A Randomized Dormant Ensemble Kalman Filter

Sampling Errors: An Alternative Look

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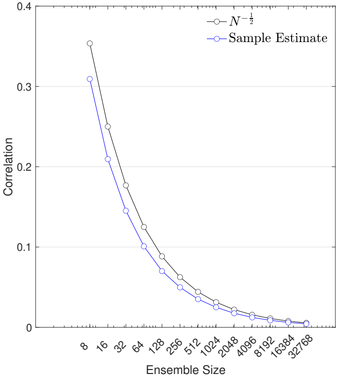
Sampling Errors in the EnKF

- *Bluntly:* Basic statistics tells us small ensemble sizes just won't cut it



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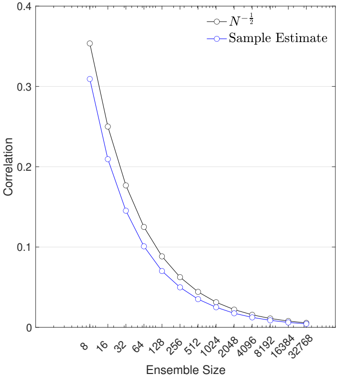

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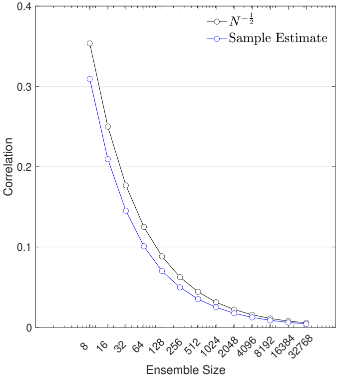

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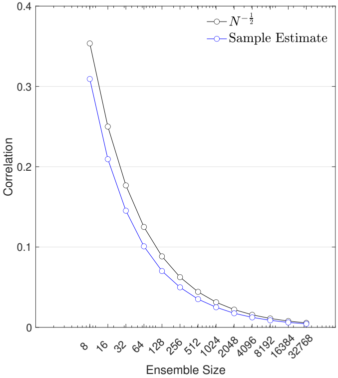
- Can become even more complex in non-Gaussian and non-linear regimes



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1. **Localization:** Localize the impact of the observations to nearby state variables only (Houtekamer and Mitchell, 2001)
2. **Inflation:** Ensemble state covariance is increased by linearly inflating each scalar component of the state while preserving the mean (Pham et al. 1998) :

$$
\begin{aligned}
\widetilde{x}_i &\leftarrow \sqrt{\lambda}\,(x_i - \overline{x}) + \overline{x}, \\
\widehat{\sigma}^2 &= \lambda \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2 ,
\end{aligned}
$$

where $\lambda > 1$ is an inflation factor and $i = 1, 2, \ldots, N$.

- Adaptive forms (Anderson 2007, 2009; El Gharamti 2018)
- Posterior (El Gharamti 2019), RTPS (Whitaker and Hamill, 2012)

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- Large inflation can cause issues for ocean models

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RESEARCH ARTICLE

Quarterly Journal of the Royal Meteorological Society RMetS

A global coupled ensemble data assimilation system using the Community Earth System Model and the Data Assimilation Research Testbed

Alicia R. Karspeck[1,4] | Gokhan Danabasoglu[1] | Jeffrey Anderson[1] | Svetlana Karol[2] | Nancy Collins[1] | Mariana Vertenstein[1] | Kevin Raeder[1] | Tim Hoar[1] | Richard Neale[1] | Jim Edwards[1] | Anthony Craig[3]

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- When the observations are assimilated serially (e.g., DART, GSI), the dormant subset can change for each observation

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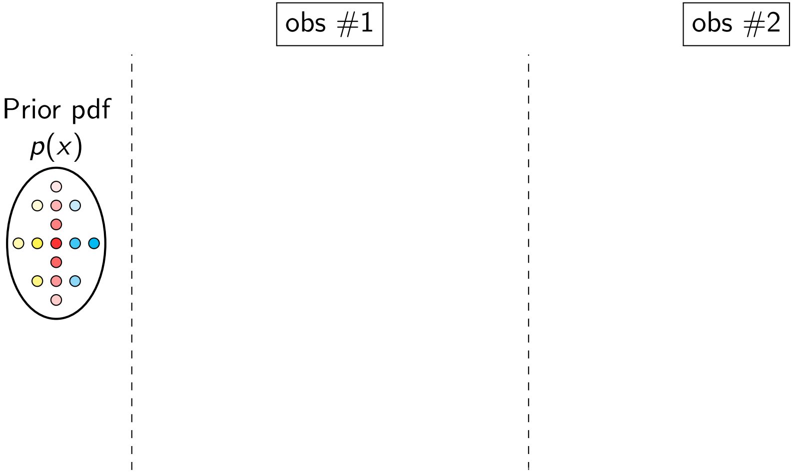

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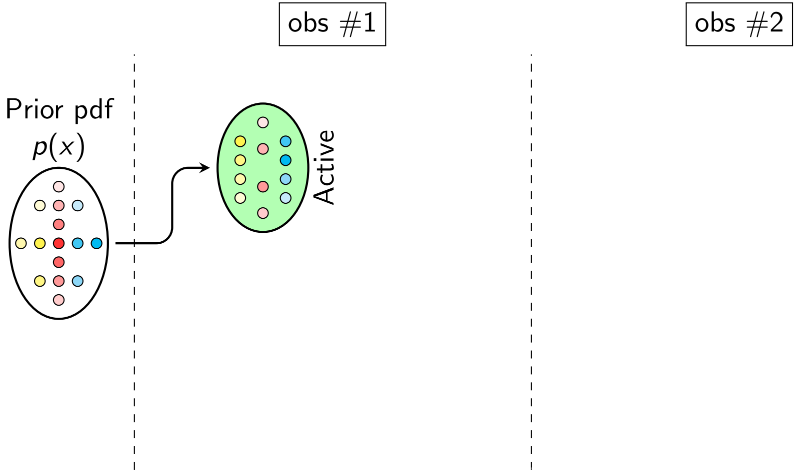

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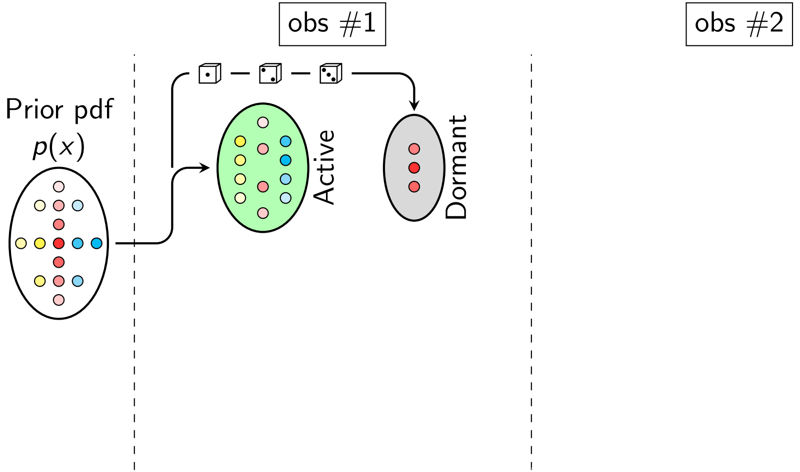

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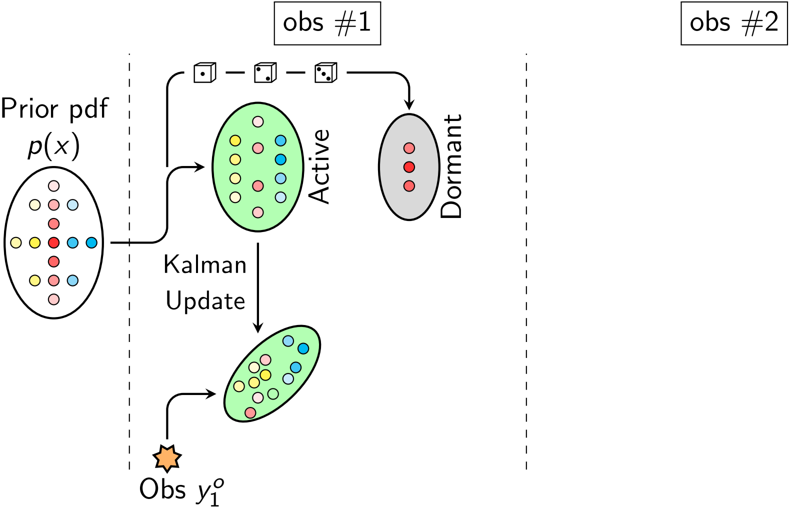

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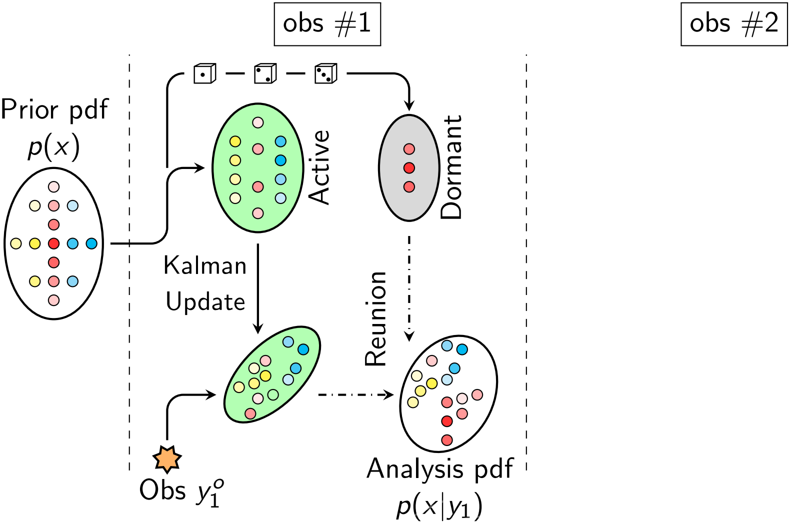

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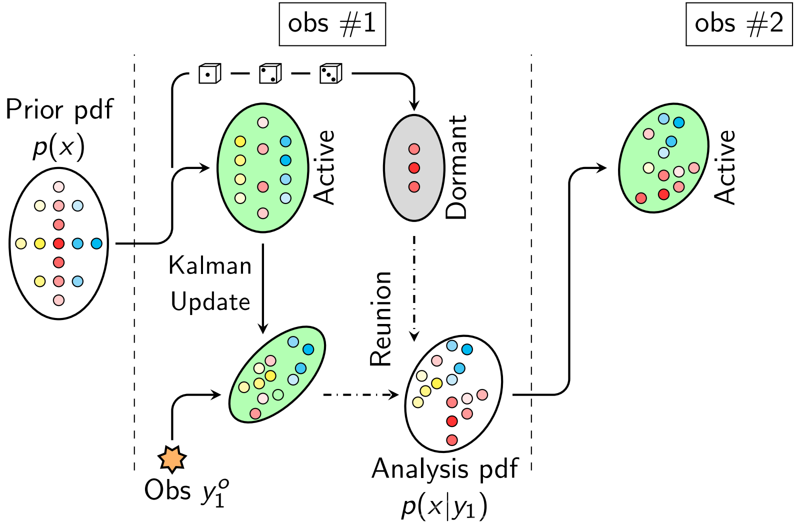

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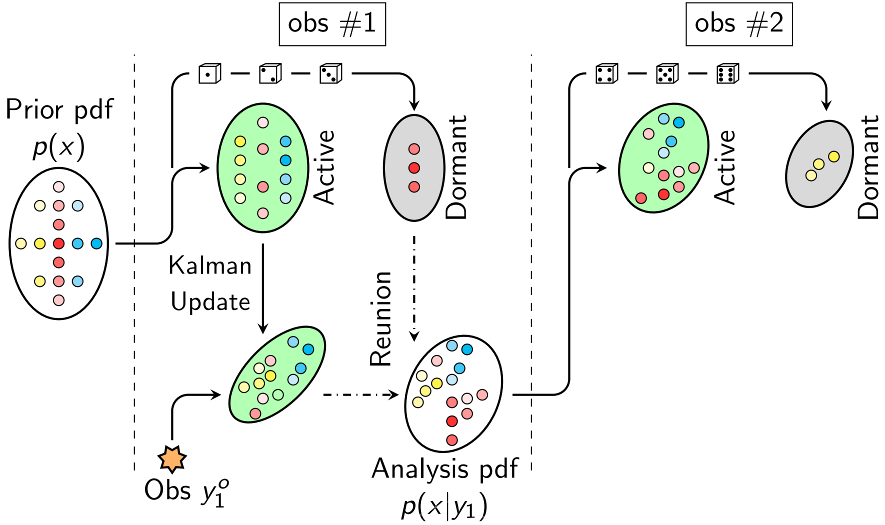

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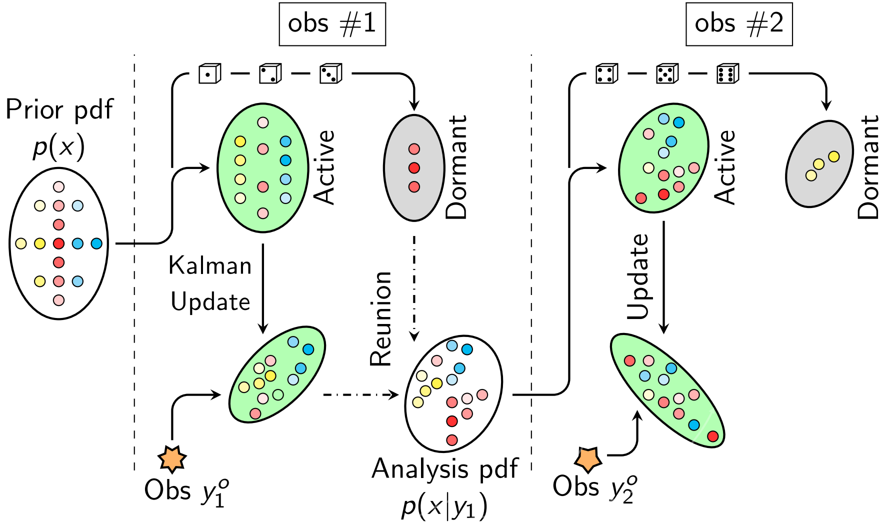

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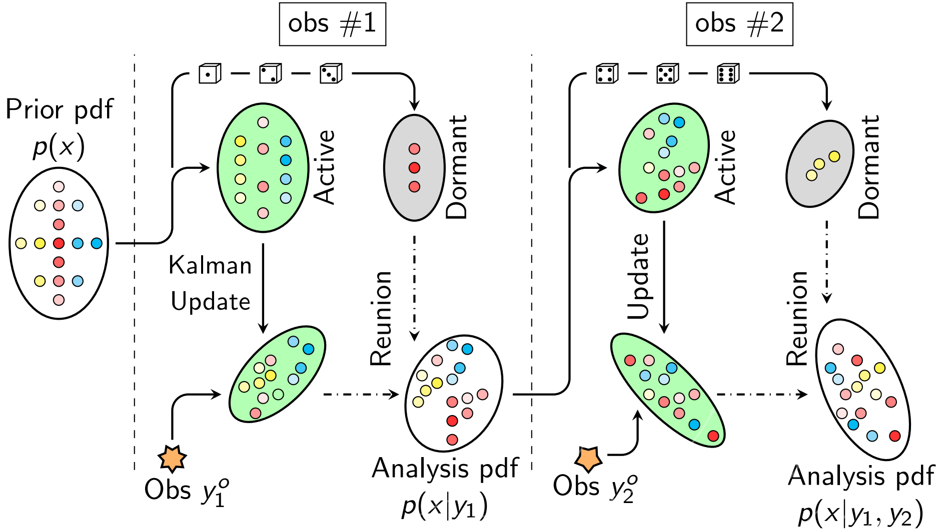
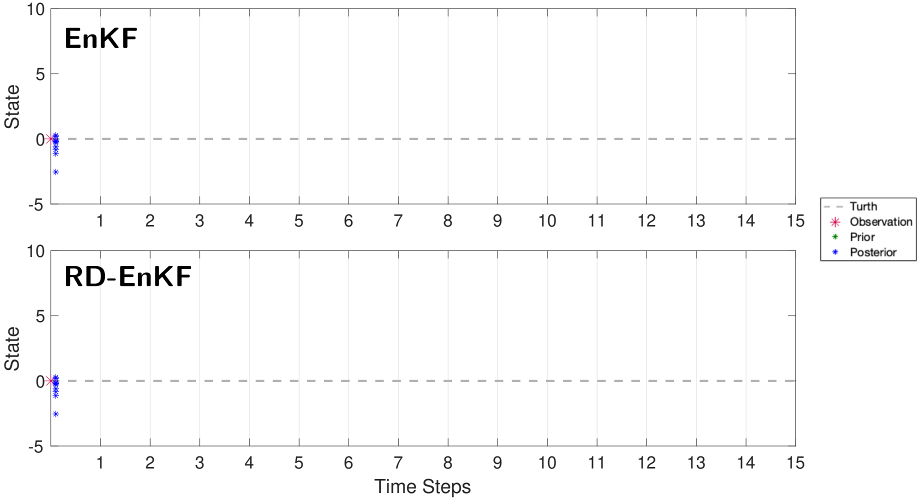


A Scalar Example

10 members, *biased* linear model, **EnKF** vs **RD-EnKF** ($\alpha = 10\%$)



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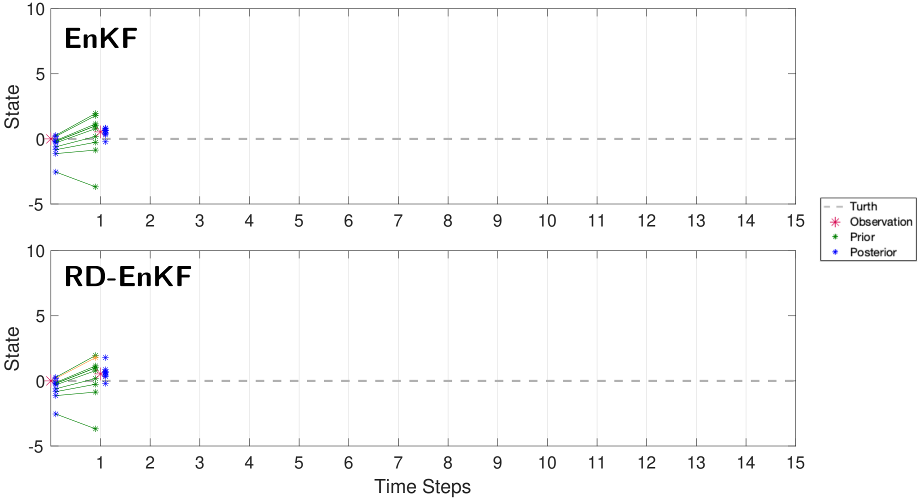

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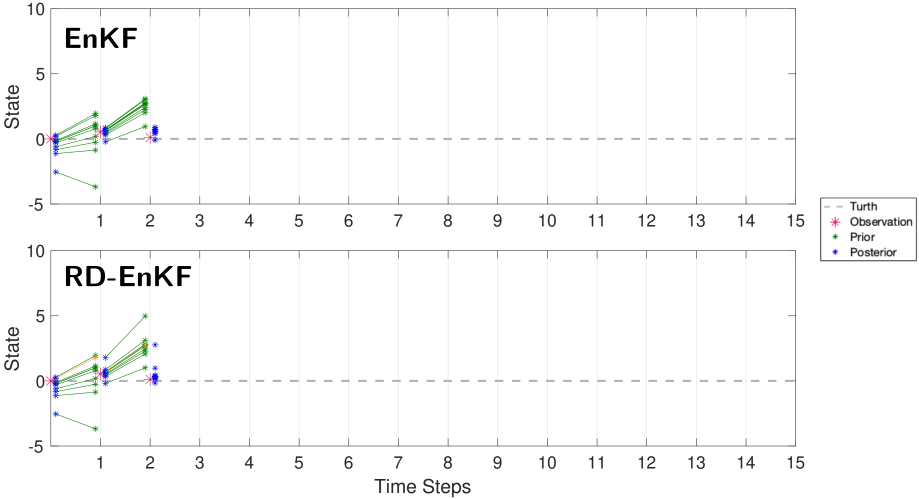

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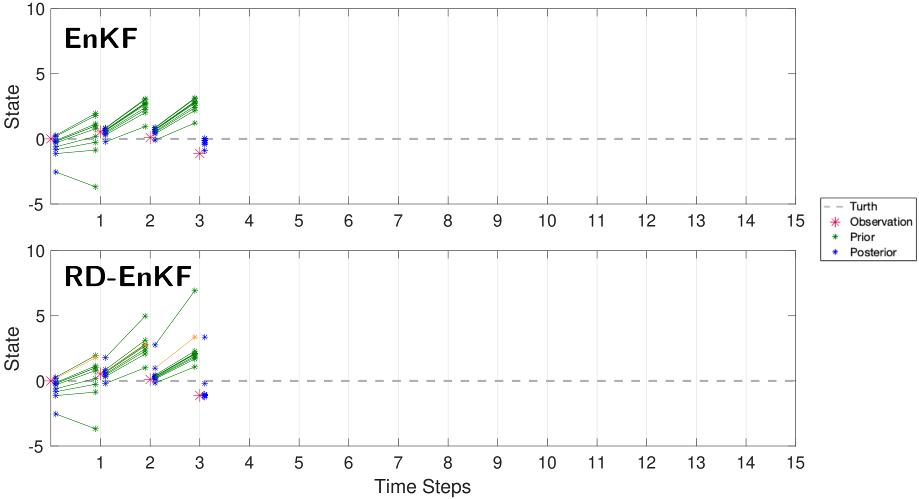

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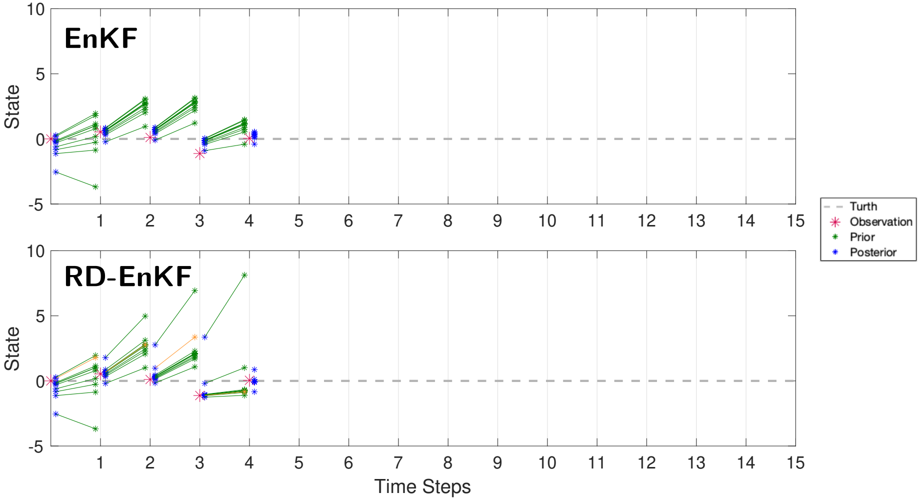

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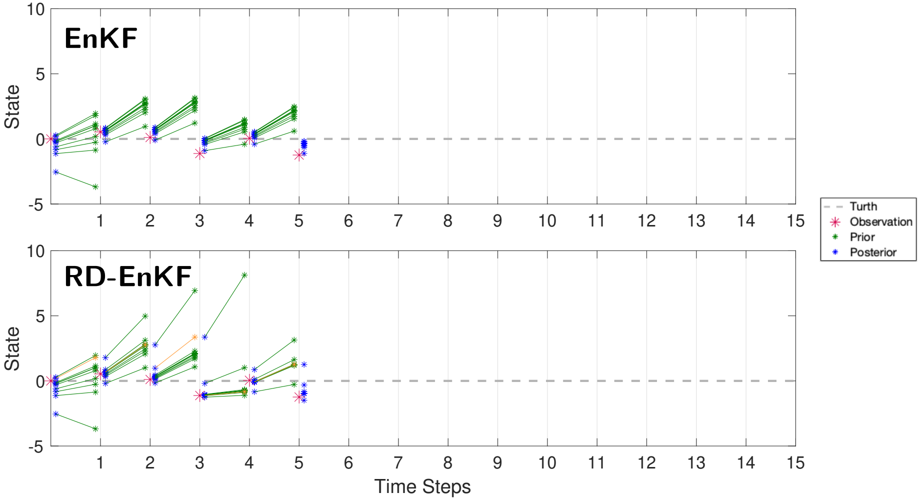

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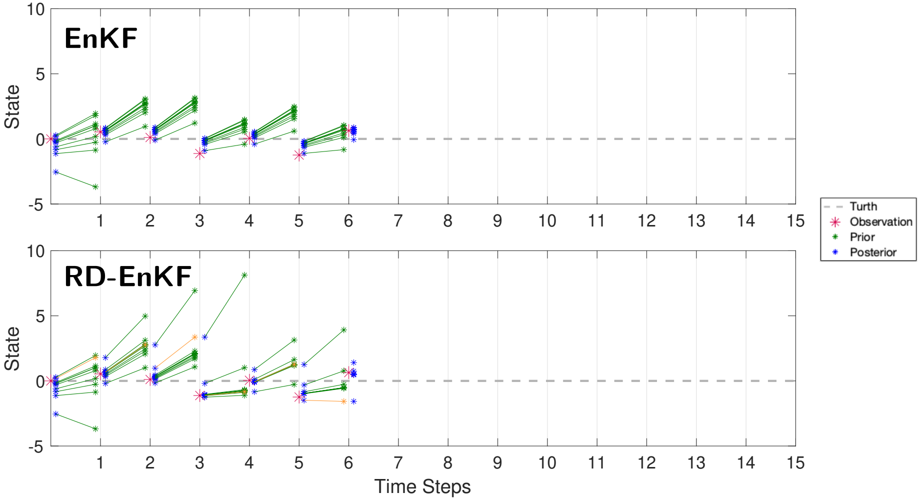

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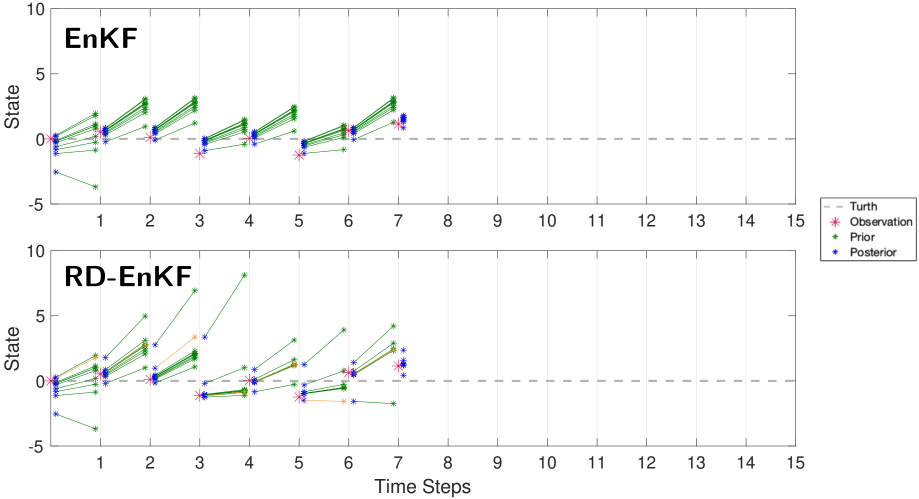

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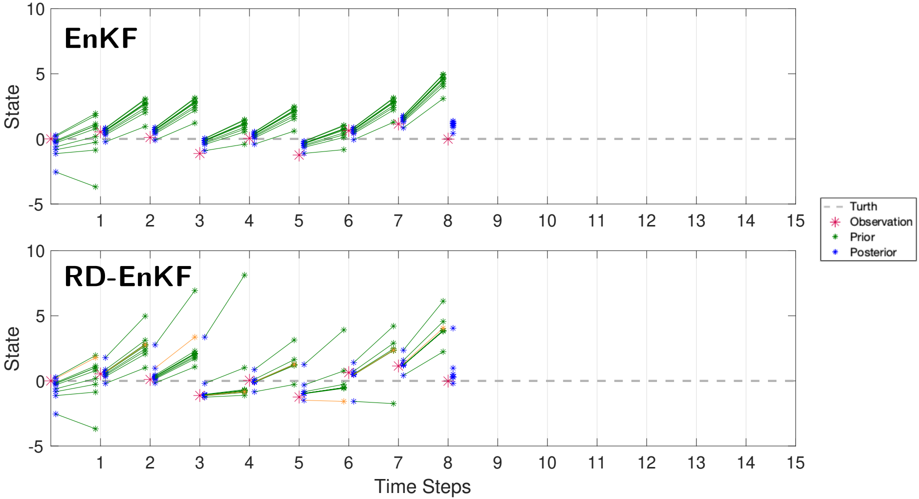

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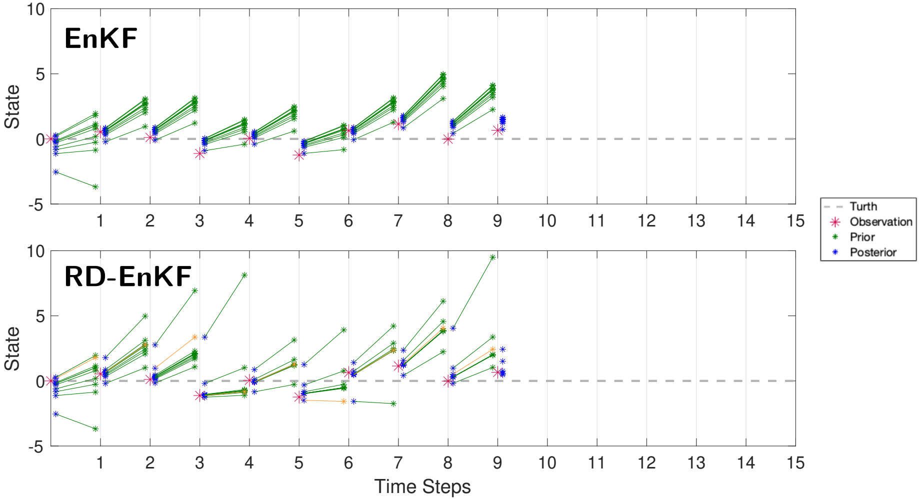

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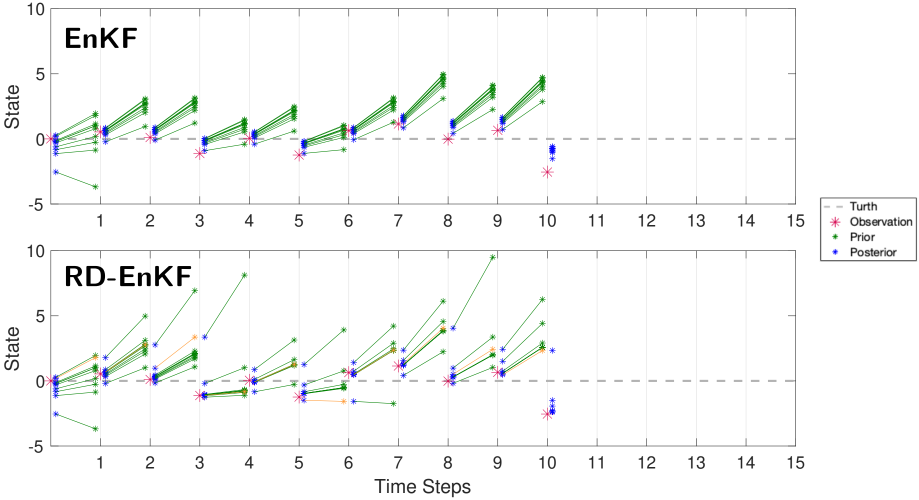

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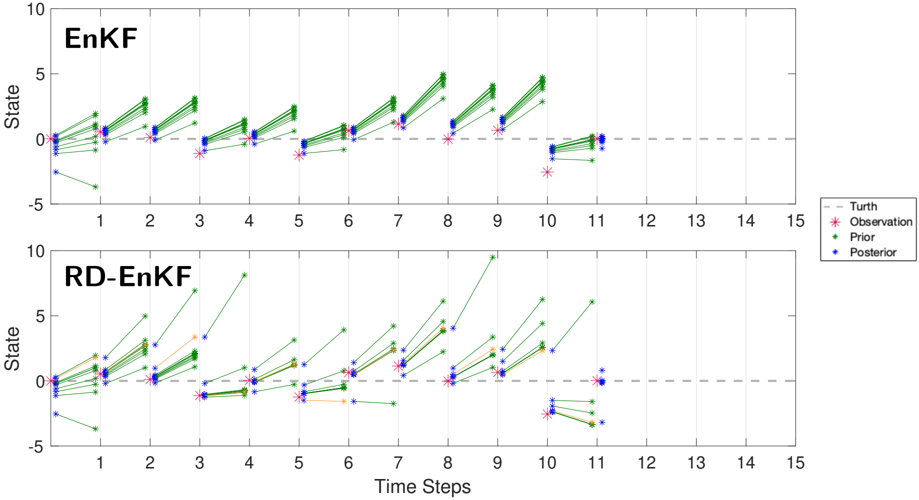

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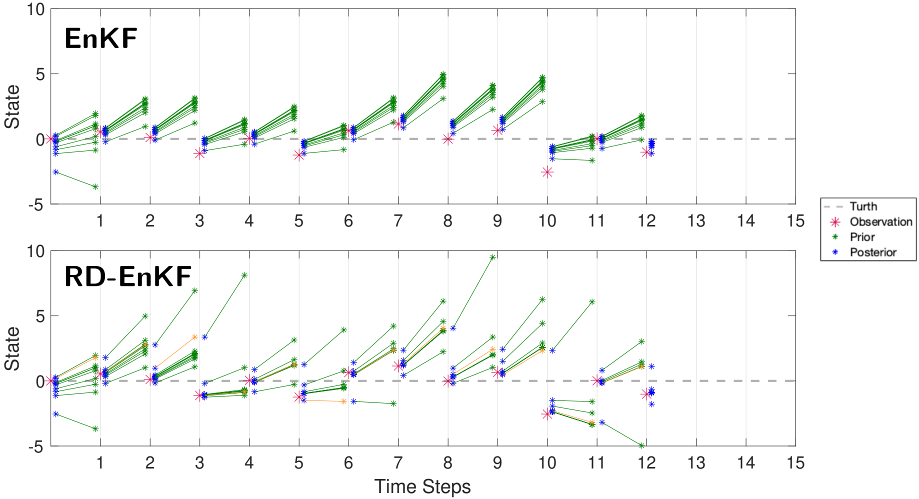

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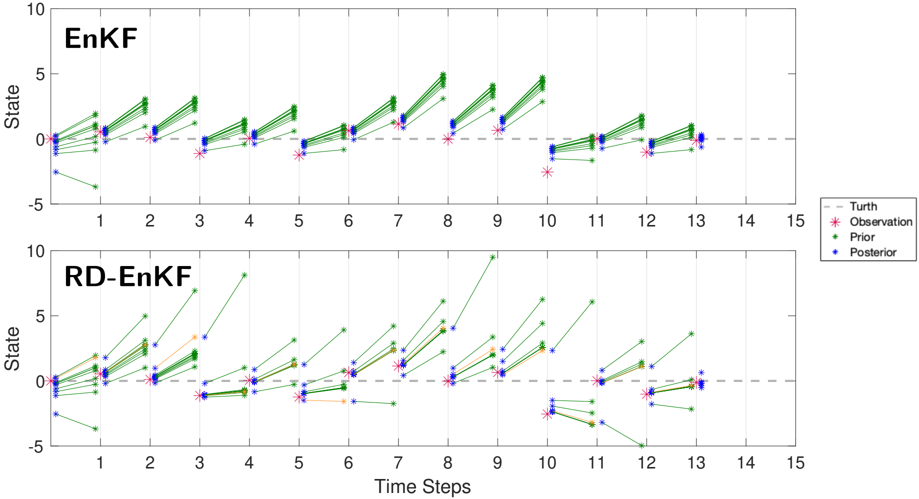

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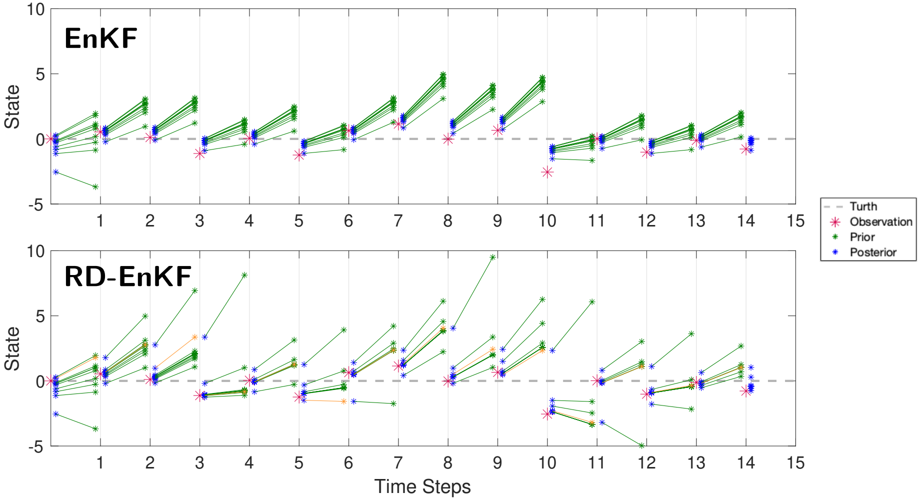

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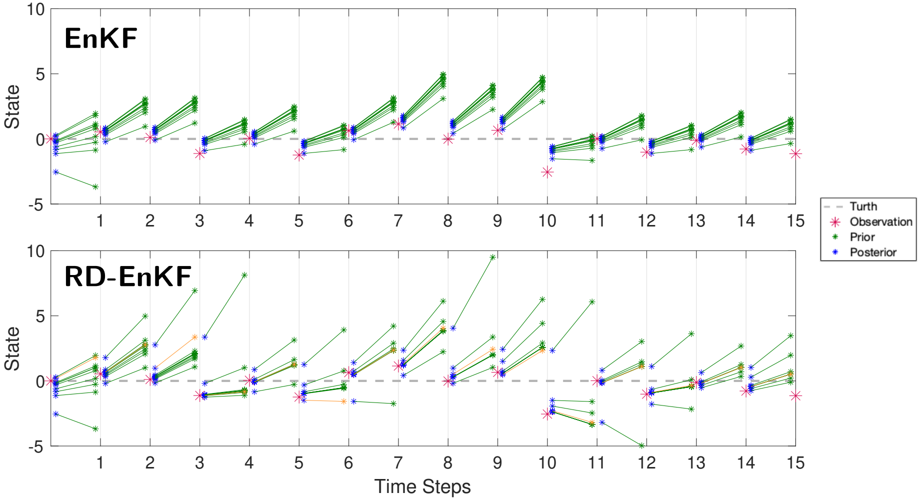

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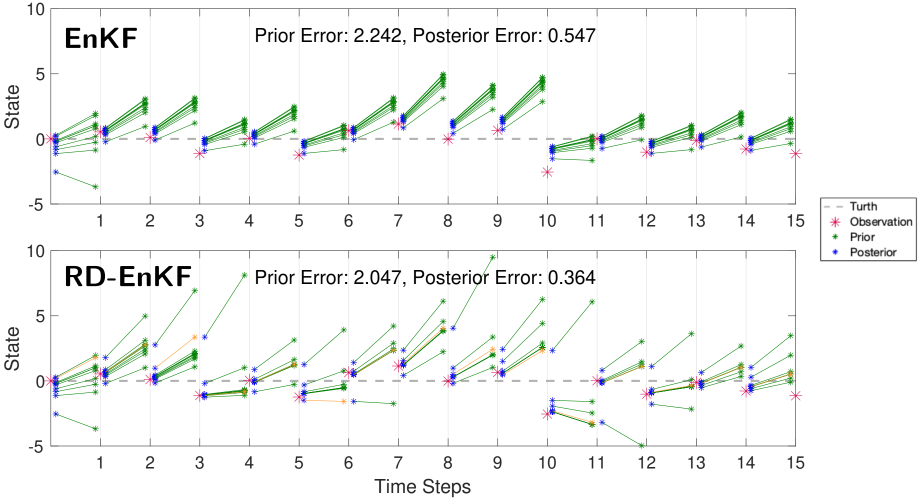

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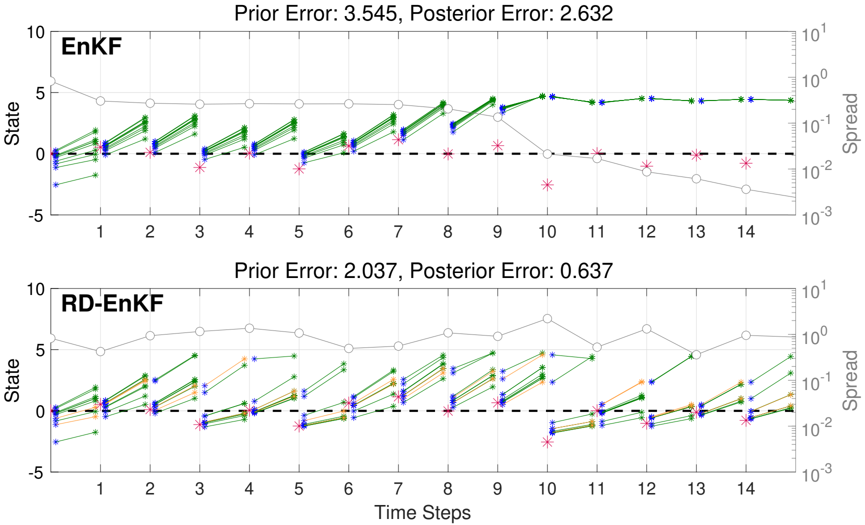
10 members, *biased & nonlinear* model, **EnKF** vs **RD-EnKF** ($\alpha = 20\%$)



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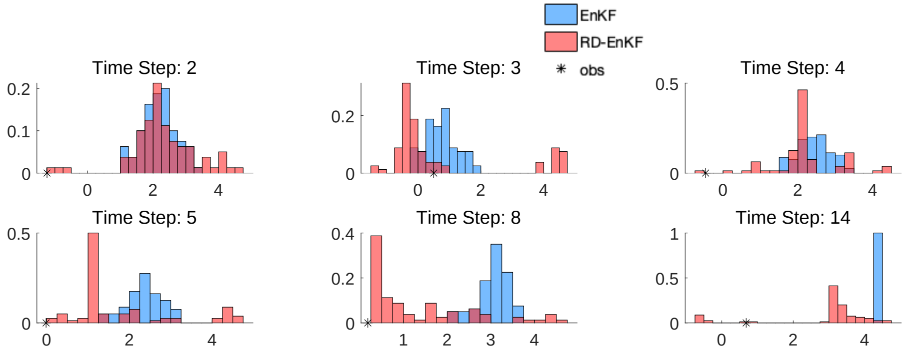
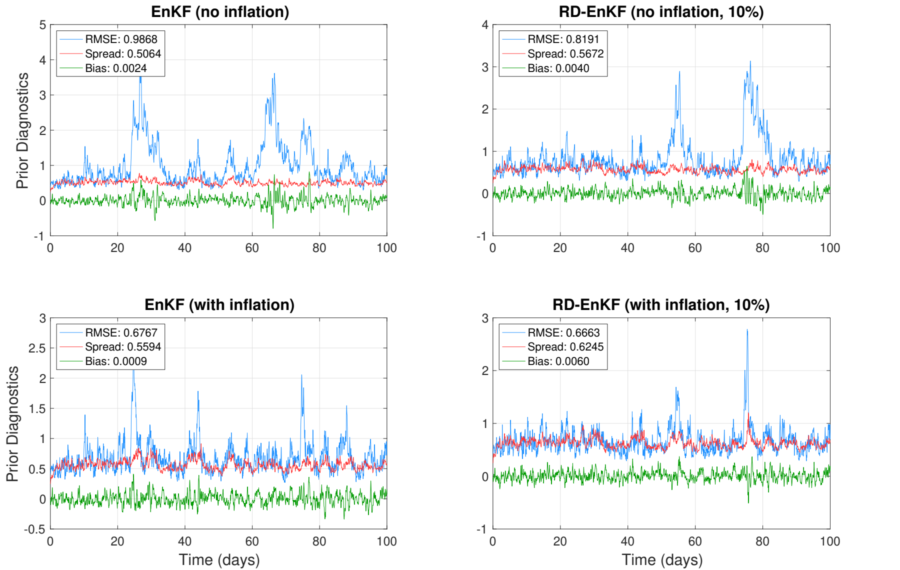


Experiments using Lorenz'96



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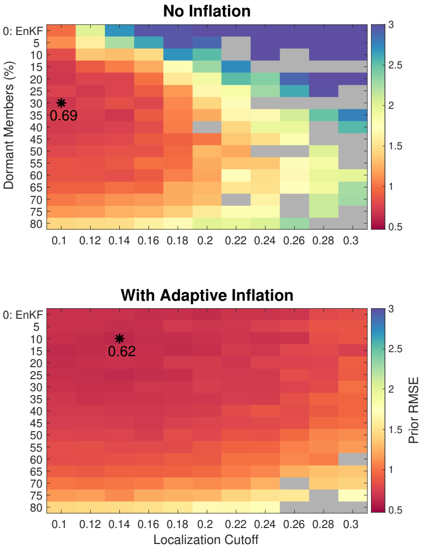
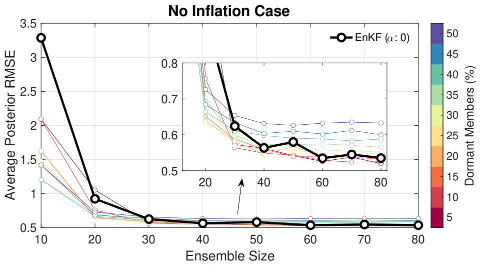
L96: Localization Sensitivity



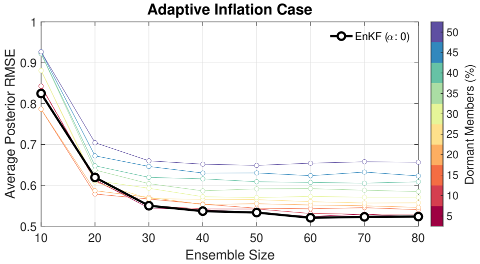
- Without inflation, the most accurate prior estimates are obtained with a `cutoff` of 0.1 radians and α : 30%
- **RD-EnKF** performs well even in poorly localized regimes
- Adaptive prior inflation stabilizes the performance and improves the accuracy
- **RD-EnKF** performance *degrades* as α increases

L96: Ensemble Size Sensitivty

- Without inflation and $N = 10$, **RD-EnKF** outperforms the standard **EnKF** for all tested dormancy rates
- For $N = 80$, **RD-EnKF** still outperforms the **EnKF** using $\alpha \in [5, 10]\%$



- With adaptive inflation, the **RD-EnKF** is only more accurate than the **EnKF** for small ensemble sizes; $N \leq 20$



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L96: Comparison to RTPS

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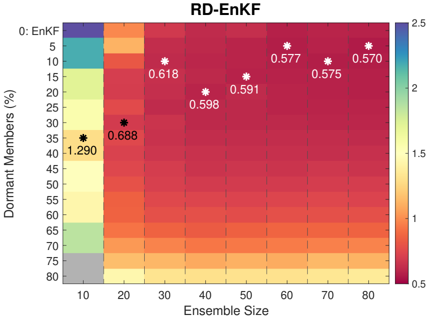

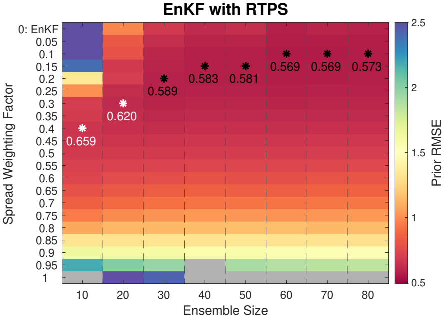

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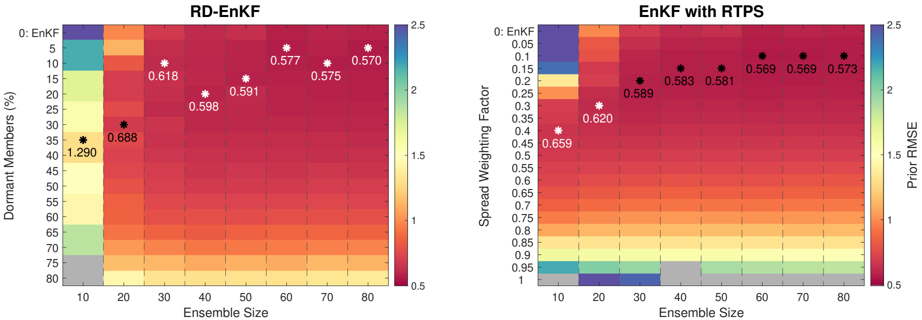
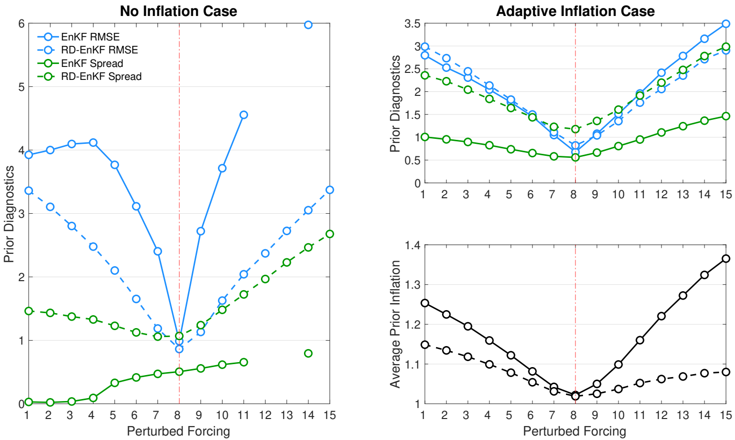


- For small ensemble sizes (i.e., $N \leq 20$), **RD-EnKF** estimates are less accurate than those obtained with **RTPS**
- Ensemble spread retained by the **RD-EnKF** is consistently larger than the **RTPS**

L96: Model Errors



- Without inflation: **RD-EnKF** is *significantly* better than the **EnKF**
- With inflation: Both schemes perform equally well. The **RD-EnKF** uses less inflation but still yields larger ensemble variability

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- Extensive testing: Ocean DA application
- Different ways to randomly (or not) select the dormant members