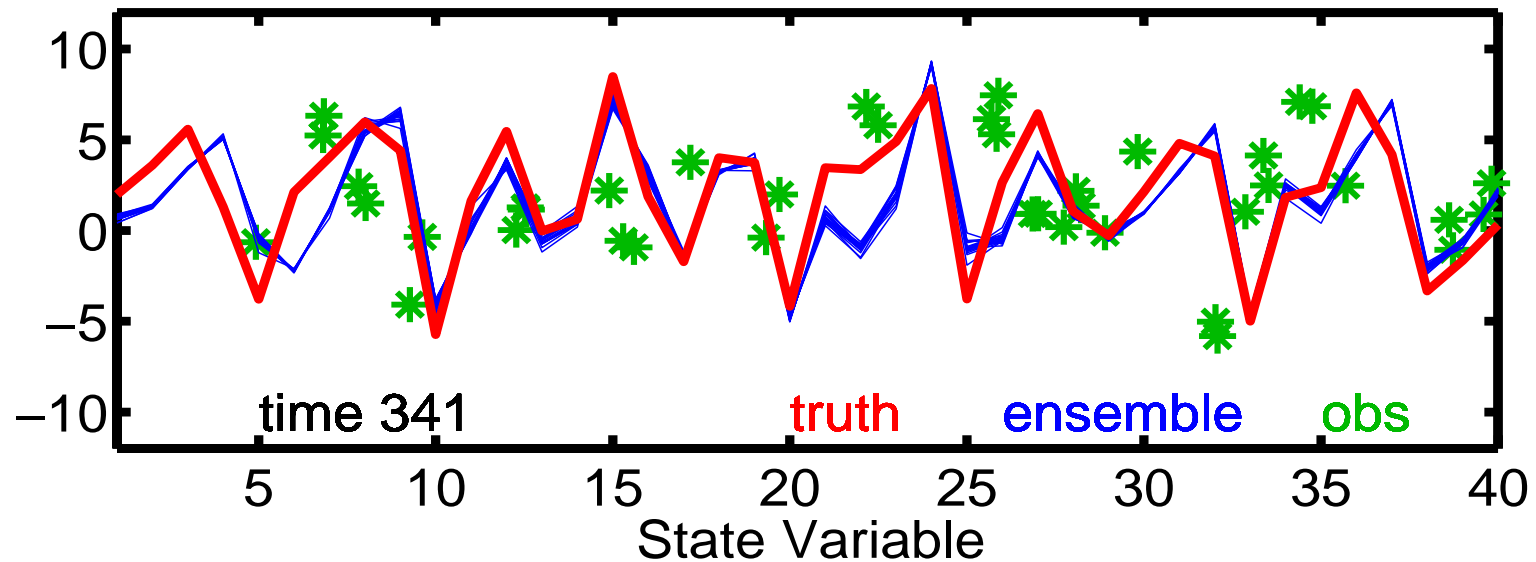


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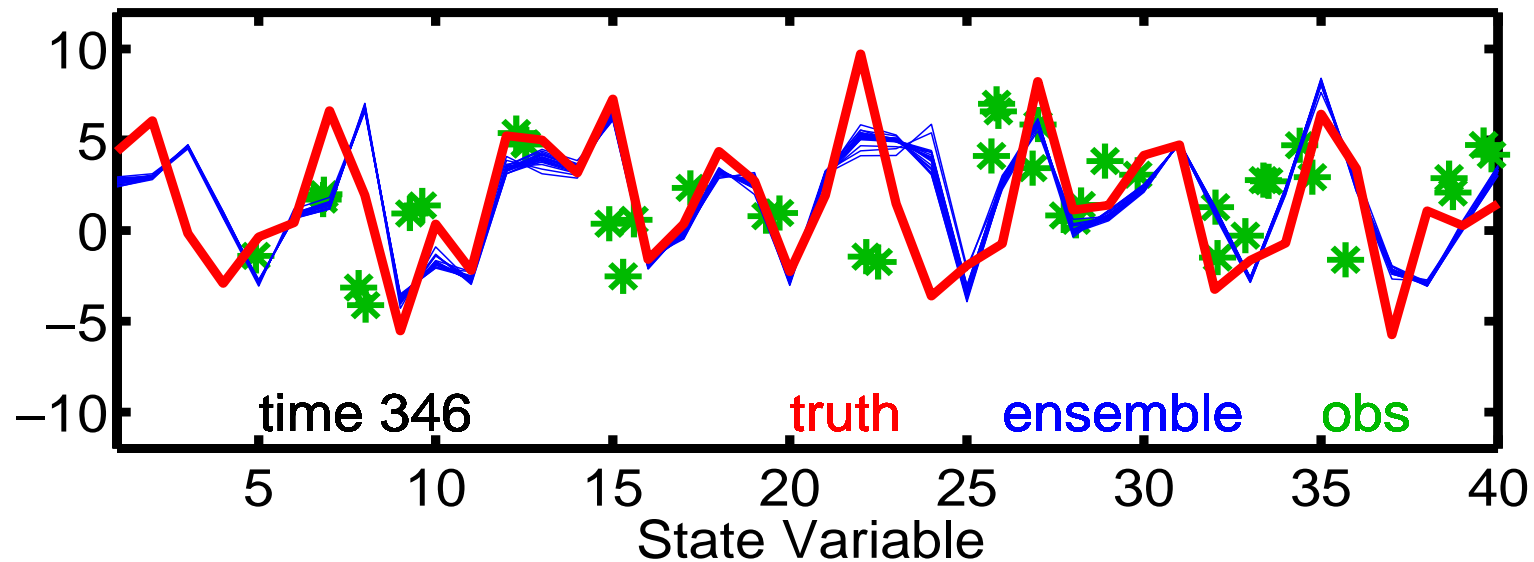


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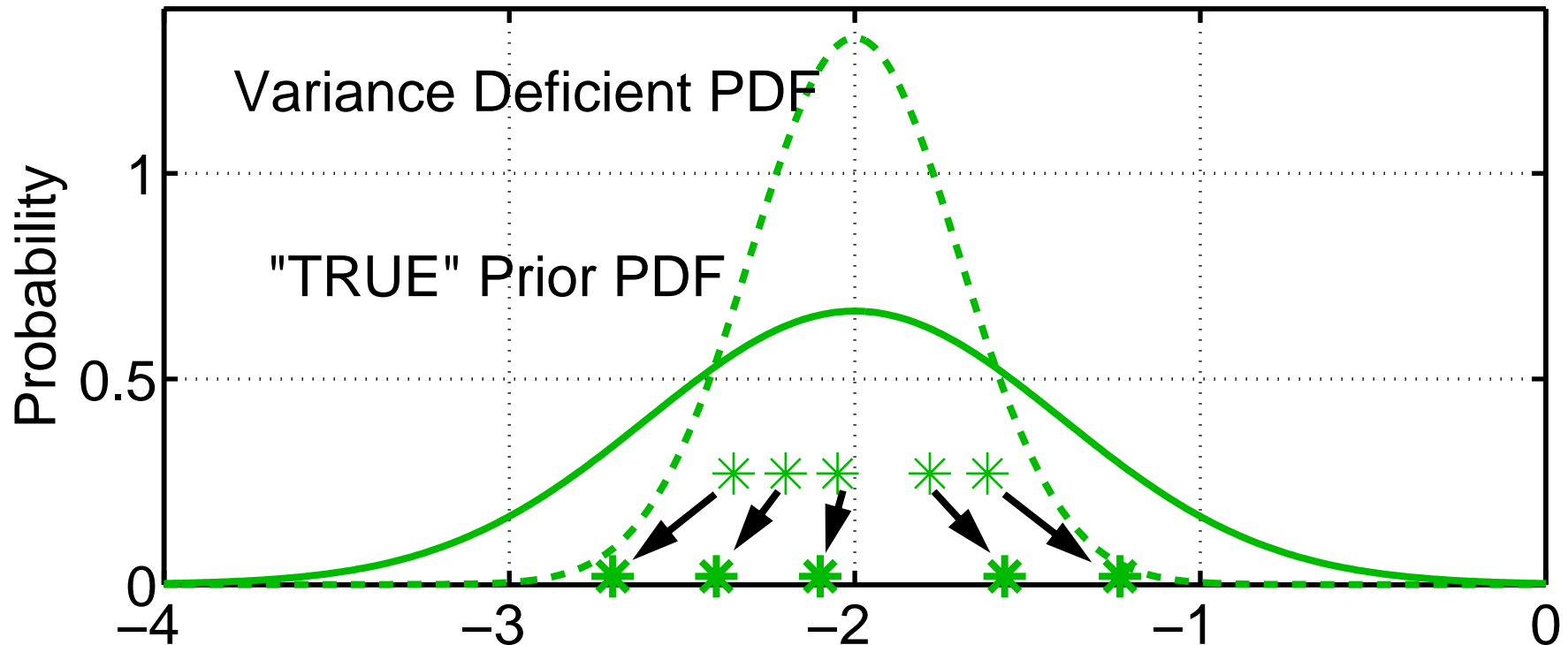
In assimilating model, use $F = 6$.



This isn't working again!
It will just keep getting worse.

Model/Filter Error; Filter Divergence and Variance Inflation

1. History of observations and physical system => 'true' distribution.
2. Sampling error, some model errors lead to insufficient prior variance.

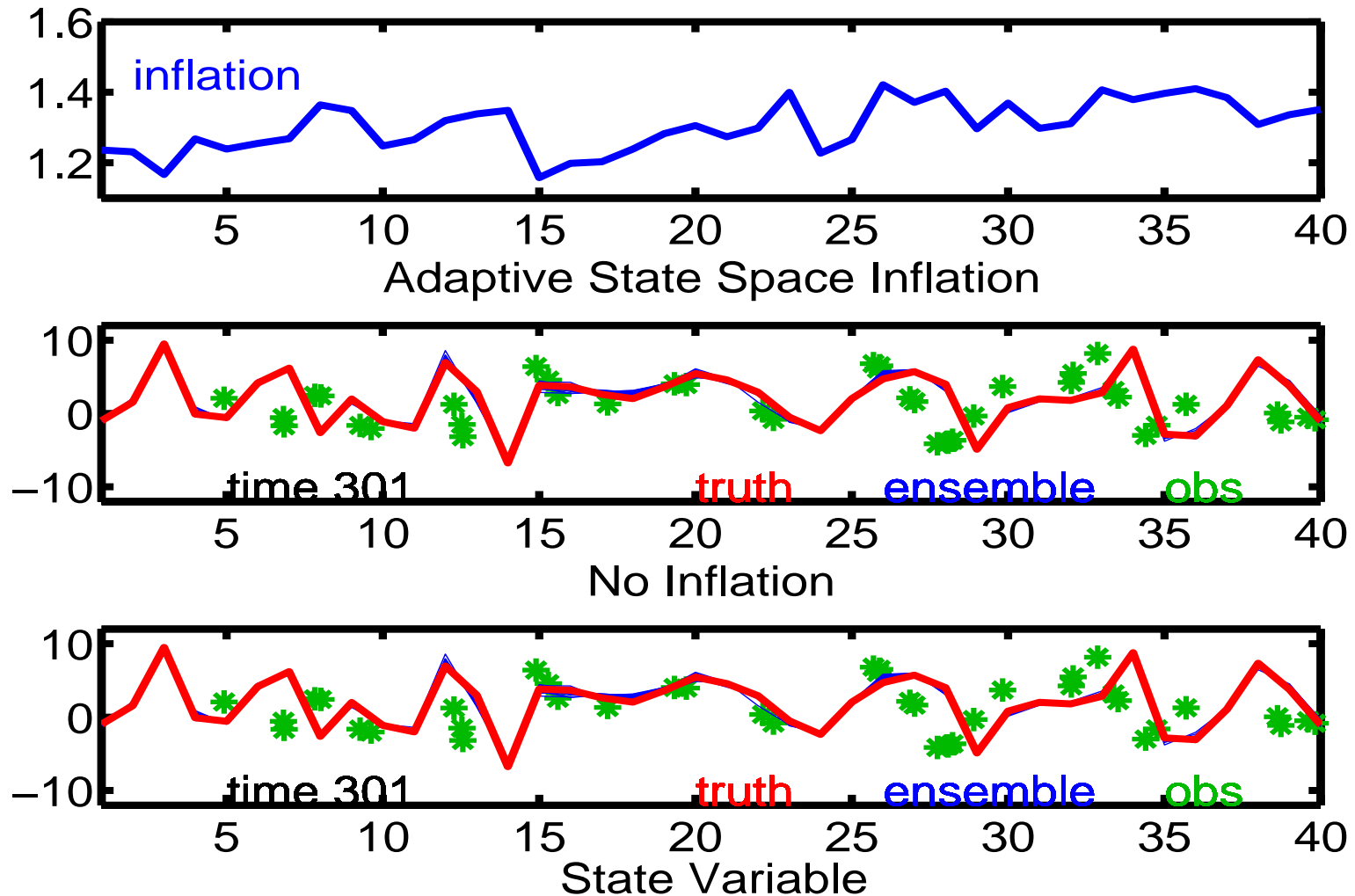


3. Naive solution is Variance inflation: just increase spread of prior
4. For ensemble member i , $inflate(x_i) = \sqrt{\lambda}(x_i - \bar{x}) + \bar{x}$.

Assimilating with Inflation in presence of model error

Inflation is a function of state variable and time.

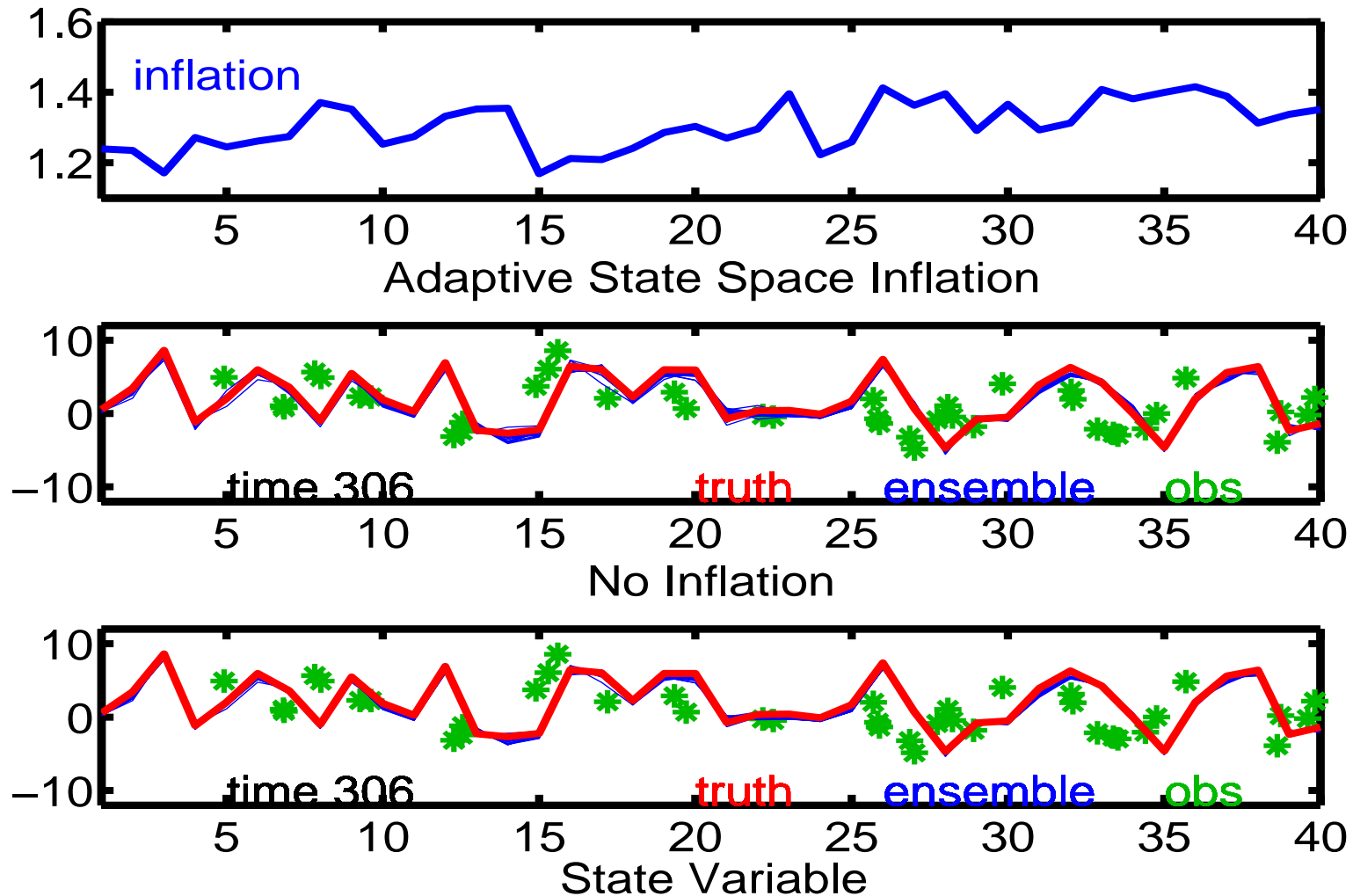
Automatically selected by adaptive inflation algorithm.



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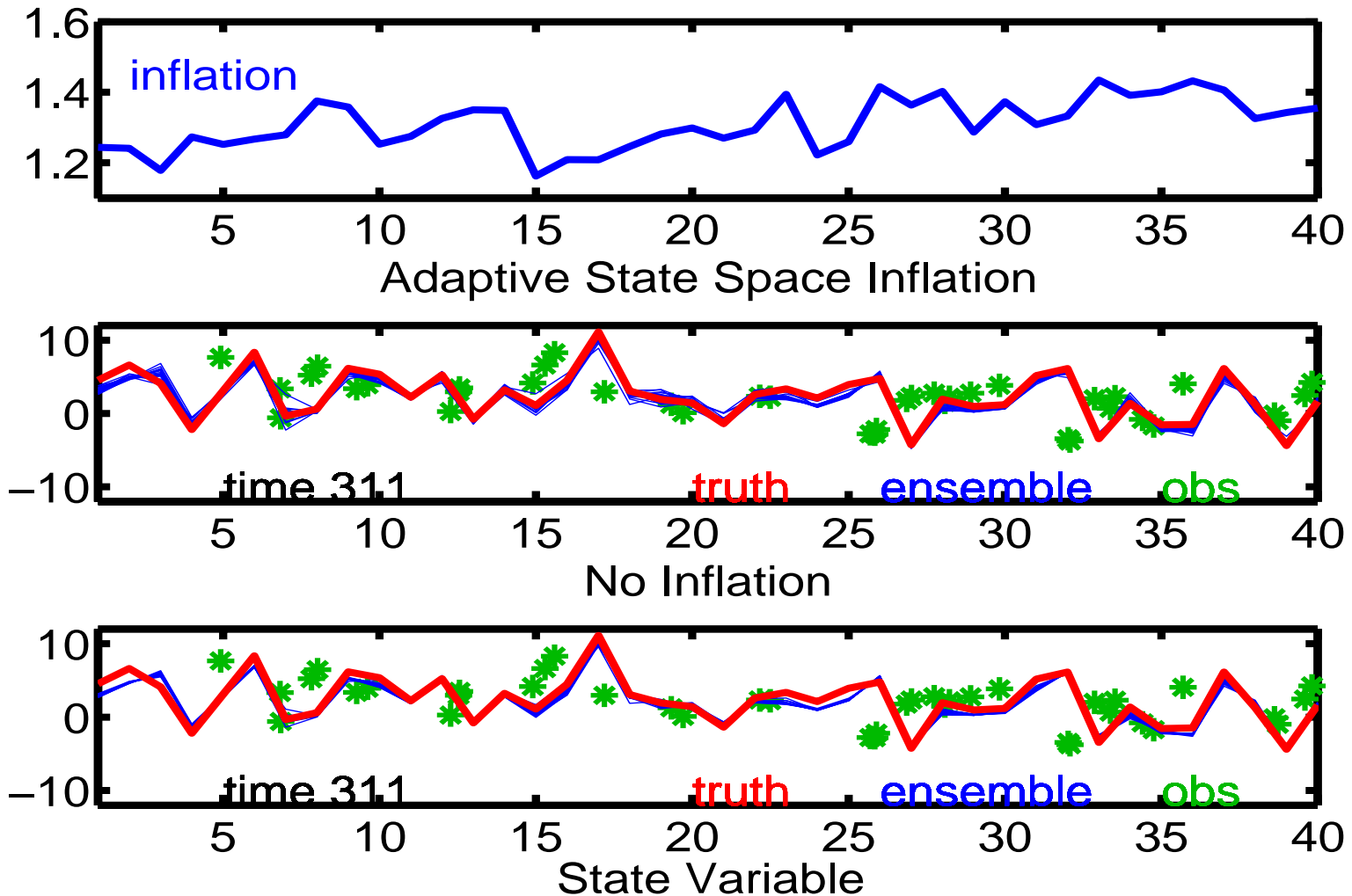
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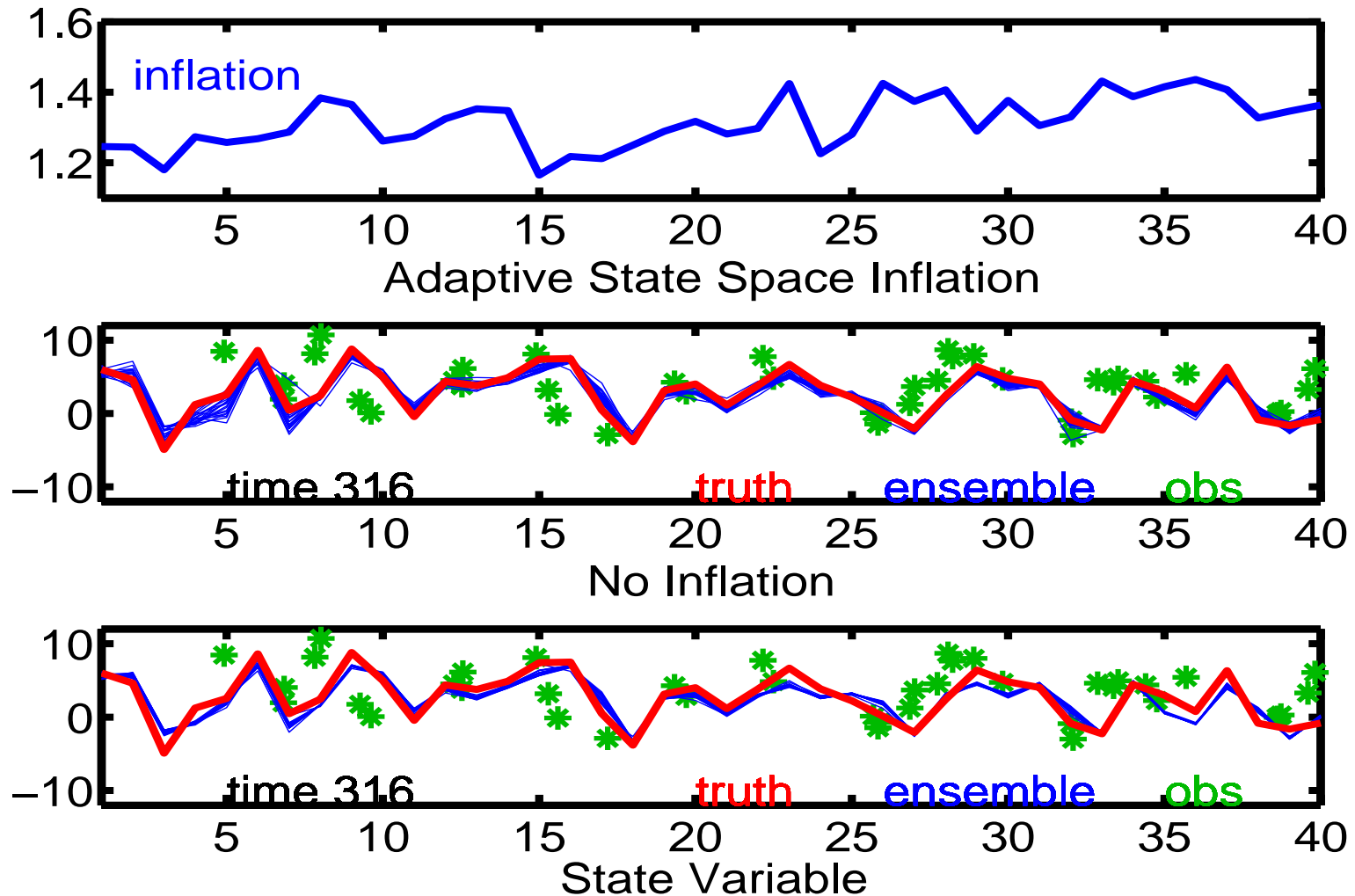
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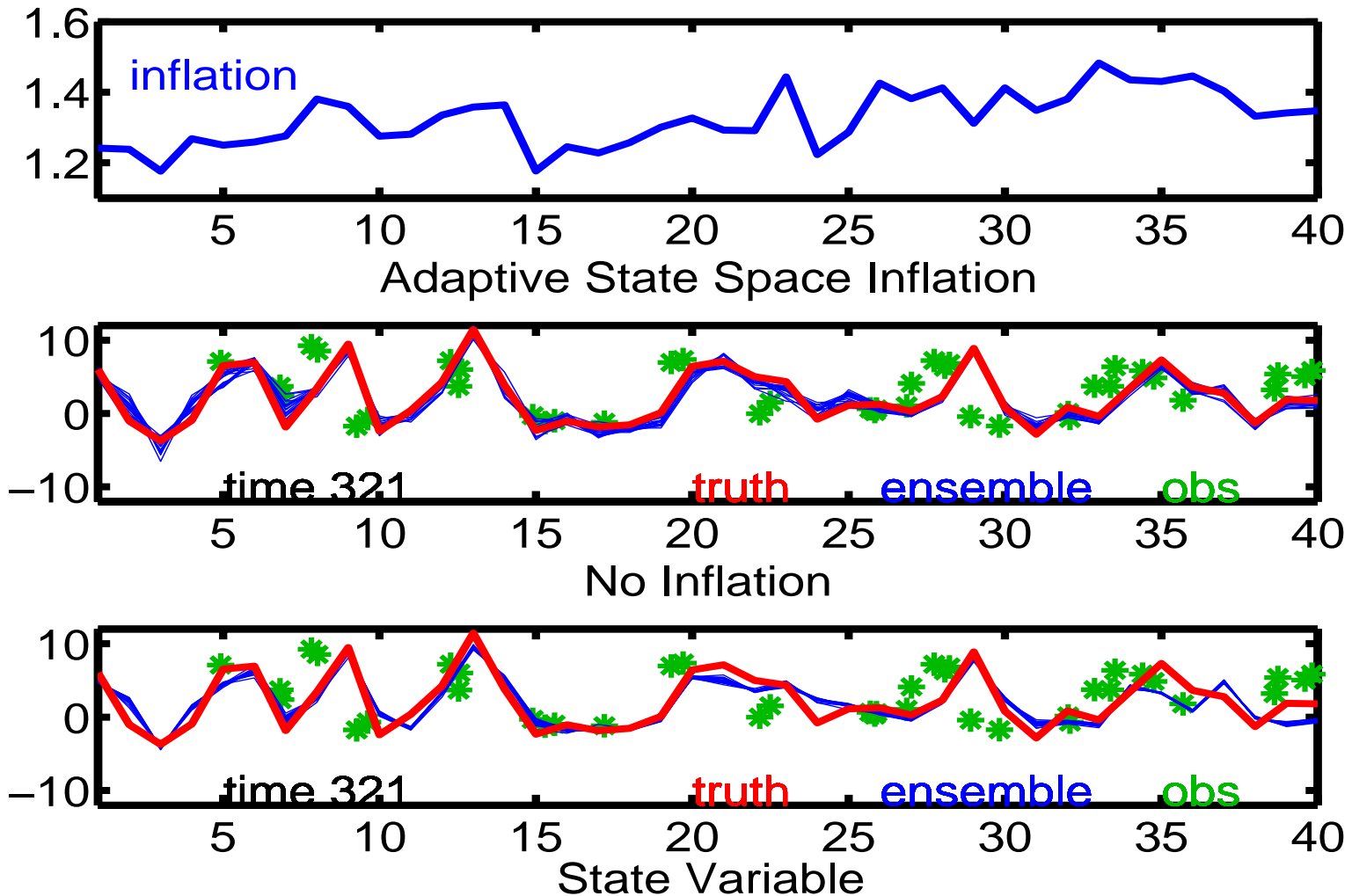
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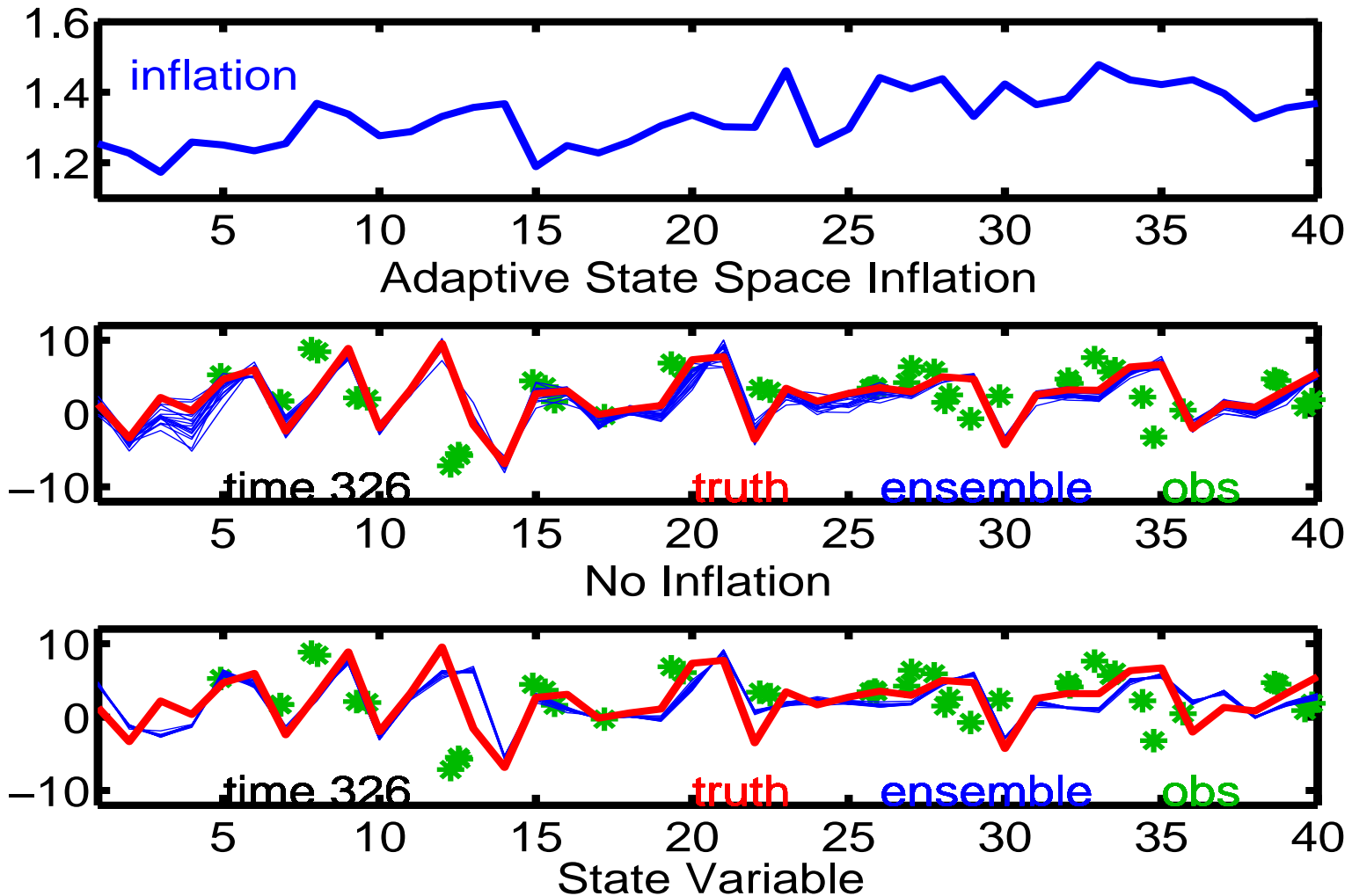
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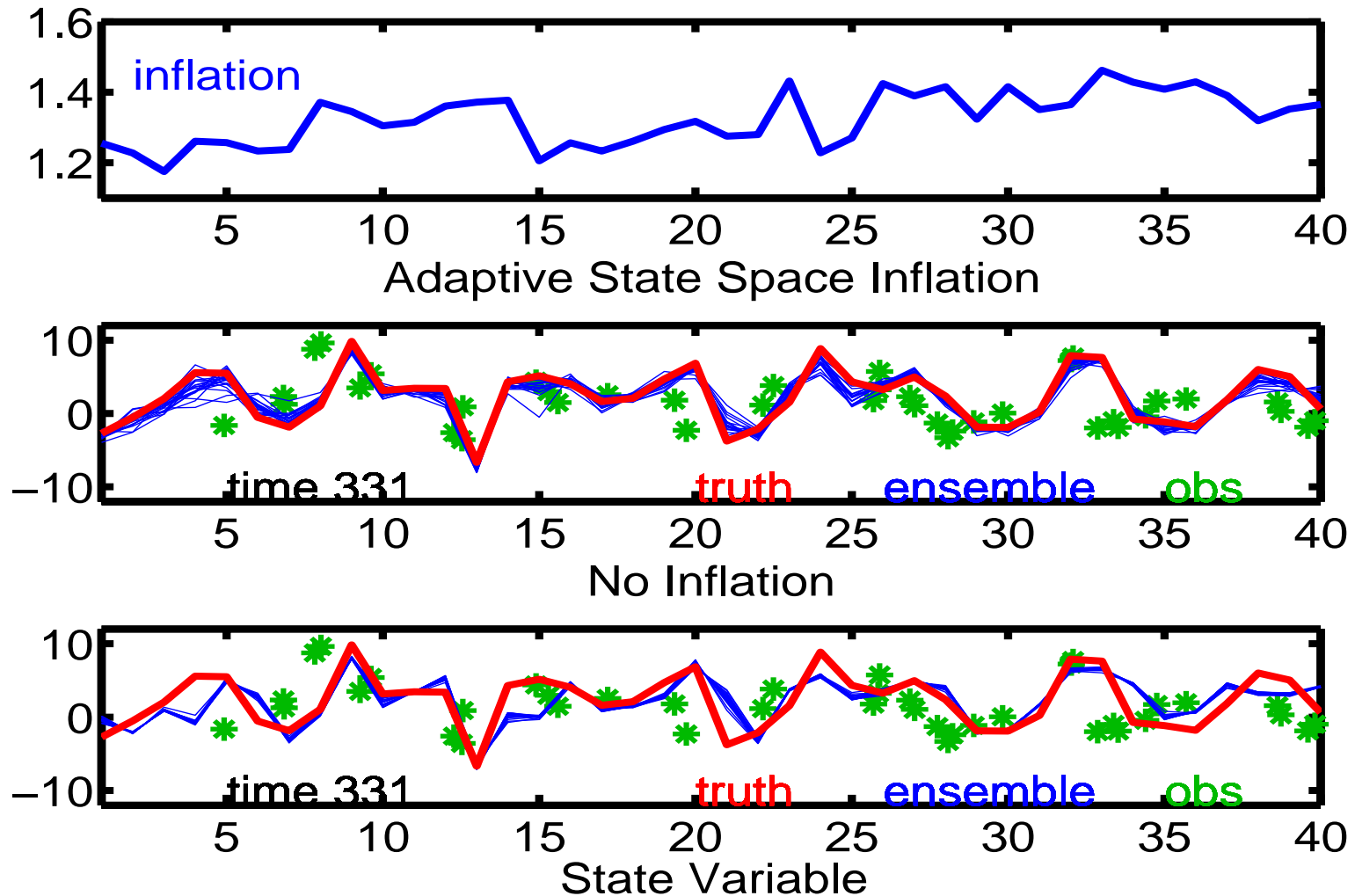
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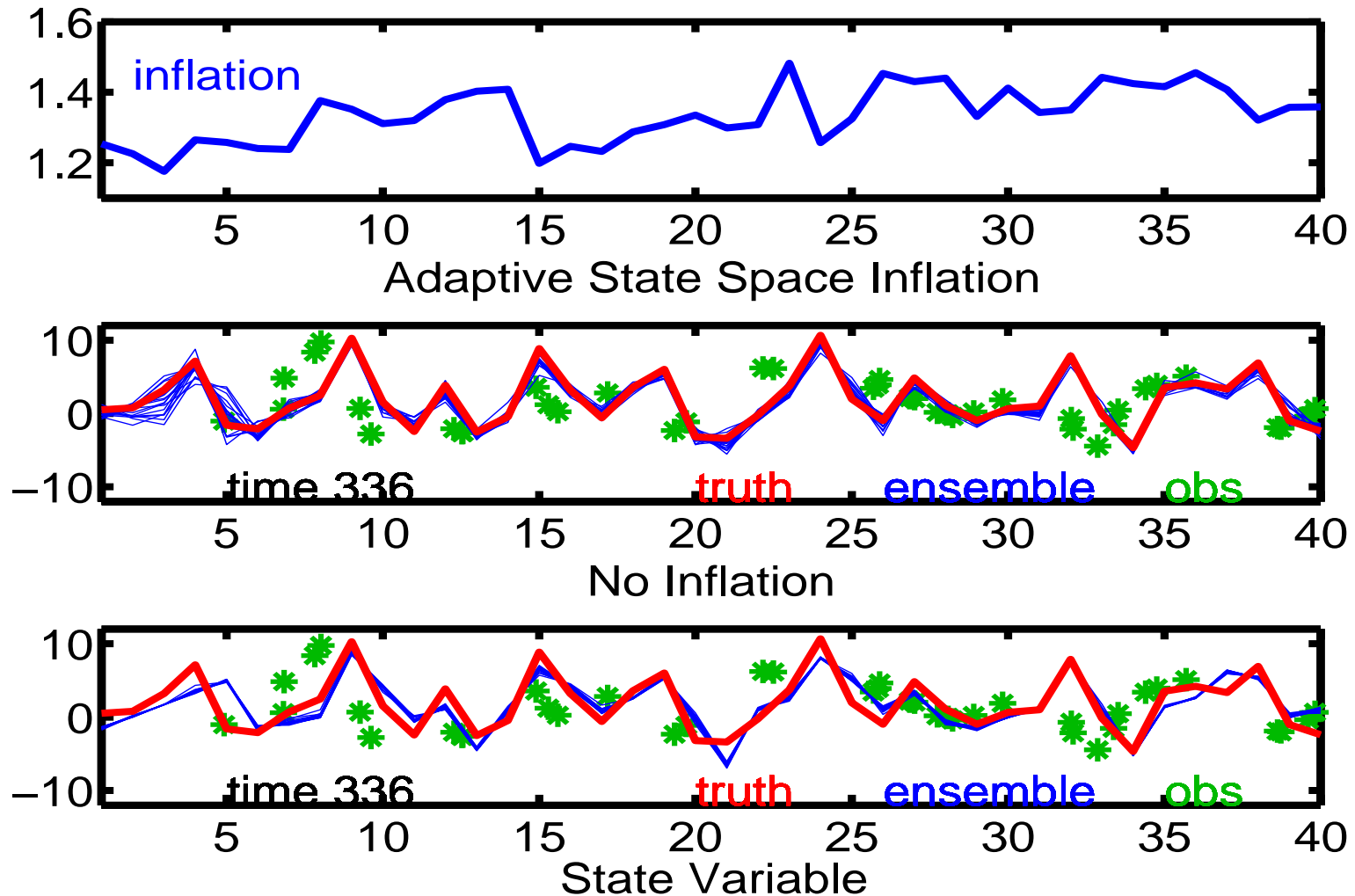
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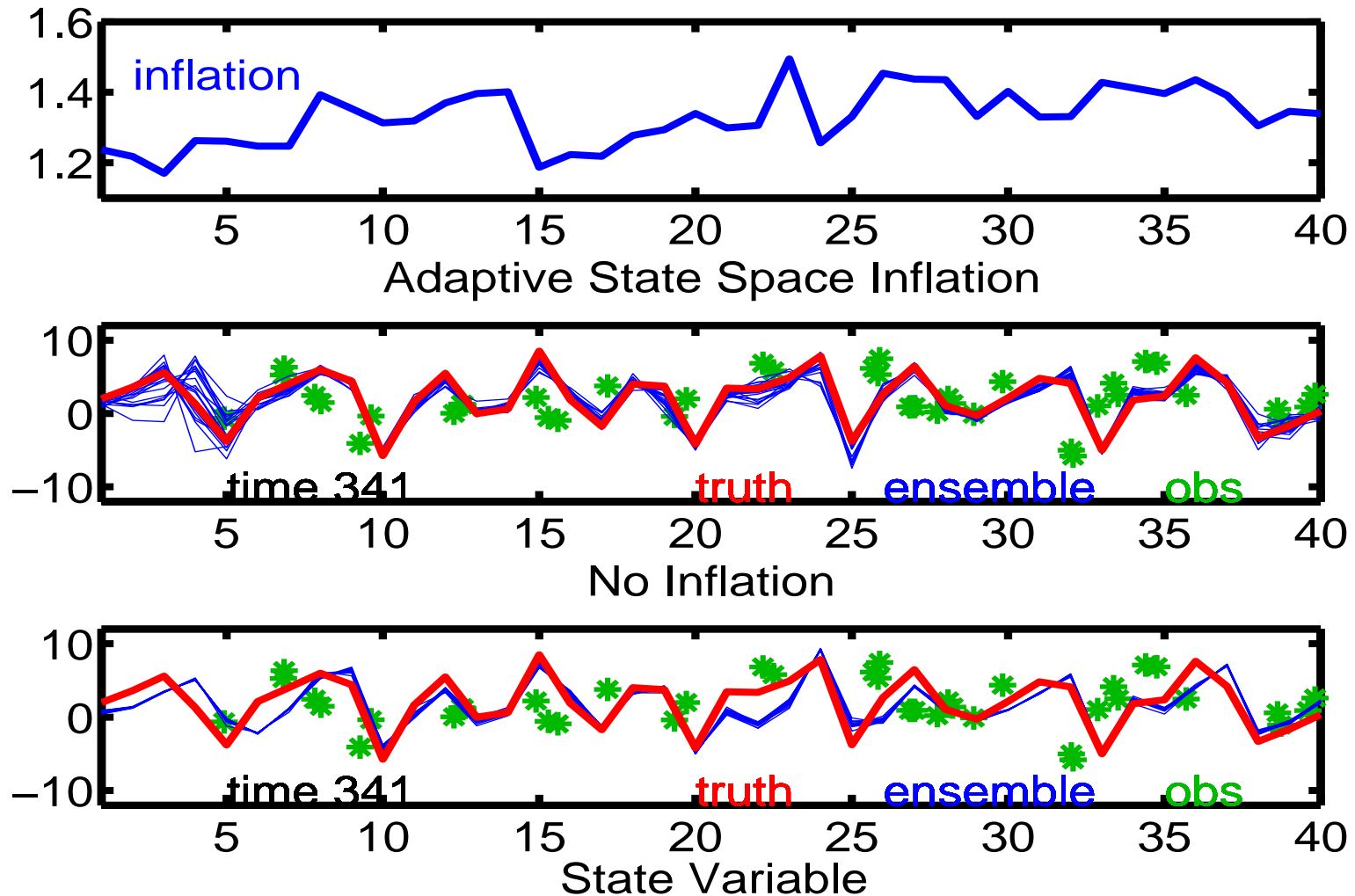
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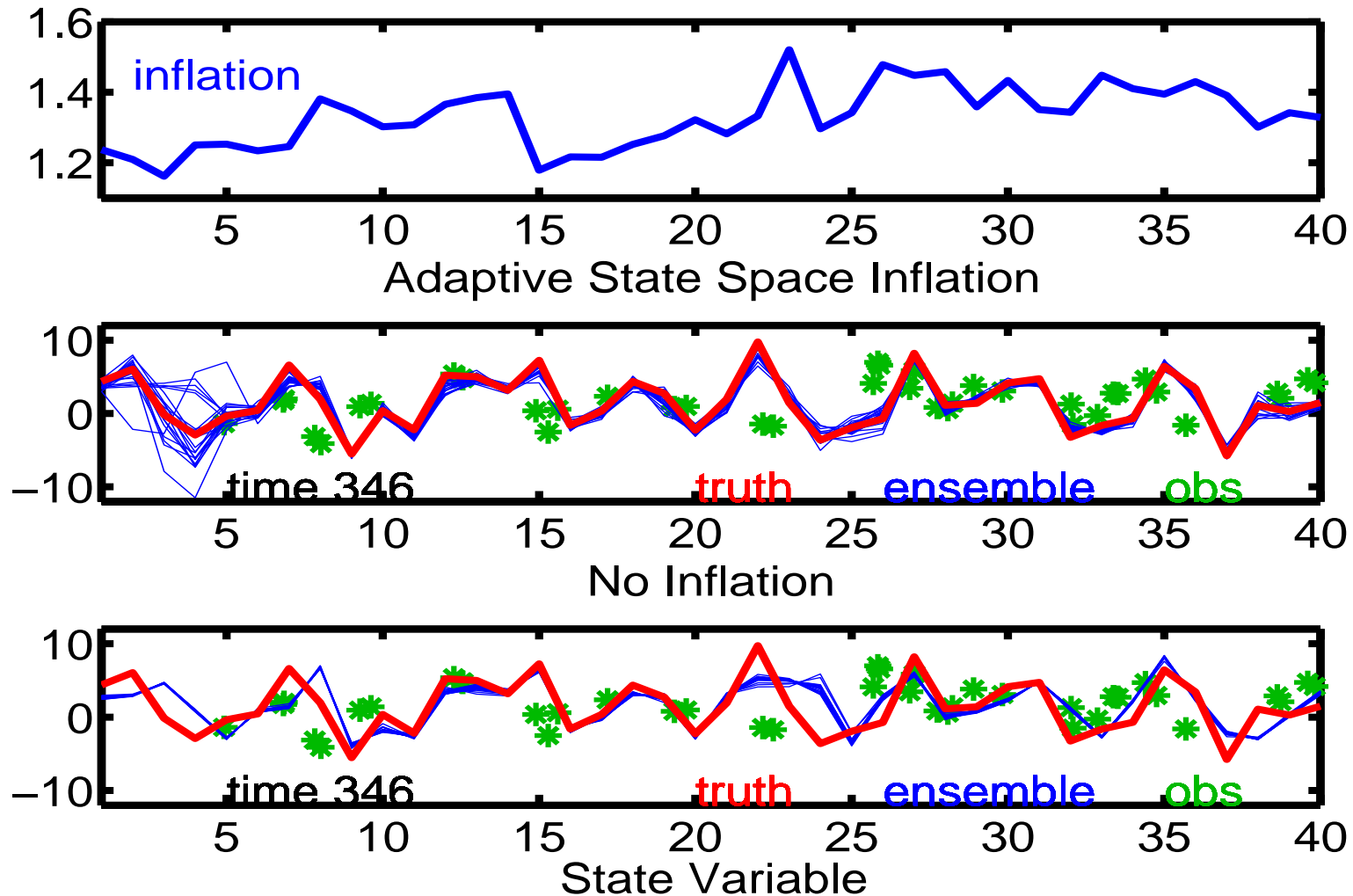


Assimilating with Inflation in presence of model error

Inflation is a function of state variable and time.

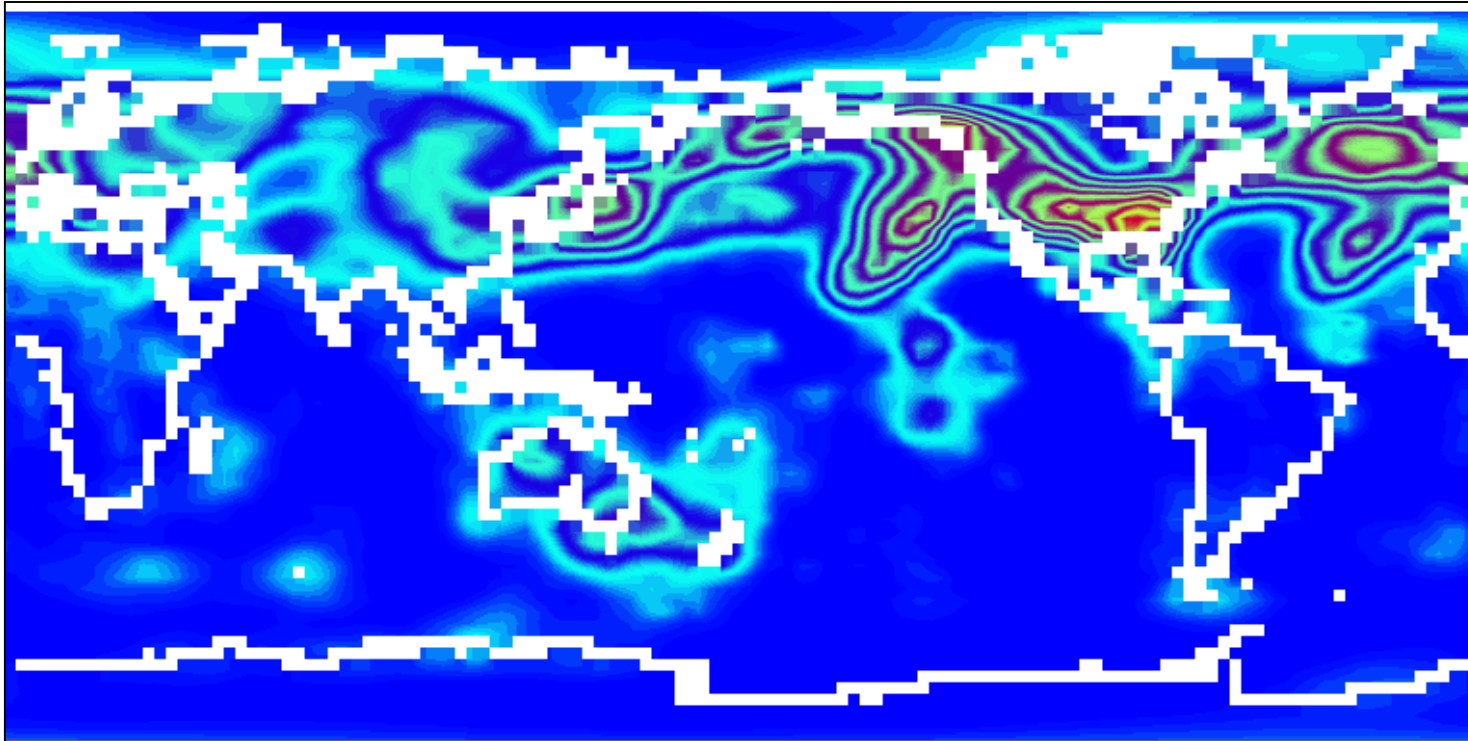
Automatically selected by adaptive inflation algorithm.

It can work, even in presence of severe model error.



Adaptive Inflation from CAM 'Operational' Assimilation

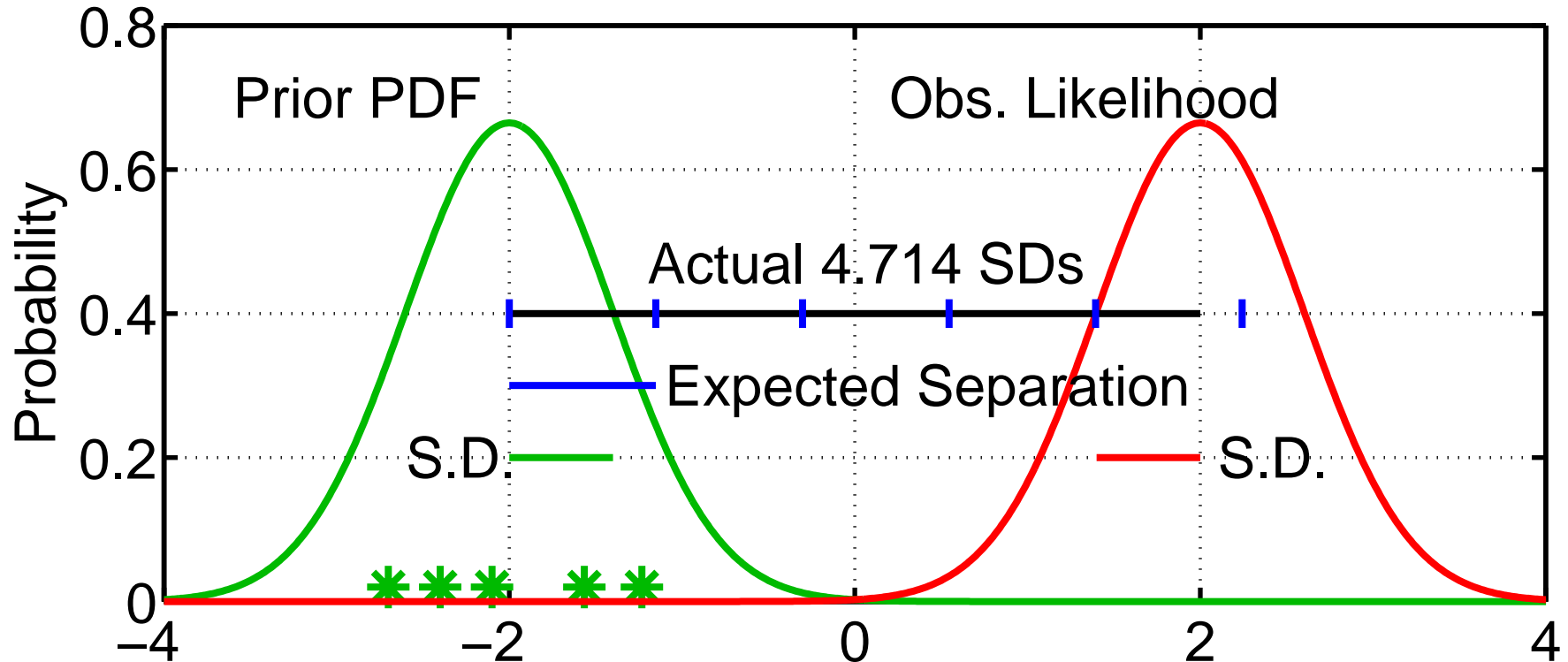
Mean inflation (range 1 to 3) for 500mb Temperature



This field is very complicated, depends on model details and obs.
Adaptive inflation tool automatically produces this.
Allows filter to work well with significant model (and other) errors.

Adaptive Inflation for Ensemble Filtering

1. For observed variable, have estimate of prior-observed inconsistency.



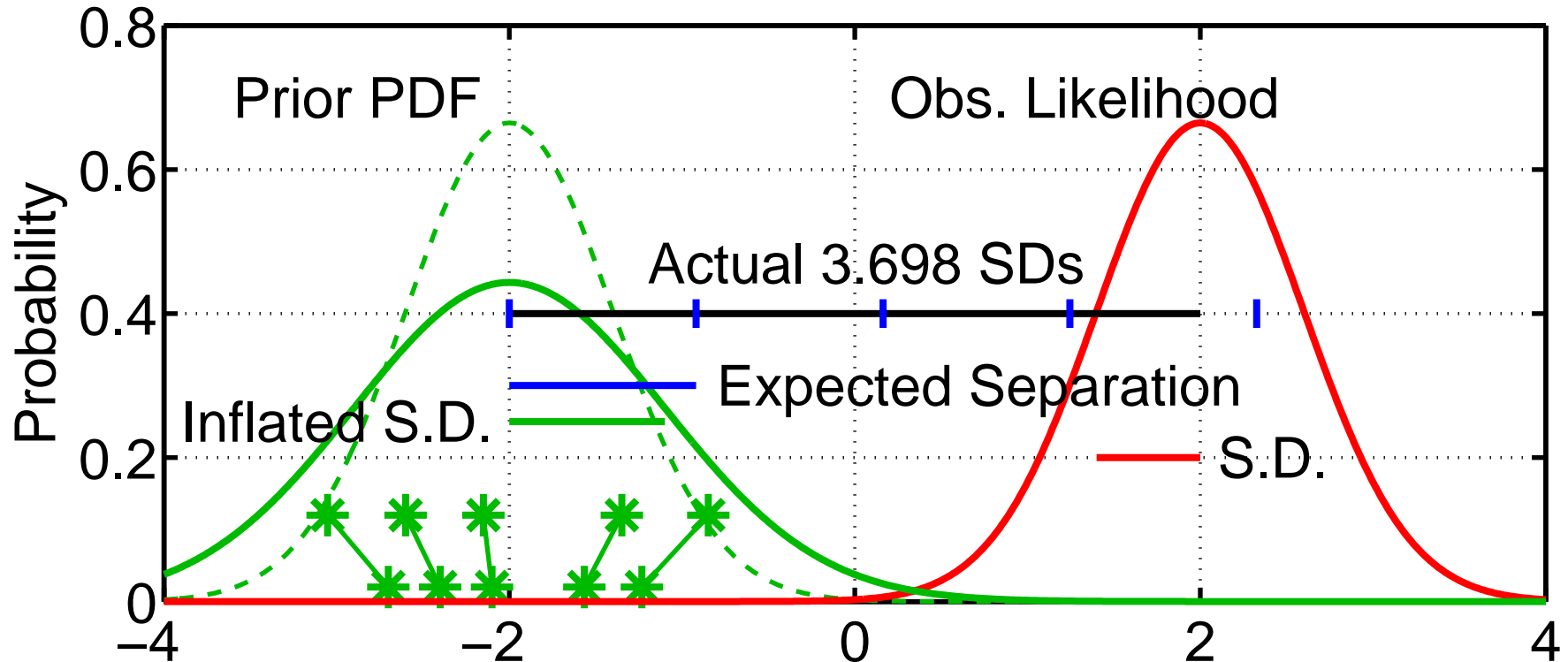
2. Expected(prior mean - observation) = $\sqrt{\sigma_{prior}^2 + \sigma_{obs}^2}$.

Assumes that prior and observation are supposed to be unbiased.

Is it model error or random chance?

Adaptive Inflation for Ensemble Filtering

1. For observed variable, have estimate of prior-observed inconsistency.



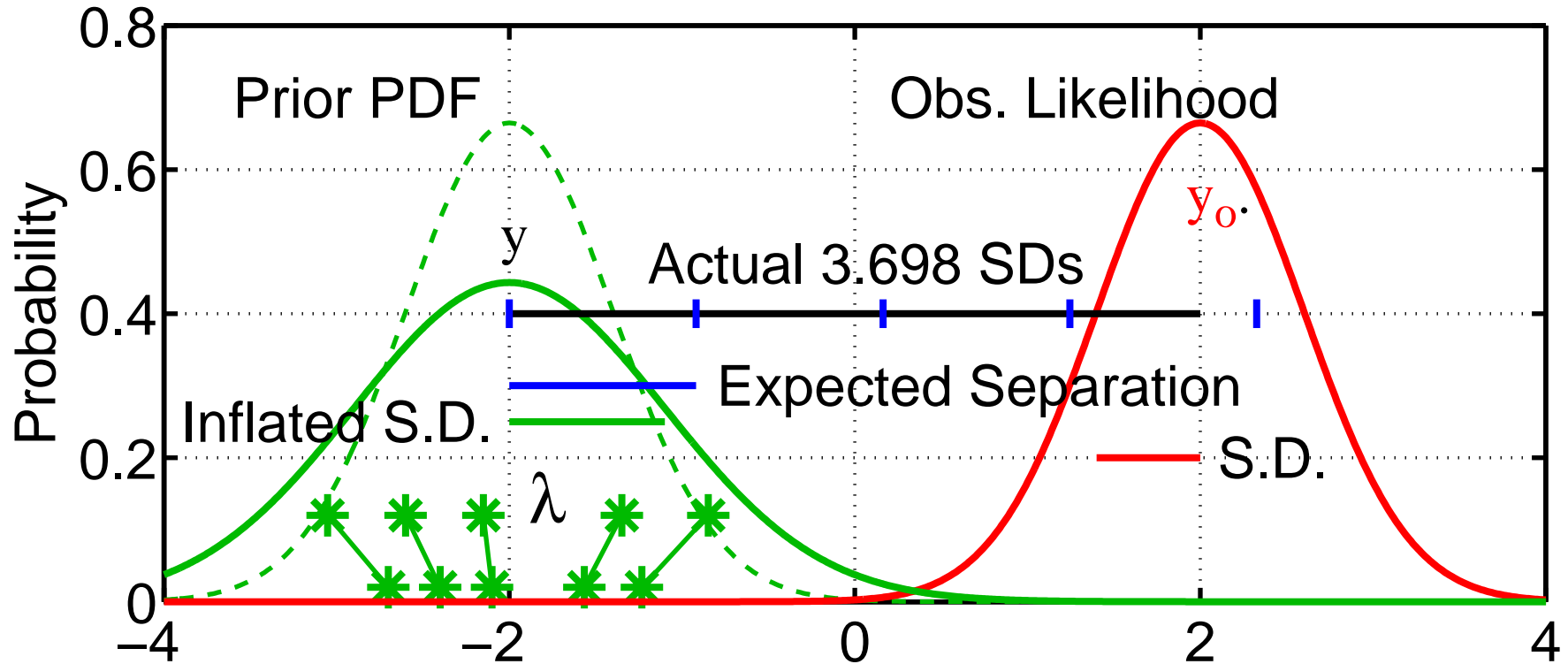
2. Expected(prior mean - observation) = $\sqrt{\sigma_{prior}^2 + \sigma_{obs}^2}$.

3. Inflating increases expected separation.

Increases 'apparent' consistency between prior and observation.

Adaptive Inflation for Ensemble Filtering

1. For observed variable, have estimate of prior-observed inconsistency.

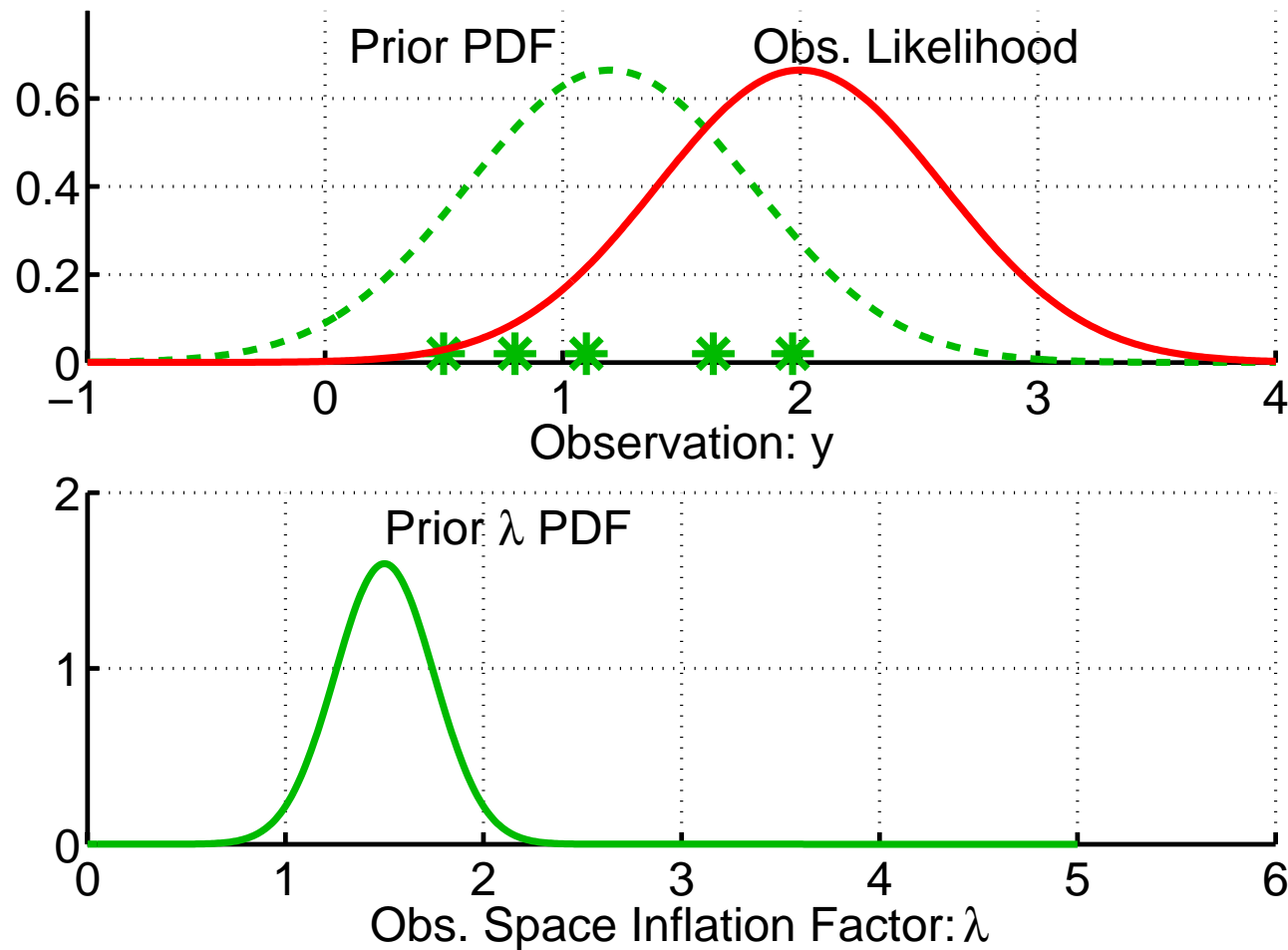


Distance, D , from prior mean y to obs. is $N\left(0, \sqrt{\lambda\sigma_{prior}^2 + \sigma_{obs}^2}\right) = N(0, \theta)$

Prob. y_o is observed given λ : $p(y_o|\lambda) = (2\pi\theta^2)^{-1/2} \exp(-D^2/2\theta^2)$

Adaptive Inflation for Ensemble Filtering

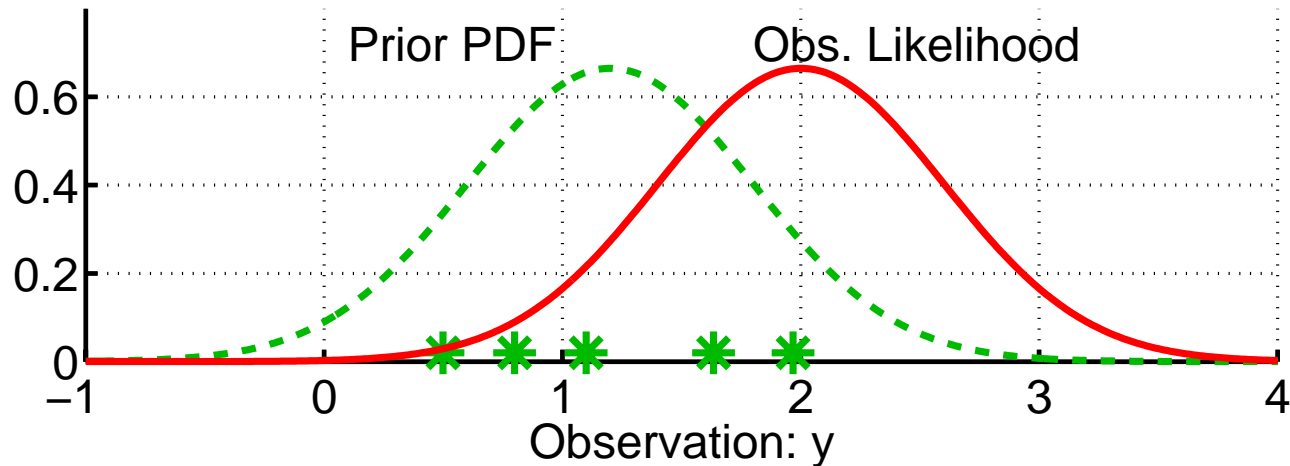
Use Bayesian statistics to get estimate of inflation factor, λ .



Assume prior is gaussian; $p(\lambda|Y_{prev}) = N(\bar{\lambda}_p, \sigma_{\lambda,p}^2)$.

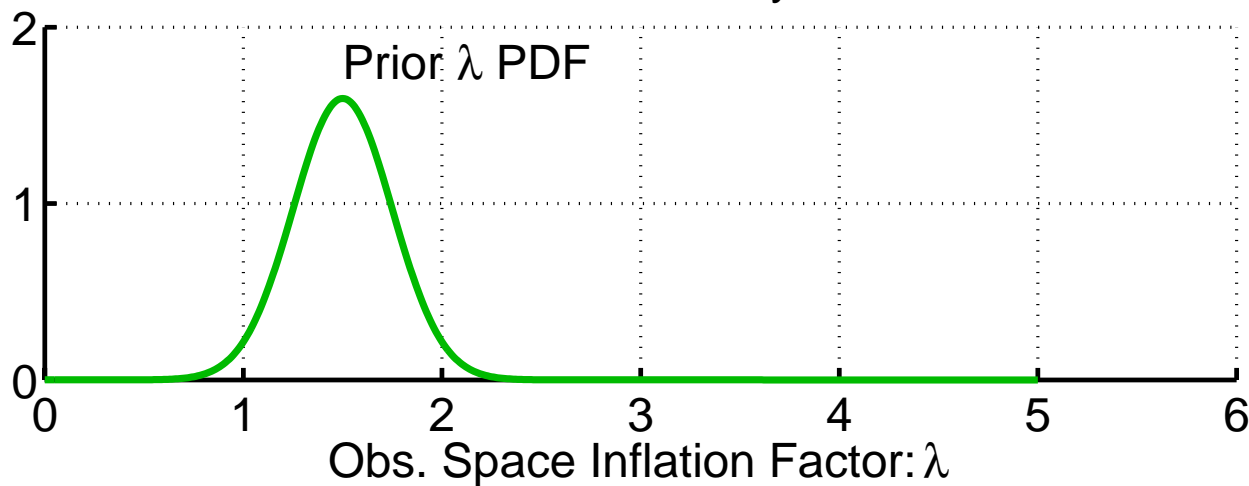
Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



We've assumed a gaussian for prior

$$p(\lambda|Y_{prev})$$

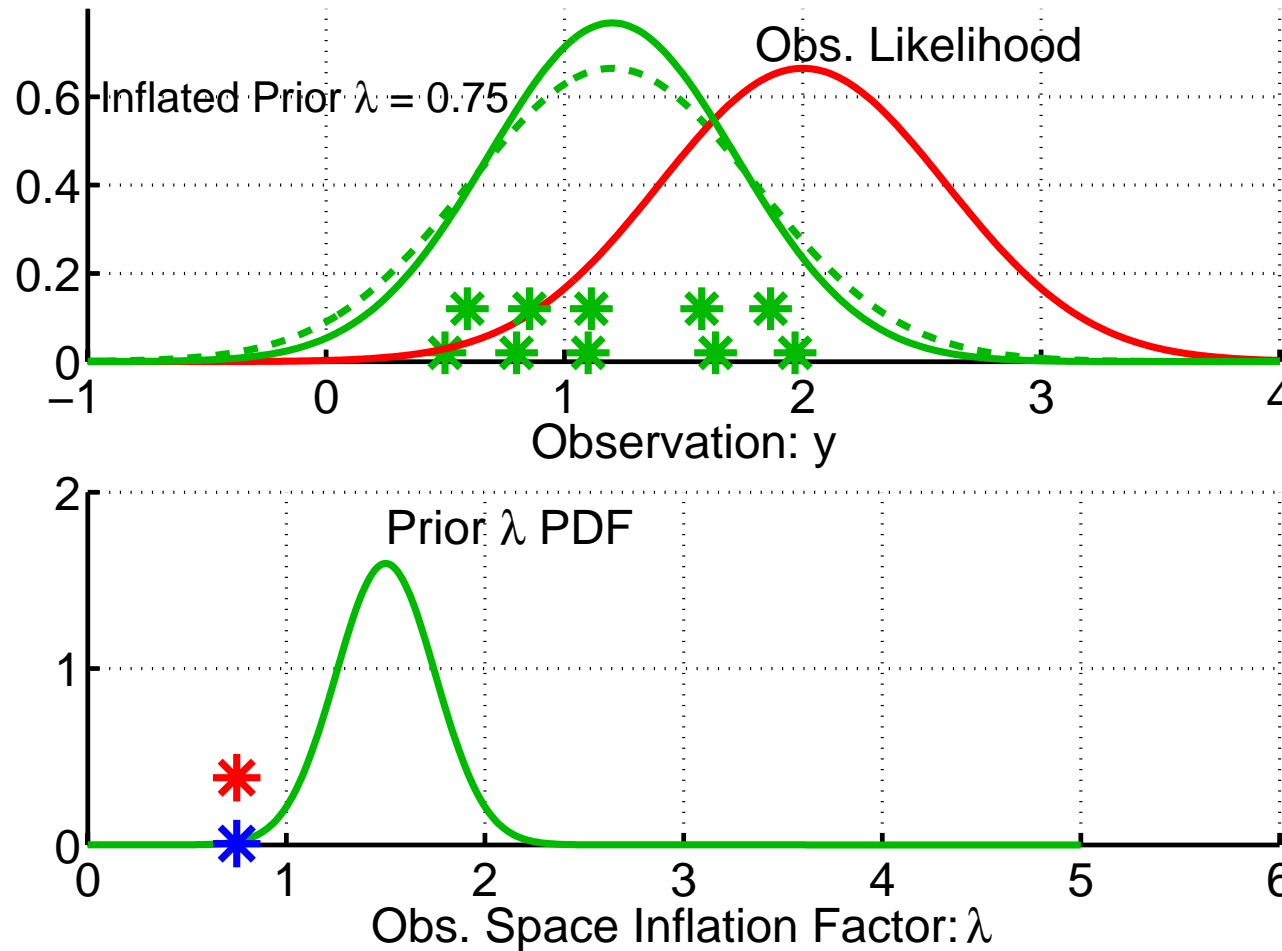


Recall that $p(y_o|\lambda)$ can be evaluated from normal PDF.

$$p(\lambda|Y_{prev}, y_o) = p(y_o|\lambda)p(\lambda|Y_{prev})/normalization.$$

Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



Get $p(y_o | \lambda = 0.75)$
from normal PDF.

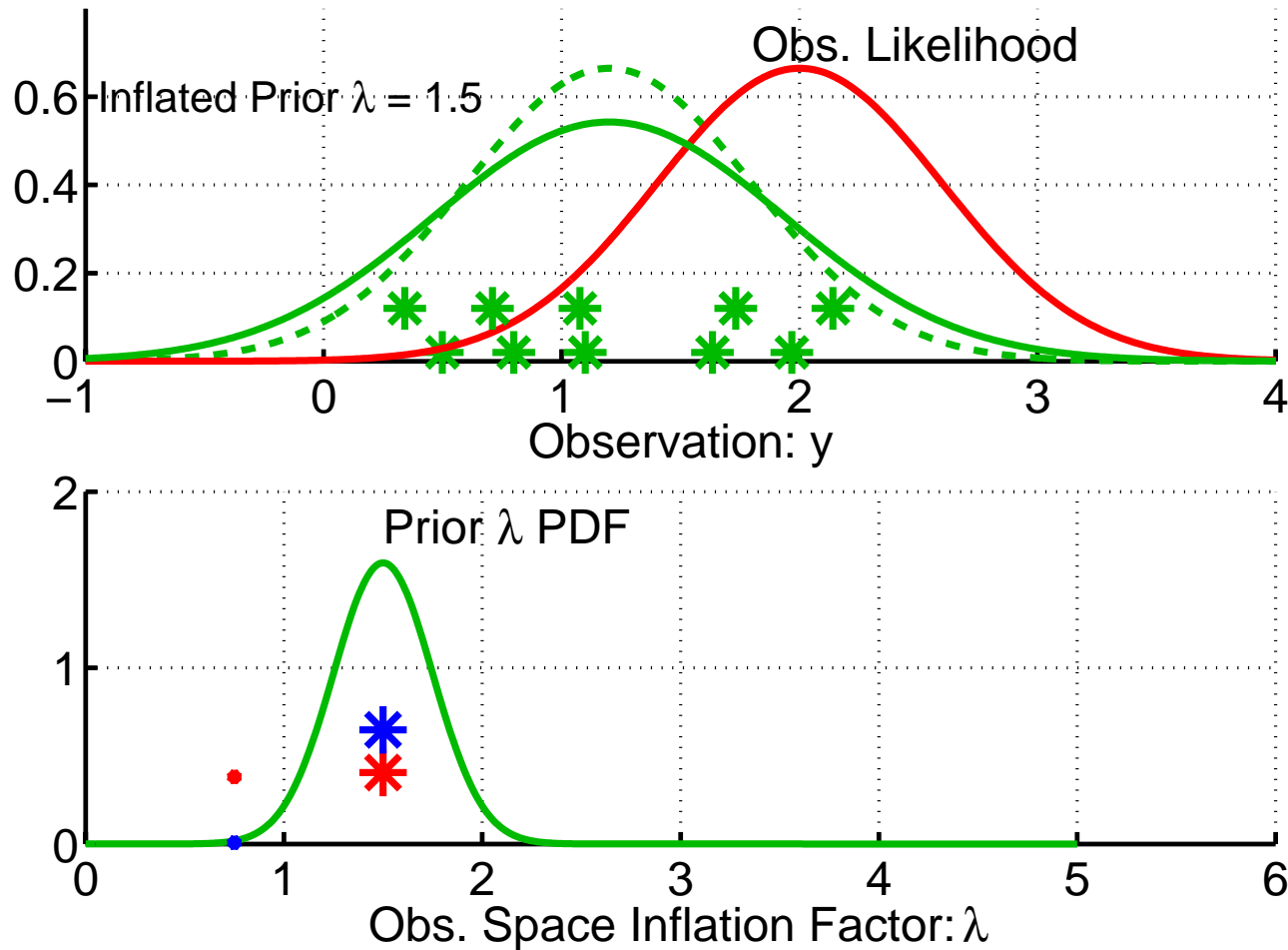
Multiply by
 $p(\lambda = 0.75 | Y_{prev})$
to get

$$p(\lambda = 0.75 | Y_{prev}, y_o)$$

$$p(\lambda | Y_{prev}, y_o) = p(y_o | \lambda) p(\lambda | Y_{prev}) / \textit{normalization} .$$

Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



Get $p(y_o|\lambda = 1.5)$
from normal PDF.

Multiply by

$$p(\lambda = 1.5|Y_{prev})$$

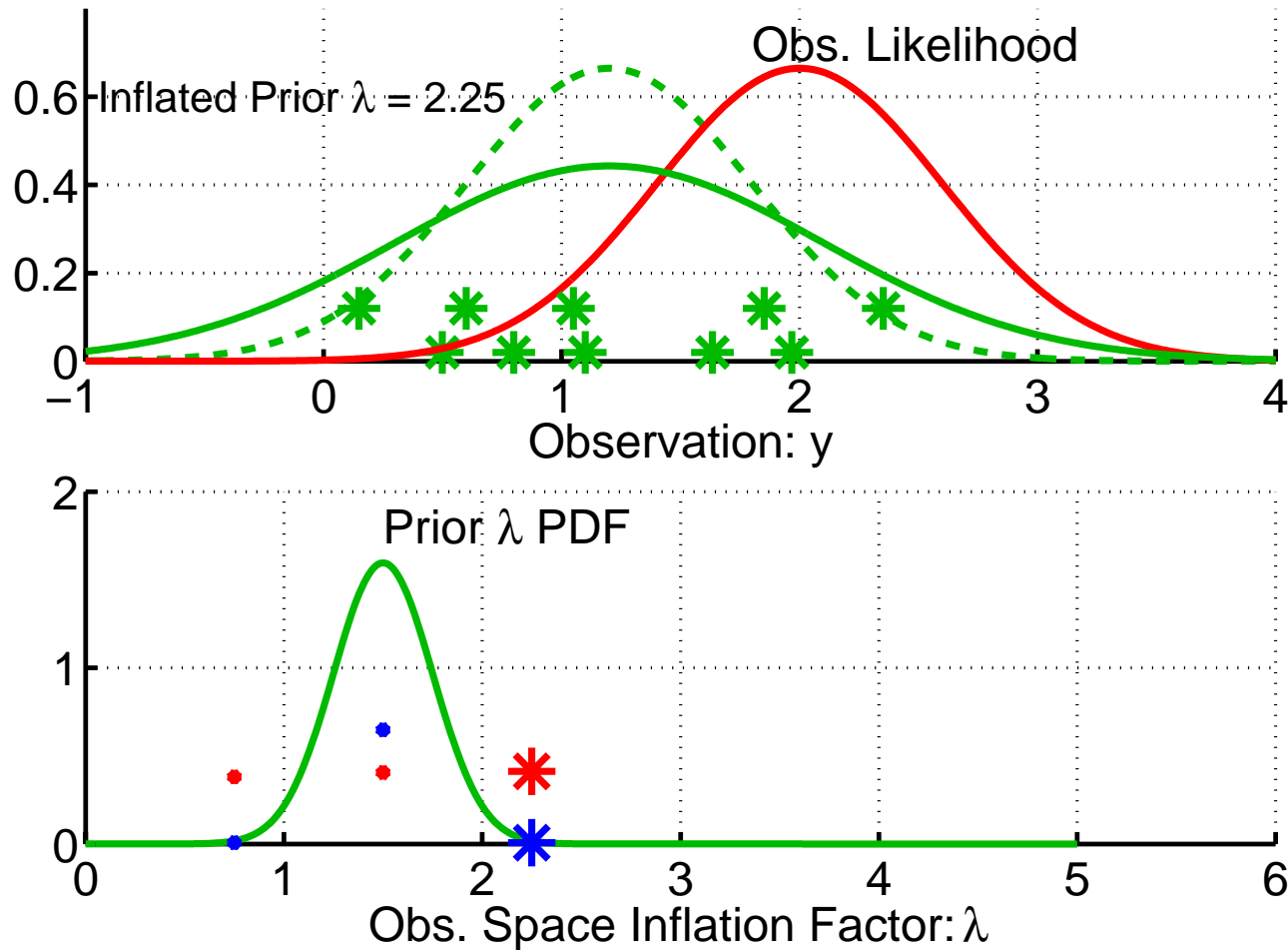
to get

$$p(\lambda = 1.5|Y_{prev}, y_o)$$

$$p(\lambda|Y_{prev}, y_o) = p(y_o|\lambda)p(\lambda|Y_{prev})/normalization .$$

Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



Get $p(y_o|\lambda = 2.2)$
from normal PDF.

Multiply by

$$p(\lambda = 2.2|Y_{prev})$$

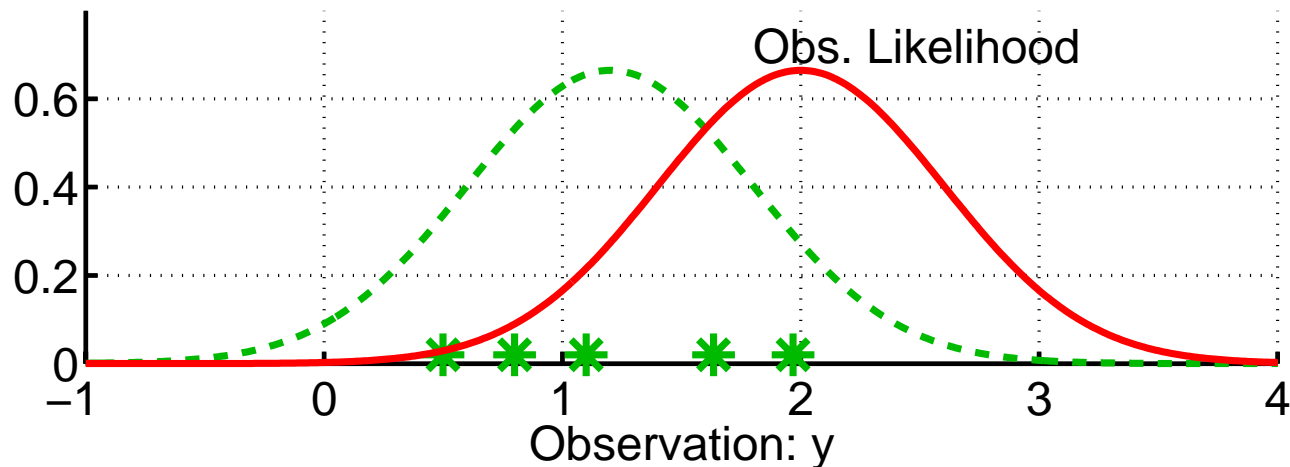
to get

$$p(\lambda = 2.2|Y_{prev}, y_o)$$

$$p(\lambda|Y_{prev}, y_o) = p(y_o|\lambda)p(\lambda|Y_{prev})/normalization .$$

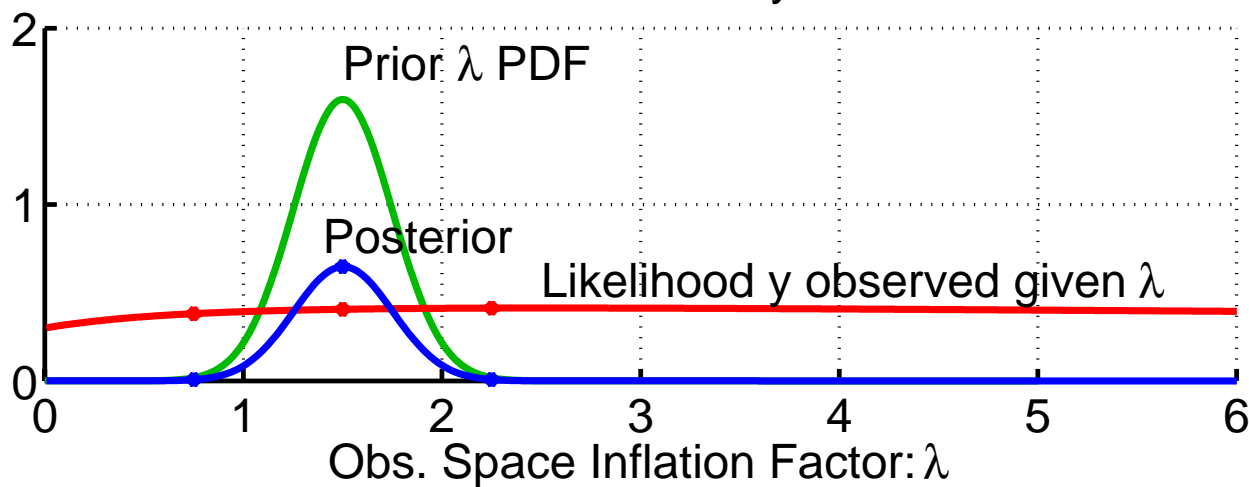
Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



Repeat for a range of values of λ .

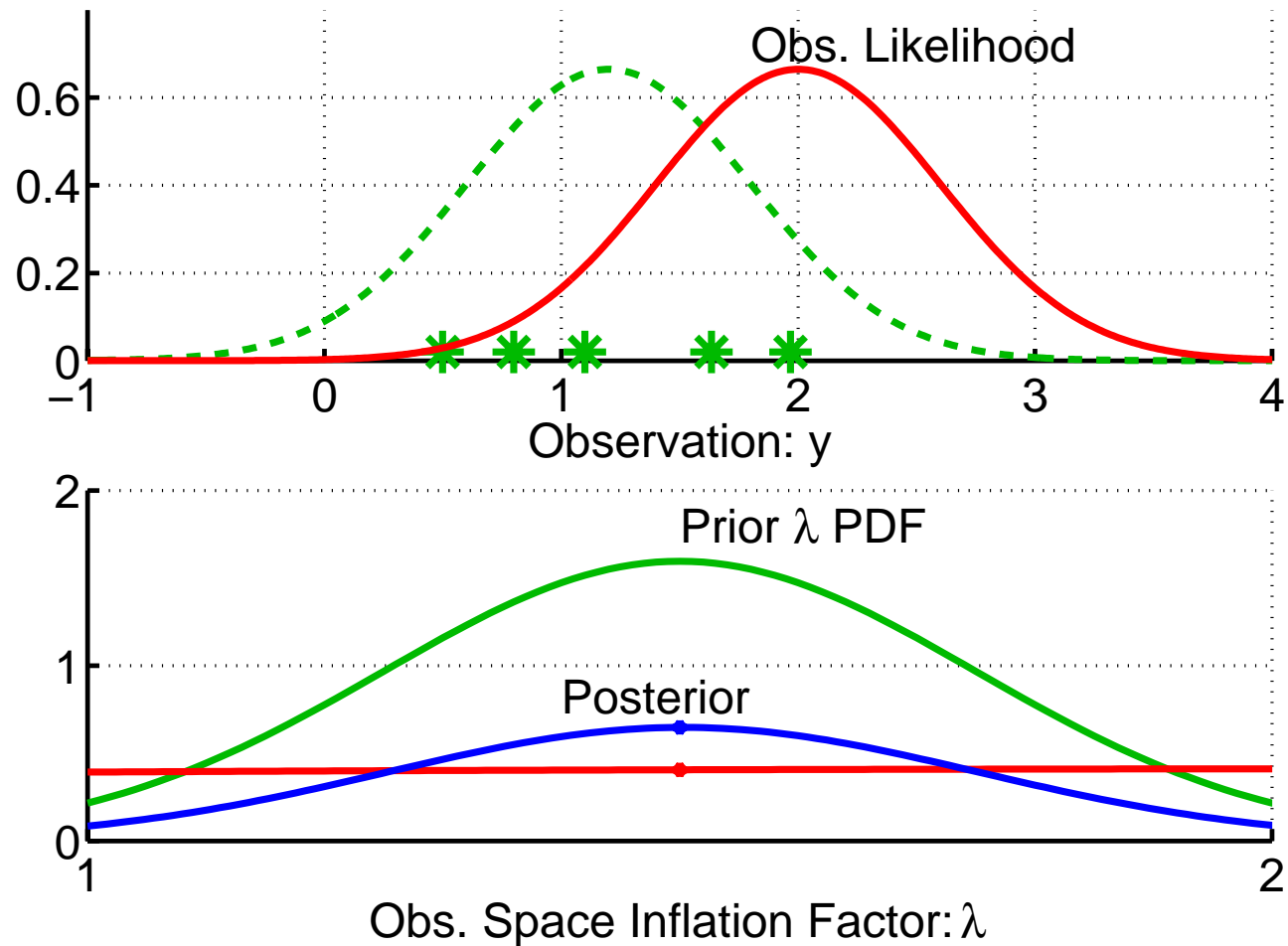
Now must get posterior in same form as prior (gaussian).



$$p(\lambda|Y_{prev}, y_o) = p(y_o|\lambda)p(\lambda|Y_{prev})/normalization .$$

Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



Very little information about λ in a single observation.

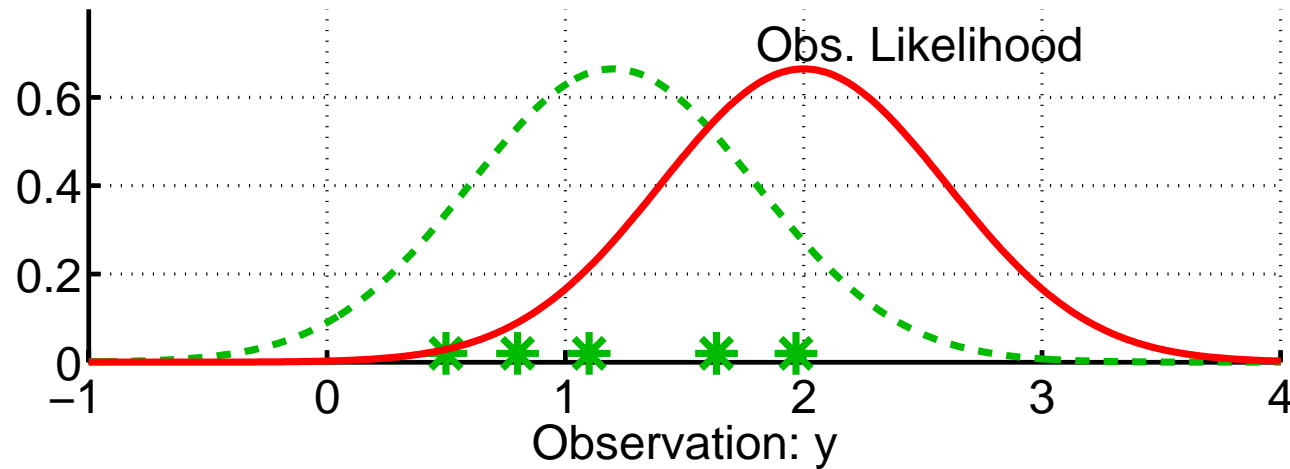
Posterior and prior are very similar.

Normalized posterior indistinguishable from prior.

$$p(\lambda | Y_{prev}, y_o) = p(y_o | \lambda) p(\lambda | Y_{prev}) / \textit{normalization} .$$

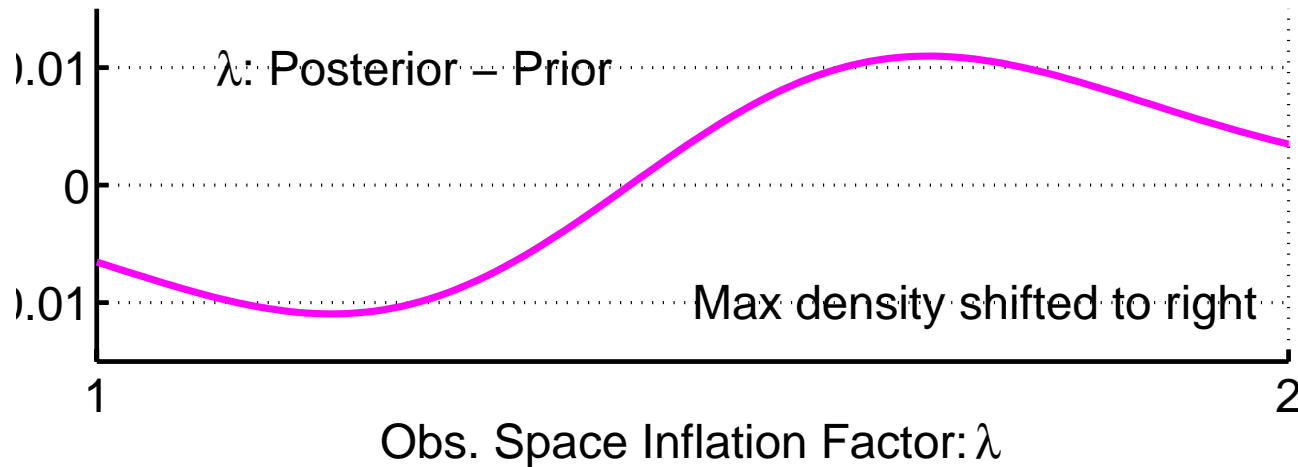
Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



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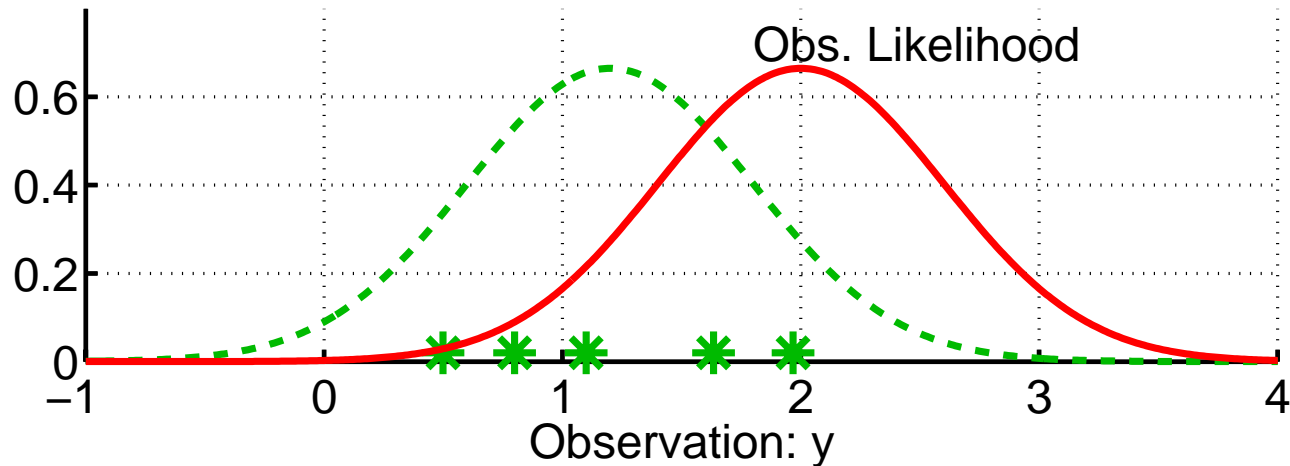


Difference shows slight shift to larger values of λ .

$$p(\lambda | Y_{prev}, y_o) = p(y_o | \lambda) p(\lambda | Y_{prev}) / \textit{normalization} .$$

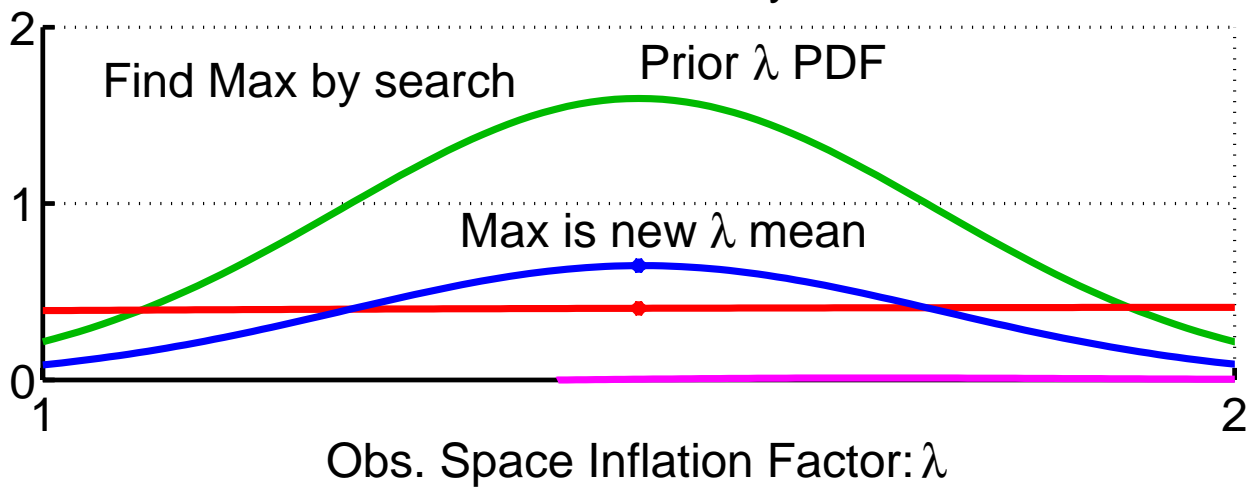
Adaptive Inflation for Ensemble Filtering

Use Bayesian statistics to get estimate of inflation factor, λ .



One option is to use Gaussian prior for λ .

Select max (mode) of posterior as mean of updated Gaussian.



Do a fit for updated standard deviation.

$$p(\lambda|Y_{prev}, y_o) = p(y_o|\lambda)p(\lambda|Y_{prev})/normalization .$$

Adaptive Inflation for Ensemble Filtering

A. Computing updated inflation mean, $\bar{\lambda}_u$.

Mode of $p(y_o|\lambda)p(\lambda|Y_{prev})$ can be found analytically!

Solving $\partial[p(y_o|\lambda)p(\lambda|Y_{prev})]/\partial\lambda = 0$ leads to 6th order poly in θ

This can be reduced to a cubic equation and solved to give mode.

New $\bar{\lambda}_u$ is set to the mode.

This is relatively cheap compared to computing regressions.

Adaptive Inflation for Ensemble Filtering

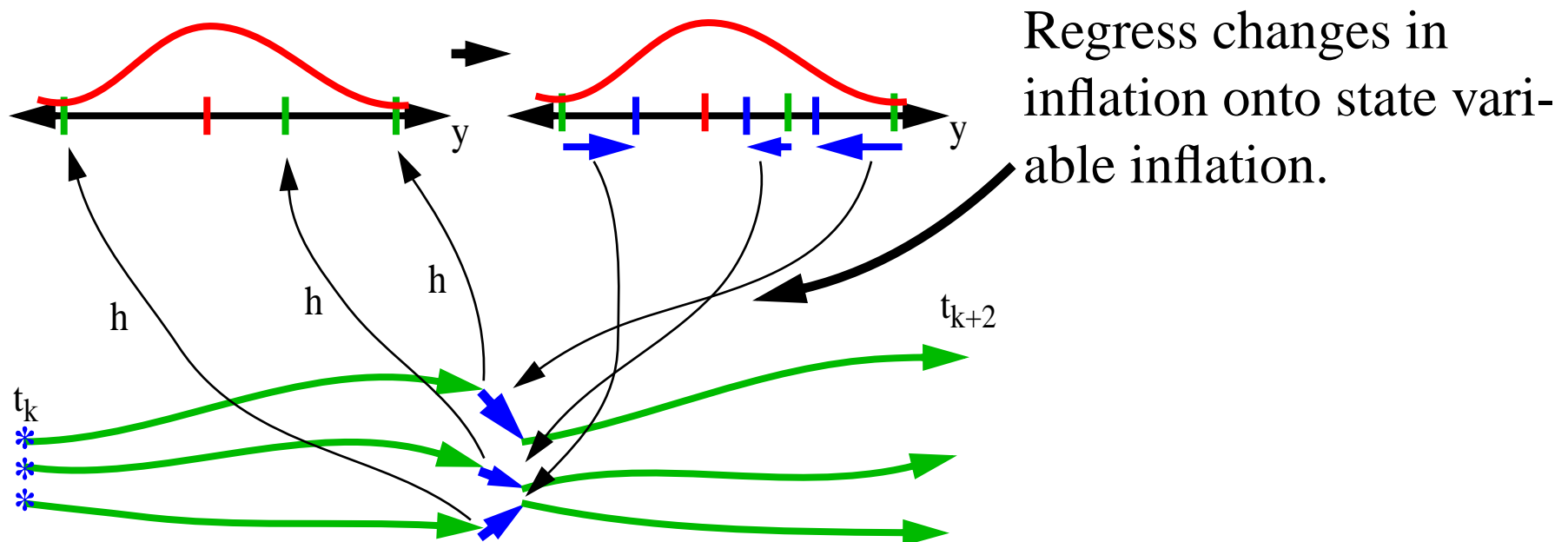
- A. Computing updated inflation variance, $\sigma_{\lambda, u}^2$
1. Evaluate numerator at mean $\bar{\lambda}_u$ and second point, e.g. $\bar{\lambda}_u + \sigma_{\lambda, p}$.
 2. Find $\sigma_{\lambda, u}^2$ so $N(\bar{\lambda}_u, \sigma_{\lambda, u}^2)$ goes through $p(\bar{\lambda}_u)$ and $p(\bar{\lambda}_u + \sigma_{\lambda, p})$.
 3. Compute as $\sigma_{\lambda, u}^2 = -\sigma_{\lambda, p}^2 / 2 \ln r$ where $r = p(\bar{\lambda}_u + \sigma_{\lambda, p}) / p(\bar{\lambda}_u)$.

State Space Adaptive Inflation

Computations so far adapt inflation for observation space.

What is relation between observation and state space inflation?

Have to use prior ensemble observation/state joint distribution.



Spatially varying adaptive inflation algorithm:

Have a distribution for λ at each time for each state variable, $\lambda_{s,i}$.

Use prior correlation from ensemble to determine impact of $\lambda_{s,i}$ on prior variance for given observation.

If γ is correlation between state variable i and observation then

$$\theta = \sqrt{[1 + \gamma(\sqrt{\lambda_{s,i}} - 1)]^2 \sigma_{prior}^2 + \sigma_{obs}^2}.$$

Equation for finding mode of posterior is now full 12th order:

Analytic solution appears unlikely.

Can do Taylor expansion of θ around $\lambda_{s,i}$.

Retaining linear term is normally quite accurate.

There is an analytic solution to find mode of product in this case!

Hierarchical Bayesian Methods for Adaptive Filters: Summary

1. Localization:

Run an ensemble of ensembles.

Use regression coefficient signal-to-noise ratio to minimize error.

2. Inflation:

Use each observation twice.

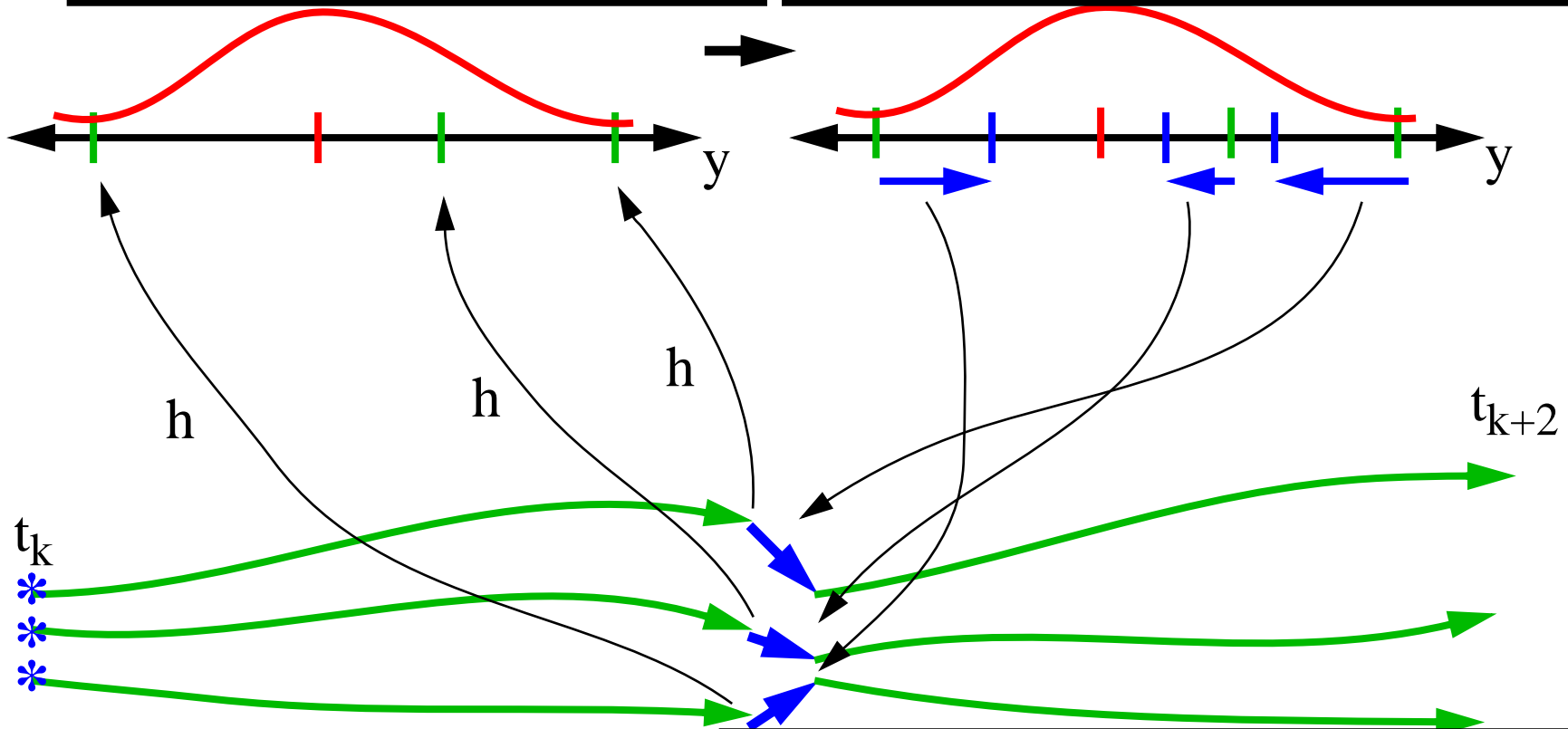
Once to adjust parameter (inflation) of filter system.

Second time to adjust mean and variance of estimate.

Speed is of the Essence: A Parallel Sequential Filter.

2. h for each ensemble is independent.
All done in parallel.

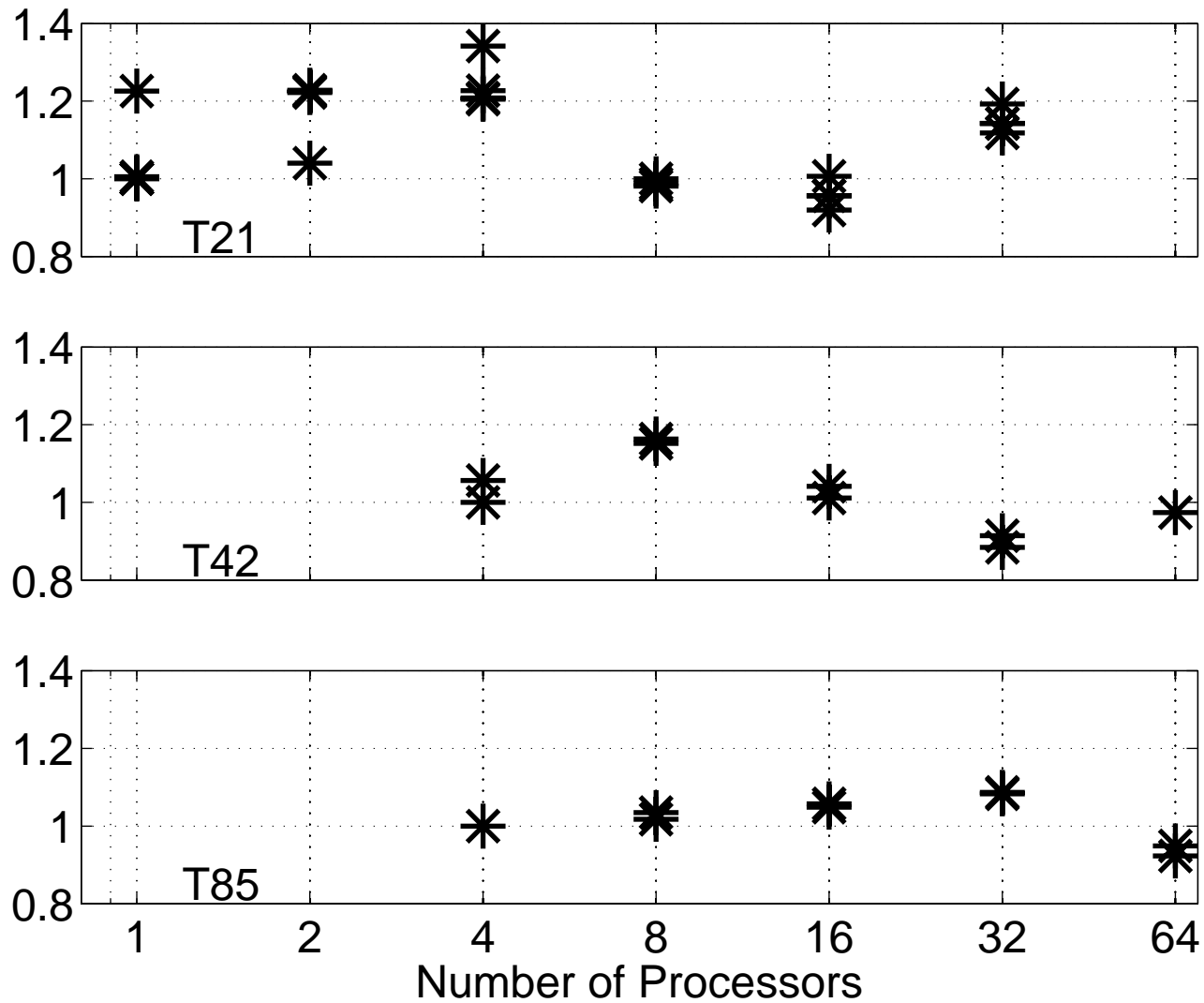
3. Designated process computes increments.
Broadcasts increments to all processes.



1. Advances Embarrassingly Parallel.
Each is completely independent.

4. State variables randomly assigned to processes.
Each process does regression for its state.
Completely independent.

Speed is of the Essence: A Parallel Sequential Filter.



Normalized Time for CAM Assimilations (1 is perfect scaling).

Simple for Users to Get Started with DART

1. Incorporating existing model requires handful of interfaces.
 - A. No need for linear tangents or adjoints.
 - B. Finite Volume CAM added in 1-month by postdoc.
 - C. Recently received unsolicited low-order model.

2. Adding observations also straightforward.
 - A. Only need forward operator (map state to expected observation).
 - B. No linear tangents or adjoints.
 - C. Several different GPS operators added in weeks.

DART compliant models (largest set ever with assim system)

1. Many low-order models (Lorenz63, L84, L96, L2004,...).
2. Global 2-level PE model (from NOAA/CDC).
3. CGD's CAM 2.0, 3.0, 3.1 (global spectral model).
- 3a. CGD's CAM 3.1 FV (global finite volume model) with chemistry.
4. GFDL AM GCM (global grid point model).
5. MIT GCM (from Jim Hansen MIT; configured for annulus).
6. WRF model (regional prediction grid point, MMM).
- 6a. WRF column physics model (Josh Hacker RAL).
7. NCEP GFS (operational global spectral; assisted by NOAA/CDC).
8. GFDL MOM3/4 (global grid point ocean model).
9. ACD's ROSE model (upper atmosphere with chemistry).
10. Cane-Zebiak 5 (tropical ocean/atmosphere, Alicia Karspeck CGD).
11. EPA's CMAQ dispersion model (U. Chicago).

Also models from outside geophysics.

This allows for a hierarchical approach to filter development.

DART compliant Forward Operators and Datasets

Many linear and non-linear forward operators for low-order models.

U, V, T, Ps, Q, dewpoint for realistic models.

Radar reflectivity, doppler velocity, GPS refractivity for realistic models.

Mopitt CO retrievals.

Can ingest observations from reanalysis or operational BUFR files.

Can create synthetic (perfect model) observations for any of these.

Ensemble Assimilation Research Challenges

1. Assimilating when gaussian assumption breaks down.
 - Discrete structures (thunderstorms).
 - Bounded quantities (mixing ratio).

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 - Direct assimilation of satellite radiances.

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References on Filter Algorithms:

1. Deterministic ensemble filters:

Anderson, J., “An ensemble adjustment Kalman filter for data assimilation,” *Monthly Weather Review*, **129**, 2884-2903, 2001.

2. Sequential filter algorithms:

Anderson, J., “A local least squares framework for ensemble filtering,” *Monthly Weather Review*, **131**, 634-642, 2003.

3. Inflation:

Anderson, J., and S. L. Anderson, “A Monte Carlo implementation of the non-linear filtering problem to produce ensemble assimilations and forecasts,” *Monthly Weather Review*, **127**, 2741-2758, 1999.

Anderson, J., “An adaptive covariance inflation error correction algorithm for ensemble filters”, To appear in *Tellus A*, 2007.

4. Adaptive localization:

Anderson, J., “Exploring the need for localization in ensemble data assimilation using an hierarchical ensemble filter”, to appear in *Physica D*, 2007.

5. Parallel algorithm:

Anderson, J. L., and N. Collins, “Scalable implementations of ensemble filter algorithms for data assimilation. To appear in *J. Atmos. Ocean. Tech. A.*, 2007.