# A Comparison of Three Kalman Filters Using a Large Atmospheric General Circulation Model Ensemble

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## Filter Methodology

This study tests three filters: the Ensemble Kalman Filter (EnKF; Evensen, 2003), the Ensemble Adjustment Kalman Filter (EAKF; Anderson, 2003), and the Ensemble Kalman Filter with exact second order perturbation sampling (EnKF-esops; Hoteit et al., 2015). Derivation of EnKF-esops is as follows.

Kalman posterior covariance:

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b$$

Stochastic EnKF posterior covariance:

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b\,(\mathbf{I} - \mathbf{KH})^{\mathrm{T}} + \mathbf{K}\frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon})\,(\varepsilon_i - \bar{\varepsilon})^{\mathrm{T}}\mathbf{K}^{\mathrm{T}} + (\mathbf{I} - \mathbf{KH})\frac{1}{N-1}\sum_{i=1}^{N}(x_i^b - \bar{x}^b)(\varepsilon_i - \bar{\varepsilon}^b)^{\mathrm{T}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\frac{1}{N-1}\sum_{i=1}^{N}(\varepsilon_i - \bar{\varepsilon}^b)(x_i^b - \bar{x}^b)^{\mathrm{T}}(\mathbf{I} - \mathbf{KH})^{\mathrm{T}}$$

To match KF's covariance, sample $\varepsilon_i$ using a second order draw:

$$\bar{\varepsilon} = 0, \frac{1}{N-1}\sum_{i=1}^{N}\varepsilon_i\varepsilon_i^{\mathrm{T}} = \mathbf{R}$$

cross-correlations vanish:

$$\sum_{i=1}^{N}\varepsilon_i\left(x_i^b - \bar{x}^b\right)^T = 0$$

Since observations are assimilated serially in the Data Assimilation Research Testbed (Anderson et al., 2009) the code is augmented for EnKF-esops so that one rank is removed from the background perturbation matrix before analysis: $x_i^b \leftarrow x_i^b$-$(\tilde{\mathbf{X}}^b w)w_i$ where $w$ is an eigenvector of $\left(\tilde{\mathbf{X}}^b\right)^{\mathrm{T}}\tilde{\mathbf{X}}^b$. Perturb the j$^{\text{th}}$ observation, $y_i = y_j^o + s_j\sqrt{(N-1)R_j}w_i$ and update $w$:

$$w_i = \frac{\sqrt{(N-1)R_j}w_i - (\mathbf{H}_j\mathbf{x}_i^a - \mathbf{H}_j\bar{\mathbf{x}}^a)}{\sqrt{\sum_{i=1}^{N}(\mathbf{H}_j\mathbf{x}_i^a - \mathbf{H}_j\bar{\mathbf{x}}^a)^2 + (N-1)R_j}}$$

## Computational Setup

The filters are tested using large ensembles of an atmospheric general circulation model, the Community Atmosphere Model 6, which is the atmospheric component of the Community Earth System Model (CESM). The experiment is conducted with three different ensemble sizes of 250, 500 and 1000 members.

Sampling error correction (Anderson, 2012) is applied and inverse-gamma adaptive prior inflation (Gharamti, 2018) is used.

The experiments are computed on Shaheen II, a supercomputer at King Abdullah University of Science and Technology, using a 12 million core-hour allocation.


### Porting CESM to Shaheen II

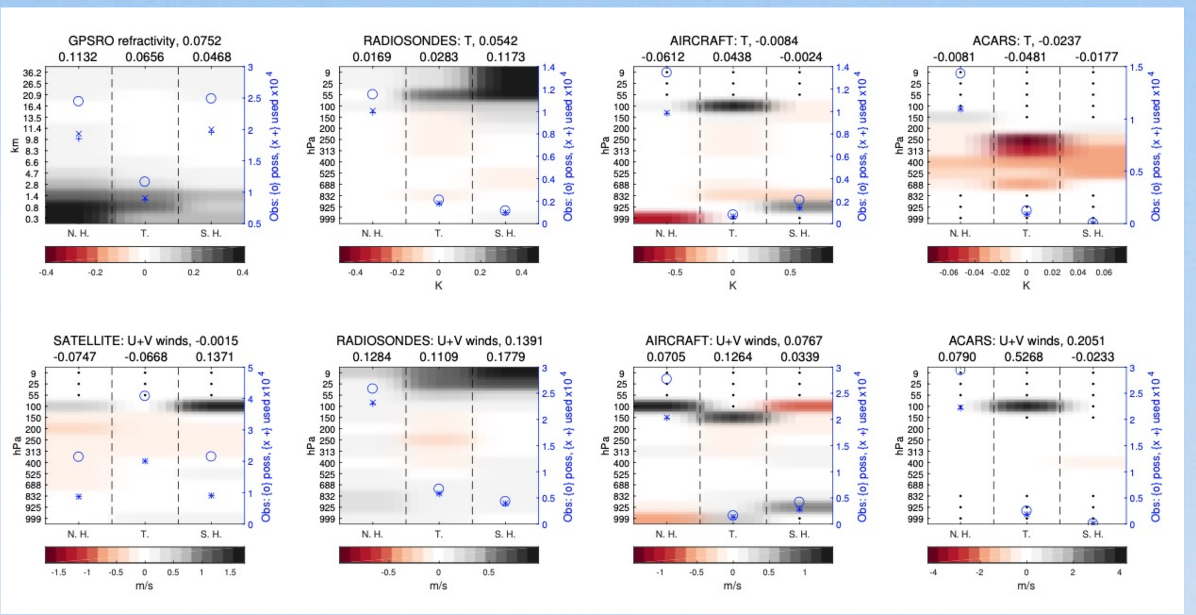
CESM has many dependencies and its constituent models do not produce bitwise reproducible output across different systems.

To ensure a successful port of CESM to Shaheen II, the researchers completed the CESM ensemble consistency verification (Baker et al., 2015).

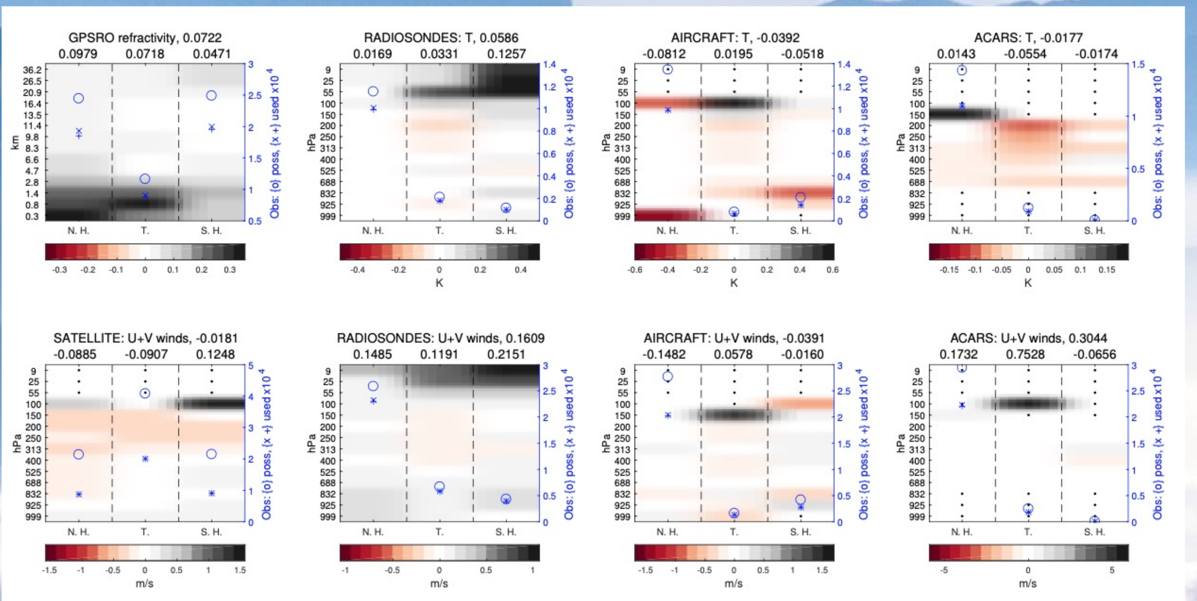
The researchers thank KAUST HPC staff for their dedicated support of the experiment.

## Results

The plots below show spatially averaged differences in RMSE for various observation types. The top panel compares EnKF versus EnKF-esops, while the second compares EAKF versus EnKF-esops.



RMSE EnKF – EnKF-esops
*(EnKF-esops outperforms EnKF shaded black)*



RMSE EAKF – EnKF-esops
*(EnKF-esops outperforms EAKF shaded black)*

## References

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