A Comparison of Three Kalman Filters Using a Large Atmospheric General Circulation Model Ensemble

B.K. Johnson,[1] M. Gharamti,[1] & I. Hoteit[2]

[1] National Center for Atmospheric Research [2] King Abdullah University of Science and Technology

This study tests three filters: the Ensemble Kalman Filter (EnKF; Evensen, 2003), the Ensemble Adjustment Kalman Filter (EAKF; Anderson, 2003), and the Ensemble Kalman Filter with exact second order perturbation sampling (EnKF-esops; Hoteit et al., 2015). Derivation of EnKF-esops is as follows.

Kalman posterior covariance:

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b$$

Stochastic EnKF posterior covariance:

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b\,(\mathbf{I} - \mathbf{KH})^{\mathrm{T}} + \mathbf{K}\frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon})\,(\varepsilon_i - \bar{\varepsilon})^{\mathrm{T}}\mathbf{K}^{\mathrm{T}} + (\mathbf{I} - \mathbf{KH})\,\frac{1}{N-1}\sum_{i=1}^{N}(x_i^b - \bar{x}^b)(\varepsilon_i - \bar{\varepsilon}^b)^{\mathrm{T}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\,\frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon}^b)(x_i^b - \bar{x}^b)^{\mathrm{T}}(\mathbf{I} - \mathbf{KH})^{\mathrm{T}}$$

To match KF's covariance, sample ε_i using a second order draw:

$$\bar{\varepsilon} = 0, \frac{1}{N-1}\sum_{i=1}^{N}\varepsilon_i\varepsilon_i^{\mathrm{T}} = \mathbf{R}$$

cross-correlations vanish:

$$\sum_{i=1}^{N}\varepsilon_i\left(x_i^b - \bar{x}^b\right)^T = 0$$

References

Anderson, J. L., 2003: A Local Least Squares Framework for Ensemble Filtering. *Monthly Weather Review*, **131**, 634–642.

Evensen, G., 2003: The Ensemble Kalman Filter: theoretical formulation and practical implementation. *Ocean Dynamics*, **53**, 343–367.

Hoteit, I., D.-T. Pham, M. E. Gharamti, and X. Luo, 2015: Mitigating Observation Perturbation Sampling Errors in the Stochastic EnKF. *Monthly Weather Review*, **143**, 2918–2936.

In the Data Assimilation Research Testbed observations are assimilated serially. When augmenting code for EnKF-esops, one rank is removed from the background perturbation matrix before analysis.

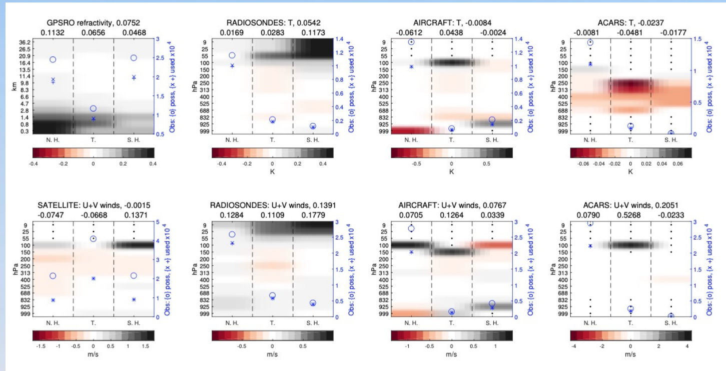
$x_i^b \leftarrow x_i^b$-$(\widetilde{\mathbf{X}}^b w)w_i$ where w is an eigenvector of $(\widetilde{\mathbf{X}}^b)^{\mathrm{T}}\widetilde{\mathbf{X}}^b$. Perturb the j$^{\text{th}}$ observation,

$y_i = y_j^o + s_j\sqrt{(N-1)R_j w_i}$ and update w:

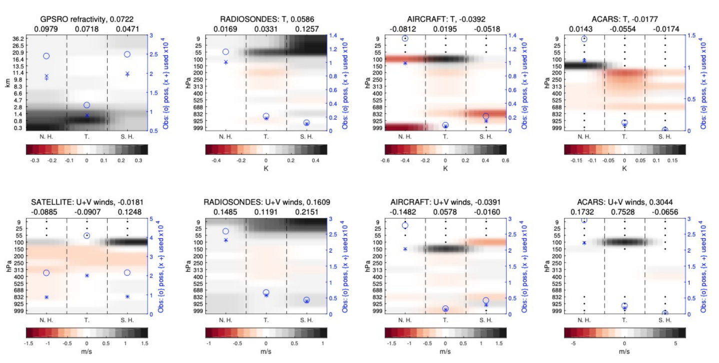
$$w_i = \frac{\sqrt{(N-1)R_j}w_i - (\mathbf{H}_j\mathbf{x}_i^a - \mathbf{H}_j\mathbf{x}^a)}{\sqrt{\sum_{i=1}^{N}(\mathbf{H}_j\mathbf{x}_i^a - \mathbf{H}_j\mathbf{x}^a)^2 + (N-1)R_j}}$$

The experiments were conducted on Shaheen II, a supercomputer at King Abdullah University of Science and Technology, using a 12 million core-hour allocation. The researchers thank KAUST HPC staff for their dedicated support of the experiment.

The filters were tested using large ensembles of an atmospheric general circulation model, the Community Atmosphere Model 6. Ensemble sizes ranged from 250-1000 ensemble members.



RMSE EnKF – EnKF-esops
(black is where EnKF-esops outperforms EnKF)



RMSE EAKF – EnKF-esops
(black is where EnKF-esops outperforms EAKF)