



Methods for Computing Localization of Observation Impacts in Ensemble Kalman Filters

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1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.



2. Get prior ensemble sample of observation, y = h(x), by applying forward operator **h** to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially. 3. Get observed value and observational error distribution from observing system.



Schematic of an Ensemble Filter for Geophysical Data Assimilation

4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



Schematic of an Ensemble Filter for Geophysical Data Assimilation

5. Use ensemble samples of *y* and each state variable to linearly regress observation increments onto state variable increments.



6. When all ensemble members for each state variable are updated, there is a new analysis. Integrate to time of next observation ...



Regress y increments onto each state component x_i.







Localization is Required for Most Applications

- Localization multiplies regression.
- > Increments for N ensemble samples of x are: $\Delta x_{i,n} = \alpha b \Delta y_{n,}$ n=1, ..., N.
- b is sample regression coefficient.
- $\succ \alpha$ is a localization (normally between 0 and 1).





Defining a Localization Function

Localization for a given (y, x) might be a function of:

Metadata for (y, x) such as: Separation between (y, x), Kind of observation y (temperature, wind, ...), Kind of state variable x.





Method 1: Best Tuned Gaspari-Cohn

- Function of 'separation' between observation y and state x.
- Length scale defined by halfwidth parameter.





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Method 2: Optimized Localization

- ➤ Get optimal localization:
- ➢ Minimize RMSE in OSSE a posteriori.
- Initial guess is best Gaspari Cohn.
- Tricky optimization problem:
 Expensive (many long OSSEs), Noisy,
 Possible multiple minima.





Method 3: Global Group Filter

Run a group of quasi-independent ensembles.







Method 3: Global Group Filter



Run 200 groups for 3000 steps.

Do least squares fit for localization for each separation subset.

Resulting localization for each separation used with single

ensemble.

Expensive to initialize.





Method 4: Correlation Error Reduction

- > Assume all errors are due to ensemble sampling error.
- Focus on regression b=r(σ_x/σ_y), r is correlation, σ_x is standard deviation of state, σ_y is standard deviation of observation.

Estimates of standard deviation are unbiased (but estimates of ratio are biased, not discussed here).

Only correct sampling error in the correlation.





Define localization function for subsets of (y, x) pairs.

Examples:

- > All pairs separated by a given distance range,
- All pairs where y is a temperature and x is a u-wind separated by a given distance range.





Method 4: Correlation Error Reduction

Overview of Algorithm:

- Estimate 'background' correlation distribution for each separation subset.
- Use current sample correlation from assimilation and associated sampling error uncertainty.
- Combine current correlation with background.
- ➢ Get 'localized' correlation to update state x.





Lorenz96 40-Variable Localization Subset Definition

Define subsets of (y, x) pairs by separation:

Example: state x is 3 grid intervals from observation y, (dx = 3).



Estimate localization distribution for each separation.





Identity observations, error variance 1.

Assimilate every 12th model timestep.

20-member ensemble.





Method 4: Correlation Error Reduction Algorithm



Start with prior estimate of correlation for a separation (dx = 3 here) between obs and state variable.







Ensemble sample correlation between observation and state prior is part of standard ensemble algorithm.

Sample correlation here is 0.2.







Likelihood for this sample correlation and ensemble size is computed off-line ahead of time.

It is probability of true correlation given the sample correlation.

Note skew to the left.





Method 4: Correlation Error Reduction Algorithm



Posterior is normalized product of prior and likelihood.

This is Bayes rule.







Use mean value of posterior correlation, (0.1228 here) in the regression to update state.

An equivalent localization is 0.1228 / 0.2 = 0.614.







Update prior correlation distribution by adding a small constant times the posterior and normalizing.

Results here use 0.001 for this constant.

This is the only free parameter in the algorithm.

This is NOT Bayes rule.

























































































































































































Identity observations, error variance 1.

Assimilate every 12th model timestep.

20-member ensembles.

All cases use same adaptive inflation settings.





Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10







Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10, 20







Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10, 20, 40







Ensemble Size 10





Ensemble Size 10

Plus Best Gaspari Cohn





Ensemble Size 10, 20

Plus Best Gaspari Cohn





Ensemble Size 10, 20, 40

Plus Best Gaspari Cohn





Identity observations, error variance 16.

Assimilate every model timestep.

20-member ensembles.

All cases use same adaptive inflation settings.





Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10







Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10, 20







Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases Ensemble Size 10, 20, 40





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Ensemble Size 10





Ensemble Size 10, 20





Ensemble Size 10, 20, 40





Lorenz96 Identity Observations Summary (N=20)







Lorenz96 Identity Observations Summary (N=20)





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Each observation is average of grid point plus its nearest 8 neighbors on both side; total of 17 points. (Something like a radiance observation.)





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Each observation is average of grid point plus its nearest 8 neighbors on both side; total of 17 points. (Something like a radiance observation.)

Error variance 0.0625.

Assimilate every standard model timestep.





Case 3: Integral Observations

Compare Correlation Error Reduction to Gaspari Cohn Localization Ensemble Size 20





Approximate Localization: Observing Average of 17 States

Ensemble Size 20





Approximate Localization: Observing Average of 17 States

Ensemble Size 20

Plus localization for observations





Lorenz96 Integral Observations Summary (N=20)





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Work with Lili Lei and Jeff Whitaker

Find localization that gives least error compared to known true state in OSSE.

No a priori relation to sampling error.







Have output from an OSSE. Know prior ensemble and truth for each state variable.







Have output from an OSSE.

Know prior ensemble and truth for each state variable. Can get truth & prior ensemble for any potential observations.







Estimate localization for set of observations and subset of state variables.

e.g. state variables at various horizontal distances from observations.





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Example: how to localize impact of temperature observations (4 shown) on a U state variable that is between 600 and 800 km distant.





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Given observational error variance, can compute expected ensemble mean increment for state. Plot this vs prior state truth - ensemble mean.





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Do this for all state variables in subset.





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Do this for all state variables in subset.





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Do this for all state variables in subset.







Find a least squares fit.

Slope is α .

Least squares minimizes:

$$\sum \left[\left(x_{truth} - \overline{x} \right) - \alpha \hat{b} \Delta \overline{y} \right]^2$$

Same as minimizing
$$\sum \left[\left(\overline{x} + \alpha \hat{b} \Delta \overline{y} \right) - x_{truth} \right]^2$$

Posterior mean





Find α that minimizes the RMS difference between the posterior ensemble mean for x and the true value over this subset.

This can be computed from the output of the OSSE.

Can then use this localization in a new OSSE for all (y, x) in the subset.

Call the values of localization for all subsets an Empirical Localization Function (ELF).







Use ELF for next OSSE.





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- 1. Gaspari Cohn: Must be tuned.
- 1. Optimal: Very expensive to tune.
- 2. Global Group Filter: Not as expensive. Assumes sampling erroneous model can determine localization.
- Correlation Error Reduction: Moderately expensive. Assumes sampling error leads to need for localization. Include background information.
- 4. Empirical Localization Functions: Expensive to tune. Fits best analysis a posteriori. Not optimal for cycled.

































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Evolution of surface pressure field every 12 hours. Has baroclinic instability: storms move east in midlatitudes.





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Evolution of surface pressure field every 12 hours. Has baroclinic instability: storms move east in midlatitudes.







30x60 horizontal grid, 5 levels.

Surface pressure, temperature, wind components. 28,800 variables.





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Low-Order Dry Dynamical Core: Observations



Observe every 12 hours for 200 days. Observe all 16 variables in 180 columns shown. Error SD: Ps=2hPa, T=3K, winds=3m/s.





Method 4: CER for Low-Order Dry Dynamical Core

Limit observation impacts to a given halfwidth:

- Reduces computational costs.
- > Limits number of very small correlation pairs.





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Prior RMSE for Surface Pressure, 20 Member Ensemble





Prior RMSE for Level 3 Temperature, 20 Member Ensemble







Prior RMSE for Level 3 Wind, 20 Member Ensemble





Prior RMSE for Level 3 Wind, 20 Member Ensemble



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Localization of Level 3 T Observation on Level 3 T State







Localization of Level 3 T Observation on Level 3 T State





Localization of Level 3 T Observation on Level 3 T State





Localization of Level 3 U Observation on PS State





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Localization of PS Observation on Level 3 U State





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Localization of PS Observation on Level 3 V State





Method 5: ELF in CAM5 AGCM

Lili Lei did most of the ELF work.



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Conduct OSSEs in DART/CAM system (Raeder et al. 2012).

CAM5 model:

- Atmospheric component of the Community Earth System Model version 1 (CESM1; Gent et al. 2011)
- Finite volume grid with approximately 2° resolution (94x144) and 30 vertical levels
- Default configuration of the Atmospheric Model Intercomparison Project (AMIP; Gates 1992) protocol

Data assimilation system:

- Ensemble adjustment Kalman filter (EAKF; Anderson 2001) in DART
- Spatially- and temporally-varying state space adaptive inflation (Anderson 2009)
- GC localization as the default




RMSE with GC Localization



RMSEs for temperature and zonal wind are averaged globally (GL), in the southern hemisphere (SH), tropics (TP) and northern hemisphere (NH).

GC0.4 has smallest globally averaged RMSE, so 0.4 is chosen as the best halfwidth.

Some RMSEs computed for SH, TP and NH separately are smallest for other halfwidths; tuning the GC halfwidth is complex.





Horizontal and Vertical ELFs for CAM



Empirical localizations (black dots) computed separately for temperature, u&v winds at ten levels (30 dots per distance).

A z-test to assess significance.

A cubic spline (blue line) gives final localization function (ELFSP).

The horizontal ELFSP is smaller than the GC0.2 and GC0.4 at small separations and has a wider tail than GC0.2 and GC0.4.

The vertical ELFSP is much broader than the GC0.2 and GC0.4.

The horizontal and vertical ELFSPs are used in a subsequent OSSE (ELFOne).



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Horizontal/vertical ELFs varying by geographic regions



Horizontal and vertical ELFSPs are computed for SH, TP and NH separately.

Horizontal and vertical ELFSPs varying by region are used in a subsequent OSSE (ELFReg).





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Temperature RMSE Averaged in NH and TP



ELFReg has slightly smaller temperature RMSE than ELFOne in NH and SH.

ELFReg has smaller temperature RMSE than ELFOne in TP.

ELFReg has smaller globally averaged RMSE than ELFOne.





Method 5: ELFs in WRF Regional Model

Is different localization needed for different weather? Raining versus not raining.





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ELFs in WRF Regional Model

WRF model V3.3.1 :

- CONUS domain with horizontal grid spacing 15 km, 40 vertical layers and model top at 50 hPa
- Model physics: RRTMG long wave and short wave radiation schemes, Thompson 2-moment microphysics scheme, Noah land surface model, MYJ PBL scheme, and Tiedtke cumulus scheme

Data assimilation system:

- EAKF in DART
- Spatially- and temporally-varying state space adaptive inflation
- GC localization of halfwidth 0.1 radians as the default





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ELFs in WRF Regional Model



The ELFs for non-precipitating regions (ELFNP) have similar shape to GC0.1, but ELFNP of u-wind is smaller than GC0.1 for small separations.

The ELFs for precipitating regions (ELFP) are narrower than GC0.1 and ELFNP.

The correlation coefficient of ELF for precipitating regions decreases faster than for non-precipitating regions.





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ELFs in WRF Regional Model



The vertical ELFs for precipitating regions generally have larger localizations.

The vertical ELFP of temperature decreases more quickly with height than for u- and v-winds between 4 and 10 km.

The correlation coefficient of ELF for precipitating regions is larger.





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Exp. name	Applied localization function
GC0.1	GC localization function with half-width of 0.1 radians.
ELFOneA	One horizontal and one vertical ELFF that are computed from the output of GC0.1.
ELFOnePN	From the output of GC0.1, two horizontal and two vertical ELFFs that vary with precipitating and non-precipitating regions.
ELFObsPN	From the output of GC0.1, one horizontal and one vertical ELFF of temperature and one horizontal and one vertical ELFF of u- and v-wind for precipitating regions, and similarly four ELFFs for non-precipitating regions.





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Average Temperature RMSE for GC0.1 and ELFs



ELFOneA yields slightly smaller (but statistically significant) RMSE than GC0.1.

ELFOnePN has slightly smaller (but statistically significant) RMSE than GC0.1 and ELFOneA, thus the advantages of varying localization for precipitation and non-precipitating regions are demonstrated.

But the localization functions varying by observation types (ELFObsPN) do not show additional benefits than ELFOnePN.





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Conclusions

- Many methods for estimating localization.
- Often produce similar results.
- Good localization important for small ensembles.
- May not matter so much for tails, larger ensembles.
- Methods can guide localization for remote sensing.
- Automated methods mostly ignore balance issues.
- For operational NWP, may be small but important.





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Learn more about DART at:



www.image.ucar.edu/DAReS/DART

Anderson, J., Hoar, T., Raeder, K., Liu, H., Collins, N., Torn, R., Arellano, A., 2009: *The Data Assimilation Research Testbed: A community facility.* BAMS, **90**, 1283—1296, doi: 10.1175/2009BAMS2618.1



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