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Adaptive Prior Inflation for Ensemble Filters: Application to a Large-Scale Atmospheric Model

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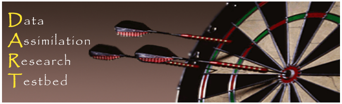

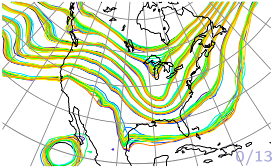

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- Others: EnTLHF [Luo and Hoteit 2011], EnKF-N [Bocquet et al. 2015]

1.2 Inflation and Innovation Statistics

Given a scalar variable with sample x_i and observation y

$$x_b = \frac{1}{N_e}\sum_{i=1}^{N_e} x_i, \qquad \widehat{\sigma_b}^2 = \frac{1}{N_e - 1}\sum_{i=1}^{N_e}(x_i - x_b)^2 \tag{1}$$

Following Desroziers et al. (2005)

$$\begin{aligned} d &= y - x_b = \varepsilon_o + (x_t - x_b) = \varepsilon_o + \varepsilon_b, && (2)\\ \mathbb{E}(d) &= \mathbb{E}(\varepsilon_o) + \mathbb{E}(\varepsilon_b) = 0, && (3)\\ \mathbb{E}\left(d^2\right) &= \mathbb{E}\left(\varepsilon_o^2\right) + \mathbb{E}\left(\varepsilon_b^2\right) + 2\mathbb{E}(\varepsilon_o\varepsilon_b) = \sigma_o^2 + \sigma_b^2. && (4) \end{aligned}$$

Impose $\sigma_b^2 = \lambda_o \widehat{\sigma_b}^2$. Assuming a correctly specified σ_o^2

$$\Rightarrow \boxed{\lambda_o = \frac{\mathbb{E}\left(d^2\right) - \sigma_o^2}{\widehat{\sigma_b}^2}} \tag{5}$$

1.3 Anderson (2009), A09 hereafter

$$p\left(\lambda|d\right) \propto p\left(d|\lambda\right) \cdot p\left(\lambda\right) \tag{6}$$

- Prior marginal distribution: $N\left(\lambda_b, \sigma^2_{\lambda_b}\right)$

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- Prior marginal distribution: $N\left(\lambda_b, \sigma^2_{\lambda_b}\right)$
- Likelihood: $d \sim N\left(0, \theta^2\right)$, with $\theta^2 = \lambda_o^k \widehat{\sigma_b}^2 + \sigma_o^2$
  - Spread the information across all variables

$$\begin{aligned} r &= corr(x^o, x^k) \quad k = 1, 2, \ldots, N_x & (7) \\ \lambda_o^k &= \left[\gamma\left(\lambda_b^k - 1\right) + 1\right]^2, \quad \gamma = \kappa|r| & (8) \end{aligned}$$

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  - $p(d|\lambda)$ is not Gaussian in λ!
- Posterior:

$$p(\lambda|d) \propto \frac{1}{2\pi\theta\sigma_{\lambda_b}} \exp\left[-\frac{(\lambda - \lambda_b)^2}{2\sigma^2_{\lambda_b}} - \frac{d^2}{2\theta^2}\right] \tag{9}$$

2.1 Enhanced Scheme: The Likelihood

- σ_o^2 is incorrectly specified, or when $\lambda_o < 0$ (likelihood peak)?

$$\lambda_o = \frac{\mathbb{E}\left(d^2\right) - \sigma_o^2}{\widehat{\sigma_b}^2}$$

- Here, assume the distance to be a random variable:

$$\underbrace{\frac{1}{N_e}\sum_{i=1}^{N_e} d_i^2}_{\approx \left(\frac{1}{N_e}\sum_{i=1}^{N_e} d_i\right)^2 + \mathbb{V}(d)} = \sigma_o^2 + \sigma_b^2 + \frac{N_e - 1}{N_e}\widehat{\sigma_b}^2, \qquad (10)$$

where $d_i = \varepsilon_o + \varepsilon_b - \widetilde{x}_i$ and $\mathbb{V}(d)$: innovation sample variance.

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- Modifies the inflation likelihood:

$$\boxed{\lambda_o^* = \frac{\mathbb{E}\left(d^2\right) - \sigma_o^2}{\widehat{\sigma_b}^2} + \frac{1}{N_e} = \lambda_o + \frac{1}{N_e}} \qquad (11)$$

2.2 Enhanced Scheme: The Prior

- Instead of a Gaussian, describe the inflation prior by an inverse Gamma (IG) distribution. Why?
  - Restriction: to positive and not very close to zero values
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$$p\left(\lambda\right) = \frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)}\lambda^{-\alpha-1}\exp\left[-\frac{\beta}{\lambda}\right] \tag{12}$$

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  - Start with a Gaussian $N\left(\lambda_b, \sigma^2_{\lambda_b}\right)$. Use mean and variance parameters to find α and β

$$\lambda_b = \frac{\beta}{\alpha+1} \equiv \mathrm{Mode}_{\mathrm{IG}} \tag{13}$$

$$\sigma^2_{\lambda_b} = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}, \qquad \alpha > 2 \tag{14}$$

  - Cubic equation (single positive root), (i) find β, (ii) deduce α

2.3 Enhanced Scheme: The Posterior

- The new posterior *is assumed IG*

$$\frac{\beta^{\alpha}\lambda^{-\alpha-1}}{\sqrt{2\pi}\theta\Gamma(\alpha)}\exp\left[-\frac{d^2}{2\theta^2}-\frac{\beta}{\lambda}\right] \tag{15}$$

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- To find the updated inflation or the mode, i.e., λ_u

$$\left(1-\frac{\lambda_b}{\beta}\right)\lambda^2+\left(\frac{\bar{\ell}}{\ell'}-2\lambda_b\right)\lambda+\left(\lambda_b^2-\frac{\bar{\ell}}{\ell'}\lambda_b\right)=0 \tag{16}$$

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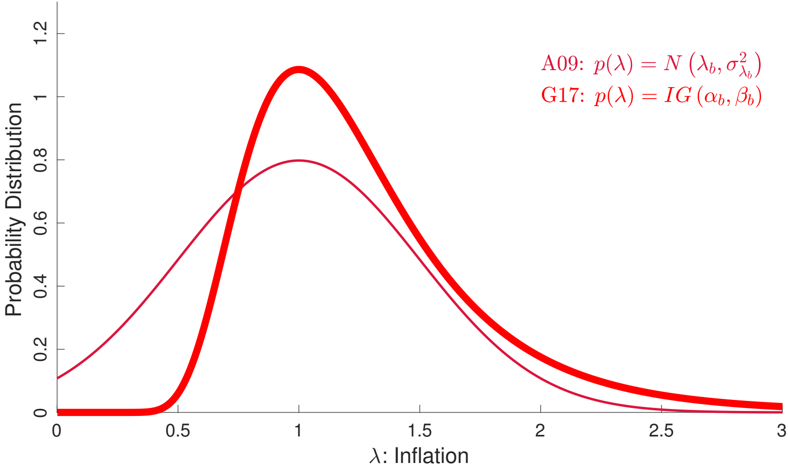
- Posterior variance can be numerically obtained. It can both increase & decrease
- In DART, the user only deals with Gaussian input/output inflation fields
- Lower bound can be set to zero (allow for deflation)

2.3 Enhanced Scheme: G17 hereafter

- Example: *from an L63 DA run*

$\sigma_b^2 = 2.4$, $\sigma_o^2 = 5 \times 10^{-2}$, $d^2 = 0.09$, $N_e = 10$
$\lambda_b = 1$, $\sigma_{\lambda_b}^2 = 0.25$



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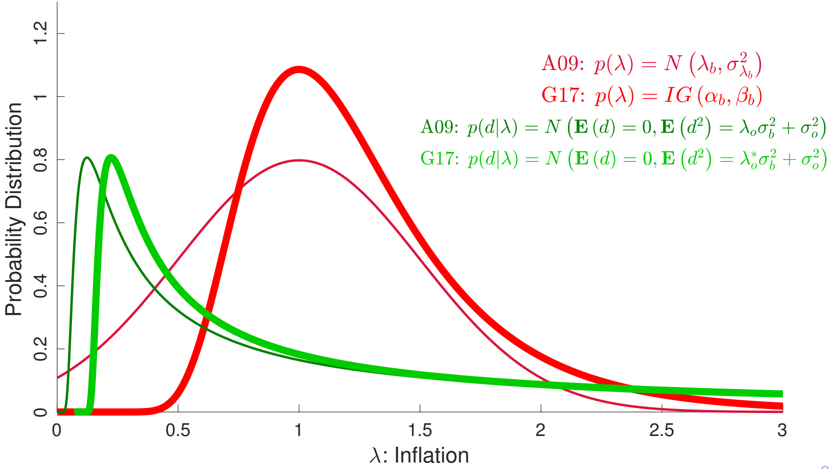

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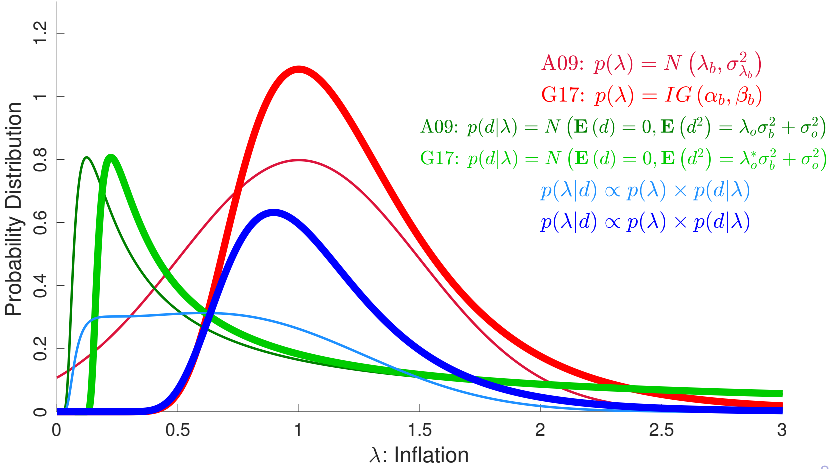

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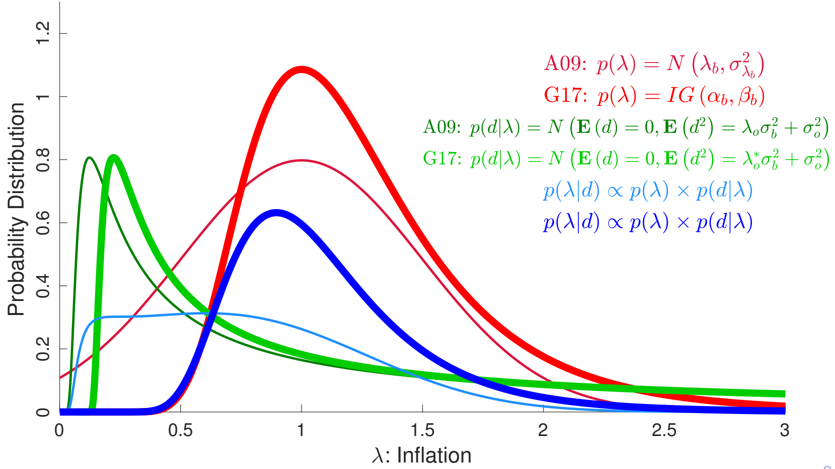
$\lambda_b = 1$, $\sigma_{\lambda_b}^2 = 0.25$ $\Rightarrow$ using G17: $\lambda_u = 0.90$



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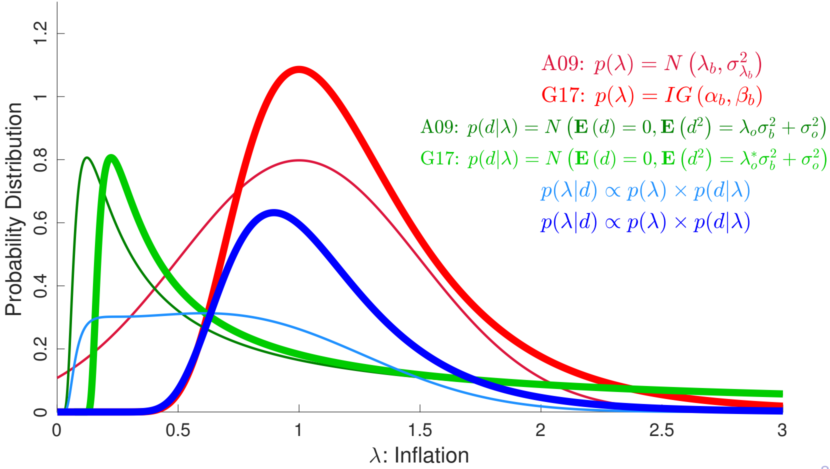
- **Gharamti, M. E.** (2017). Enhanced Adaptive Inflation Algorithm for Ensemble Filters. *Monthly Weather Review*, in press.



3.1 CAM (The Community Atmosphere Model)

- version: `CESM2_0_beta05`
- resolution: $1.9^\circ \times 1.9^\circ$ FV core;
  `LAT: 96`, `LON: 144`, `LEV: 26`

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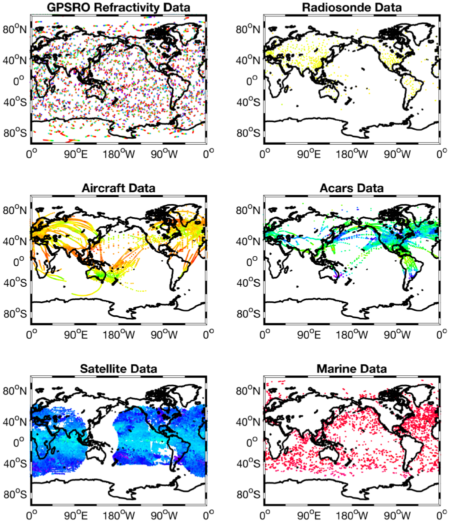
- resolution: $1.9^\circ \times 1.9^\circ$ FV core; `LAT: 96`, `LON: 144`, `LEV: 26`
- single state spinup, 80 members ensemble initialization
- DA (EAKF) between 08.16.2010 to 09.30.2010
- data available every 6 hours: wind and temperature observations from radiosondes, ACARS and aircraft along with GPS radio occultation
- Localization cutoff: 0.15 rad

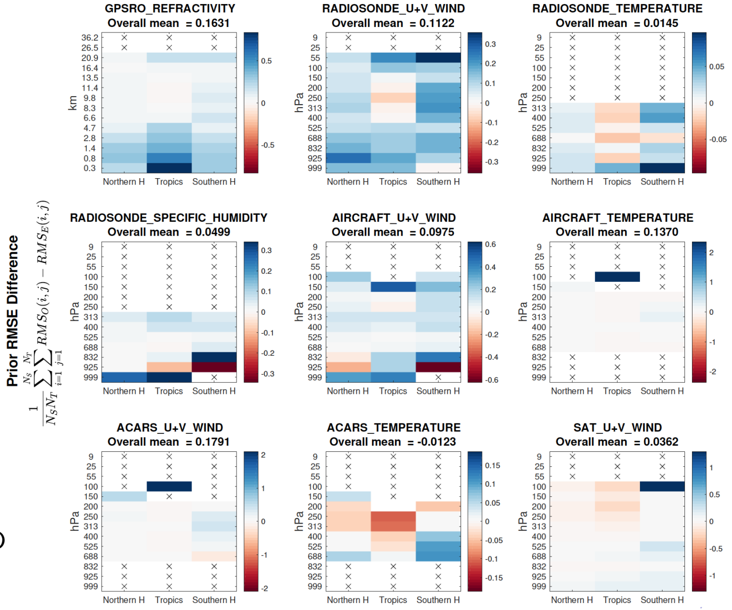


3.2 Assimilation Results: A09 vs. G17

Obs. Space Diagnostics: RMSE

- Both schemes initialized with $\lambda \sim N(1, 0.36)$
- inflation variance is fixed

- Largest improvements: Tropics & Southern H.
- improved GPSRO near surface



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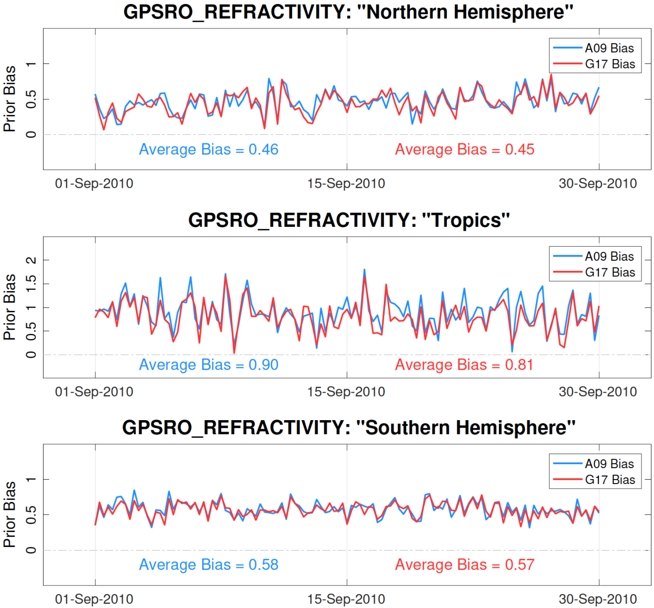
Obs. Space Diagnostics: Bias, Consistency and Profiles



- $\sim$ 10% improvements in the Tropics

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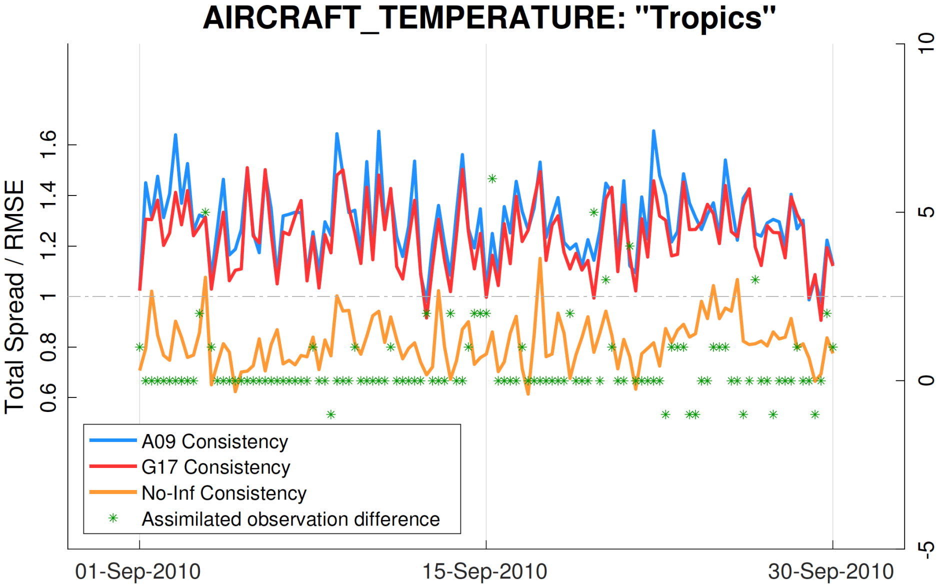

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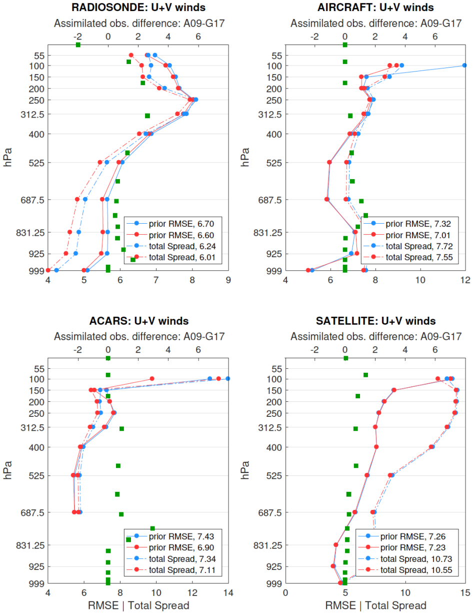
Obs. Space Diagnostics: Bias, Consistency and Profiles

- Suggested (average) improvements of wind estimates using G17
  - Radiosondes: 1.5%
  - Aircrafts: 4.24%
  - Acars: 4.95%
  - Satellite: 0.41%
- Both schemes assimilate almost the same number of observations (<1% difference)



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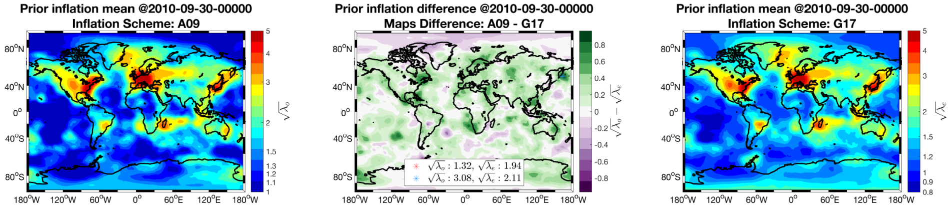
Inflation Fields and Patterns



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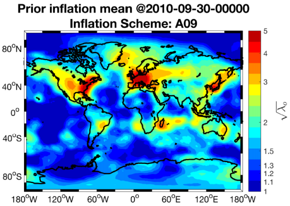

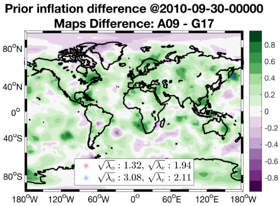

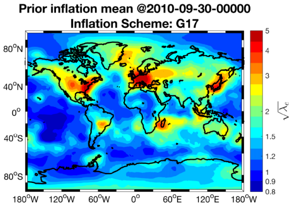

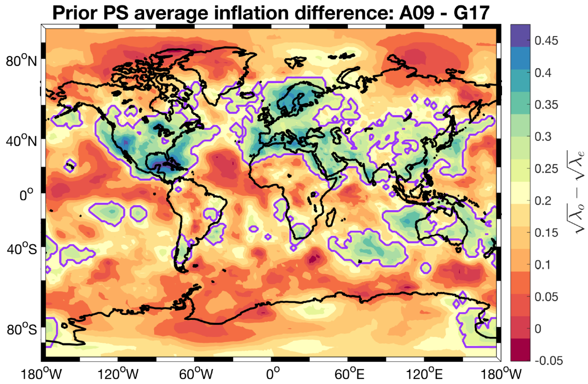

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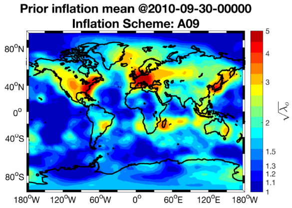

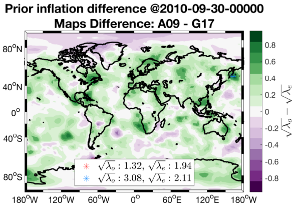

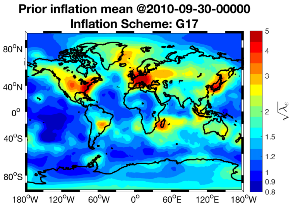

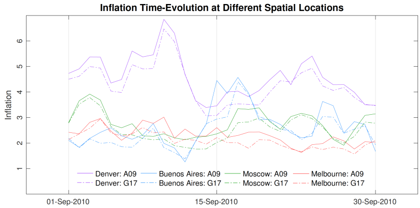

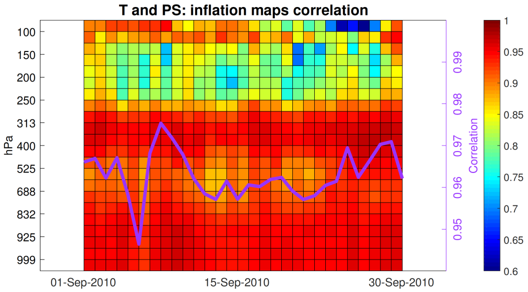

4. Conclusion

- Proposed an enhanced spatially and temporally varying adaptive prior covariance inflation
- The prior distribution is assumed IG and the likelihood density is slightly shifted to larger distances
- Improvements using the DART-CAM framework are observed for different observation types and mainly for near-surface GPSRO observations
- In the Tropics and the Southern Hemisphere, the proposed scheme outperforms the original inflation algorithm. In the Northern Hemisphere, both schemes yield comparable results
- A09 over-inflates in the N. H. G17 allows for slight deflation especially in the central Pacific. Inflation maps obtained using both schemes are highly correlated.