

Practical Implementations of the Ensemble Kalman Filter

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Building a Forecast System



The Data Assimilation Research Testbed (DART)

DART provides data assimilation 'glue' to build state-of-theart ensemble forecast systems for even the largest models.



DART Goals

Provide State-of-the-Art Data Assimilation capability to:

- Prediction research scientists,
- Model developers,
- Observation system developers,
- Who may not have any assimilation expertise.

DART Design Constraints

- Models small to huge.
- Few or many observations.
- > Tiny to huge computational resources.
- > Entry cost must be low.
- Competitive with existing methods for weather prediction: Scientific quality of results, Total computational effort.

Product of d-dimensional normals (gaussians) with means μ_1 and μ_2 and covariance matrics Σ_1 and Σ_1 is normal.

$$N(\mu_1, \Sigma_1)N(\mu_2, \Sigma_2) = cN(\mu, \Sigma)$$
(11)

Covariance:
$$\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$$
 (12)

Mean: $\mu = \Sigma(\Sigma_1^{-1}\mu_1 + \Sigma_2^{-1}\mu_2)$ (13)

Weight: $c = \frac{1}{(2\pi)^{d/2} |\Sigma_1 + \Sigma_2|^{1/2}} \exp\left\{-\frac{1}{2} [(\mu_2 - \mu_1)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_2 - \mu_1)]\right\}$

We'll ignore the weight since we immediately normalize products to be PDFs.

Recall our earlier assimilation update equation.

$$P(\mathbf{x}_{t_k}|\mathbf{Y}_{t_k}) = \frac{P(\mathbf{y}_k|\mathbf{x})P(\mathbf{x}_{t_k}|\mathbf{Y}_{t_{k-1}})}{Normalization}$$

(10)

Numerator is just product of two gaussians.

Denominator just normalizes posterior to be a PDF.



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Likelihood is gaussian, mean measured, covariance from designer. $\frac{P(\mathbf{y}_{k}|\mathbf{x})P(\mathbf{x}_{t_{k}}|\mathbf{Y}_{t_{k-1}})}{Normalization}$ $P(\mathbf{x}_{t_k}|\mathbf{Y}_{t_k}) =$ (10)0.35 0.3 0.25 0.2 0.15 0.1 0.05 0 50 0 100 80 100 60 40 20 0

Posterior is gaussian, from (11).



A Fast, Simple, Sequential Ensemble Kalman Filter

1. A one-dimensional ensemble Kalman filter.

- 2. One observed, one unobserved variable.
- 3. Ensemble Kalman Filter: A full implementation.
- 4. Making it work:
 - Localization
 - Inflation
- 5. Parameter estimation.
- 6. Some sample applications.

A One-Dimensional Ensemble Kalman Filter

Represent a prior pdf by a sample (ensemble) of N values:



Example: Predict temperature on the Nanjing campus.

A One-Dimensional Ensemble Kalman Filter

Represent a prior pdf by a sample (ensemble) of N values:



If posterior ensemble at time t_1 is $T_{1,n}$, n = 1, ..., N



If posterior ensemble at time t_1 is $T_{1,n}$, n = 1, ..., N, advance each member to time t_2 with model, $T_{2,n} = L(T_{1,n})$, n = 1, ..., N.



Same as advancing continuous pdf at time $t_1 \dots$



Same as advancing continuous pdf at time t_1 to time t_2 with model L.







Fit a Gaussian to the sample.



Get the observation likelihood.



Compute the continuous posterior PDF.



Use a deterministic algorithm to 'adjust' the ensemble.



First, 'shift' the ensemble to have the exact mean of the posterior.



First, 'shift' the ensemble to have the exact mean of the posterior. Second, linearly contract to have the exact variance of the posterior. Sample statistics are identical to Kalman filter.

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So far, we have a known likelihood for a single variable.



Now, suppose the model state has an additional variable,

temperature at Shanghai.

How should ensemble members update the additional variable?



Assume that all we know is the prior joint distribution.

One variable is observed.

What should happen to the unobserved variable?



Assume that all we know is the prior joint distribution.

One variable is observed.

Update observed variable with ensemble Kalman filter.



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Using only increments guarantees that if observation had no impact on observed variable, the unobserved variable is unchanged.

Highly desirable!






Have joint prior distribution of two variables.

Next, regress the observed variable increments onto increments for the unobserved variable.

Equivalent to first finding image of increment in joint space.



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Now have an updated (posterior) ensemble for the unobserved variable.

Fitting Gaussians shows that mean and variance have changed.



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Fitting Gaussians shows that mean and variance have changed.

Other features of the prior distribution may also have changed.

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1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.



2. Get prior ensemble sample of observation, y = h(x), by applying forward operator h to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially.

3. Get observed value and observational error distribution from observing system.



4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



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5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



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6. When all ensemble members for each state variable are updated, integrate to time of next observation ...



For linear, gaussian problem:

If, ensemble size $N>N_{crit}$

Mean and covariance are identical to Kalman Filter,

Else

Diverges.

Ncrit: Number of positive singular values in SVD of covariance matrix.

- (Ensemble) KF optimal for linear model, gaussian likelihood, perfect model.
- \succ In KF, only mean and covariance have meaning.
- Ensemble allows computation of many other statistics.
- ➤ What do they mean? Not entirely clear.
- What do they mean when there are all sorts of error? Even less clear.
- Must Calibrate and Validate results.

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Making it work:

Lorenz-96 low-order model example. 40 state variables: $X_1, X_2, ..., X_{40}$. $dX_i / dt = (X_{i+1} - X_{i-2})X_{i-1} - X_i + F.$

Acts 'something' like weather around a latitude band.



Making it work:

Lorenz-96 is sensitive to small perturbations. Introduce 20 'ensemble' state estimates. Each is perturbed for each of the 40-variables at time 0. Refer to unperturbed control integration as 'truth'.



Assimilate 'observations' from 40 random locations. Interpolate truth to station location. Simulate observational error:

Add random draw from N(0, 16) to each. Start from 'climatological' 20-member ensemble.





Making it work:

Some error sources in ensemble filters.



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Making it work: Localization

Sampling Error: Observations Impact Unrelated State Variables



Plot shows expected absolute value of sample correlation vs. true correlation.

Unrelated obs. reduce spread, increase error.

Attack with localization.

Reduce impact of observation on weakly correlated state variables.

Let weight go to zero for many 'unrelated' variables to save on computing.

Making it work: Localization



Making it work: Localization


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Some error sources in ensemble filters.



Assimilating with simulated model error. dXi / dt = (Xi+1 - Xi-2)Xi-1 - Xi + F. For truth, use F = 8. In assimilating model, use F = 6.



Time evolution for first state variable shown. Assimilating model quickly diverges from 'true' model.

Assimilating with simulated model error. dXi / dt = (Xi+1 - Xi-2)Xi-1 - Xi + F. For truth, use F = 8. In assimilating model, use F = 6.

F=8 Truth Model; F=6 assim with localization



Reduce confidence in prior to deal with model error. Use inflation.

Simply increase prior ensemble variance for each state variable. Adaptive algorithms use observations to guide this.



Inflation is a function of state variable and time. Automatically selected by adaptive inflation algorithm.



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A time-varying state-vector \mathbf{x}_t ,

Times t_k with observations: $k = 1, 2, ...; \quad t_{k+1} > t_k \ge t_0$,

Observations at t_k related to \mathbf{x}_{t_k} ; $\mathbf{y}_k = h_k(\mathbf{x}_{t_k}) + v_k$, (1)

Observation error is zero mean, normal, $v_k = N(0, \mathbf{R}_k)$, (2)

A forecast model m for the state-vector; $\mathbf{x}_{t_{k+1}} = m_{k:k+1}(\mathbf{x}_{t_k})$ (3)

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Parameters could be tuning for parameterizations, external forcing,...

Example: Sources for chemical tracers in atmosphere.

A forecast model *m* for the state-vector; $\mathbf{x}_{t_{k+1}} = m_{k:k+1}(\mathbf{x}_{t_k}; \boldsymbol{\alpha})$ (3a)

One solution: State augmentation.

Define augmented state vector $\mathbf{x}^+ = (\mathbf{x}, \boldsymbol{\alpha})$

Prediction model becomes (just a change in notation):

$$\mathbf{x}_{t_{k+1}}^+ = m_{k:k+1} \left(\mathbf{x}_{t_k}^+ \right)$$

State augmentation challenges:

In general, no time prediction model for parameters.

- If we had a prediction model, they would just have been state.
- Kalman filter prior covariance comes from prediction model.

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Prior ensembles for parameters must be specified.

- Prior sample covariance controls impact of observations on parameters.
- If prior covariance is not well-known, estimating parameters can be challenging.

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Science: A global atmospheric ensemble reanalysis. **Collaborators: Model Developers at NCAR**

O(1 million) atmospheric obs are assimilated every day.

> 500 hPa GPH Feb 17 2003



Assimilation uses 80 members of 2° FV CAM forced by a single ocean 120E (Hadley+ NCEP-OI2) and produces a very competitive reanalysis.

1998-2010 4x daily is available. Science: Do new satellite observations of cloud motion improve hurricane forecasts?

Atmospheric motion vectors from CIMMS at University of Wisconsin.

Collaborator: Ting-Chi Wu, Graduate Student, University of Miami.

Tropical Cyclones and Atmospheric Motion Vectors



Wu et al., 2014, MWR, **142**, 49–71.

Science: Where should more observations be taken to improve landfall forecasts?

Ensemble sensitivity analysis for Katrina.

Collaborator: Ryan Torn, University at Albany.

Hurricane Katrina Sensitivity Analysis



Contours are ensemble mean 48h Color shows where forecast of deep-layer mean wind. observations could help.

Identifying Model Systematic Errors



Science: Diagnosing and correcting errors in the CAM FV core. Collaborator: Peter Lauritzen, CGD.

Gridpoint noise detected in CAM/DART analysis

Ensemble Mean V at 266 hPa at 6 hours



CAM FV core - 80 member mean - 00Z 25 September 2006

Suspicions turned to the polar filter (DPF)

Ensemble Mean V at 266 hPa at 6 hours



Continuous polar filter (alt-pft) eliminated noise.

Meridional Wind Speed from Alternate Polar Filter (ALT)



Differences mostly in transition region of default filter.



- The use of DART diagnosed a problem that had been unrecognized (or at least undocumented).
- Could have an important effect on any physics in which meridional mixing is important.
- The problem can be seen in 'free runs' it is not a data assimilation artifact.
- Without assimilation, can't get reproducing occurrences to diagnose.

Science: Global Ocean data assimilation. Collaborators: Alicia Karspeck, Steve Yeager, CGD.

- Climate change over time scales of 1 to several decades has been identified as very important for mitigation and infrastructure planning.
- Need ocean initial conditions for the IPCC decadal prediction program (and maybe a crystal ball, too!).

World Ocean Database T, S observation counts.

These counts are for 1998 & 1999 and are representative.

FLOAT SALINITY FLOAT TEMPERATURE DRIFTER TEMPERATURE MOORING SALINITY MOORING_TEMPERATURE BOTTLE SALINITY BOTTLE TEMPERATURE CTD SALINITY CTD TEMPERATURE STD SALINITY STD TEMPERATURE XCTD SALINITY XCTD TEMPERATURE MBT_TEMPERATURE **XBT TEMPERATURE** APB TEMPERATURE





Physical Space: 1998/1999 SST Anomaly from HadOI-SST



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Science: Land surface analysis with DART/CLM. Collaborator: Yongfei Zhang, UT Austin.

Land surface analysis with DART/CLM:

Estimate snow water equivalent with observations of snow cover fraction from satellites (MODIS).

Assimilation of MODIS snow cover fraction

- 80 member ensemble for onset of NH winter
- Assimilate once per day
- Level 3 MODIS product regridded to a daily 1 degree grid
- Observation error variance is 0.1 (for lack of a better value)
- Observations can impact state variables within 200km
- CLM variable to be updated is the snow water equivalent "H205N0"

Standard deviation of the snow cover fraction initial conditions for Oct. 2002





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An early result: assimilation of MODIS *snowcover fraction* on total *snow water equivalent* in CLM.



Science: Regional Atmospheric Chemistry. Collaborator: Arthur Mizzi, NCAR/ACD.

WRF/Chem Chemical Weather Forecast System

- WRF-Chem Weather Research and Forecasting Model (WRF) with online chemistry.
- Meteorological Observations NOAA PREPBUFR conventional observations.
- Chemistry Observations MOPITT CO retrieval profiles (also IASI CO retrievals – results not shown).

WRF/Chem Chemical Weather Forecast System

- WRF/Chem-DART cycling with conventional meteorological observations and MOPITT CO V5 retrieval profiles.
- ≻ Continuous six-hr cycling (00Z, 06Z, 12Z, and 18Z).
- > CONUS grid with 101x41x34 grid points and 100 km resolution.
- ≻ 20-member ensemble.
- ≻ June 1 30, 2008 (112 cycles) study period.
- ≻ Full state variable/obs interaction.
- Initial and lateral chemical boundary conditions from MOZART-4 simulation.
- Emissions: Biogenic MEGAN, Anthropogenic global inventories, and Fire – Fire Inventory from NCAR (FINN).
WRF/Chem Chemical Weather Forecast System

> Two experiments:

- ♦ Exp 1: PREPBUFR conventional obs (CNTL DA).
- ♦ Exp 2: MOPITT CO retrieval profiles and PREPBUFR conventional obs (CHEM DA).

WRF/Chem Chemical Weather Forecast System





DART Science and Collaborators (8)

Science: Global Atmospheric Chemistry. Collaborators: Jerome Barre, Benjamin Gaubert, NCAR/ACD.

Uses global CAM/Chem model, 1 degree.

Have full meteorological assimilation capability already.

CAM/Chem Chemical DA System



MOPITT CO: On TERRA satellite tropospheric profiles Global coverage in 4 days Multispectral retrievals high sensitivity on surface land/day



IASI CO: On MetOpA satellite tropospheric profiles Global coverage in 1 day Only thermal infrared Sensitivity on upper PBL & mid troposphere



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CAM/Chem Chemical DA System



Control run: Met Only assimilated



IASI run: Met + IASI assimilated



MOPITT run: Met + MOPITT assimilated



Combined run: Met + MOP+ IASI assimilated



Learn more about DART at:



www.image.ucar.edu/DAReS/DART

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Anderson, J., Hoar, T., Raeder, K., Liu, H., Collins, N., Torn, R., Arellano, A., 2009: *The Data Assimilation Research Testbed: A community facility.* BAMS, **90**, 1283—1296, doi: 10.1175/2009BAMS2618.1

