Reducing the Impact of Sampling Errors in Ensemble Filters

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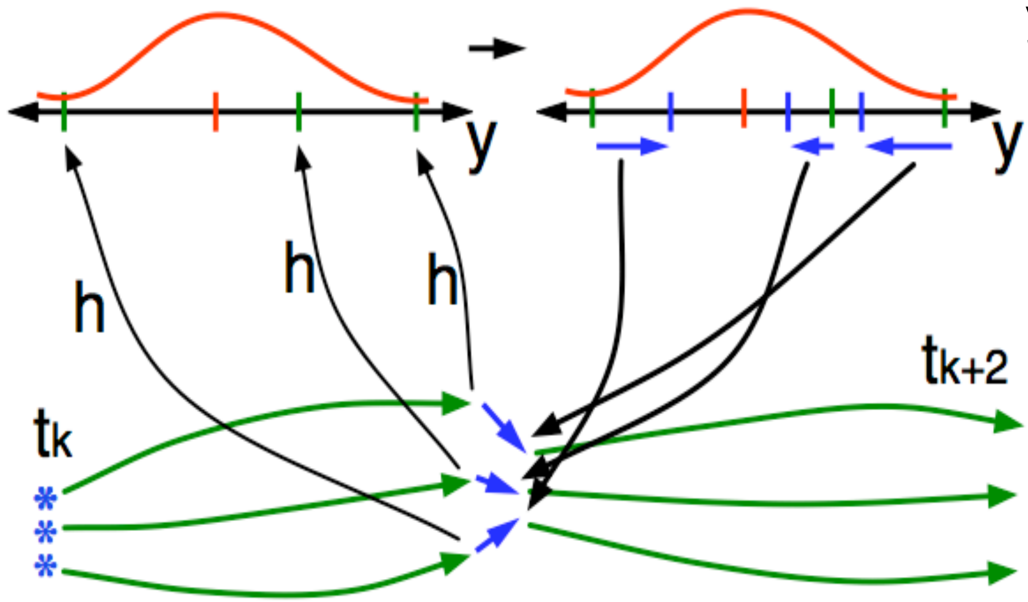
Sequential ensemble filter assimilation

- Impact of single ob y on scalar state component x_i.
- Describes problem without loss of generality.



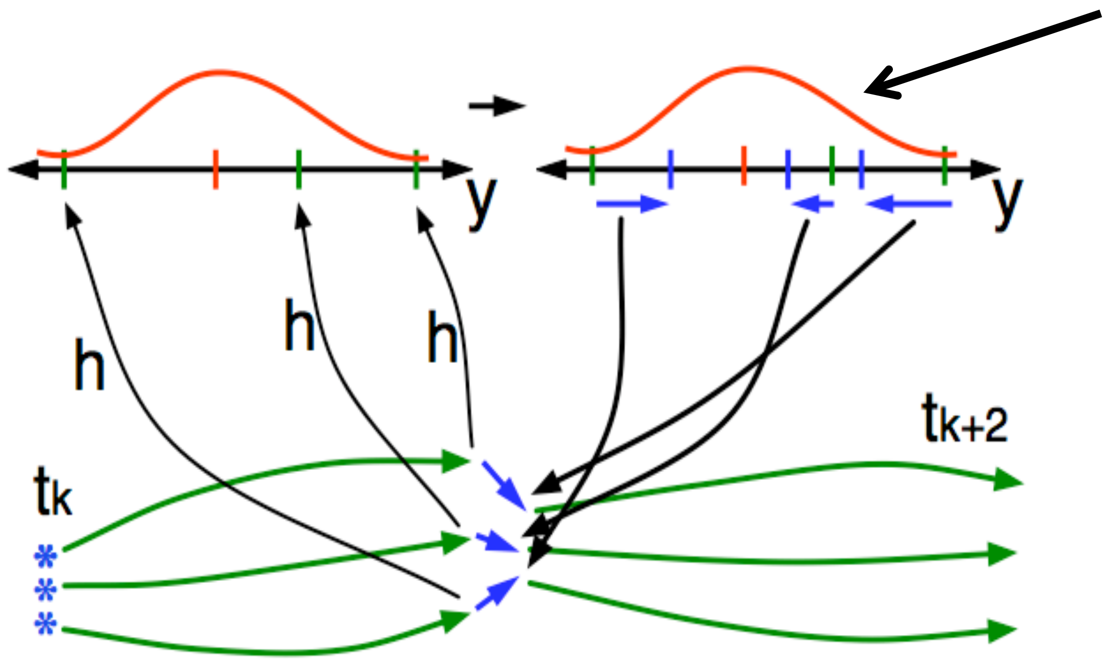
y is scalar observation.

Model state vector ensemble.

Sequential ensemble filter assimilation

- Impact of single ob y on scalar state component x_i.

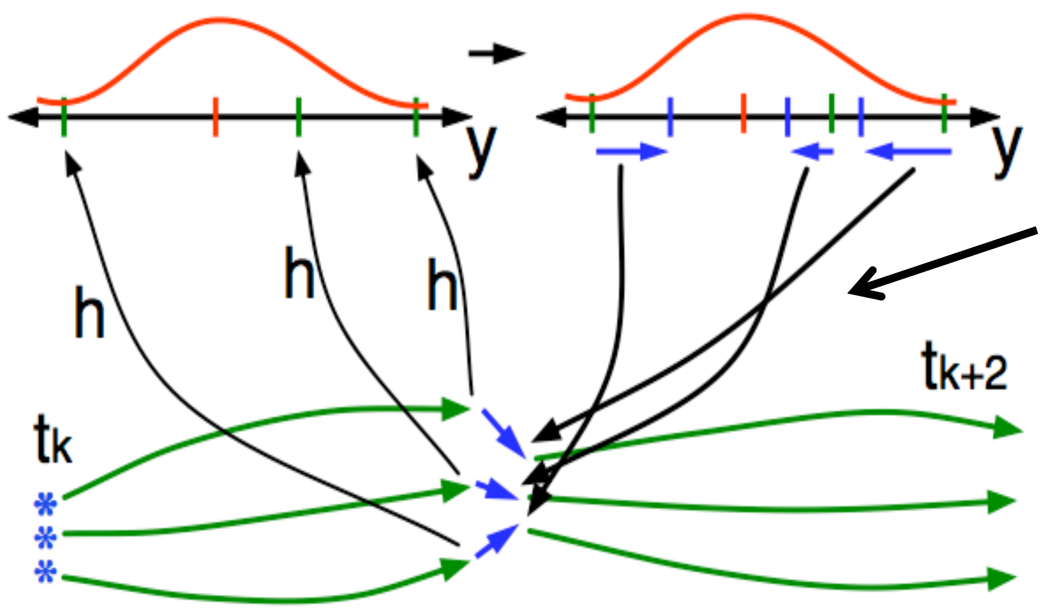
  Compute increments for prior y ensemble.
  (Details irrelevant for this talk)



Sequential ensemble filter assimilation

- Impact of single ob y on scalar state component x_i.

Regress y increments onto each component of x.



$\Delta x_{i,n} = b\Delta y_{n},$
$n=1,\ldots N$

N is ensemble size.

b is regression coefficient.

Ensemble filters are optimal and exact when…

- Model is linear,
- Observation operators h are linear,
- Observation error is gaussian,
- Ensemble size $N > N_{crit}$,
- Filter is EAKF (or similar deterministic).

Violating any of these may lead to suboptimal solution.

Most commonly observed errors are…

- Too little variance in the ensemble (often solved by inflation).
- Too much correlation including spurious correlations between unrelated distant observations (often solved by localization).

Will treat all errors as sampling errors.

Dealing with Sampling Errors

- A posteriori tuning: select parameters, run assimilation, evaluate fit to observations.
- A priori use of model only: assess sampling error using ensembles during assimilation (no use of obs).
- A priori use of observations and model: adaptive inflation (Miyoshi).
- A priori use of observations and model: adaptive localization (really hard?).

Dealing with Sampling Errors in This Talk

- Focus on errors in the regression step (Kalman gain).
- A priori methods using model only.
- In the form of a localization (next slide).

Definition of localization

- For impact of observation y on state component x_i, multiply regression coefficient by factor α

$$\Delta x_{i,n} = \alpha b \Delta y_n , \qquad n=1,\ldots N.$$

- α often function of distance between y and x_i.
- Gaspari Cohn compact pseudo-gaussian function.
- Usually α<1, correlation never too small.

Sampling error correction as a localization

Let $\hat{b}$ be ensemble sample regression.

(Hats always mean ensemble sample)

Sampling error correction as a localization

Let $\hat{b}$ be ensemble sample regression.

Assume that there is sampling error.

View $\hat{b}$ as draw from $Normal\left(\bar{b}, \sigma_b\right)$.

Find value of α that minimizes expected RMS error of $\bar{x}_i$ after regression.

This can be done analytically…

$$\alpha = \bar{b}^2 \Big/ \left(\sigma_b^2 + \bar{b}^2\right) = Q^2 \Big/ \left(1 + Q^2\right)$$

where

$$Q = \bar{b} \,/\sigma_b \ .$$

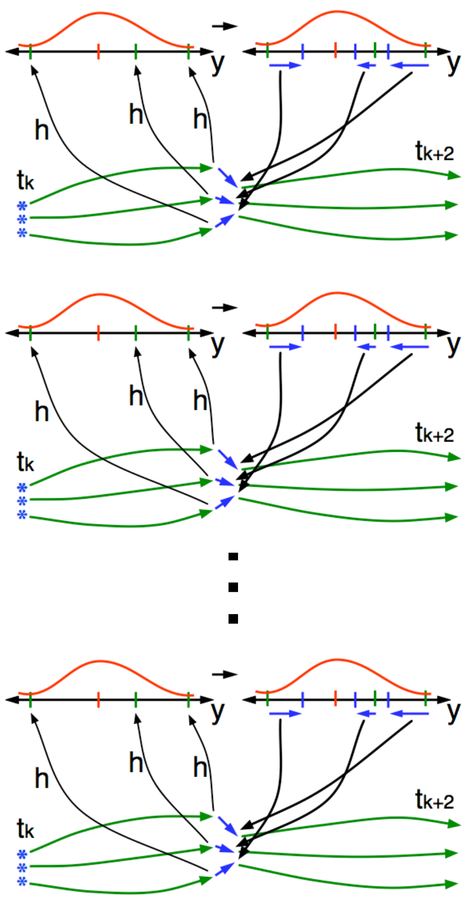
But, all I have is single estimate of $\hat{b}$.

Solution 1. Group filter:

- Run M ensembles each with N members.

Correcting Regression Sampling Error: Group Filter

Ensembles differ only in initial values (long, long ago).



$\hat{b}_1$

$\hat{b}_2$

$\vdots$

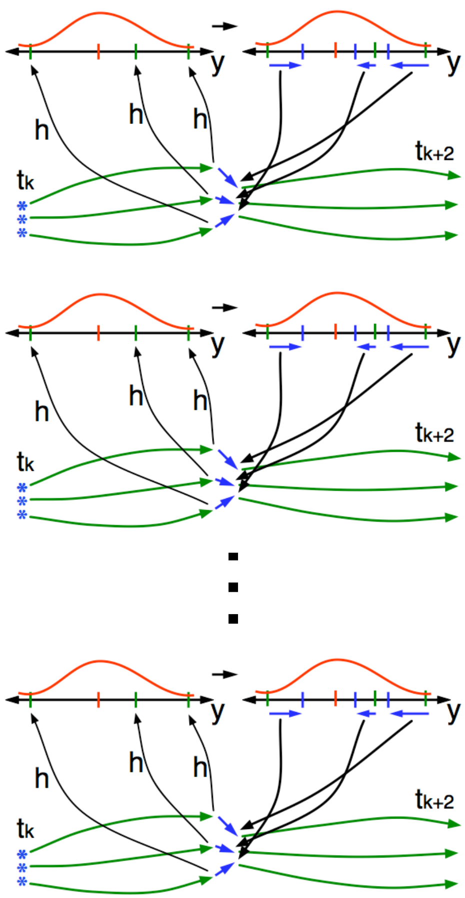
$\hat{b}_M$

Compute $\bar{b}$ and σ_b.

Compute optimal localization α for this y and x_i.

Correcting Regression Sampling Error: Group Filter

Ensembles differ only in initial values (long, long ago).



$\hat{b}_1$

$\hat{b}_2$

$\vdots$

$\hat{b}_M$

Very Expensive!

Compute $\overline{b}$ and σ_b.

Compute optimal localization α for this y and x_i.

Using additional prior information

Need for localization implies that standard ensemble is suboptimal.

Need additional prior assumptions to improve it.

Assume a prior distribution for correlation r

- Assuming prior for b is hard (it's unbounded).
- Assuming that correlation of y and x_i is U(-1, 1) when nothing else is known is natural.

Assume a prior distribution for correlation r

- Given prior U(-1, 1) and ensemble sample correlation $\hat{r}$ can compute:
  - Most likely value of ‘true’ correlation r,
  - Optimal value of α.

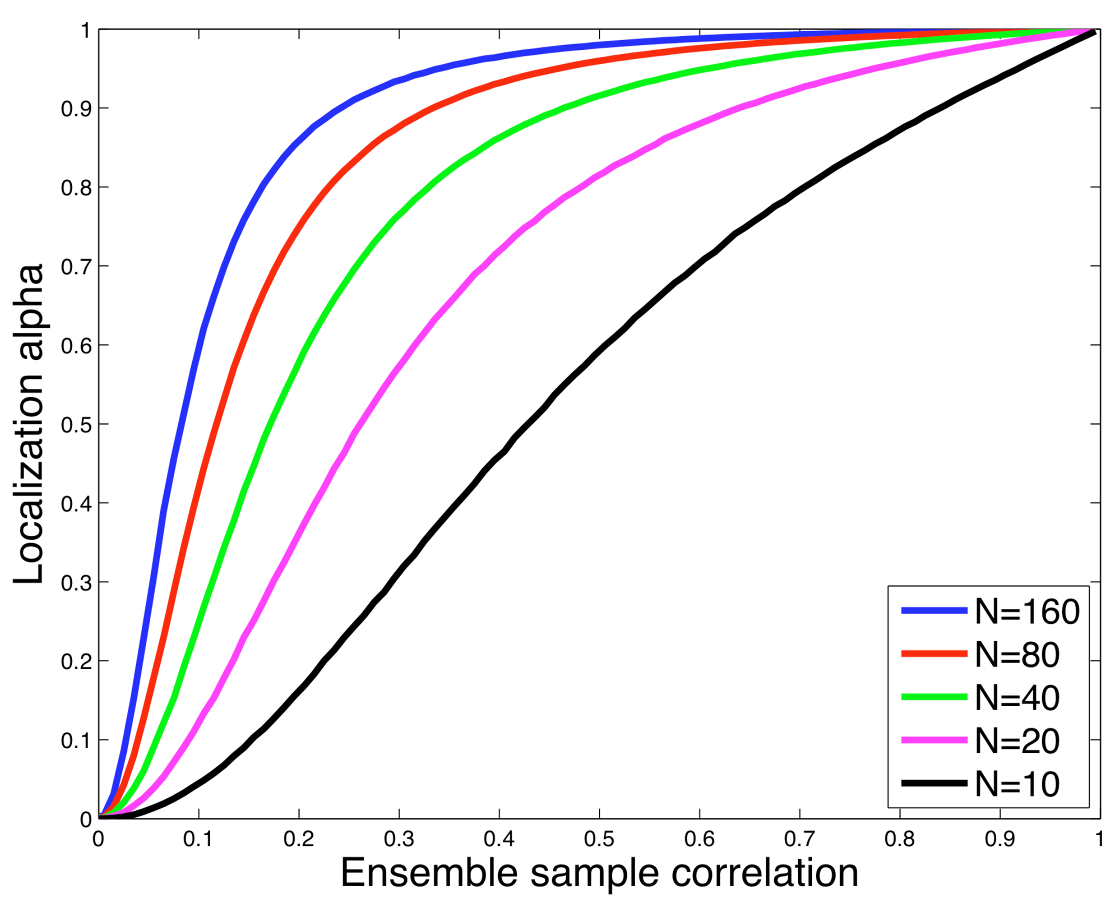
Sampling error correction algorithm:

Solution 2: Sampling error correction.

- Compute $\hat{r}$, sample correlation(y, x_i).
- Replace $\hat{r}$ with r , and localize with α.

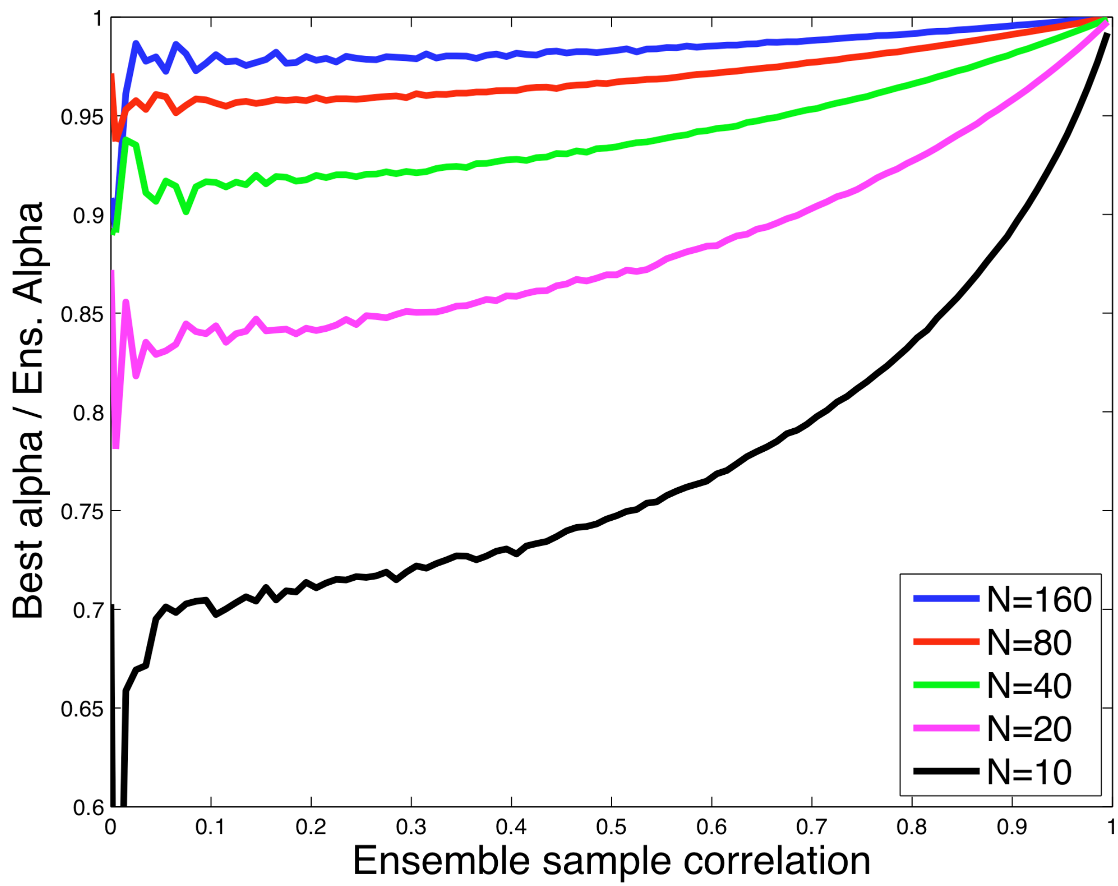
r and α are only a function of $\hat{r}$ and ensemble size so we can precompute. This is cheap!

Localization α as function of N and sample correlation $\hat{r}$.



This is precomputed for table look-up.

Most likely correlation r as function of N and $\hat{r}$.



This is precomputed for table look-up.

Test I: Separable linear model, identity obs.

$$x_{i,t+1} = (1 + c)x_{i,t} \qquad \text{i = 1, 200.}$$

Observation error variance is N(0, 1).

Has 200 degrees of freedom.

N>200 EAKF converges to optimal solution.

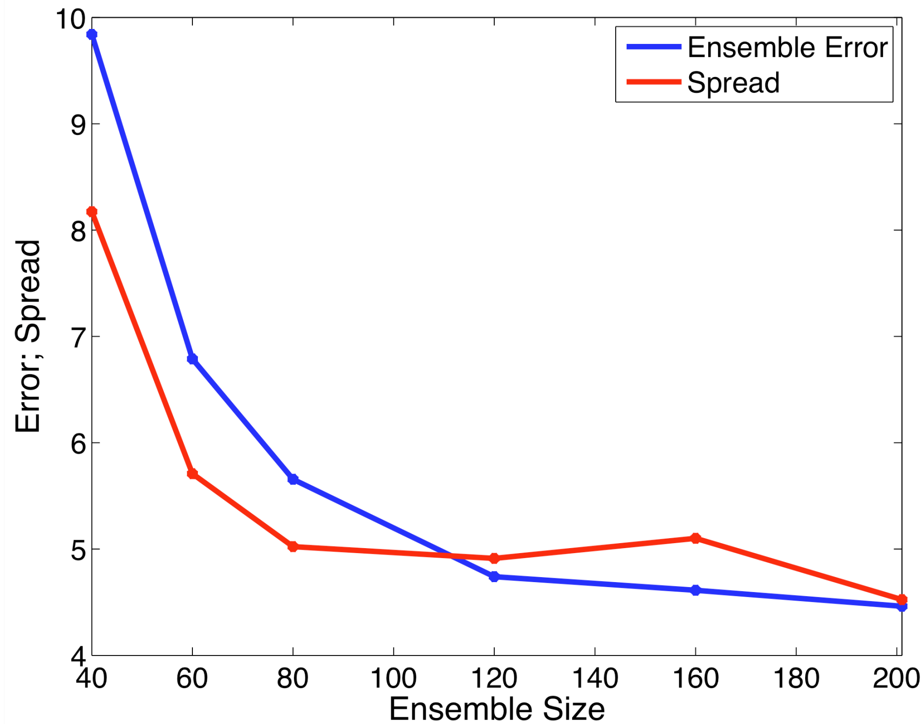
N<201 diverges without sampling error correction.

For N<201, optimal localization is delta function.

Compute time mean RMSE and α for various N.

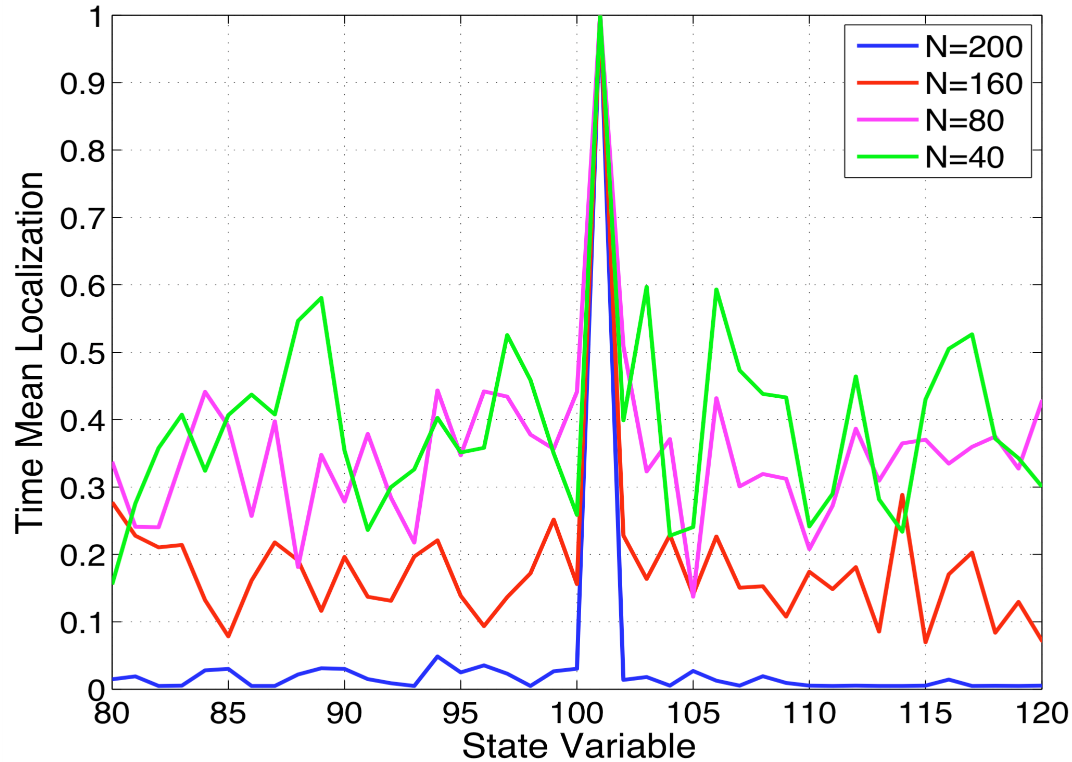
Note: Adaptive inflation used to stabilize.

Test I: Separable linear model, identity obs.



Time mean RMSE and spread as function of N.
N=201 is nearly optimal.
N<201 fails without sampling error correction .

Test I: Separable linear model, identity obs.

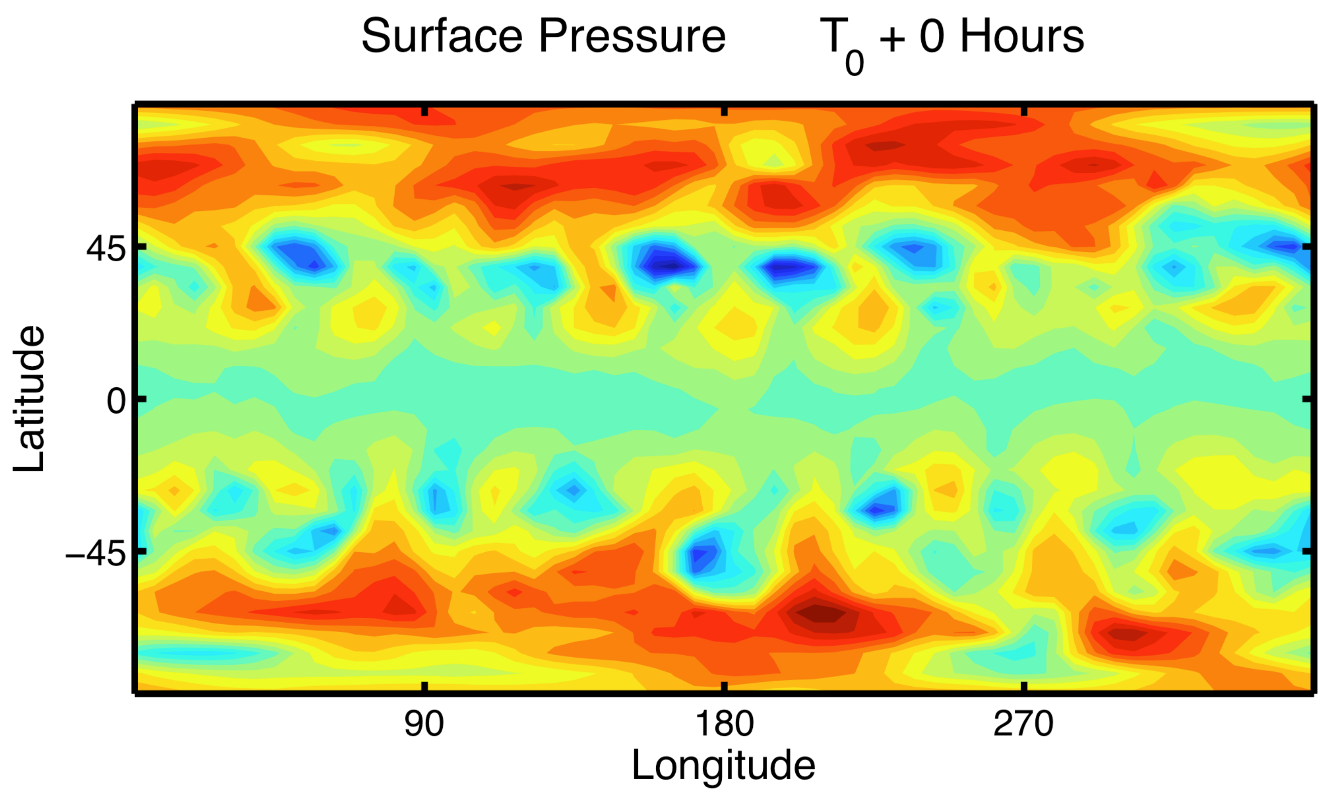


Time mean localization of observation 101.

Should be delta function.

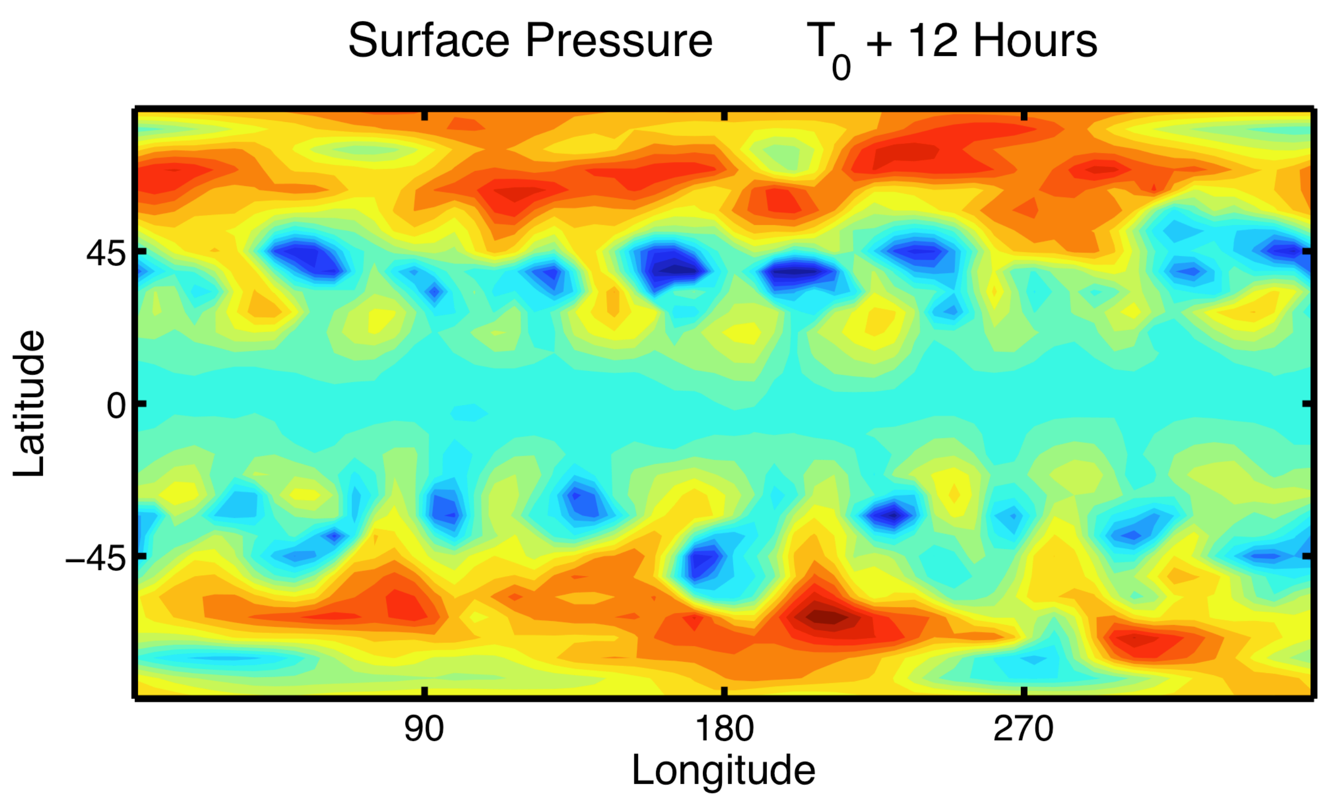
Ability to detect noise degrades as N decreases.

Test II: Low-order dry dynamical core



Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

Test II: Low-order dry dynamical core



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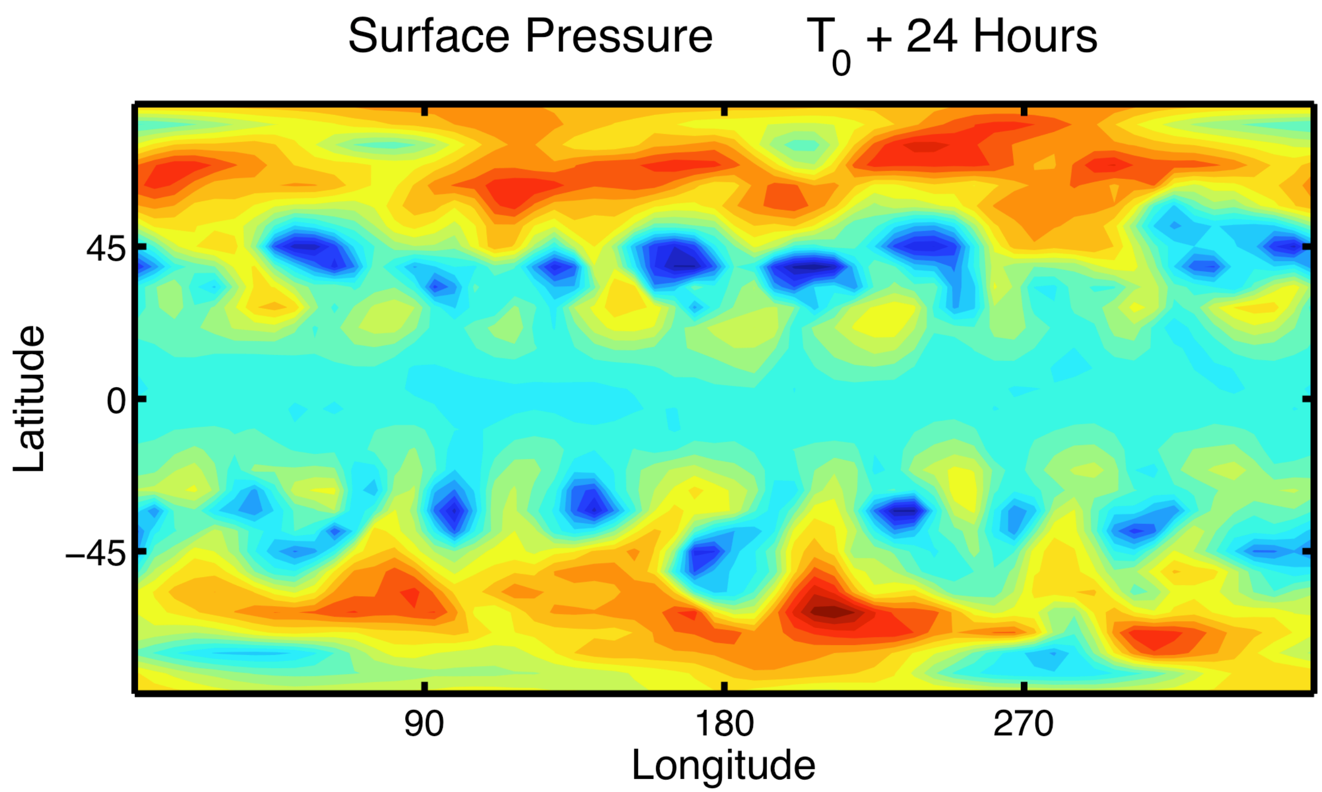

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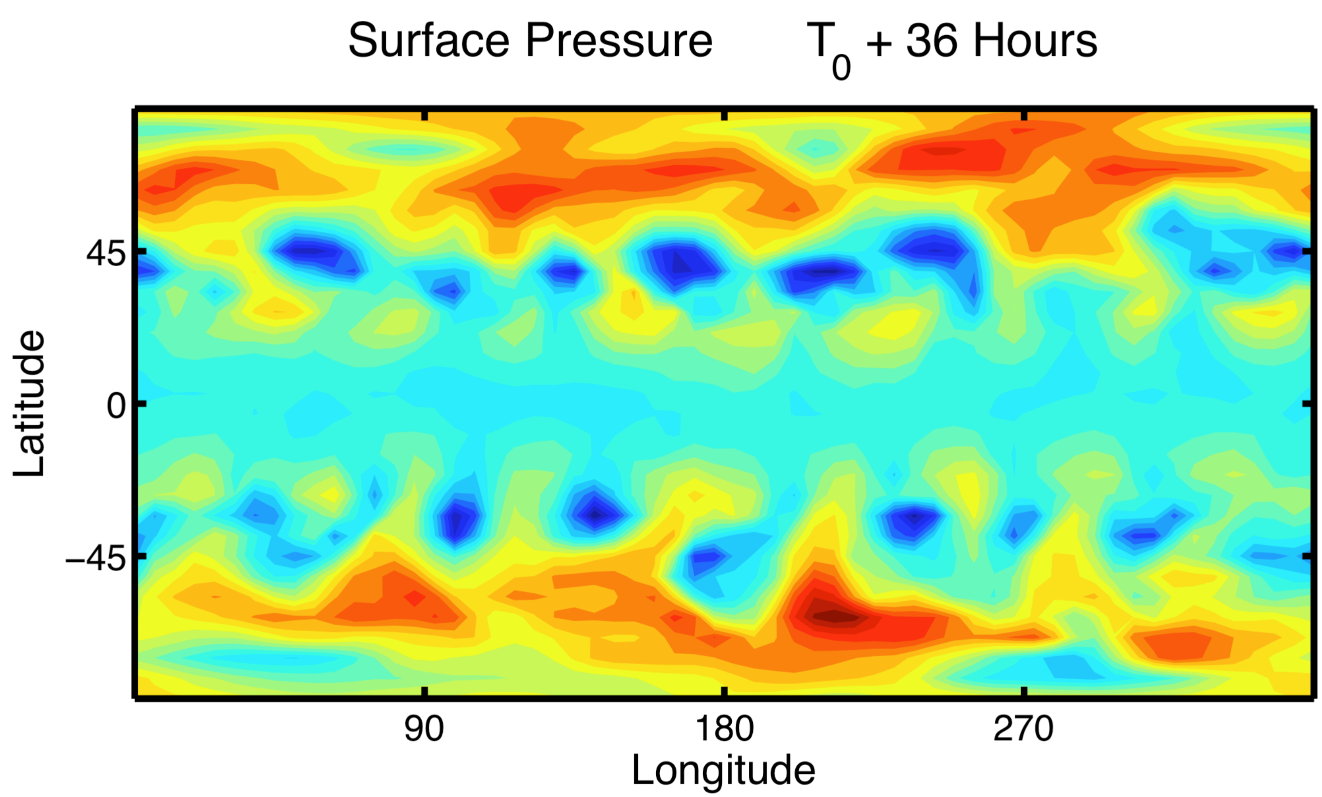

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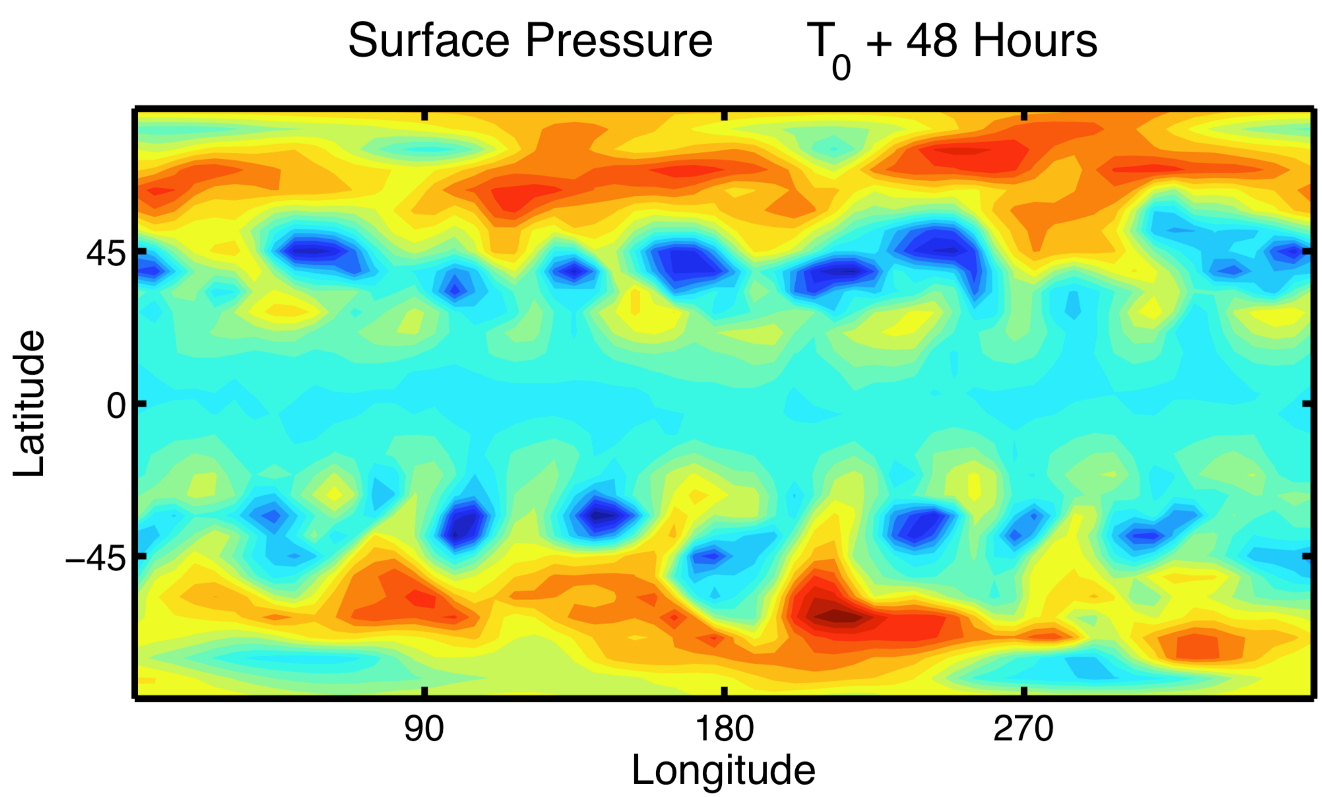

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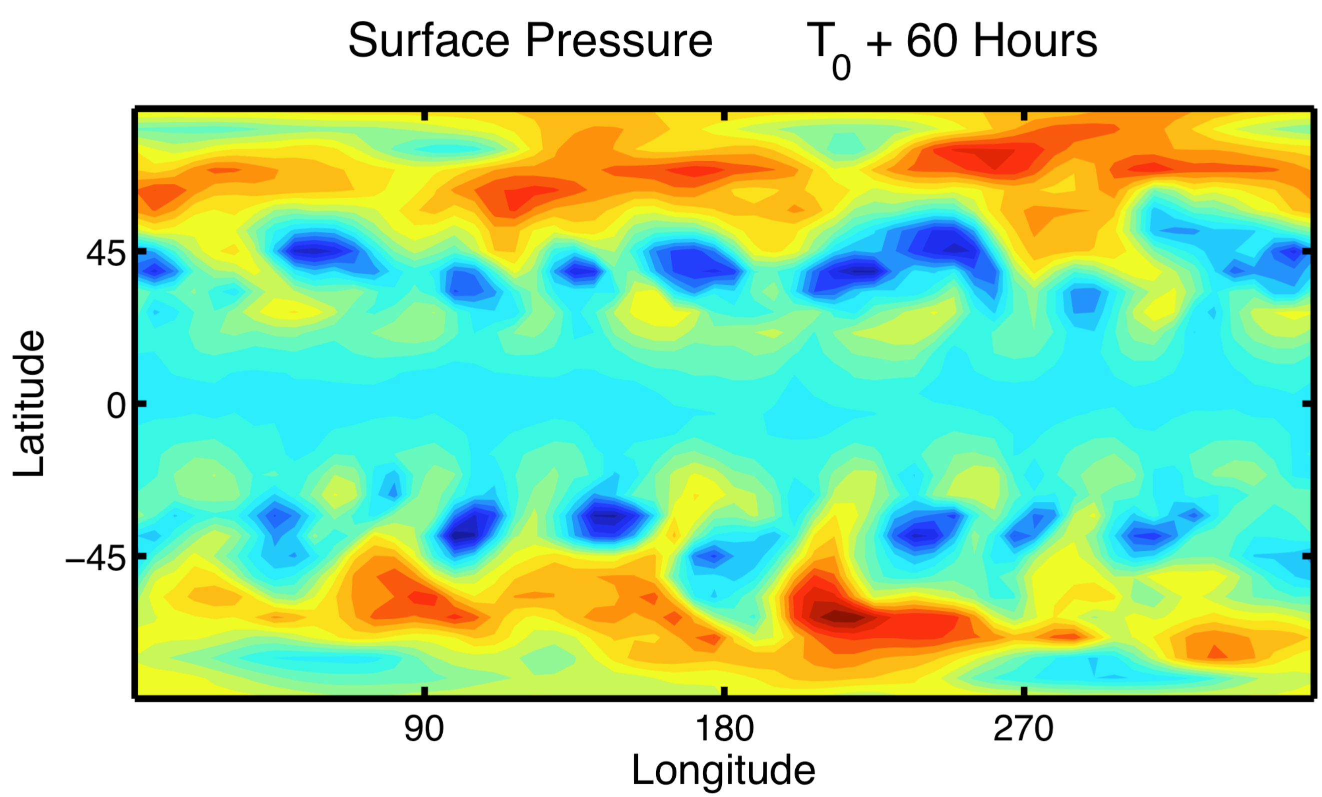

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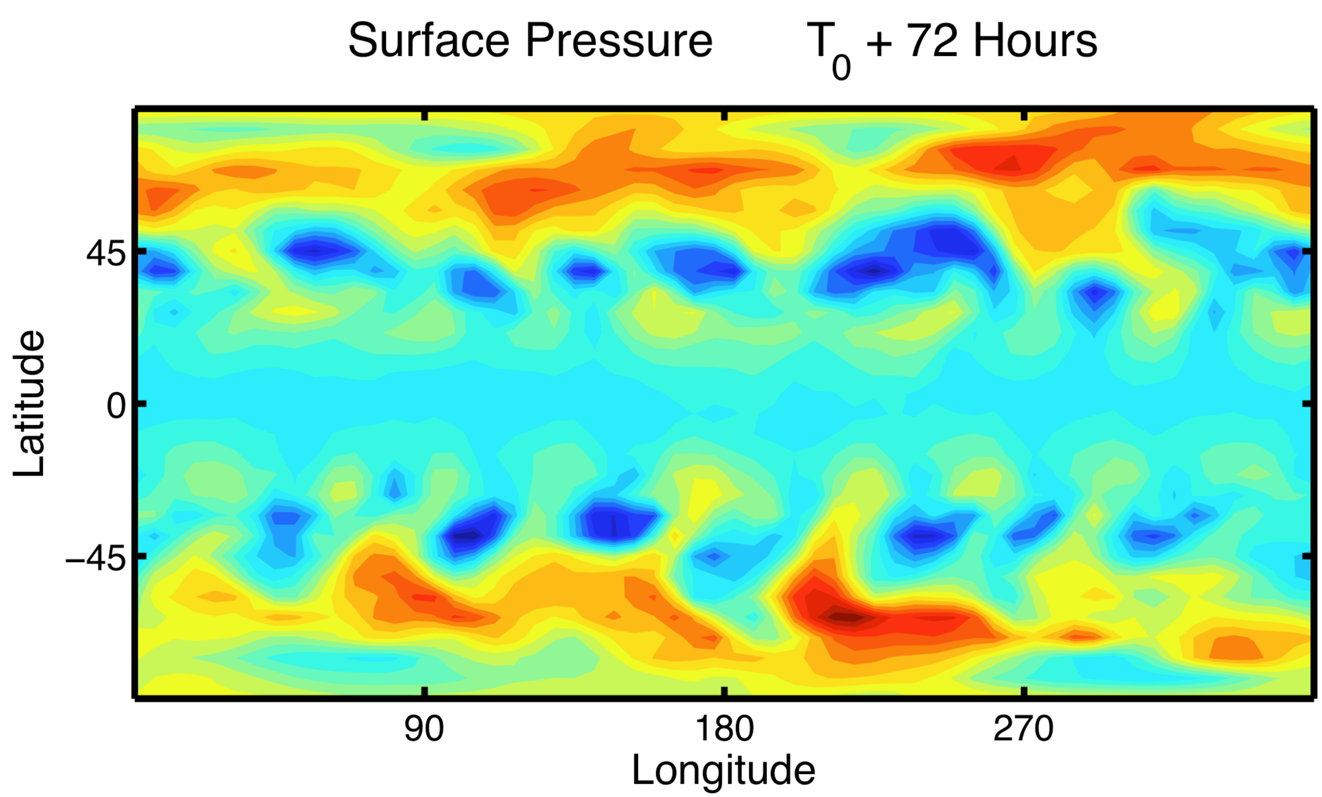

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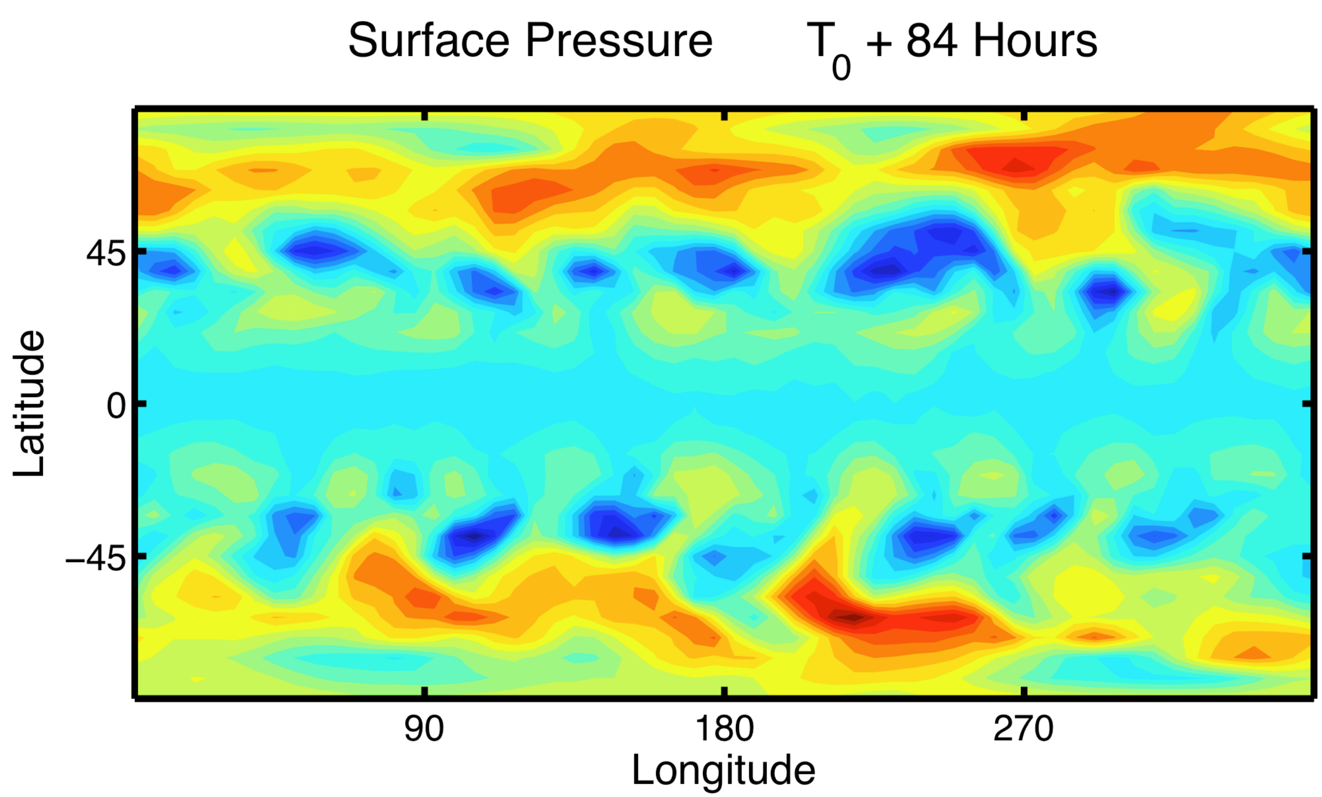

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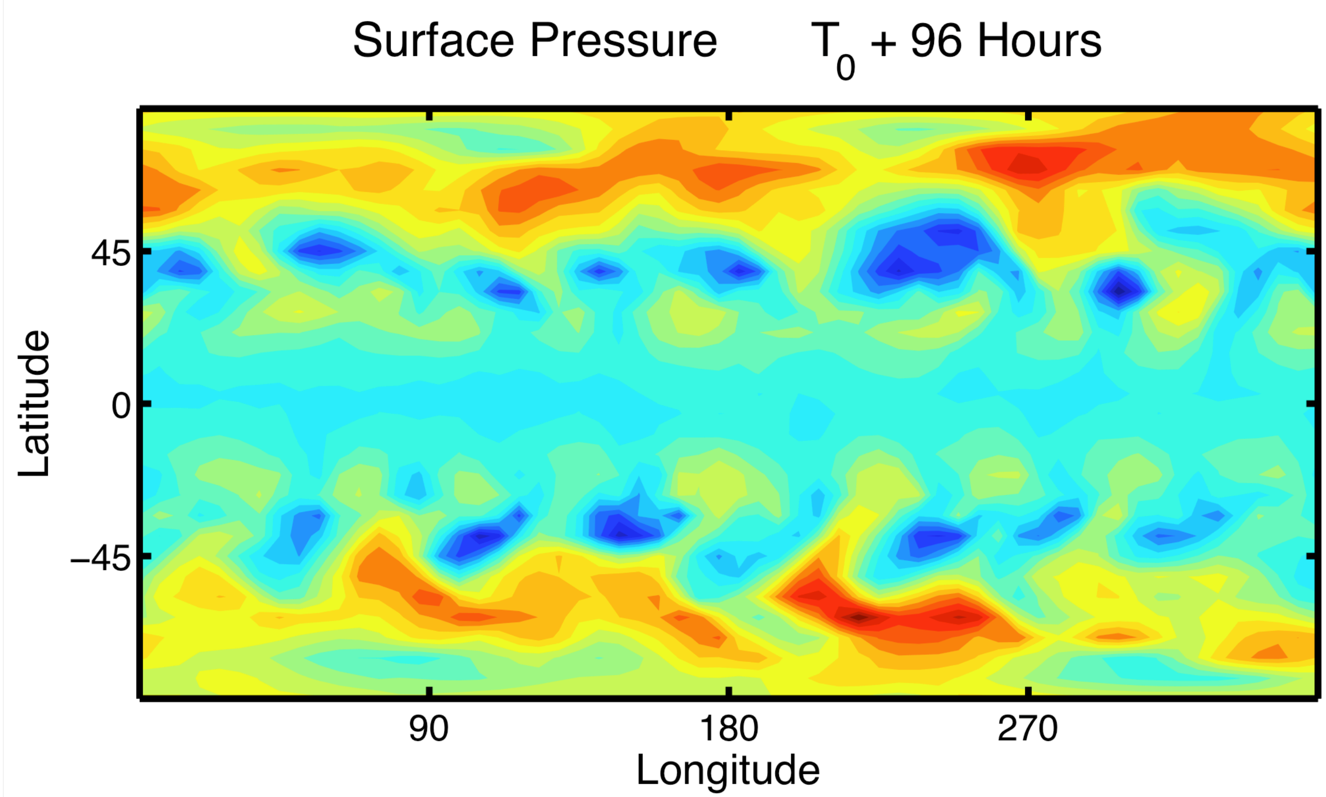

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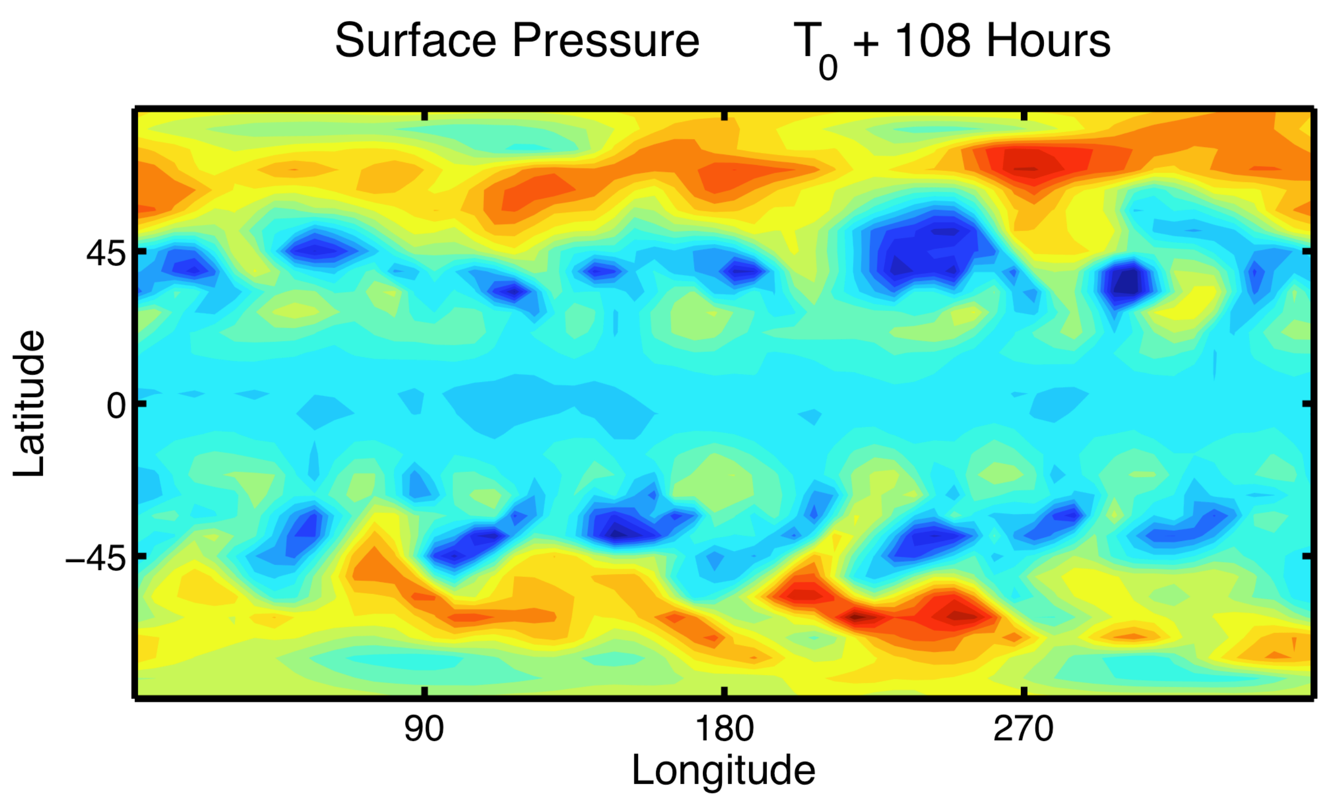

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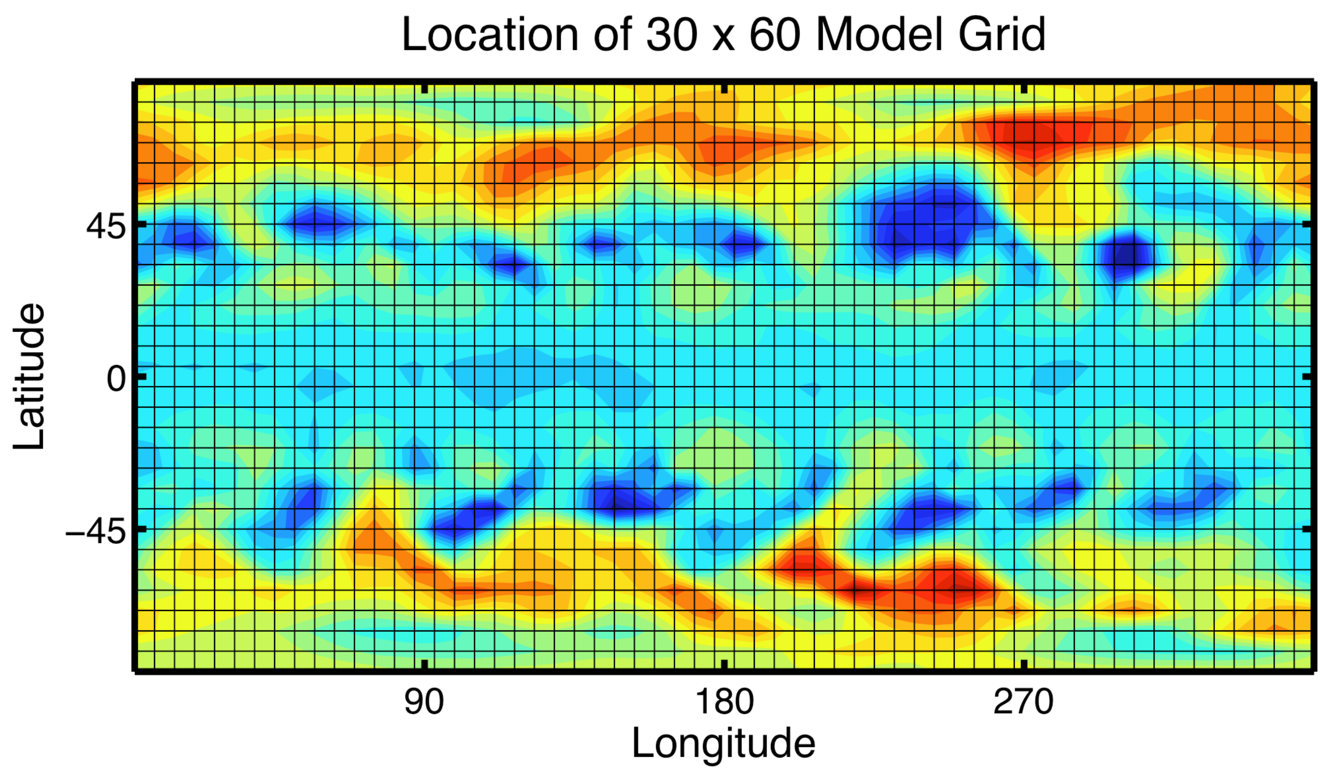
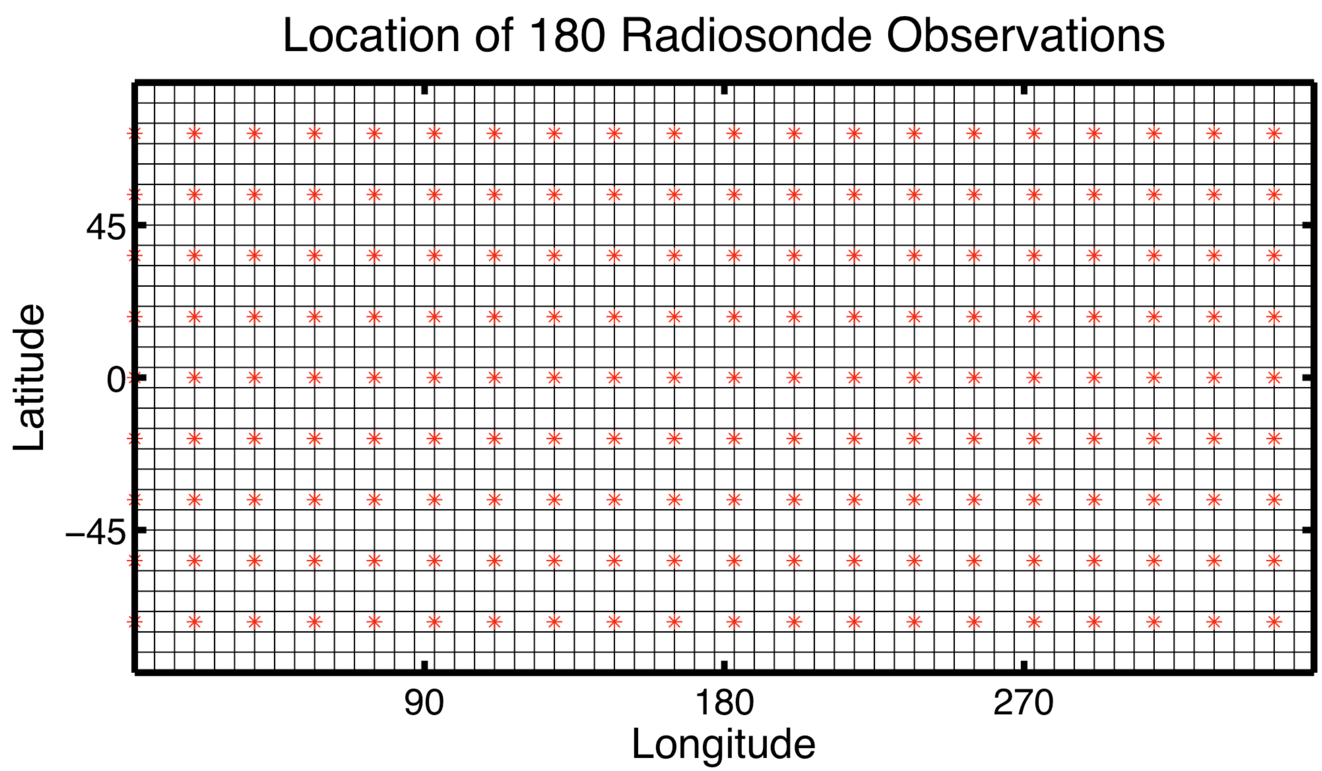


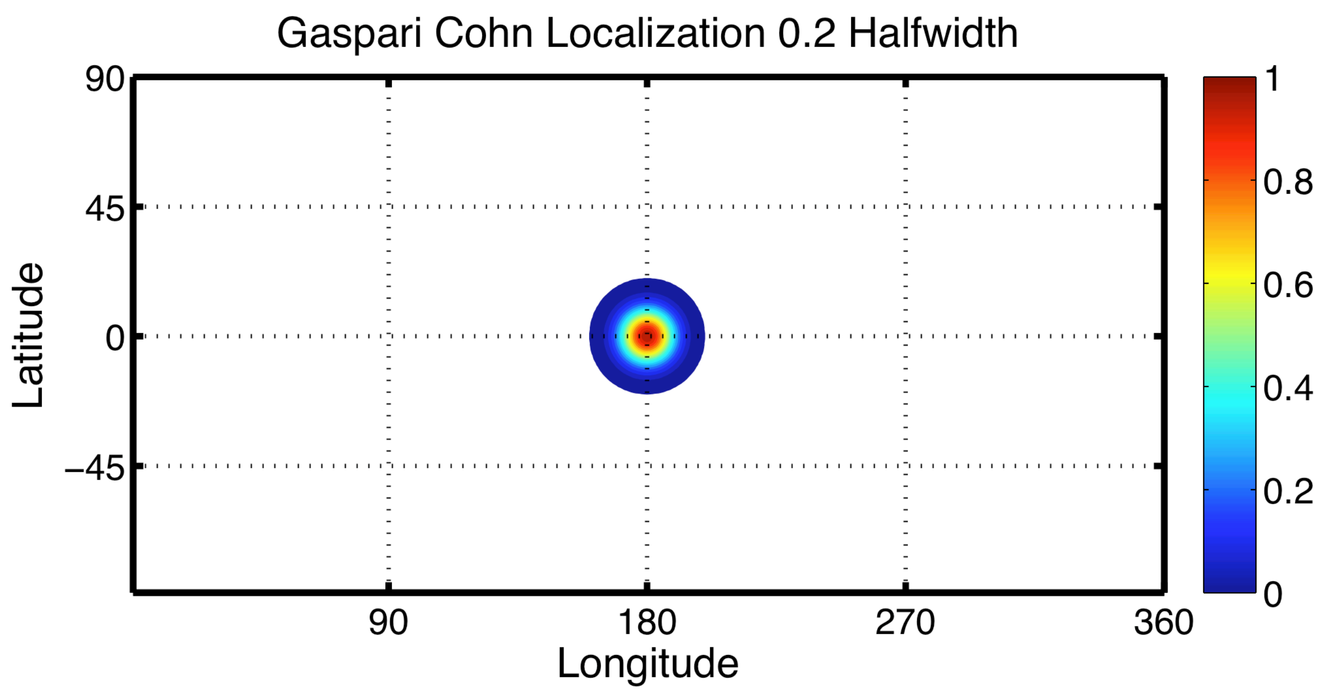
30x60 horizontal grid, 5 levels.
Surface pressure, temperature, wind components.
28,800 variables.

Test II: Low-order dry dynamical core



Observe every 12 hours.
Observe all 16 variables in 180 columns shown.
Error SD: Ps=2hPa, T=3K, winds=3m/s.

Test II: Low-order dry dynamical core



Use a range of Gaspari Cohn background localizations.
Larger ensemble sizes can use broader localizations.

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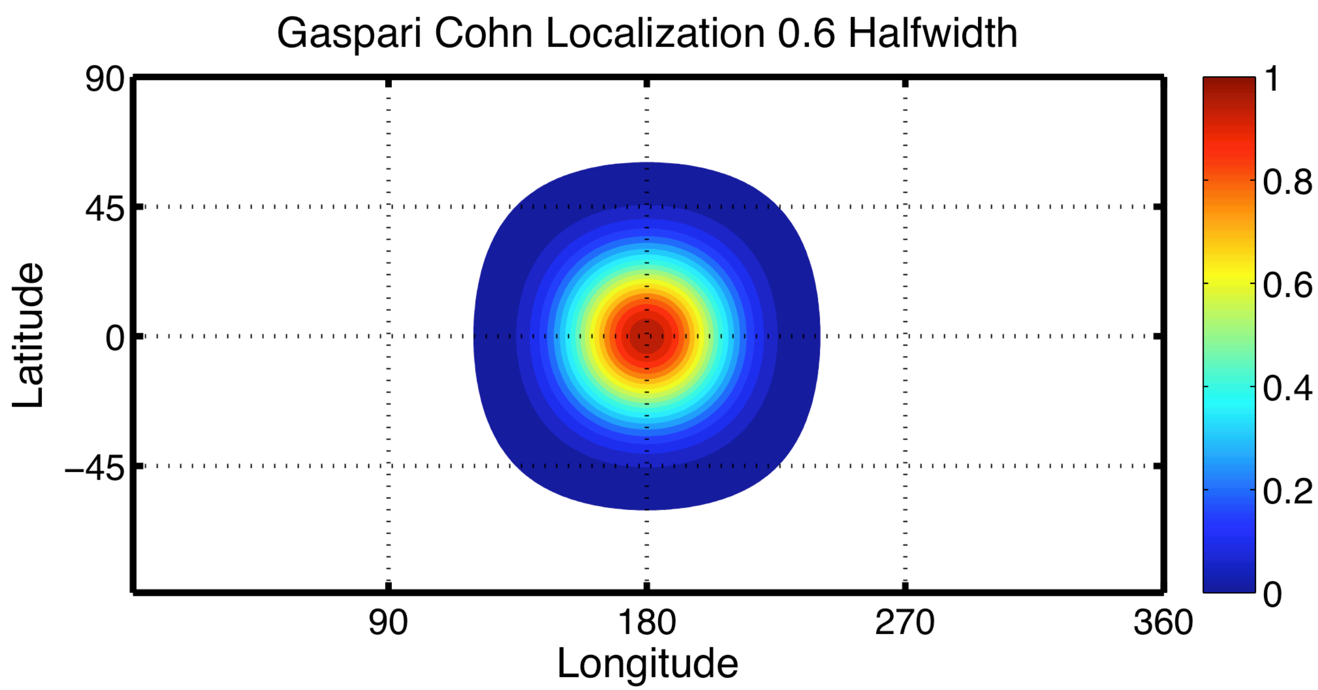

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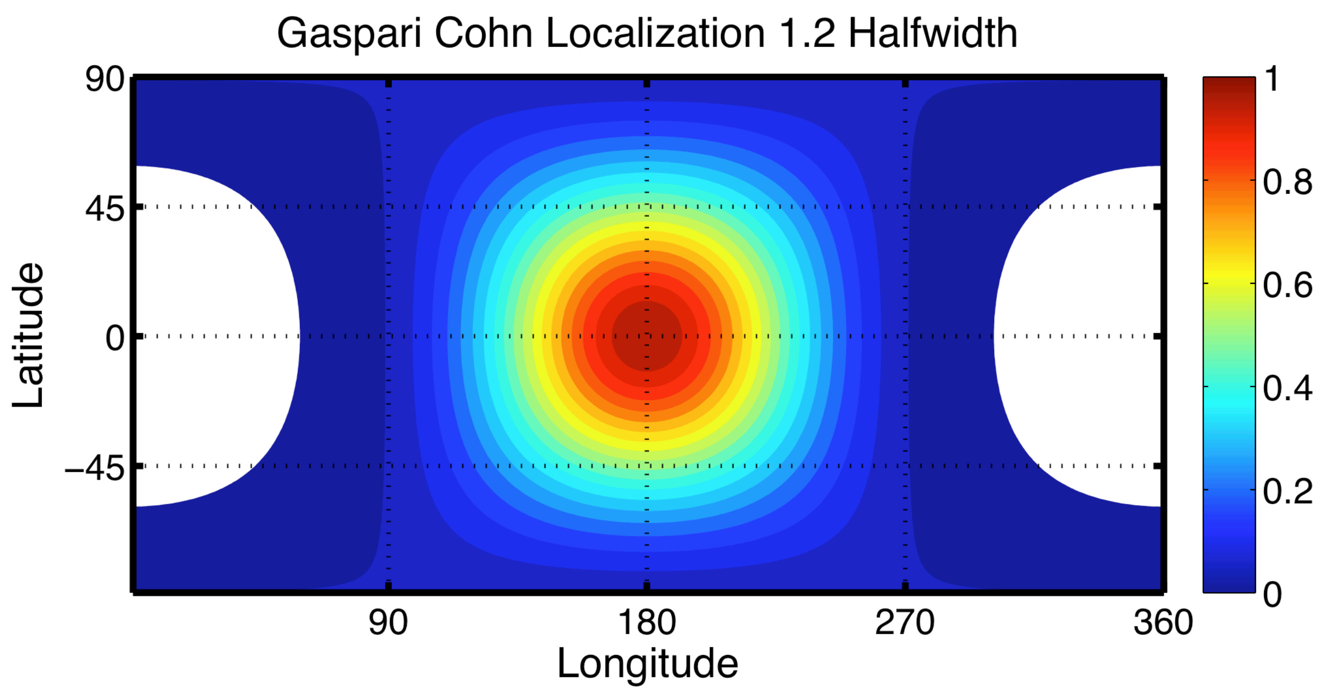

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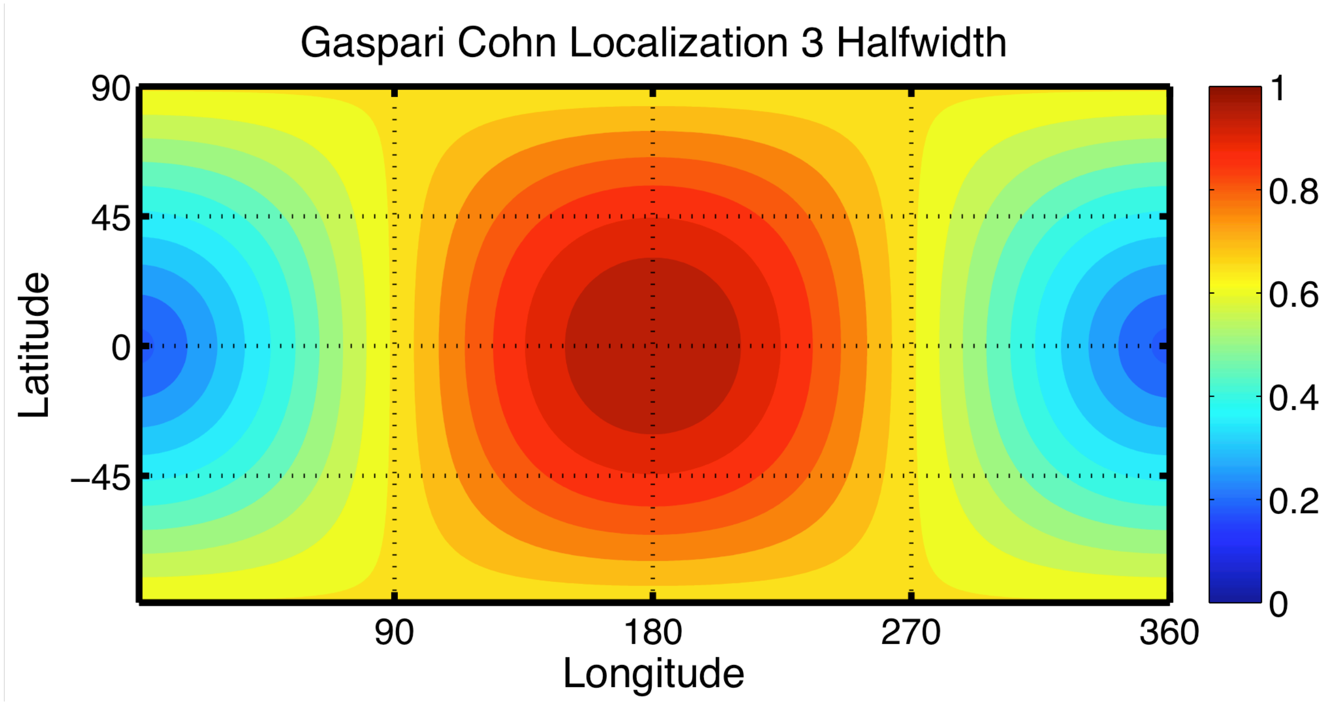


Use a range of Gaspari Cohn background localizations.
Larger ensemble sizes can use broader localizations.

Test II: Low-order dry dynamical core

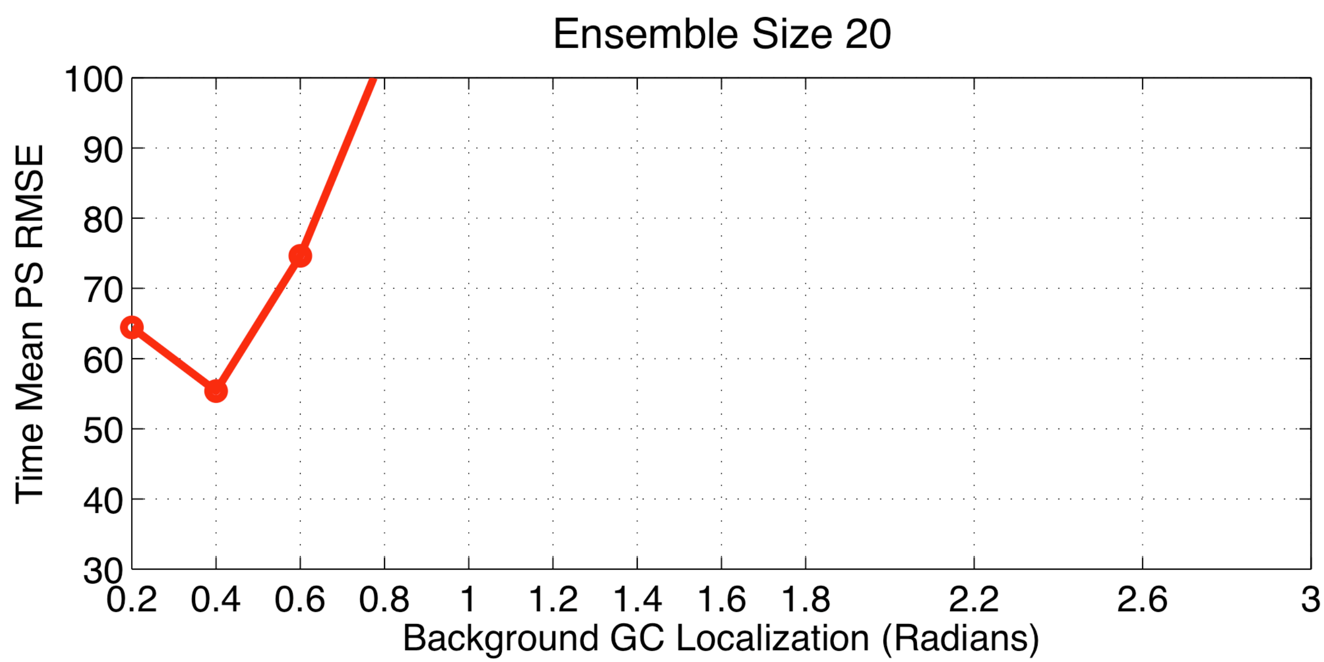
All cases assimilate same observations.
150 day period (300 assimilation cycles).

All use damped adaptive inflation with same settings.
Inflation is spatially- and temporally-varying (see Miyoshi).

Primary metric is prior RMSE for surface pressure.

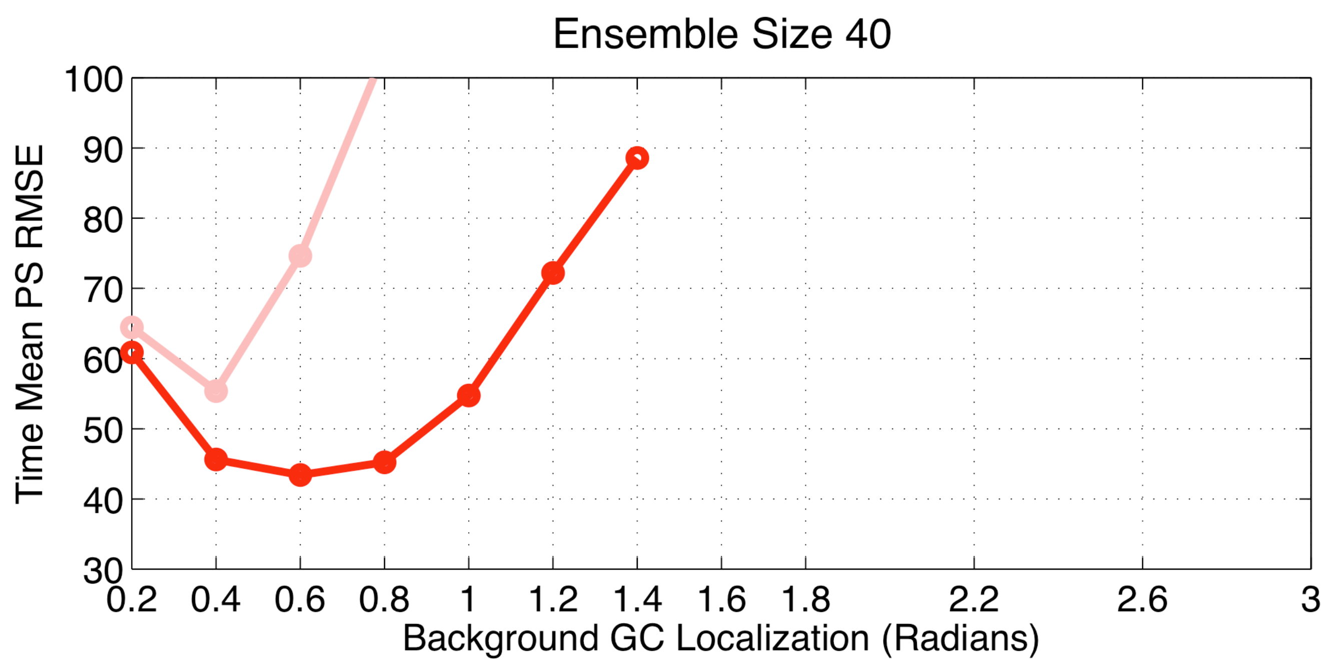
Compare base algorithm to sampling error correction.

Test II: Low-order dry dynamical core



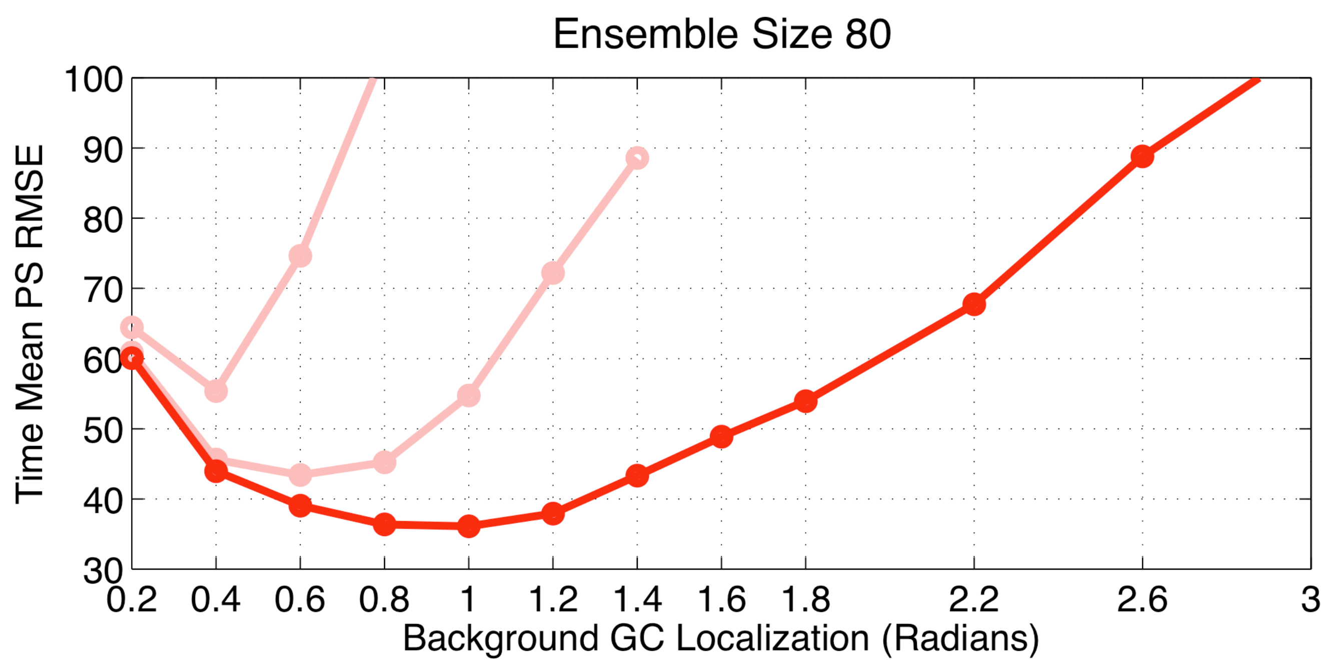
Base GC localization: Ensemble size 20.

Test II: Low-order dry dynamical core



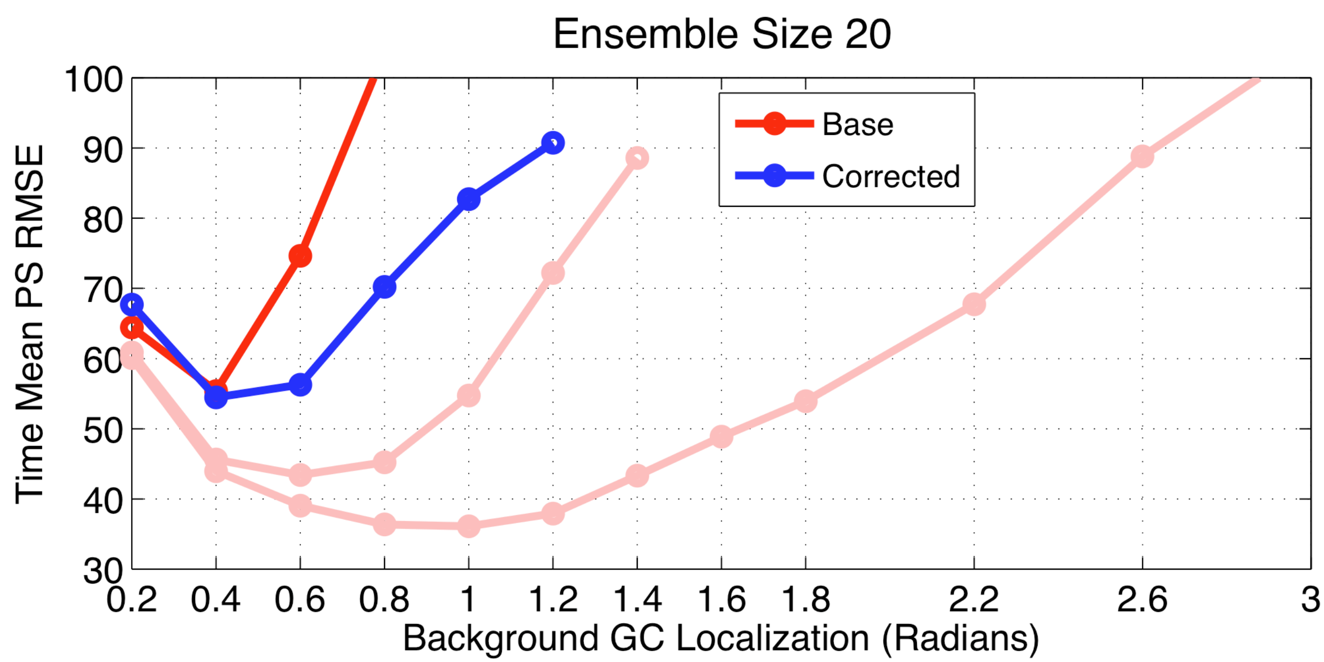
Base GC localization: Ensemble size 40.
RMSE reduced, localization broader for larger ensemble.

Test II: Low-order dry dynamical core



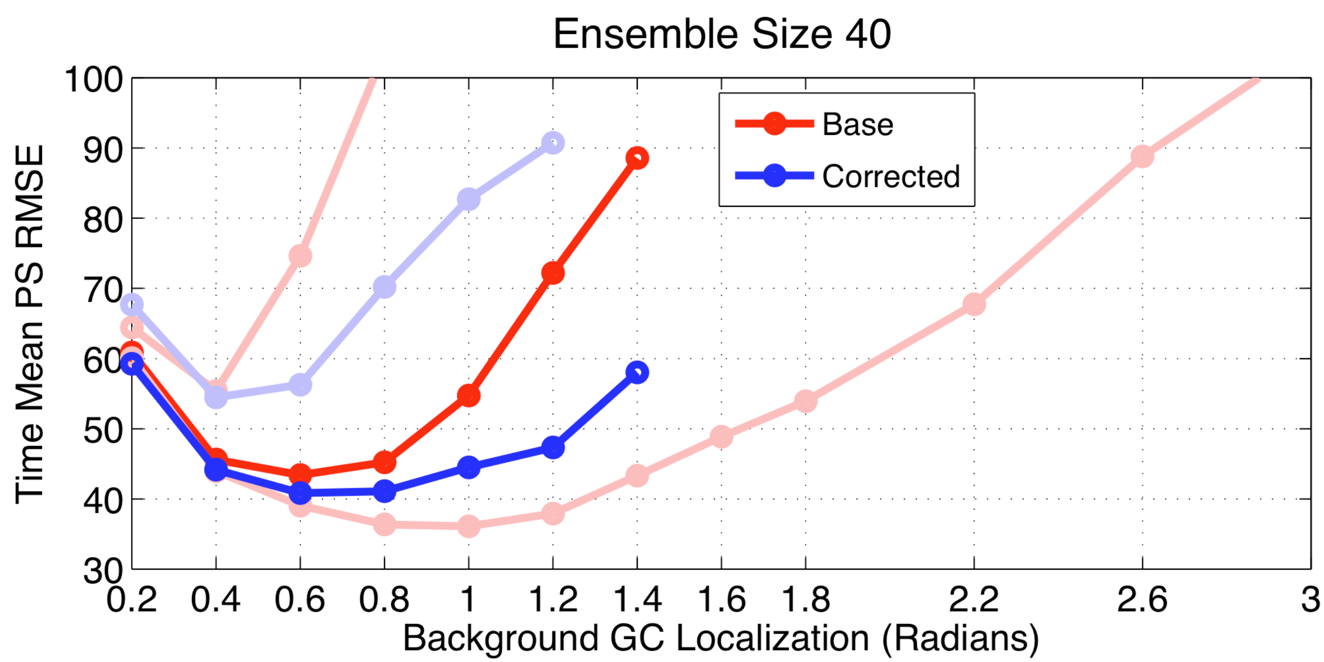
Base GC localization: Ensemble size 80.
RMSE reduced, localization broader for larger ensemble.

Test II: Low-order dry dynamical core



Sampling Error Correction: Ensemble size 20.
Has smaller RMSE for range of background GC localization.

Test II: Low-order dry dynamical core



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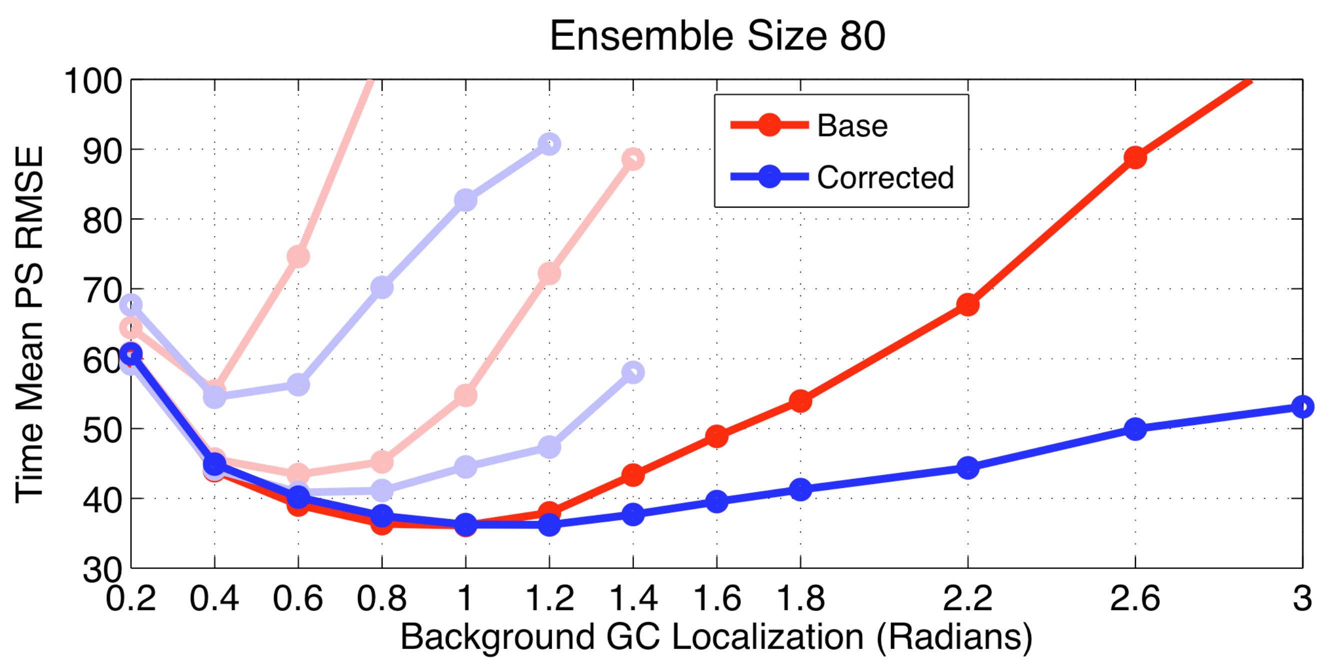

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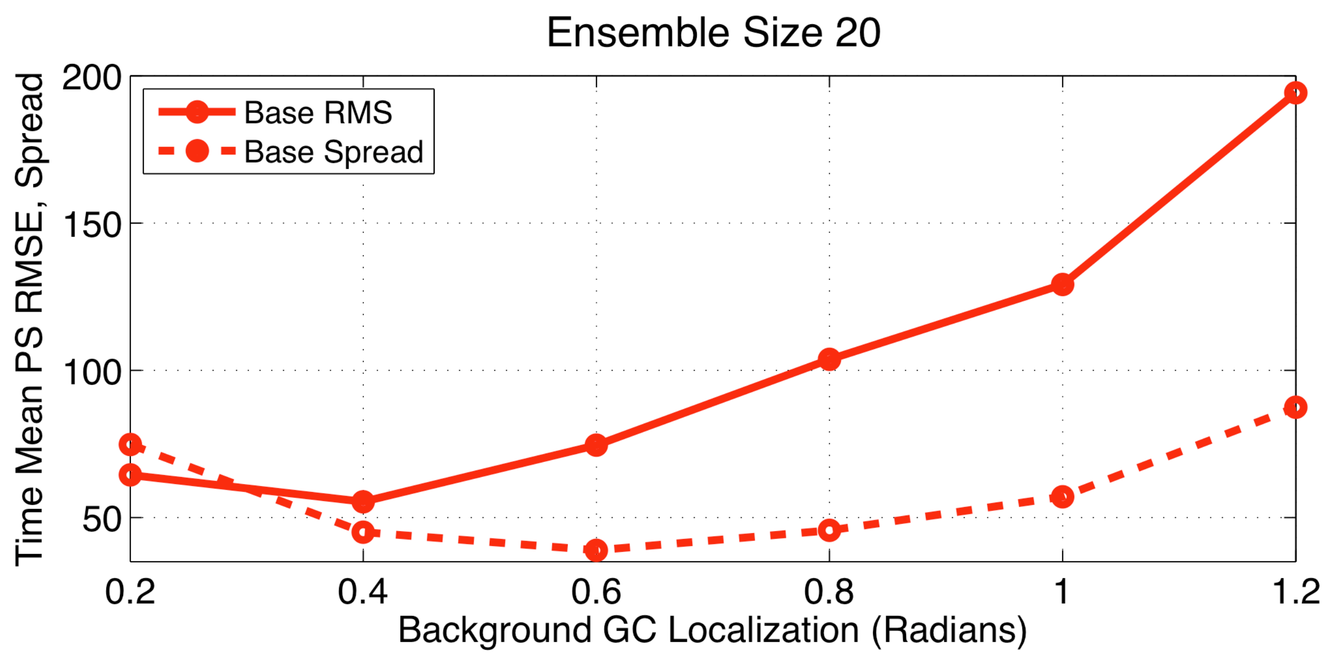
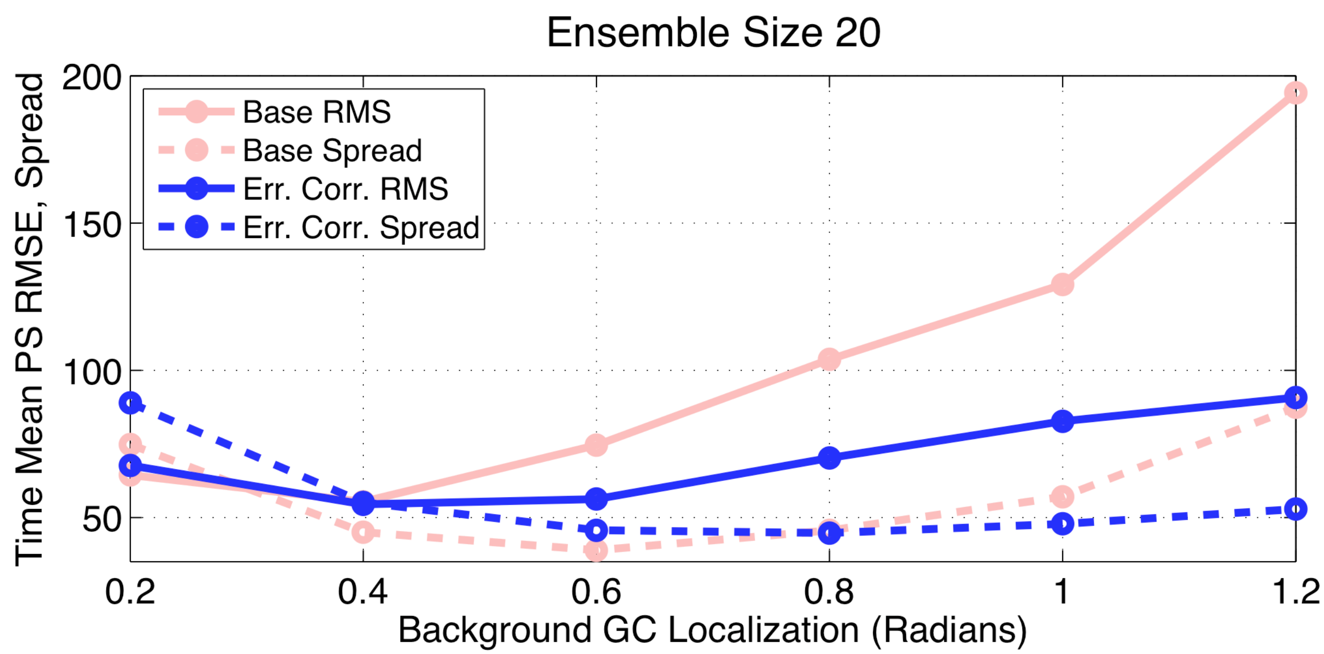


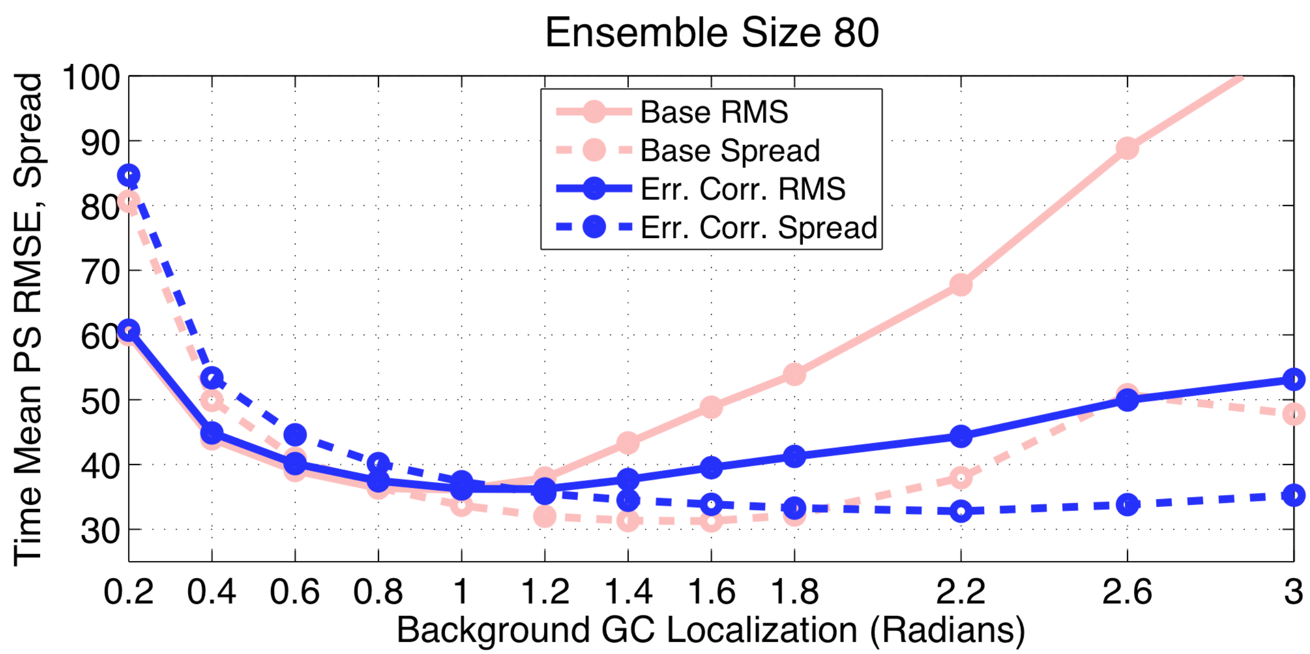
Base spread deficient for large GC, even with inflation.
This is expected as more remote observations are used.

Test II: Low-order dry dynamical core



Sample corrected spread is not as deficient.

Test II: Low-order dry dynamical core



Sample corrected spread is not as deficient.
Behavior is similar for all ensemble sizes tested.

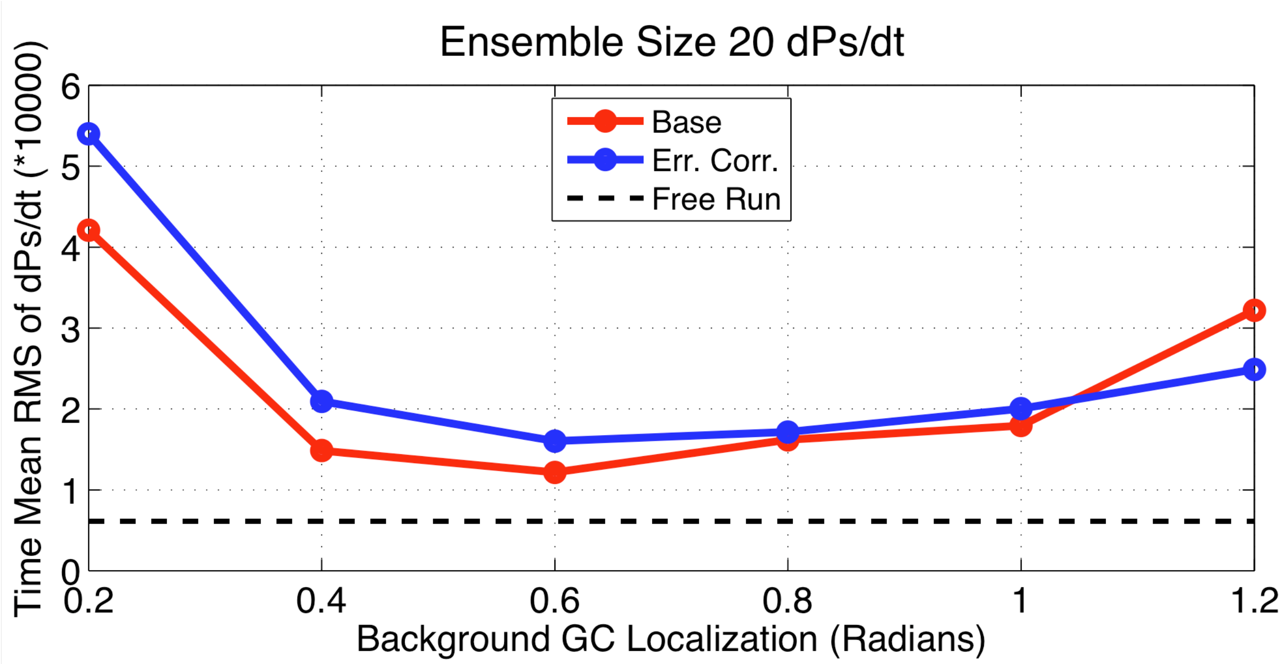
Test II: Low-order dry dynamical core

Localization associated with increased ‘imbalance’.
Transient off-attractor dynamics after assimilation.

Here, imbalance results in high-frequency gravity waves.
Measured by mean amplitude of surface pressure tendency.
Computed at first timestep after each assimilation.
Averaged globally and for all times.

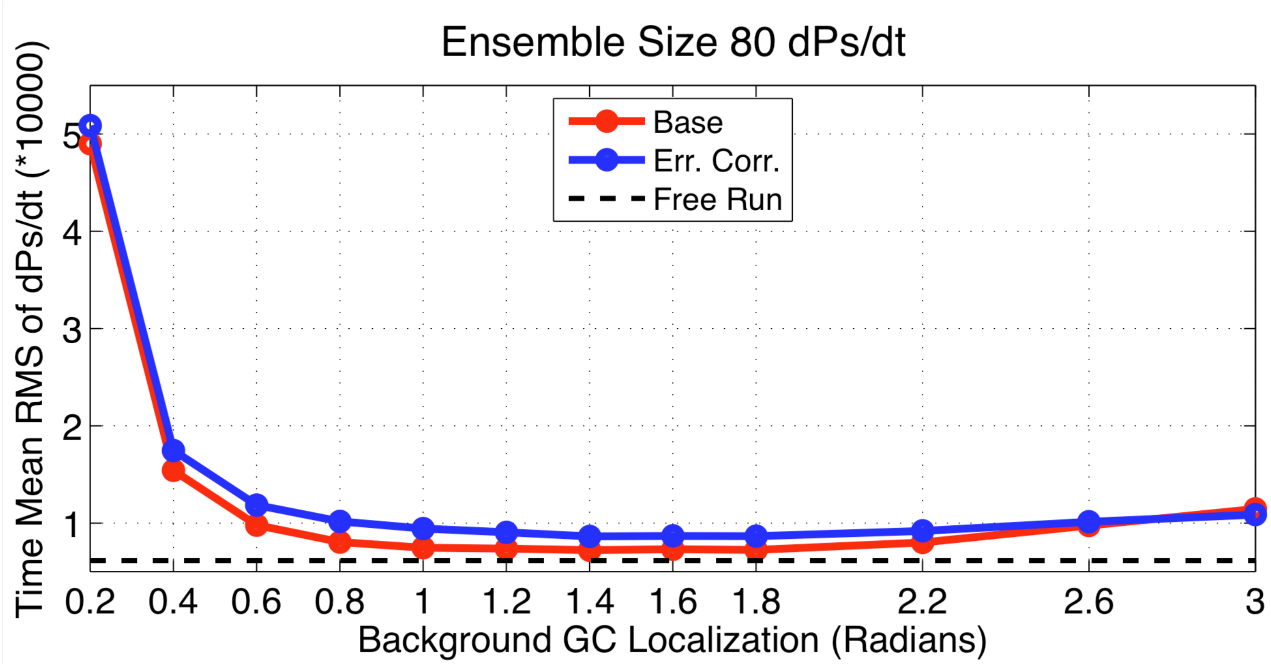
Would like this to be small compared to a free model run.

Test II: Low-order dry dynamical core



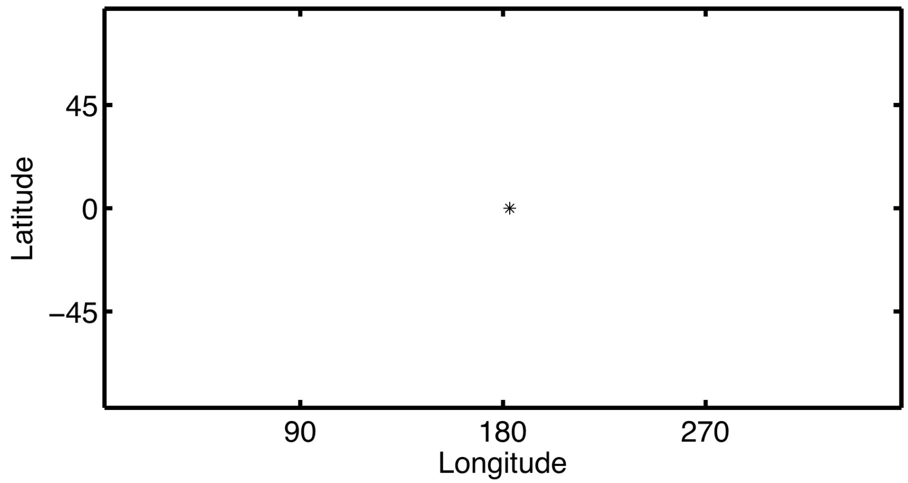
Both cases have enhanced dPs/dt for ensemble size 20.
Sampling error corrected generally worse.
dPs/dt for long free model run shown for comparison.

Test II: Low-order dry dynamical core



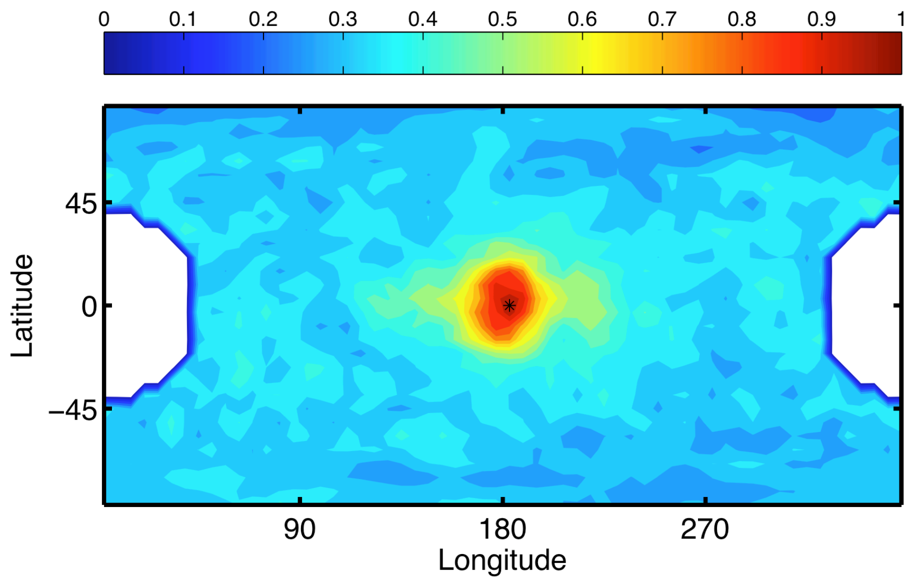
Much smaller enhancement in dPs/dt for ensemble size 80.
Sampling error corrected still generally worse.

Test II: Low-order dry dynamical core



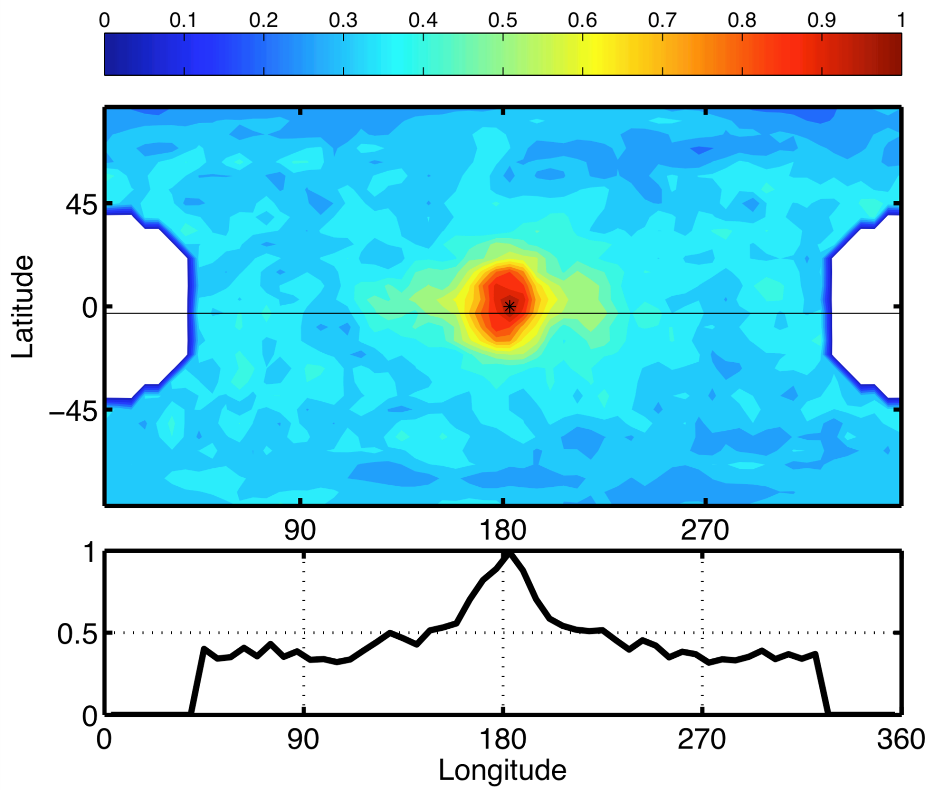
Observation of Ps on the equator.

Test II: Low-order dry dynamical core



Time mean sampling localization for equatorial Ps ob. on Ps state variables. (Ens. Size 80, GC localization 1.2).

Test II: Low-order dry dynamical core



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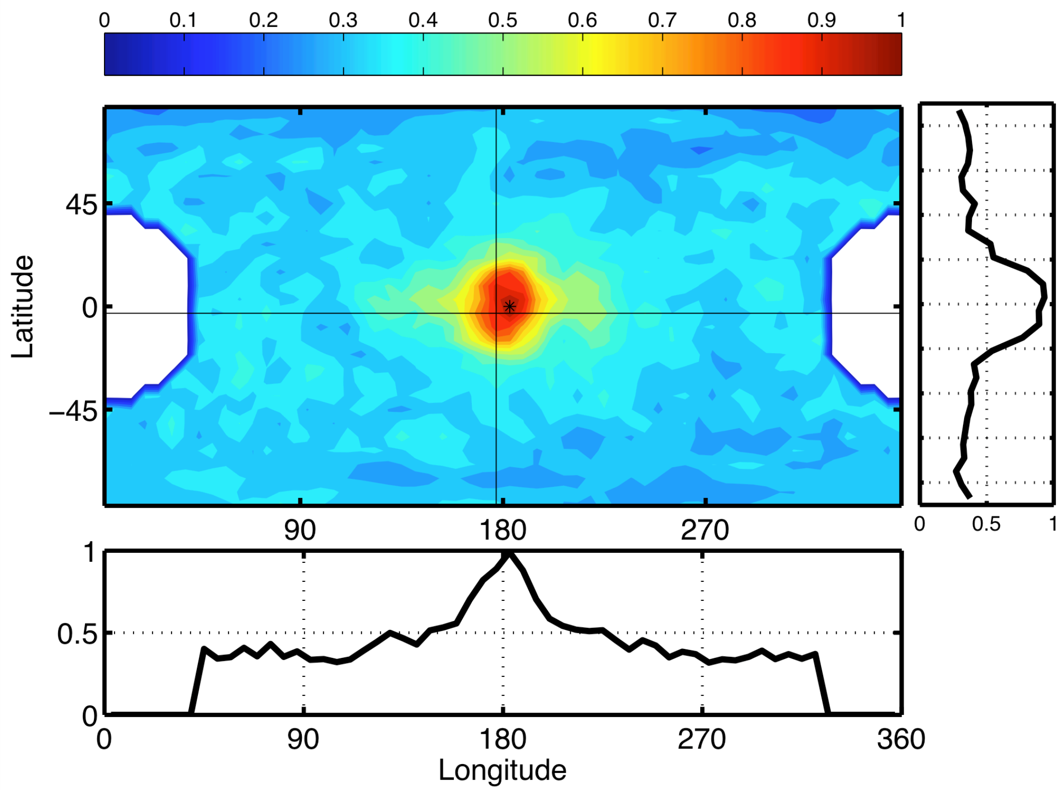
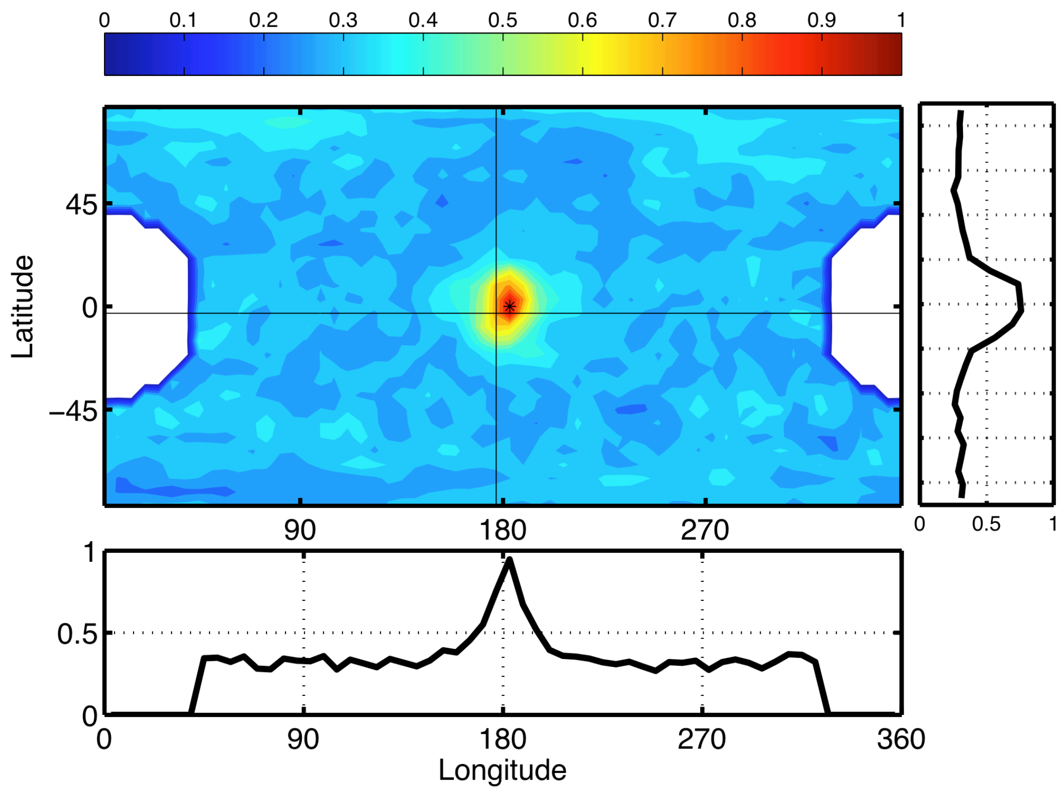


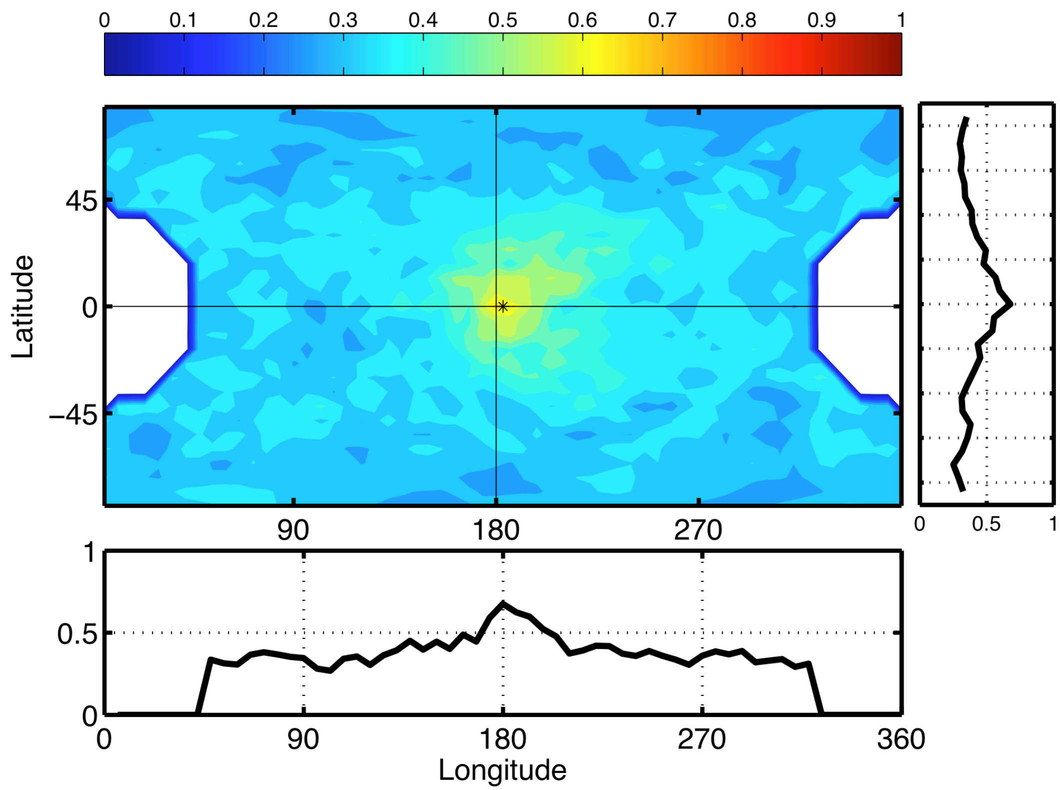
Time mean sampling localization for equatorial Ps ob. on Ps state variables. (Ens. Size 80, GC localization 1.2).
Nearly gaussian locally.

Test II: Low-order dry dynamical core



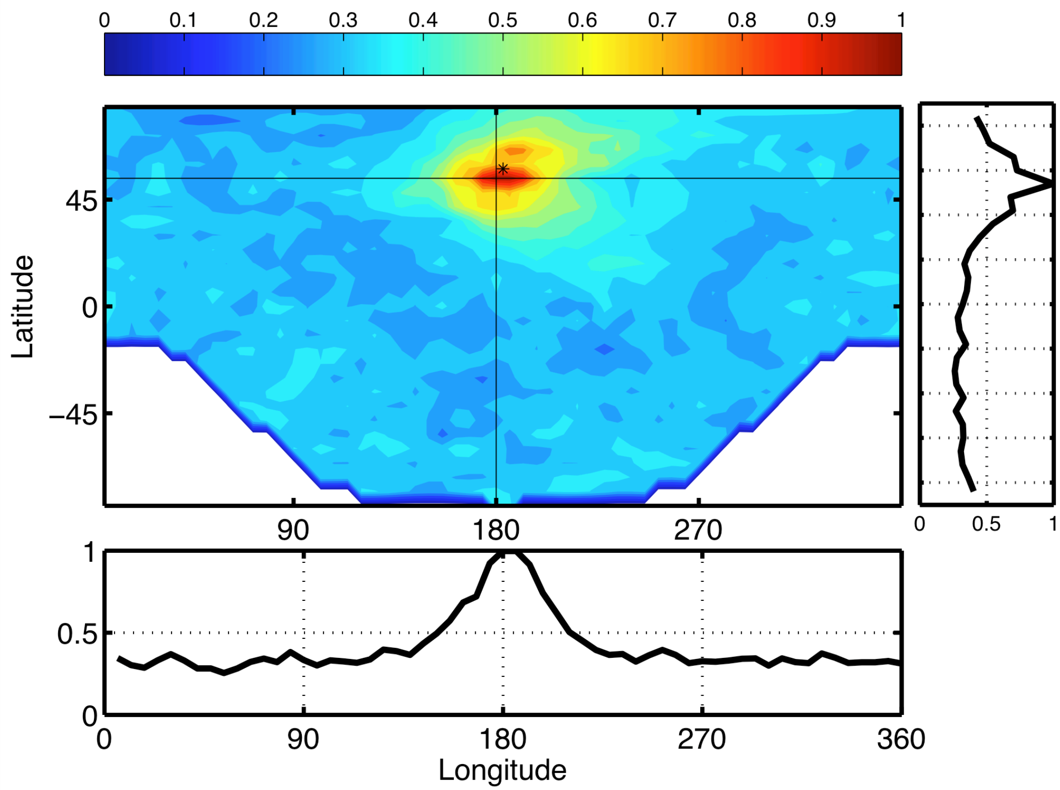
Ensemble size 20 localization of Ps ob. on Ps state.
Localization is tighter for smaller ensemble.

Test II: Low-order dry dynamical core



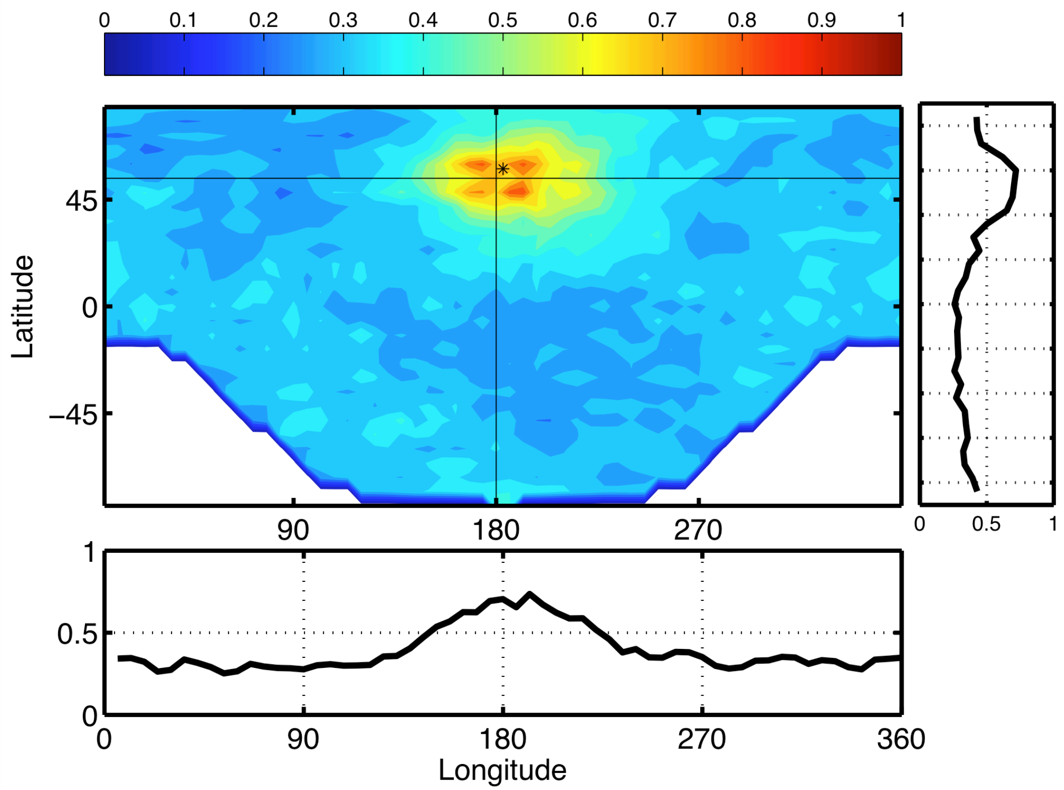
Localization for Ps ob. on zonal wind (U) state variables (N=80).
Localization much less than 1 for horizontally close.

Test II: Low-order dry dynamical core



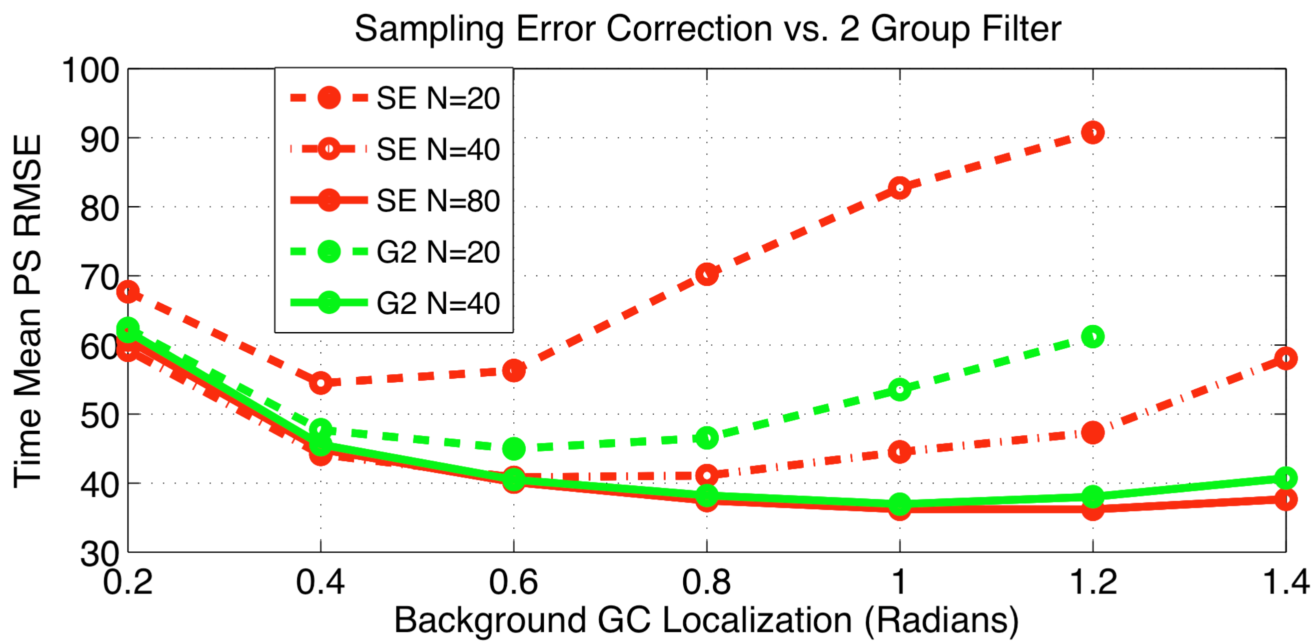
Localization for mid-level U ob. on U state variables (N=80).
Local dipole, but mostly still gaussian.

Test II: Low-order dry dynamical core



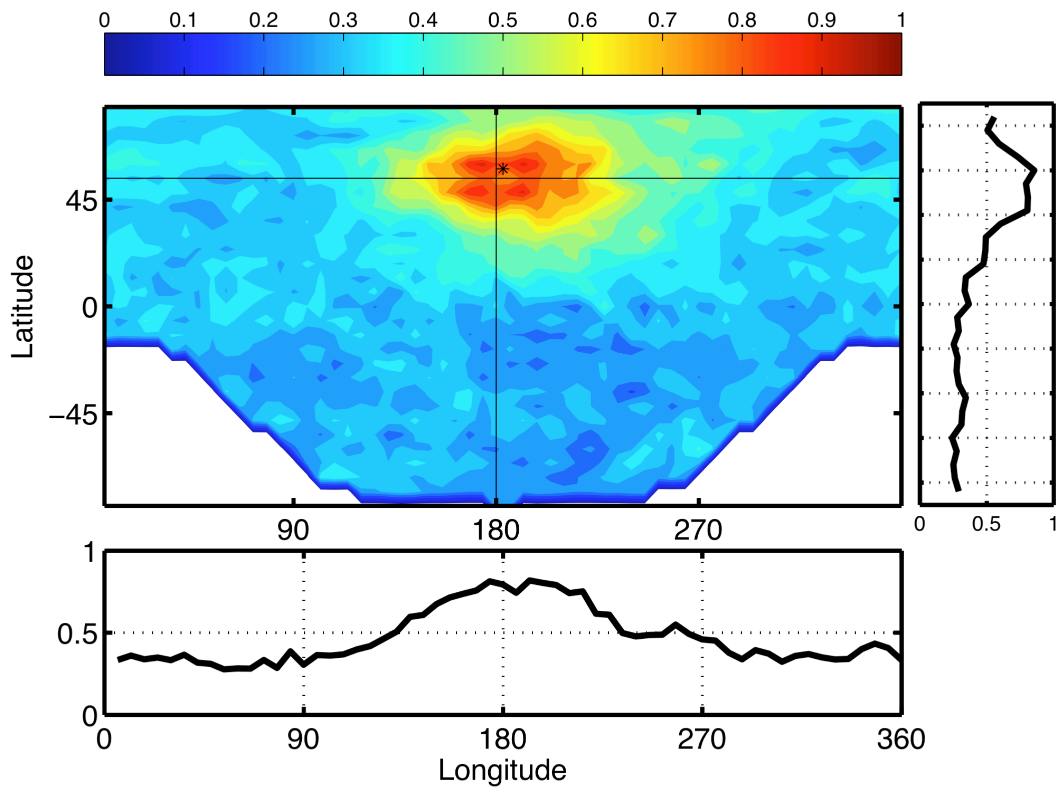
Localization for mid-level V ob. on U state variables (N=80).
Local four maxima, but mostly still gaussian.

Test II: Low-order dry dynamical core



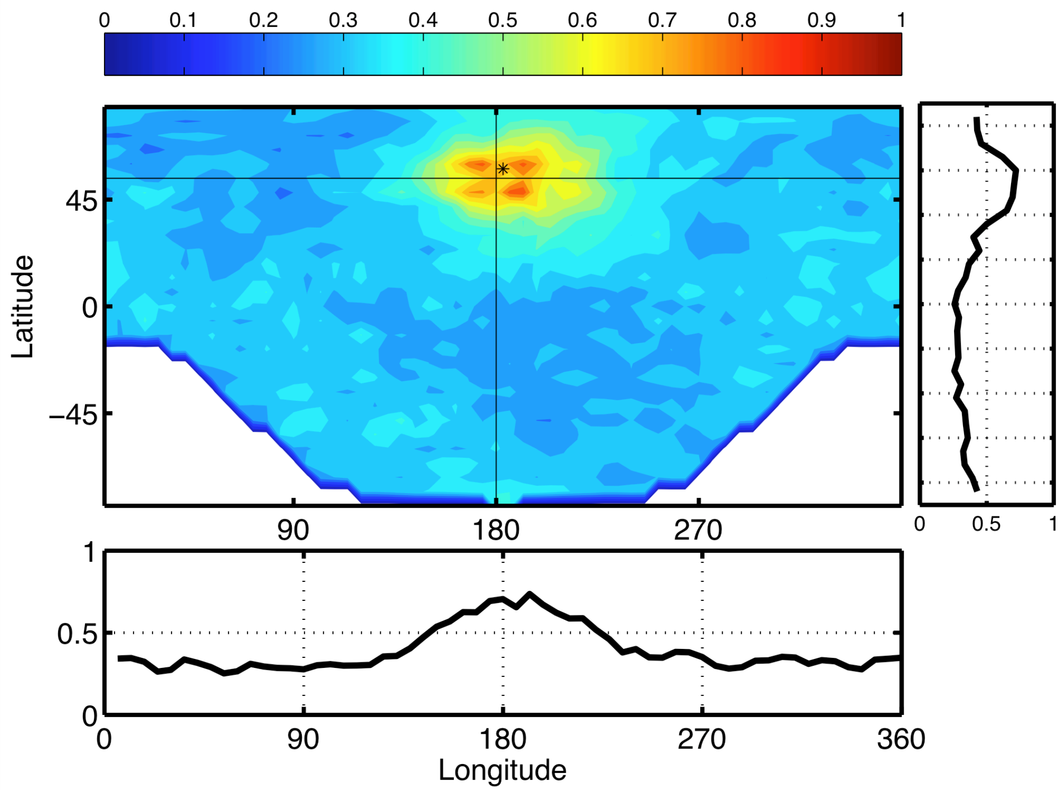
Comparison to group filter.
Two-group filters with total of 40 and 80 members aren't quite as good as same cost sampling error correction.

Test II: Low-order dry dynamical core



Localization for V ob. on U state variables from 2 groups with N=80, GC = 1.2.
Patterns similar but group values larger close to observation.

Test II: Low-order dry dynamical core



Corresponding V ob. on U state variables from sampling error correction.
Patterns similar but group values larger close to observation.

Conclusions (1)

- Implemented sampling error correction localization.
- Localization is function of sample correlation and ensemble size.
- Can be used with Gaspari Cohn prior localization.
- Improves/stabilizes performance for low-order examples.
- Helps when localization is not known a priori
- Could use priors other than uniform for correlation.

Conclusions (2)

- Group filter and sampling error localizations are similar.
- Suggests that these applications are more like a random sample than a Kalman filter.

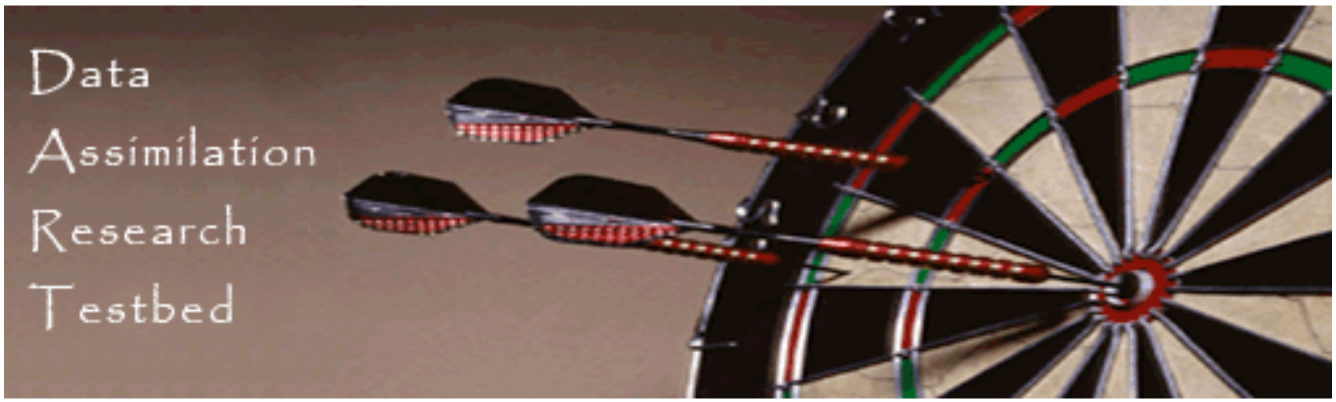
- Could build statistical models of time-mean localization.
- Would be laborious but could improve performance.

- Non-atmospheric applications might be quite different.

Conclusions (3)

- There are still lots of ways to improve EnKFs.
- Understanding of behavior in large problems is limited.
- Implications of systematic error were not addressed here.
- Using observations in concert with model is most powerful approach (and wasn't used here).

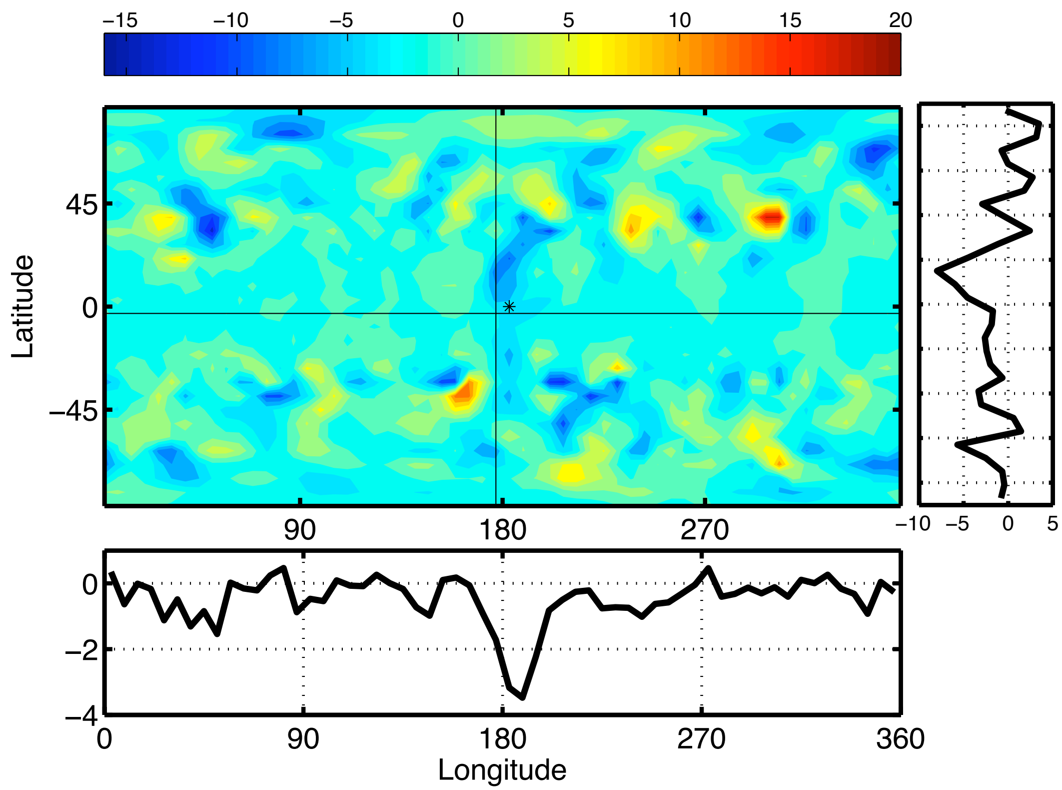
All the tools used to generate this talk are part of DART:



http://www.image.ucar.edu/DAReS/DART

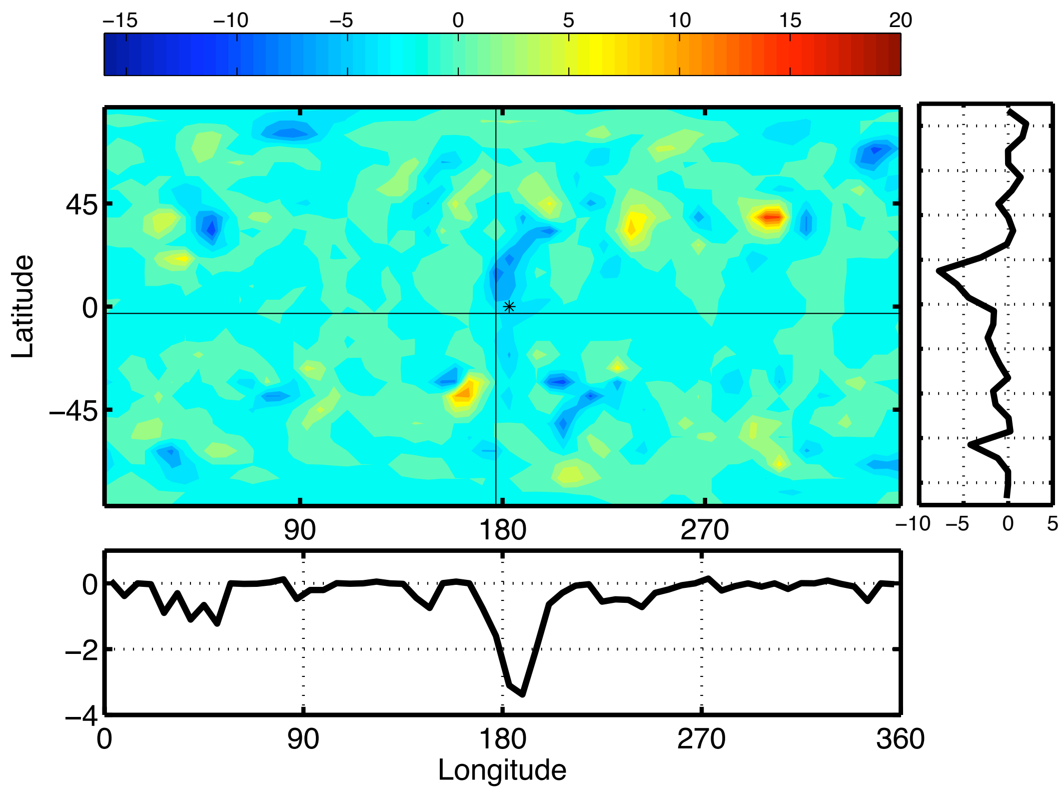
Thanks to Nancy Collins, Tim Hoar,
Kevin Raeder and Glen Romine

Test II: Low-order dry dynamical core



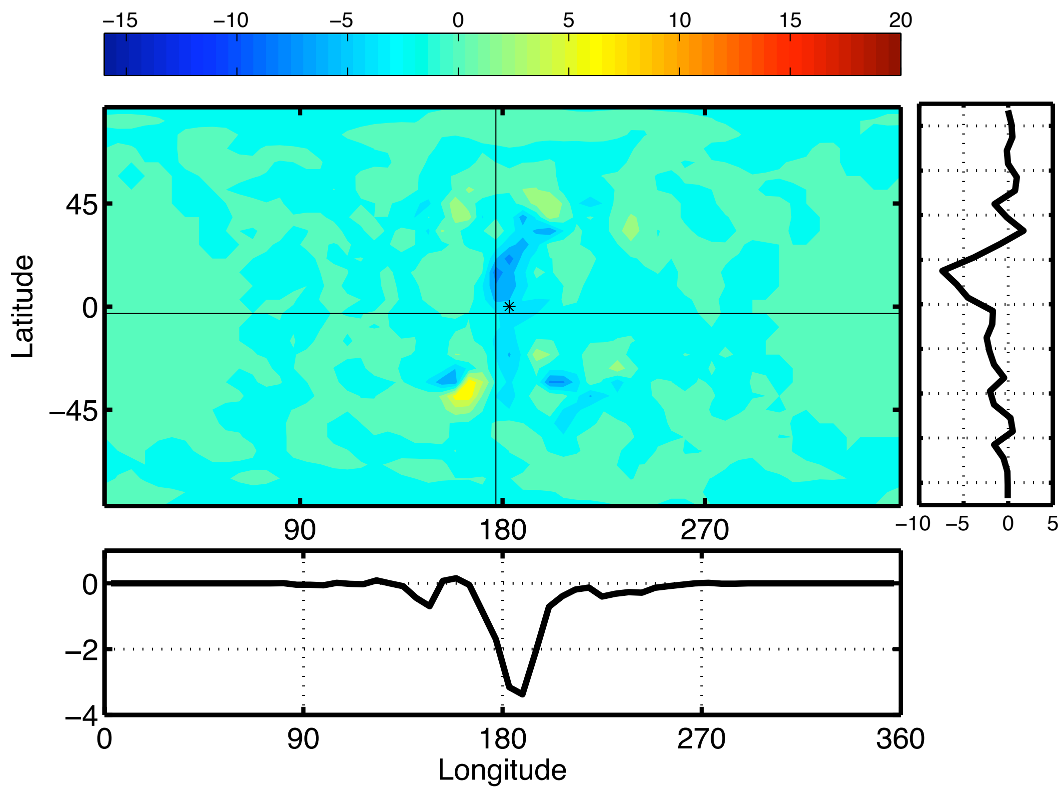
Increments from single unlocalized Ps observation on Ps.
Ensemble size 80, observation at equator, 180.

Test II: Low-order dry dynamical core



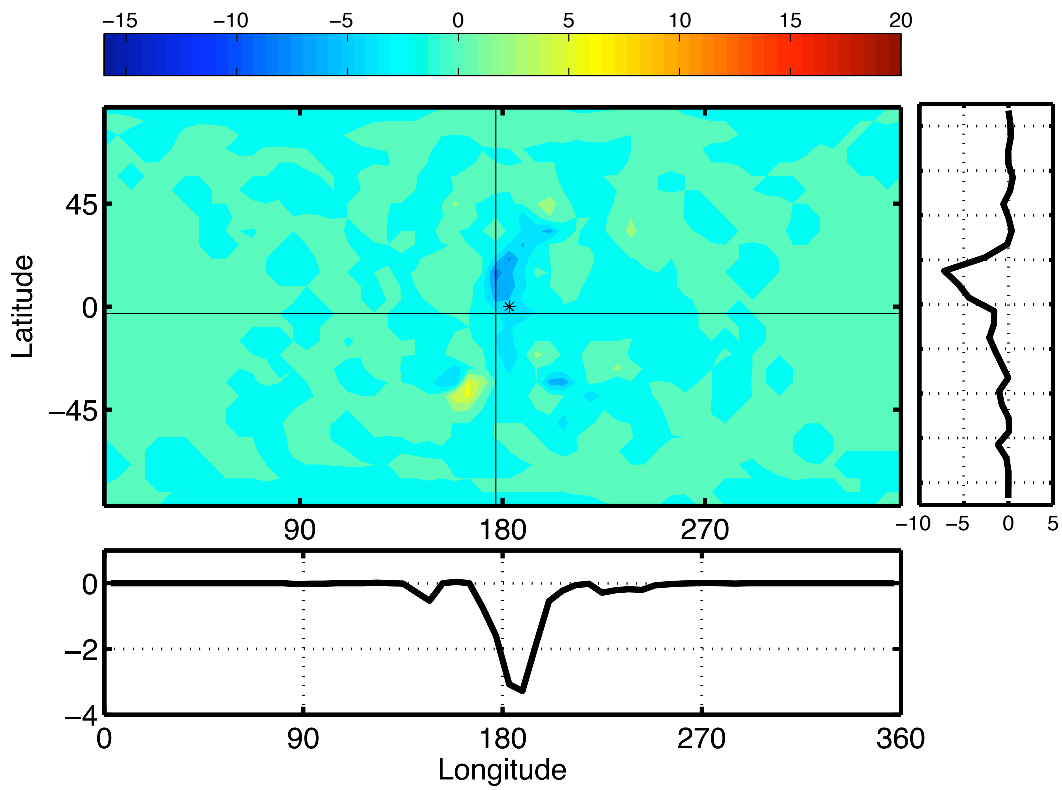
Increments with sampling error localization.
Ensemble size 80, observation at equator, 180.

Test II: Low-order dry dynamical core



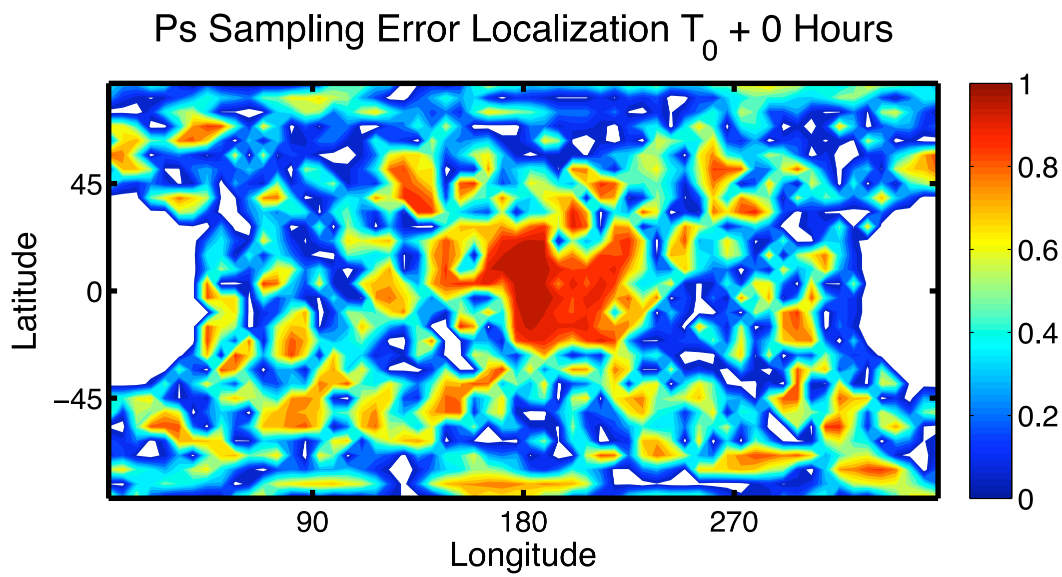
Increments GC=1.2 radians.
Ensemble size 80, observation at equator, 180.

Test II: Low-order dry dynamical core



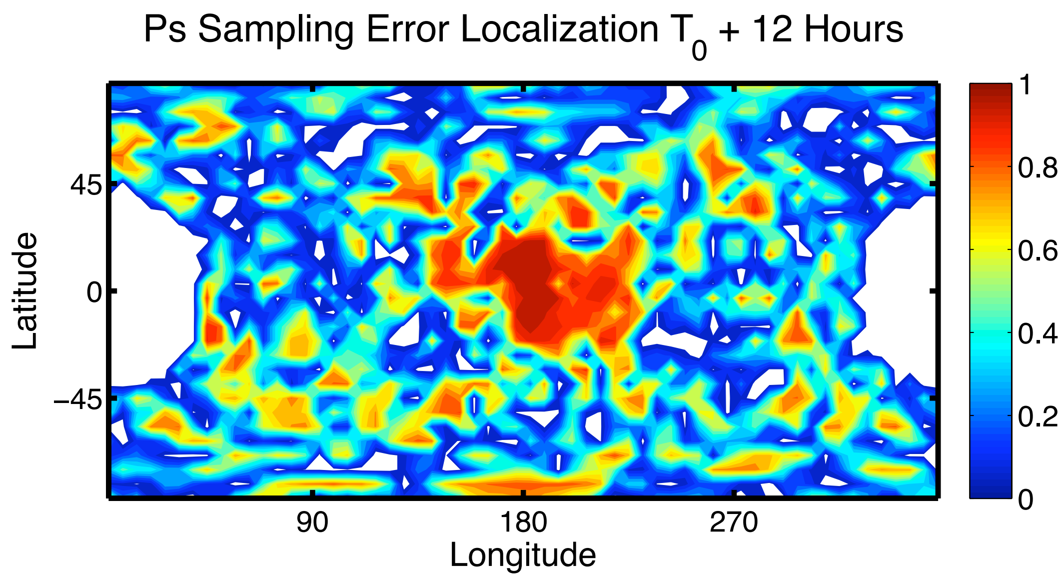
Increments GC=1.2 radians plus sampling error correction.
Ensemble size 80, observation at equator, 180.

Test II: Low-order dry dynamical core



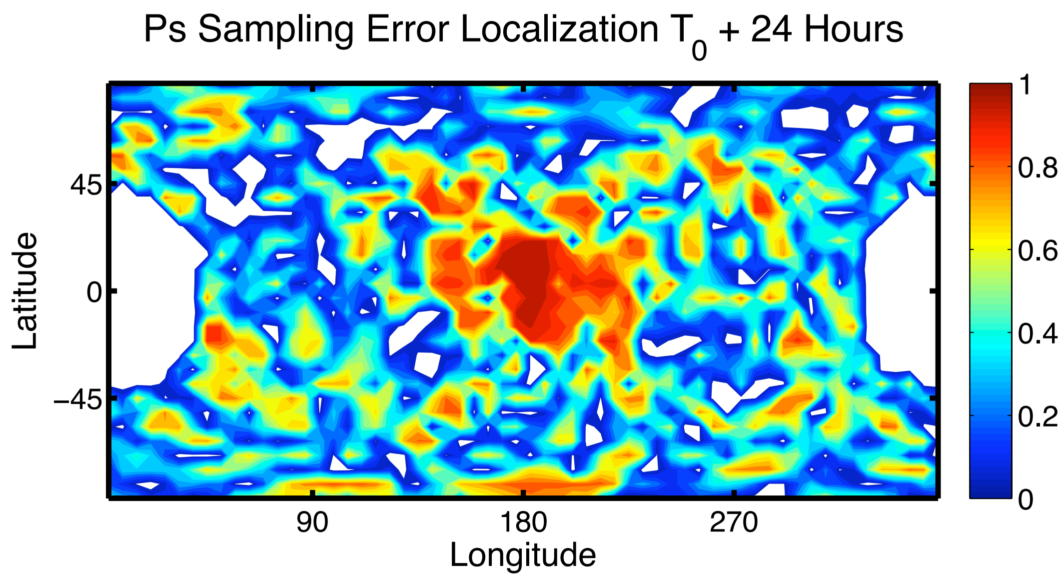
Time variation of sampling error localization on Ps field.
Ensemble size 80, Gaspari Cohn=1.2. (Full obs. set).

Test II: Low-order dry dynamical core



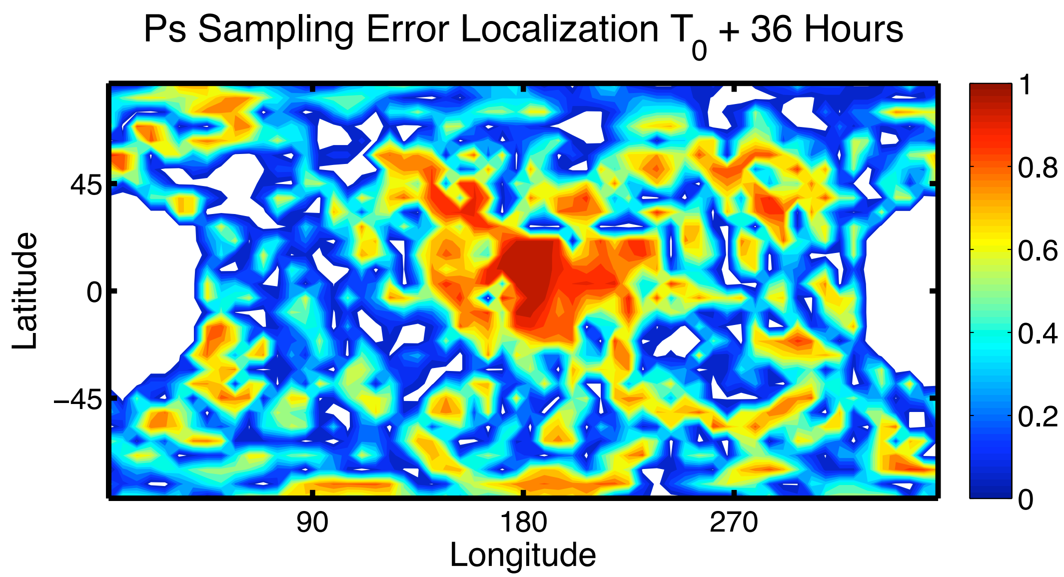
Time variation of sampling error localization on Ps field.
Ensemble size 80, Gaspari Cohn=1.2. (Full obs. set).

Test II: Low-order dry dynamical core



Time variation of sampling error localization on Ps field.
Ensemble size 80, Gaspari Cohn=1.2. (Full obs. set).

Test II: Low-order dry dynamical core



Time variation of sampling error localization on Ps field.
Ensemble size 80, Gaspari Cohn=1.2. (Full obs. set).