Assimilating Observations with Spatially and Temporally Correlated Errors in a Global Atmospheric Model

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Example: Correlated Error AR1 with Variance 1. Single Step Cov 0.999. Fixed for all cases.
Example: Correlated Error AR1 with Variance 1. Single Step Cov 0.999. Fixed for all cases. Vary uncorrelated error variance, 0.01
Example: Correlated Error AR1 with Variance 1.
Single Step Cov 0.999. Fixed for all cases.
Vary uncorrelated error variance, 0.1
Example: Correlated Error AR1 with Variance 1. Single Step Cov 0.999. Fixed for all cases. Vary uncorrelated error variance, 1.0
Example: Correlated Error AR1 with Variance 1.
Single Step Cov 0.999. Fixed for all cases. Vary uncorrelated error variance, 10.0
True trajectory is always 0.
Evolution is $x_{t+1} = 1.1x_t$
Perturbations grow exponentially in time.
Assimilating Correlated Observations

Obs1  Obs2  Obs3  Obs4  Obs5  Obs6
Exact Smoother Result. Can’t do better than this.

growth = 0.100000 m.bias = 0.000000 phi = 0.999000 sigma = 0.044710 bias = 0.000000

Uncorrelated Error Variance

1D Exponential Growth Model Results
EAKF Poor Unless Uncorrelated Error Dominates

Uncorrelated Error variance

\[ \text{RMSE} ; \text{SPREAD} \]

- Growth = 0.100000 m
- Bias = 0.000000 phi
- Sigma = 0.044710 bias = 0.000000

Exact Standard RMSE

Standard SD

Observation Number

Uncorrelated Error Variance 0.01

Total Error Correlated Error

Error

Observation Number

Uncorrelated Error Variance 0.1

Total Error

Correlated Error

Error

Observation Number

Uncorrelated Error Variance 1

Total Error

Correlated Error

Error

Observation Number
<table>
<thead>
<tr>
<th>Obs1</th>
<th>Obs2</th>
<th>Obs3</th>
<th>Obs4</th>
<th>Obs5</th>
<th>Obs6</th>
</tr>
</thead>
</table>

Two Types of Difference Observations
Unlinked Difference Observations

Unlinked Diff 1

Obs1  Obs2

Unlinked Diff 3

Obs3  Obs4

Unlinked Diff 5

Obs5  Obs6
Exact Unlinked Difference Obs Much worse.

\[ \text{growth} = 0.100000 \quad \text{mbias} = 0.000000 \quad \text{phi} = 0.999000 \quad \text{sigma} = 0.044710 \quad \text{bias} = 0.000000 \]
Unlinked Difference Observations

\[ \text{Var} \approx 2\sigma_u^2 \]

Unlinked Diff 1

Unlinked Diff 3

Unlinked Diff 5

Obs1  Obs2  Obs3  Obs4  Obs5  Obs6
Unlinked Difference Observations

\[ \text{Cov}(t, t + \Delta) \approx 0 \]

\[ \text{Var} \approx 2\sigma_u^2 \]

Unlinked Diff 1

Unlinked Diff 3

Unlinked Diff 5

Obs1, Obs2, Obs3, Obs4, Obs5, Obs6
EAKF is nearly exact for Unlinked Difference Obs.

\[ \text{growth} = 0.100000 \quad \text{mbias} = 0.000000 \quad \phi = 0.999000 \quad \sigma = 0.044710 \quad \text{bias} = 0.000000 \]

![Graph showing RMSE and Spread](image-url)
Linked Difference Observations

Linked Diff 1
Obs1
Linked Diff 2
Obs2
Linked Diff 3
Obs3
Obs4
Linked Diff 4
Obs5
Obs6
Linked Diff 5

EGU, 20 Apr., 2016
1D Exponential Growth Model Results

Exact linked Difference Obs Nearly Identical to Analytic.

growth = 0.100000 mbias = 0.000000 phi = 0.999000 sigma = 0.044710 bias = 0.000000

Exact linked Difference

Obs Nearly Identical to Analytic.
\[ Var \approx 2\sigma_u^2 \]

- Linked Diff 1
- Obs1
- Linked Diff 2
- Obs2
- Linked Diff 3
- Obs3
- Linked Diff 4
- Obs4
- Linked Diff 5
- Obs5
- Linked Diff 6
- Obs6
$Var \approx 2\sigma_u^2$

$Cov(t, t+1) \approx -\sigma_u^2$
\[ \text{Var} \approx 2\sigma_u^2 \]

\[ \text{Cov}(t, t+1) \approx -\sigma_u^2 \]

\[ \text{Cov}(\text{other}) \approx 0 \]
EAKF Linked Diff. Obs. Good when correlated error dominates.

growth = 0.100000 m bias = 0.000000 phi = 0.999000 sigma = 0.044710 bias = 0.000000
Comparison to Just Using Raw Observations

growth = 0.100000 mbias = 0.000000 phi = 0.999000 sigma = 0.044710 bias = 0.000000

Obs

Uncorrelated Error Variance

Total Error

Correlated Error
Observing System 1
3 Instruments.
Each has own
correlated error.
Observing System 2
1 Instrument
measures x, y, z each
time.
5 ensemble members.
Adaptive inflation.
Observations every 6 model timesteps.
5 ensemble members.
Adaptive inflation.
Observations every 6 model timesteps.
Difference obs better unless uncorrelated error variance dominates.

Improvement greater for single instrument.

Ensembles often under-dispersive (what a surprise!).
Lorenz 96 Model, 40-variables

Observing System 1
40 Instruments. Each has own correlated error.
Observing System 2
1 instrument
measures all 40
variables each time.
10 ensemble members.
Adaptive inflation, 0.2 halfwidth localization.
Observations every model timestep.
10 ensemble members.
Adaptive inflation, 0.2 halfwidth localization.
Observations every model timestep.
- Difference obs better unless uncorrelated error variance dominates.
- Improvement much greater for single instrument.
- Ensembles often over-dispersive.
- Dealing with time correlation harder than space correlation.
Evolution of surface pressure field every 12 hours. Has baroclinic instability: storms move east in midlatitudes.
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Low-Order Dry Dynamical Core

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Low-Order Dry Dynamical Core: Grid

30x60 horizontal grid, 5 levels.
Surface pressure, temperature, wind components.
28,800 variables.
Assimilate once per day. 0.2 radian localization. Observe each surface pressure grid point. Uncorrelated obs error variance 100 Pa.
Uncorrelated obs error variance 100 Pa.
Correlated obs error along ‘simulated polar orbiter track’.
Vary ratio of correlated to uncorrelated obs error variance.
Low-Order Dry Dynamical Core: PS Results

Global Prior Surface Pressure

- Base RMSE
- Base Spread
- Linked Diff Obs RMSE
- Linked Diff Obs Spread

Linked difference better for large correlated error. Standard better for small correlated error.
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PS RMSE Structure: Large Uncorrelated Error, Ratio 4

Base errors largest in storm tracks.

Linked difference errors largest in broad tropical band.
Base errors largest in storm tracks.

Linked difference errors largest in broad tropical band.
PS RMSE Structure: Small Uncorrelated Error, Ratio 1/4

Base errors largest in storm tracks.

Linked difference errors largest in broad tropical band.
T RMSE Structure: Small Uncorrelated Error, Ratio 1/4

Level 3 Temperature RMSE (K)

Base errors largest in tropics.

Linked difference errors have similar pattern.
• Linked difference obs better for large correlated error.

• Linked difference not sensitive to correlated error size.

• Adaptive inflation struggles with large correlated error.

• Could use base approach for uncorrelated obs, difference for correlated error obs.

• For example, base for sondes, difference for radiances.

• Difference obs allows assimilating before knowing correlated error characteristics.
Learn more about DART at:

www.image.ucar.edu/DARes/DART