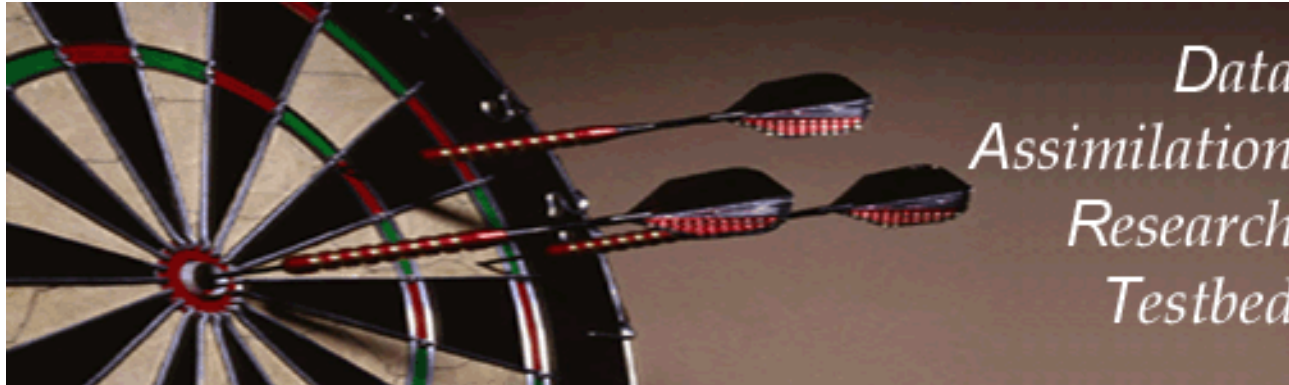


# Data Assimilation Research Testbed Tutorial

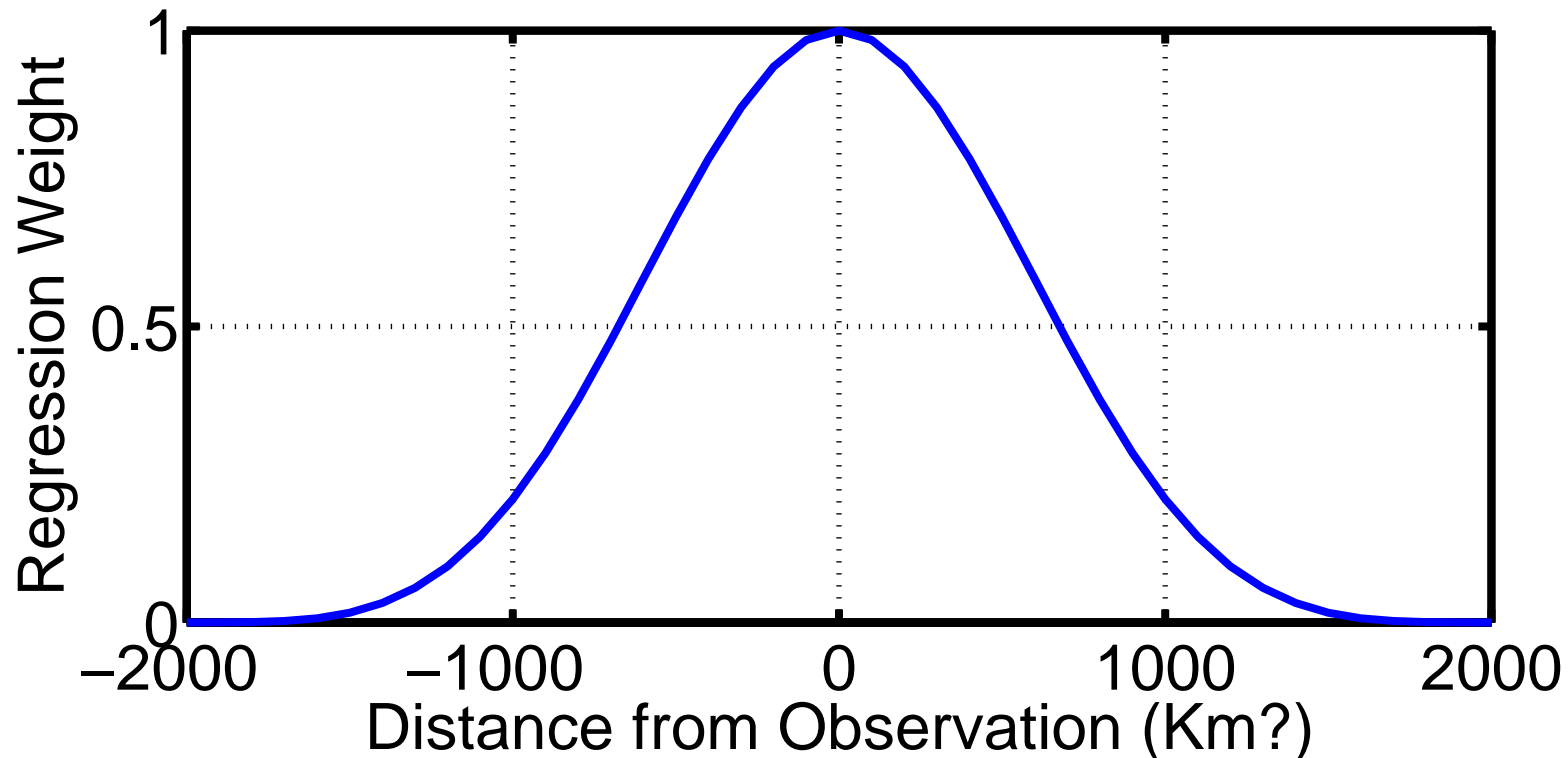


## Section 13: Hierarchical Group Filters and Localization

Version 1.0: June, 2005

## Ways to deal with regression sampling error:

3. Use additional a priori information about relation between observations and state variables.



Can use other functions to weight regression.

Unclear what *distance* means for some obs./state variable pairs.

Referred to as **LOCALIZATION**.

Localization is function of expected correlation between obs and state.

Often, don't know much about this.

Horizontal distance between same type of variable may be okay.

What is expected correlation for co-located temperature and pressure?

What about vertical localization? Looks pretty complex.

What about complicated forward operators:

Expected correlation of satellite radiance and wind component?

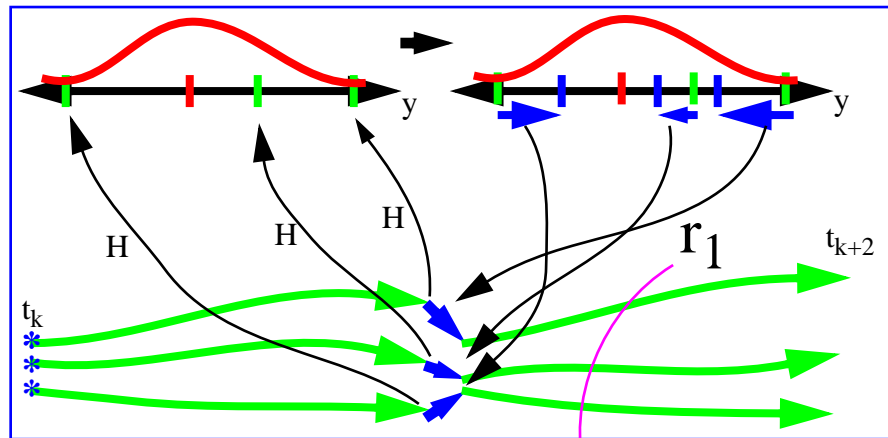
Note: DART does allow vertical localization for more complex models.

## Ways to deal with regression sampling error:

4. Try to determine the amount of sampling error and correct for it:
  - A. Could weight regressions based on sample correlation.  
Limited success in tests.  
For small true correlations, can still get large sample correl.
  - B. Do bootstrap with sample correlation to measure sampling error.  
Limited success.  
Repeatedly compute sample correlation with a sample removed.
  - C. Use hierarchical Monte Carlo.  
Have a 'sample' of samples.  
Compute expected error in regression coefficients and weight.

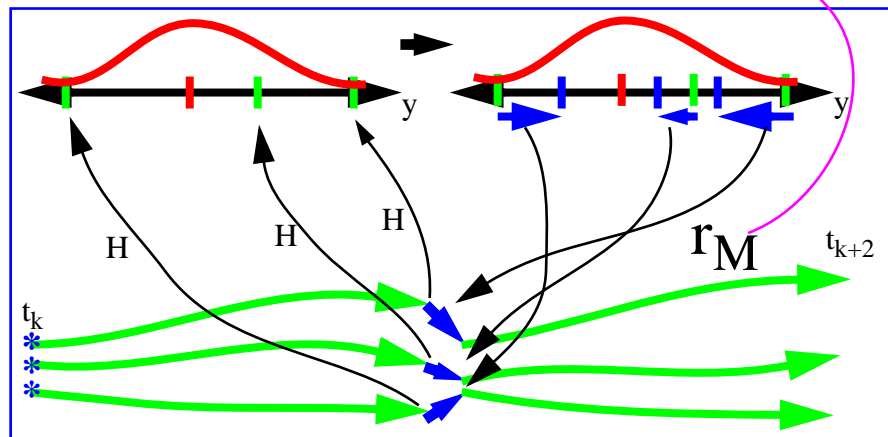
## Ways to deal with regression sampling error:

### 4C. Use hierarchical Monte Carlo: ensemble of ensembles.



M independent  
N-member  
Ensembles

Regression  
Confidence  
Factor,  $\alpha$



M groups of N-member ensembles.

Compute obs. increments for each group.

For given obs. / state pair:

1. Have M samples of regression coefficient,  $\beta$ .
2. Uncertainty in  $\beta$  implies state variable increments should be reduced.
3. Compute regression confidence factor,  $\alpha$ .

## 4C. Use hierarchical Monte Carlo: ensemble of ensembles.

Split ensemble into  $M$  independent groups.

For instance, 80 ensemble members becomes 4 groups of 20.

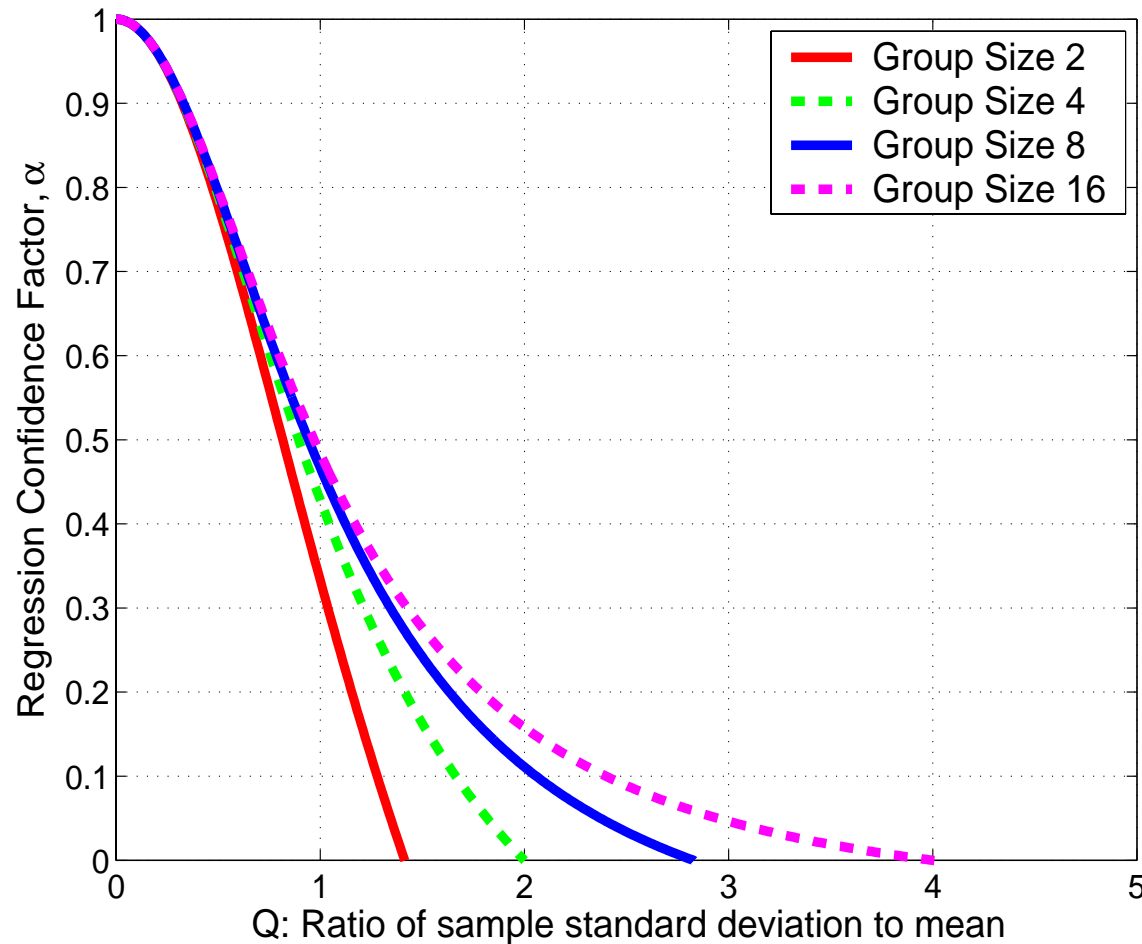
With  $M$  groups get  $M$  estimates of regression coefficient,  $\beta_i$ .

Find regression confidence factor  $\alpha$  (weight) that minimizes:

$$\sqrt{\sum_{j=1}^M \sum_{i=1, i \neq j}^M [\alpha \beta_i - \beta_j]^2}$$

Minimizes RMS error in the regression (and state increments).

## 4C. Use hierarchical Monte Carlo: ensemble of ensembles.



Weight regression by  $\alpha$ .

If one has repeated observations, can generate sample mean or median statistics for  $\alpha$ .

Mean  $\alpha$  can be used in subsequent assimilations as a localization.

$\alpha$  is function of M and  $Q = \Sigma_{\beta} / \bar{\beta}$  (sample SD / sample mean regression)

## 4C. Use hierarchical Monte Carlo: ensemble of ensembles.

Hierarchical filter controlled by setting number of groups,  $M$ .  
*num\_groups* in *filter\_nml*.

If we don't know how to localize to start with, can use groups to help.

Try splitting 80 ensemble members into 4 groups for Lorenz-96.  
(4 groups of 20 each).

Use adaptive inflation (0.05 lower bounds) to make things nice.

Can look at the time mean and median value of  $\alpha$ .

Essentially an estimate of a 'good' localization for a given observation.



## 4C. Use hierarchical Monte Carlo: ensemble of ensembles.

After running the 80 by 4 ‘group’ filter, look at plots of  $\alpha$ .

Use *plot\_reg\_factor* in matlab.

Select default input file name.

Only observations 1, 2, 3, and 4 are available:

Located at: 0.39, 0.17, 0.64, 0.86

Think about value of time median vs. time mean.

Could use time mean or median as prior localization functions.

Play around with model error again. What happens to localization?

## Lorenz 96 Experimental Design

Initial ensemble members random draws from ‘climatology’

Observations every time step

4000 step assimilations, results shown from second 2000 steps

Covariance inflation tuned for minimum RMS

4 groups of ensembles used unless otherwise noted

### EXPERIMENT SET 1:

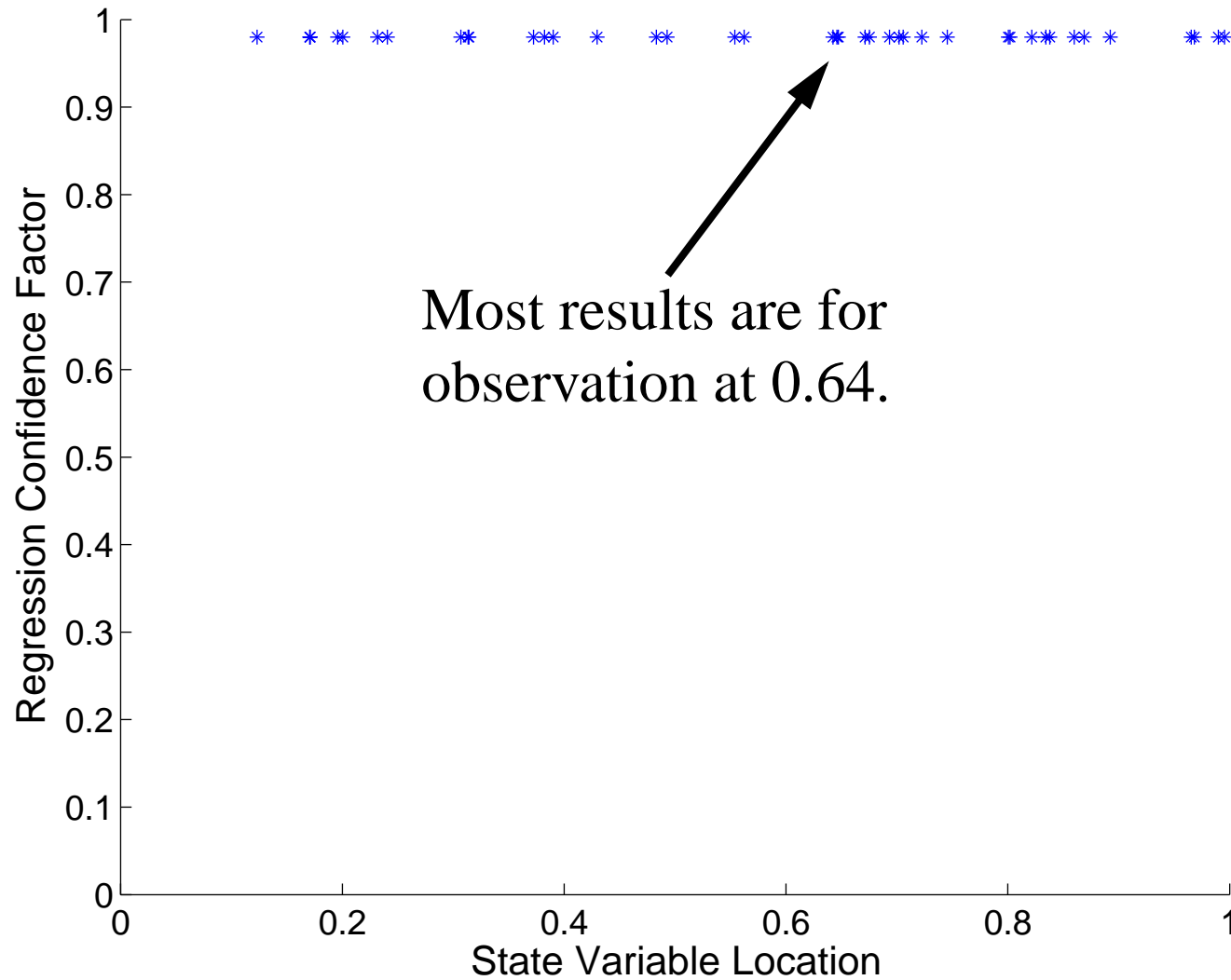
40 Randomly located observations

Error variance  $10^{-7}$  (SMALL!)

‘ERROR’ comes almost entirely from degeneracy of ensemble covariance

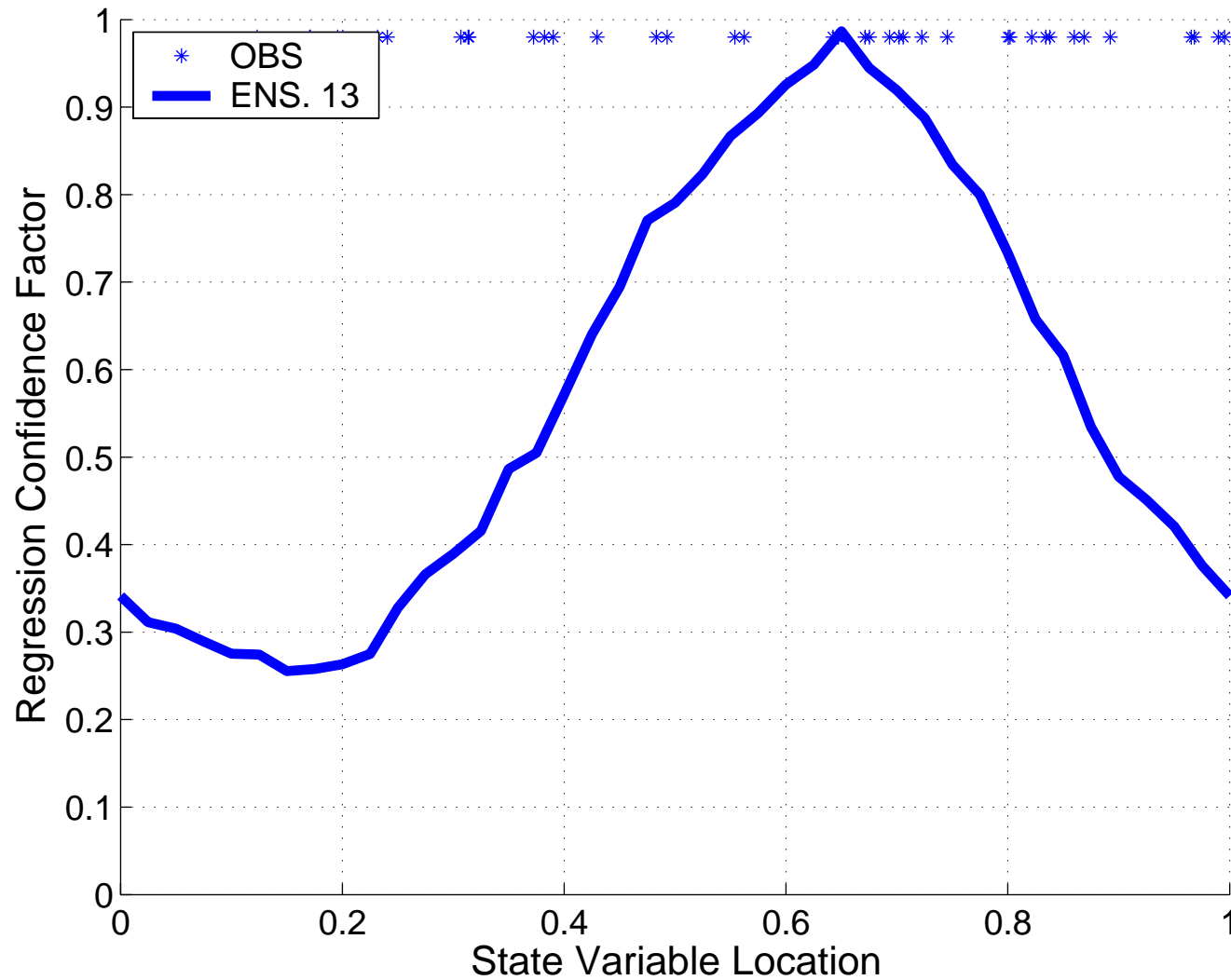
All errors shown are prior ensemble mean estimates

# Time Mean Regression Confidence Envelopes: Small Error Limit



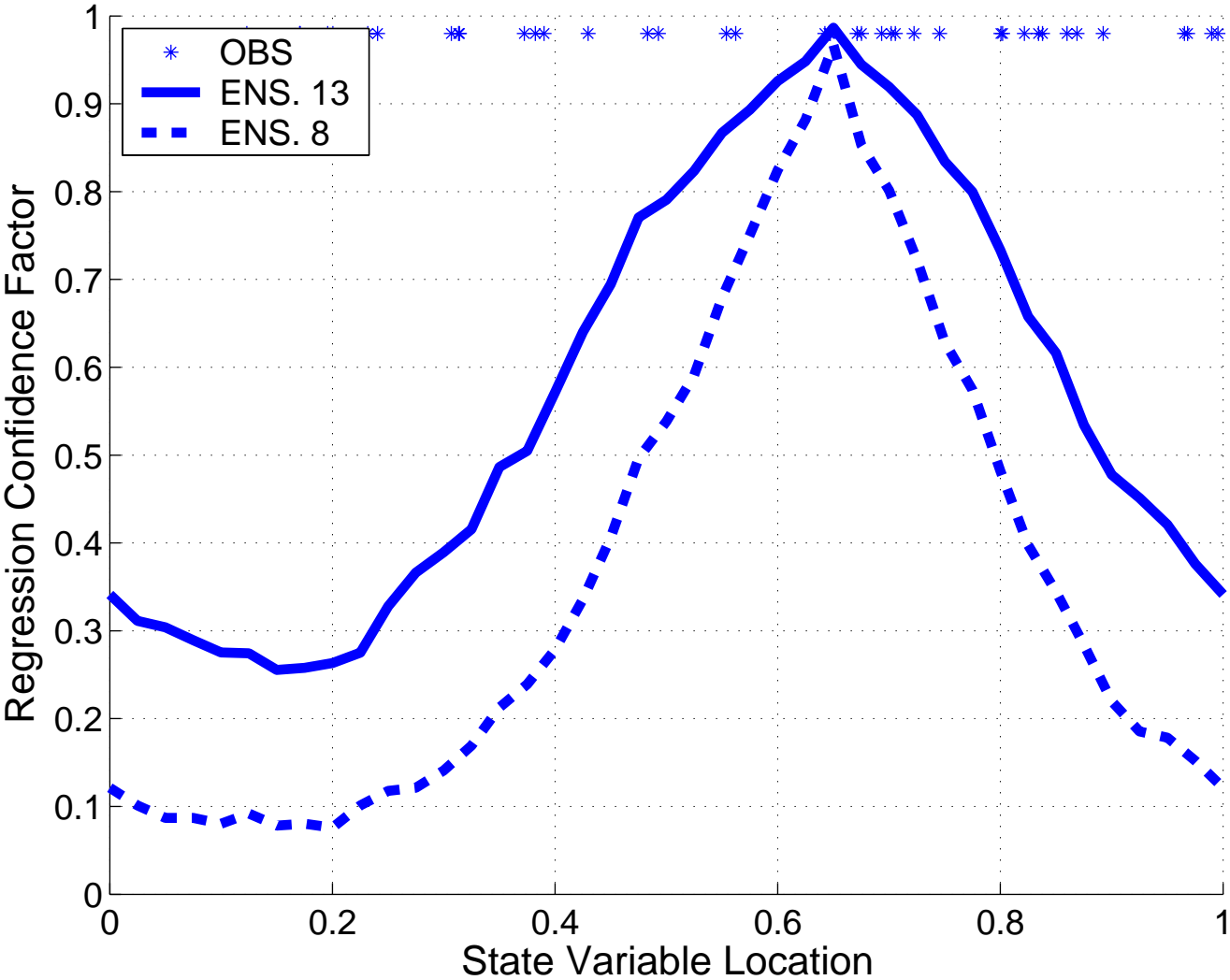
Location of 40 randomly located observations

# Time Mean Regression Confidence Envelopes: Small Error Limit



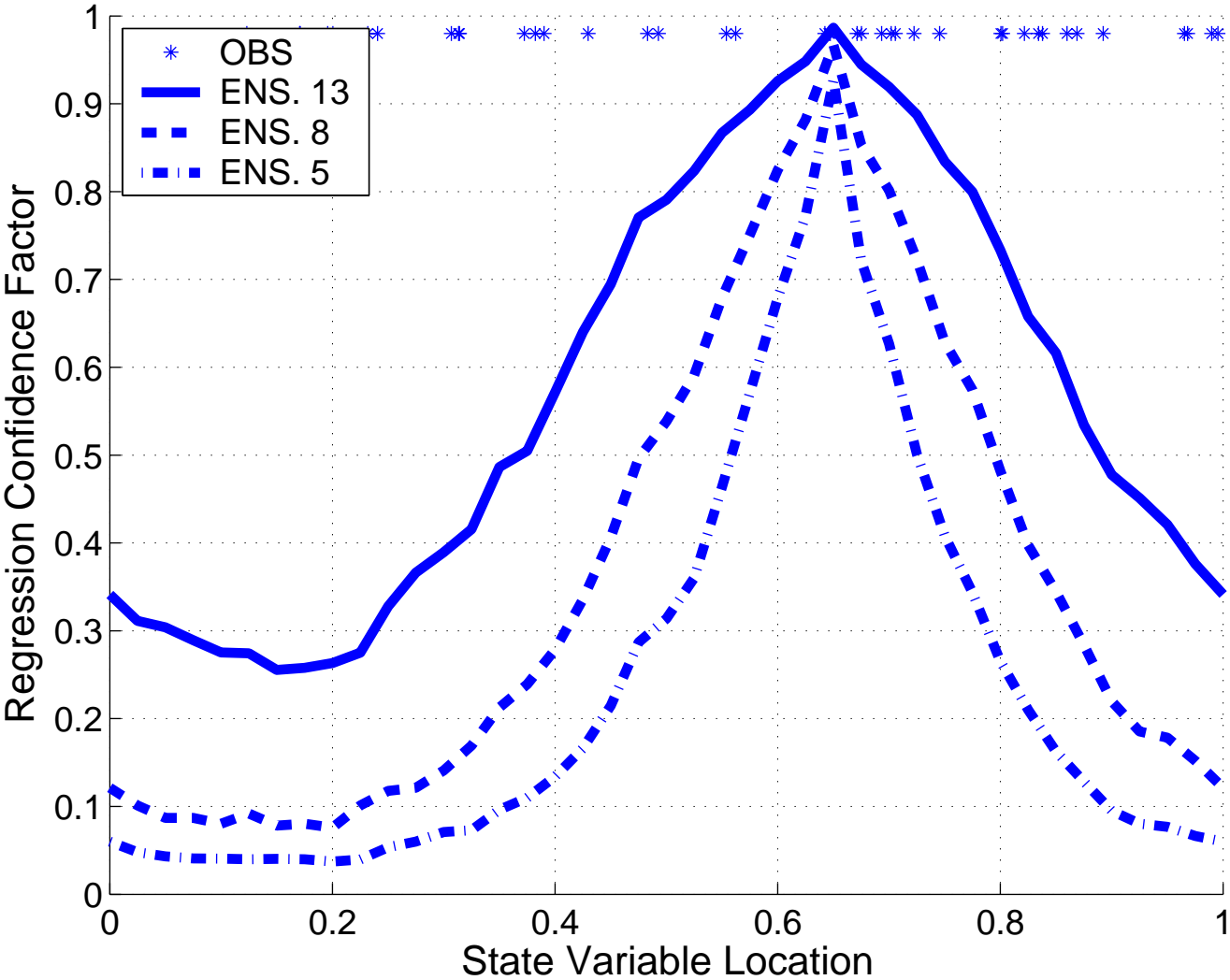
Envelope (localization) for 13 member ensemble (barely degenerate)

# Time Mean Regression Confidence Envelopes: Small Error Limit



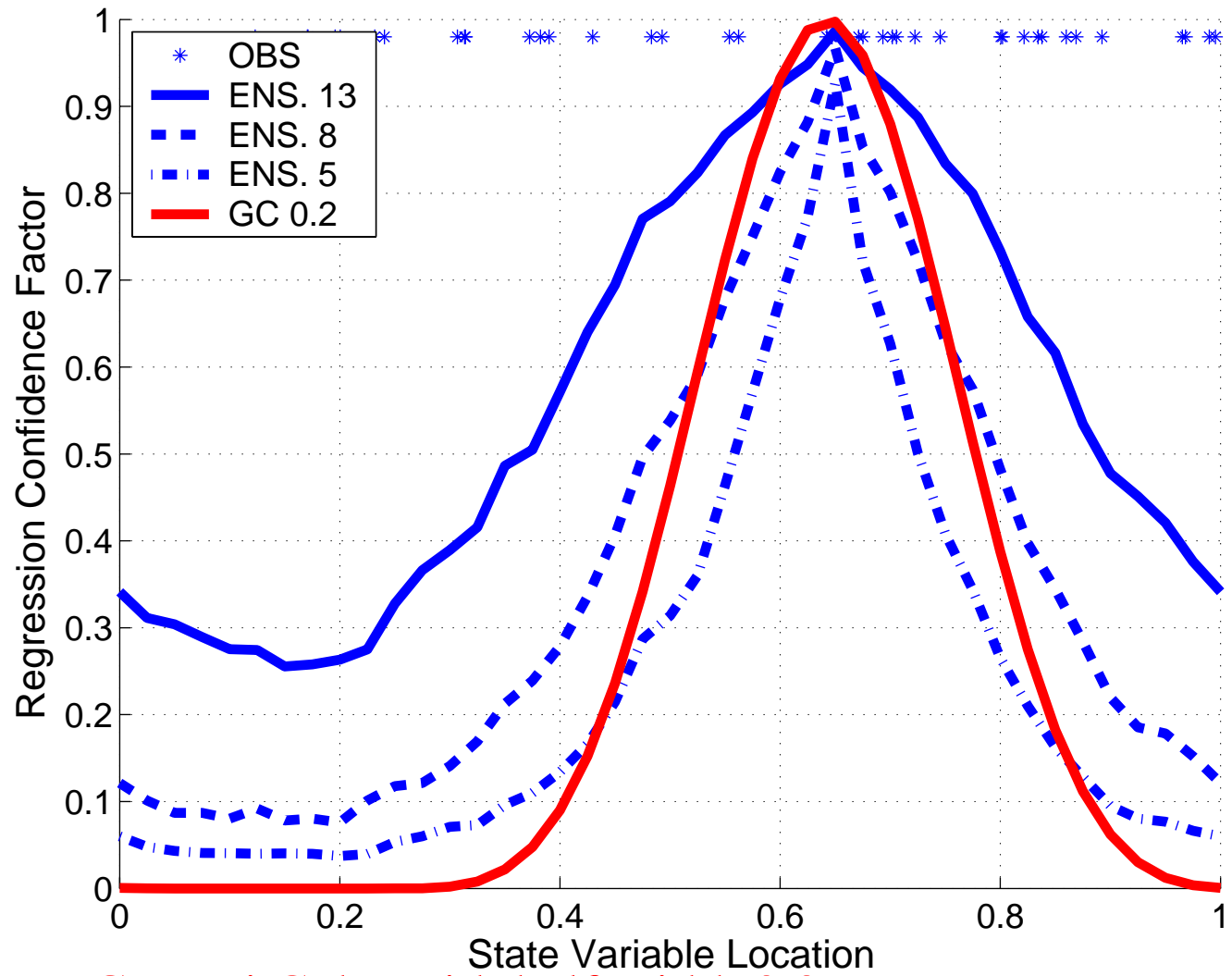
Envelope for 8 member ensemble

# Time Mean Regression Confidence Envelopes: Small Error Limit



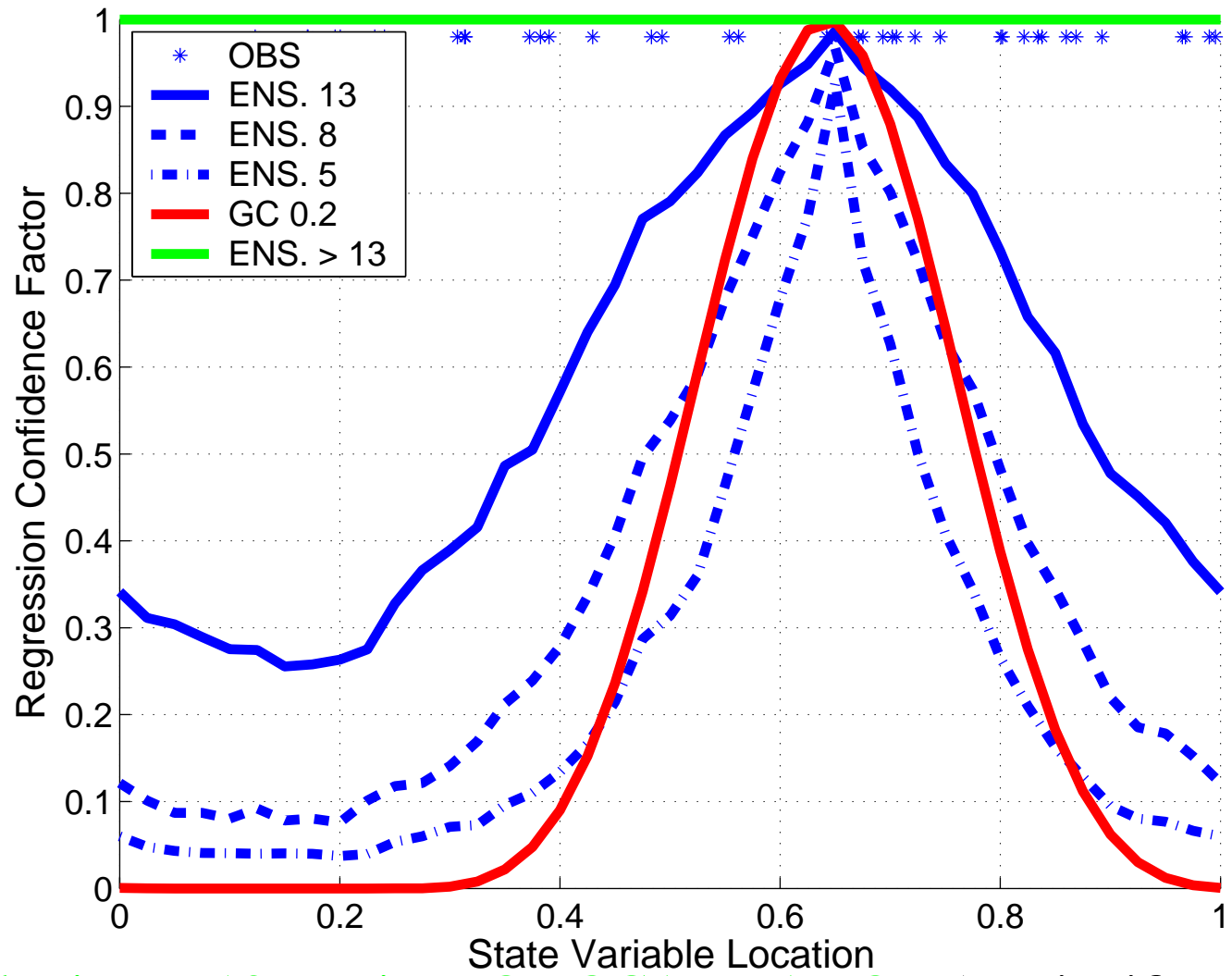
Envelope for 5 member ensemble

# Time Mean Regression Confidence Envelopes: Small Error Limit



Compare to **Gaspari Cohn with half-width 0.2**

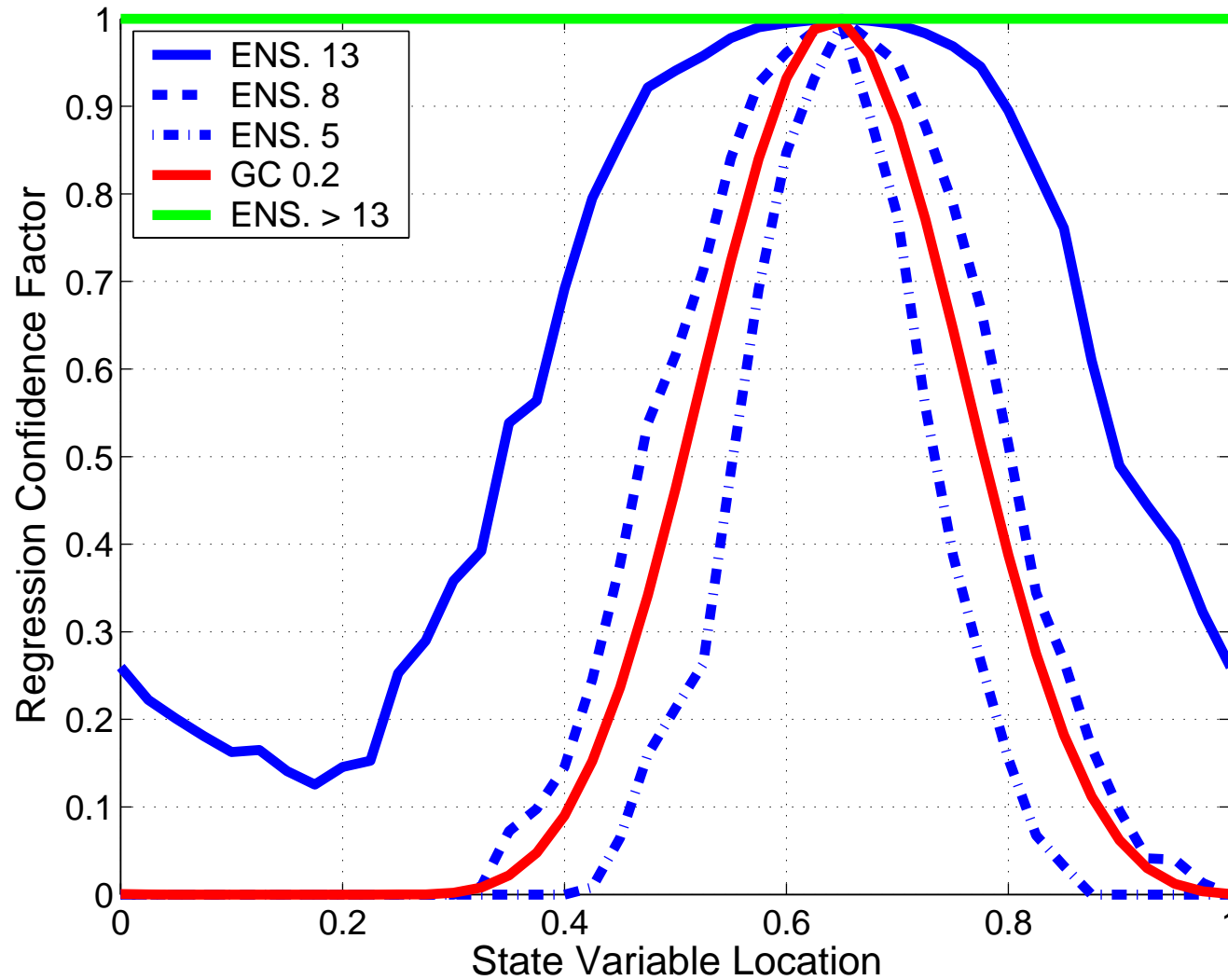
# Time Mean Regression Confidence Envelopes: Small Error Limit



Ensemble sizes > 13 require **NO LOCALIZATION** (no significant error)



# Time Median Regression Confidence Envelopes: Small Error Limit



Median reduces noise for small expected correlations

## Additional Experiments: Experimental Design

1. Base case: plain ensemble, optimized Gaspari Cohn localization half-width
2. Time mean case: plain ensemble but with localization using time mean regression confidence envelope from group assimilation
3. Time median case: as above but with time median from group

All Start from group 1 ensemble at time step 2000

Covariance inflation tuned independently for each case

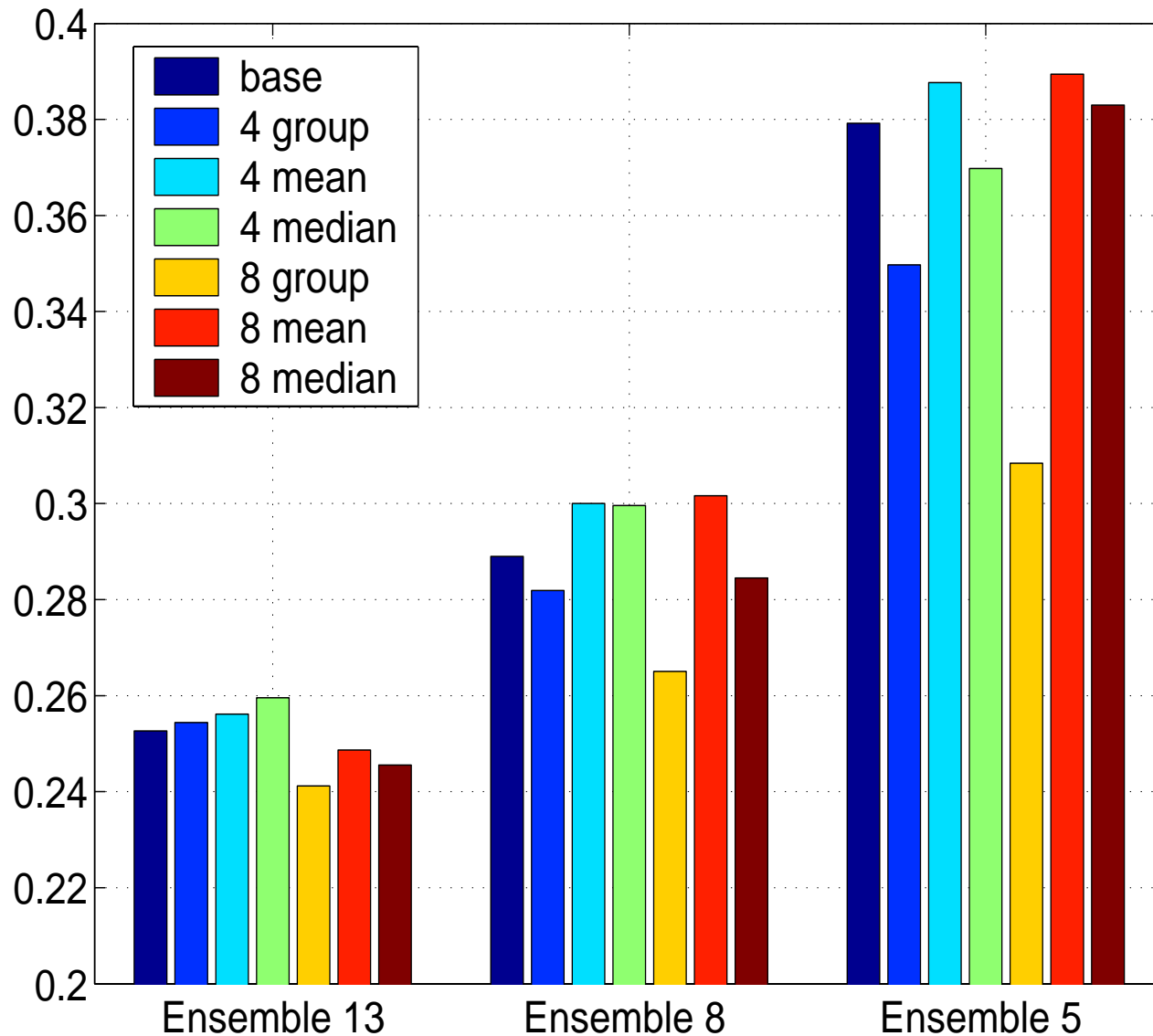
# Time Mean global mean RMS Error: Small Error Results

Error grows with reduced ensemble size

8 Group filters always best

Time mean and median close to tuned base case

Mean/median cost same as base case



## Experimental Design: Varying Observational Error Variance

Observation set as before

Observation error variance  $10^{-5}$ ,  $10^{-3}$ , 0.1, 1.0, 10.0,  $10^7$

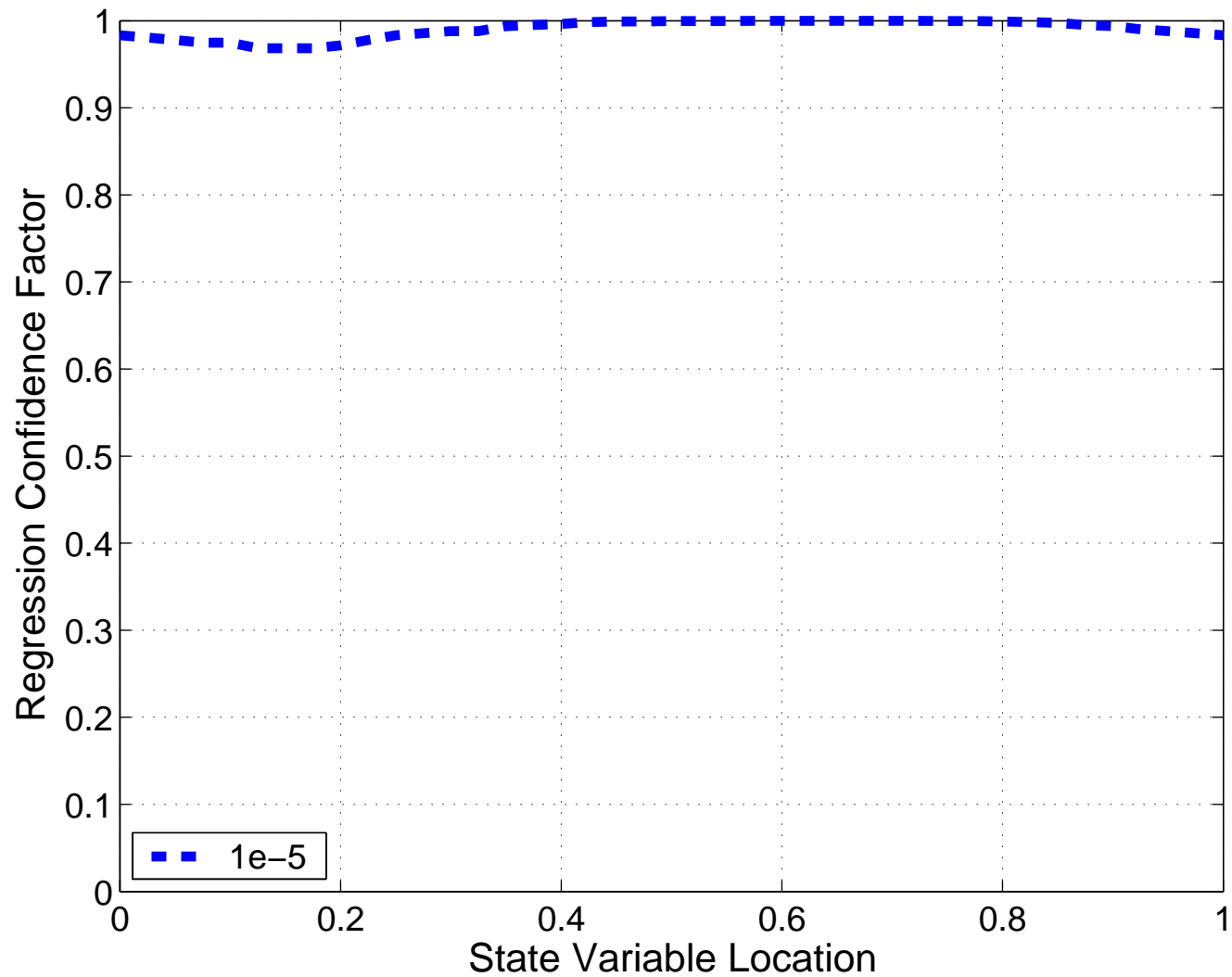
14 member ensembles; not degenerate

Error source is now from observation limitations, etc.

Claim: behavior is similar, error source is irrelevant for correction

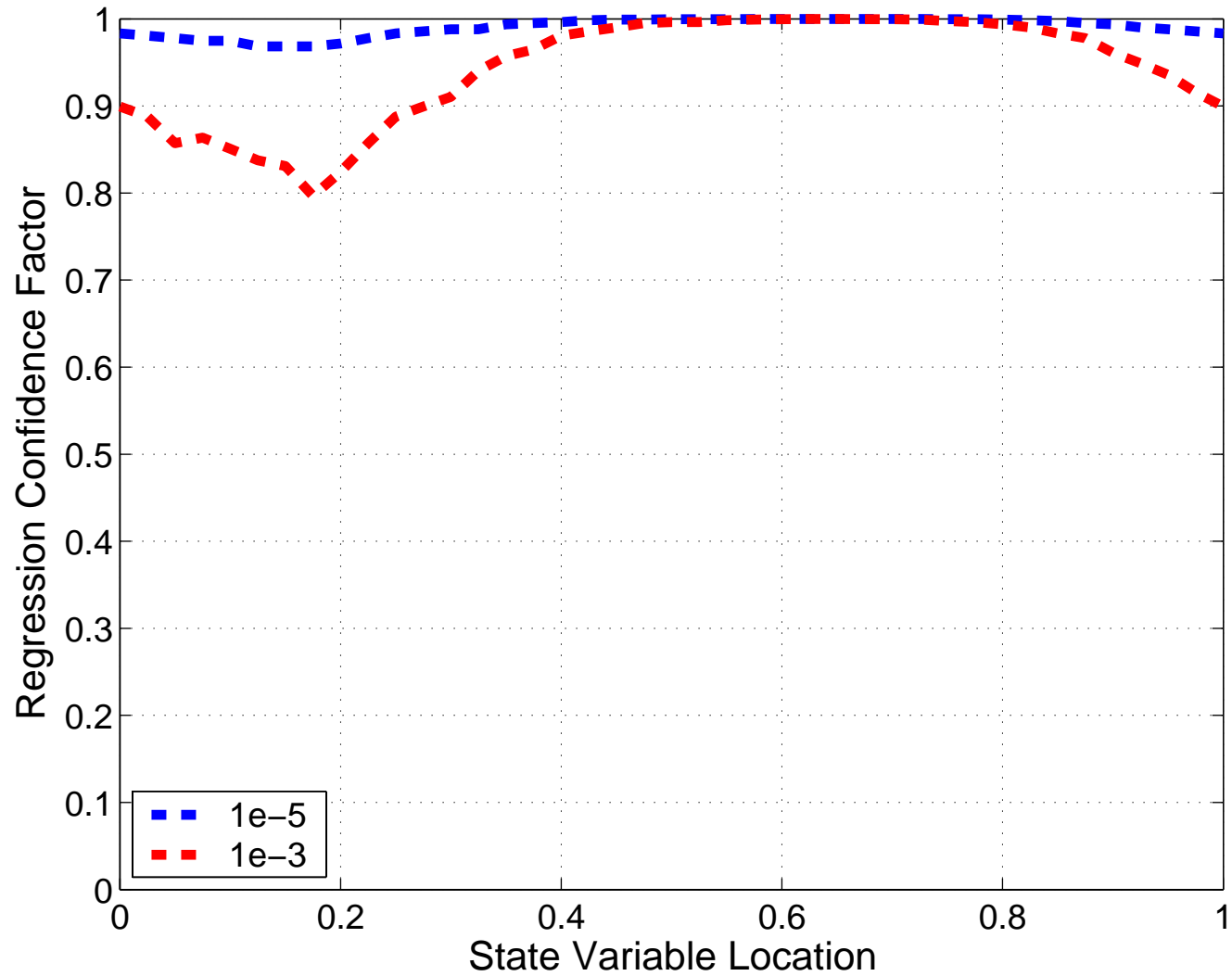
Understand degeneracy and other error sources as sampling error

## Time Median Envelopes: Varying Obs. Error Variance

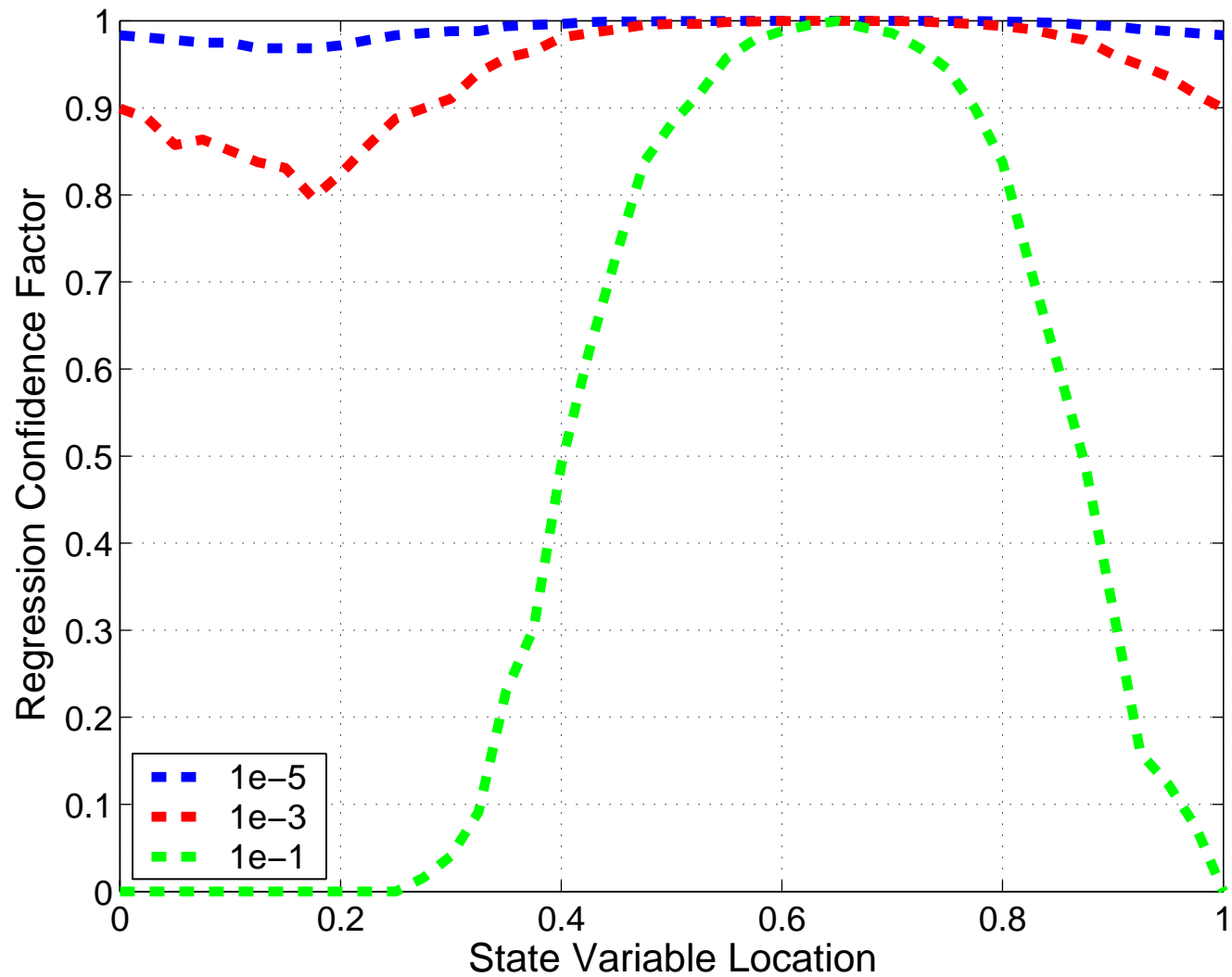


Small error implies no need for localization

# Time Median Envelopes: Varying Obs. Error Variance

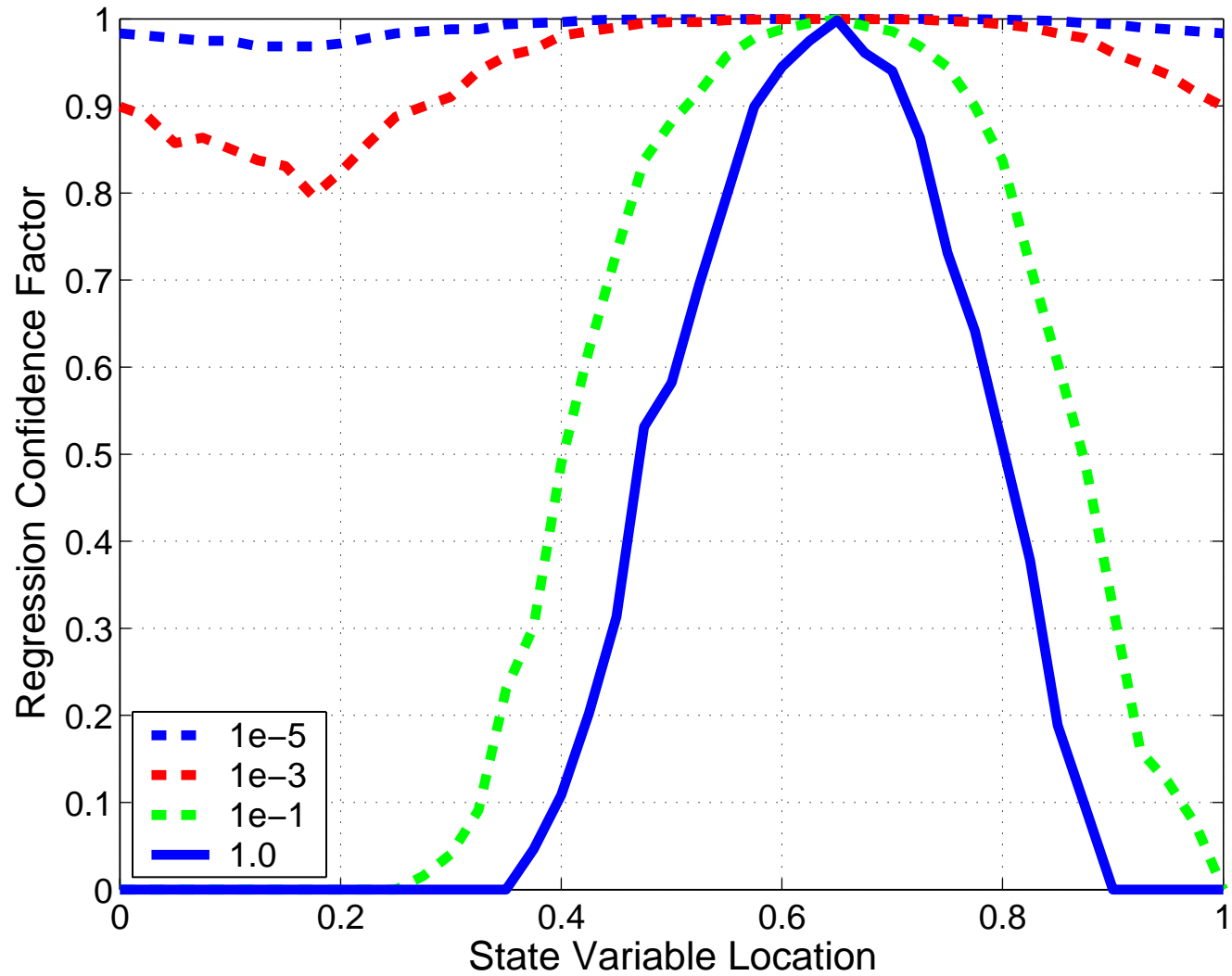


## Time Median Envelopes: Varying Obs. Error Variance



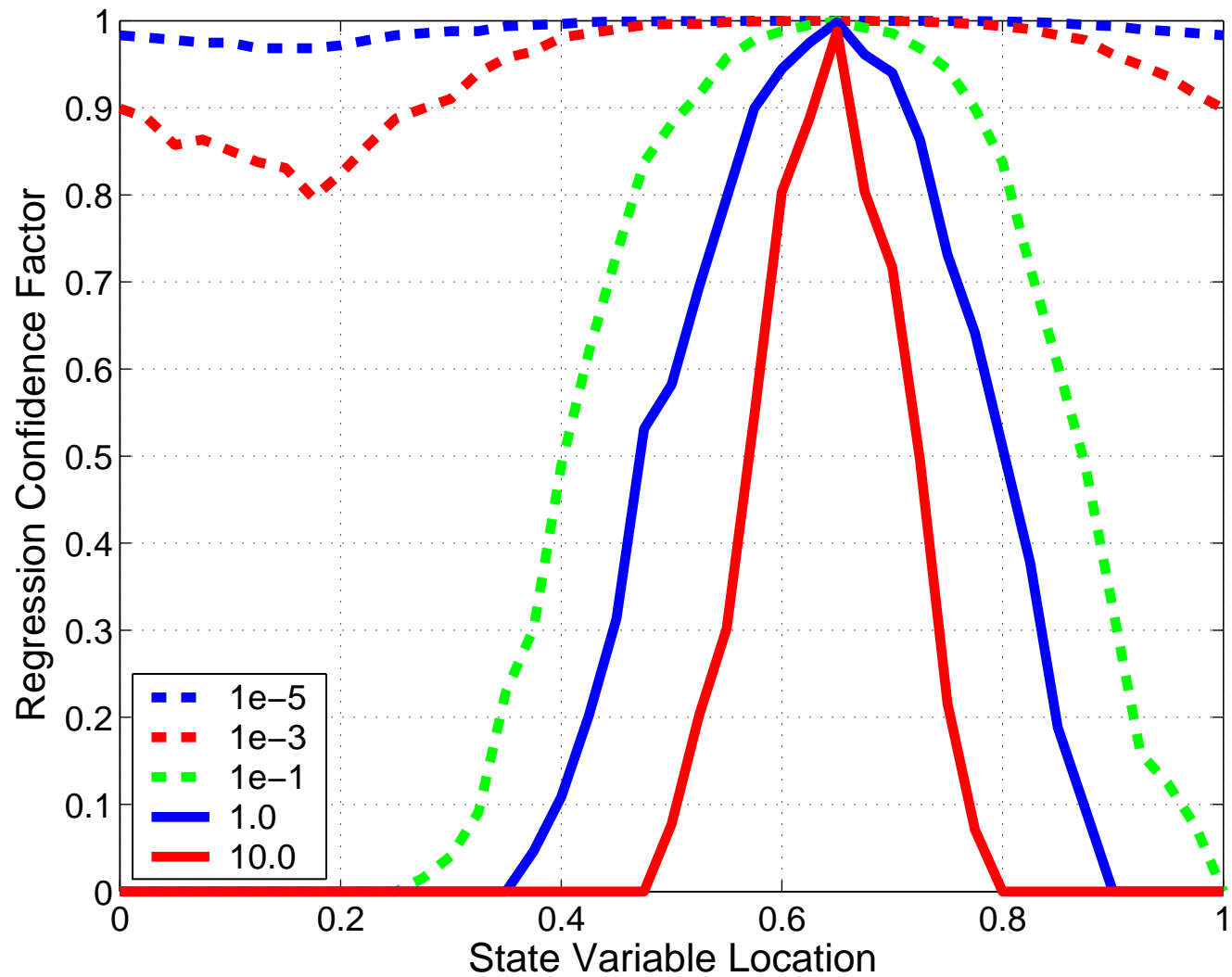
Increasing error implies increasing localization

# Time Median Envelopes: Varying Obs. Error Variance



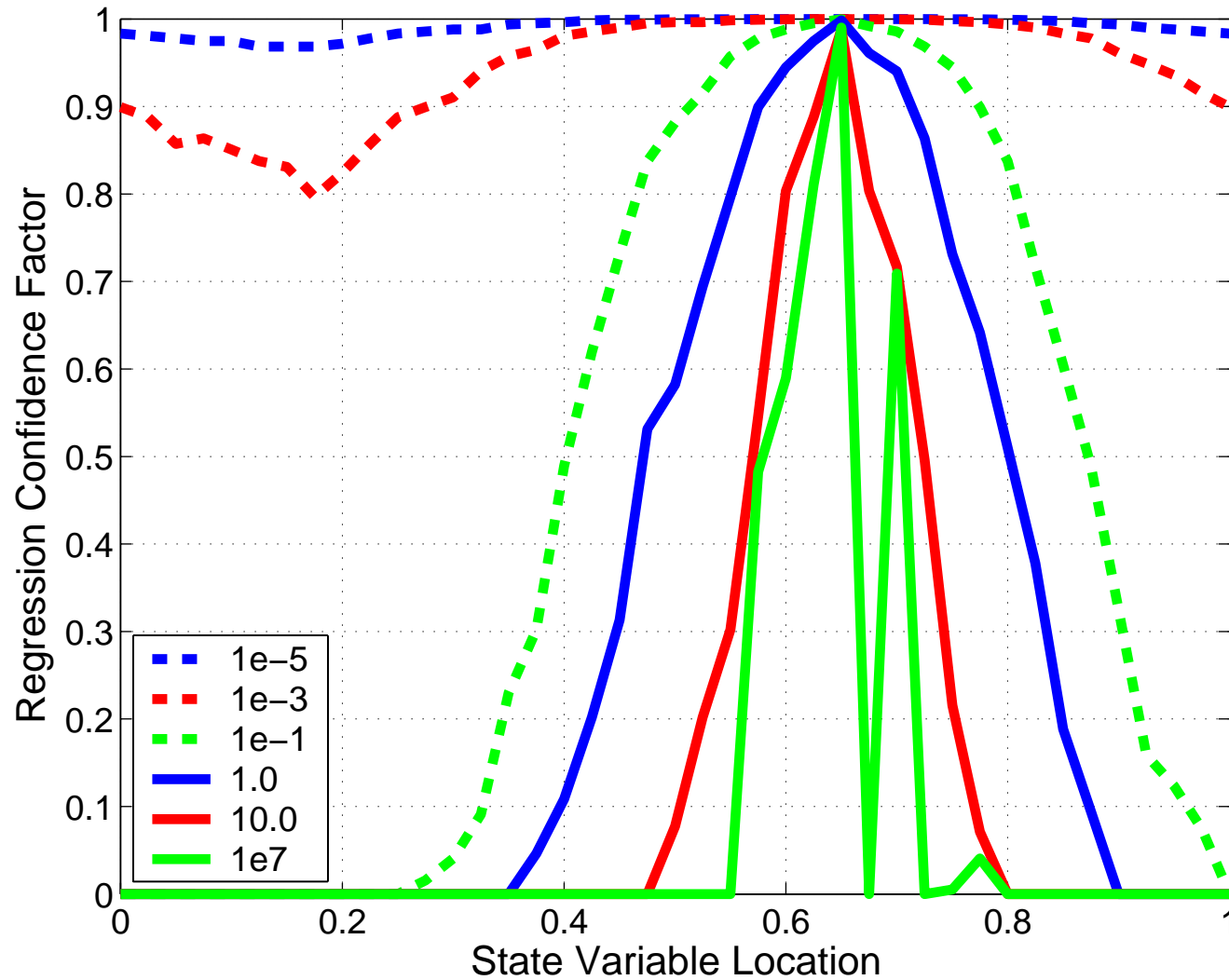


## Time Median Envelopes: Varying Obs. Error Variance



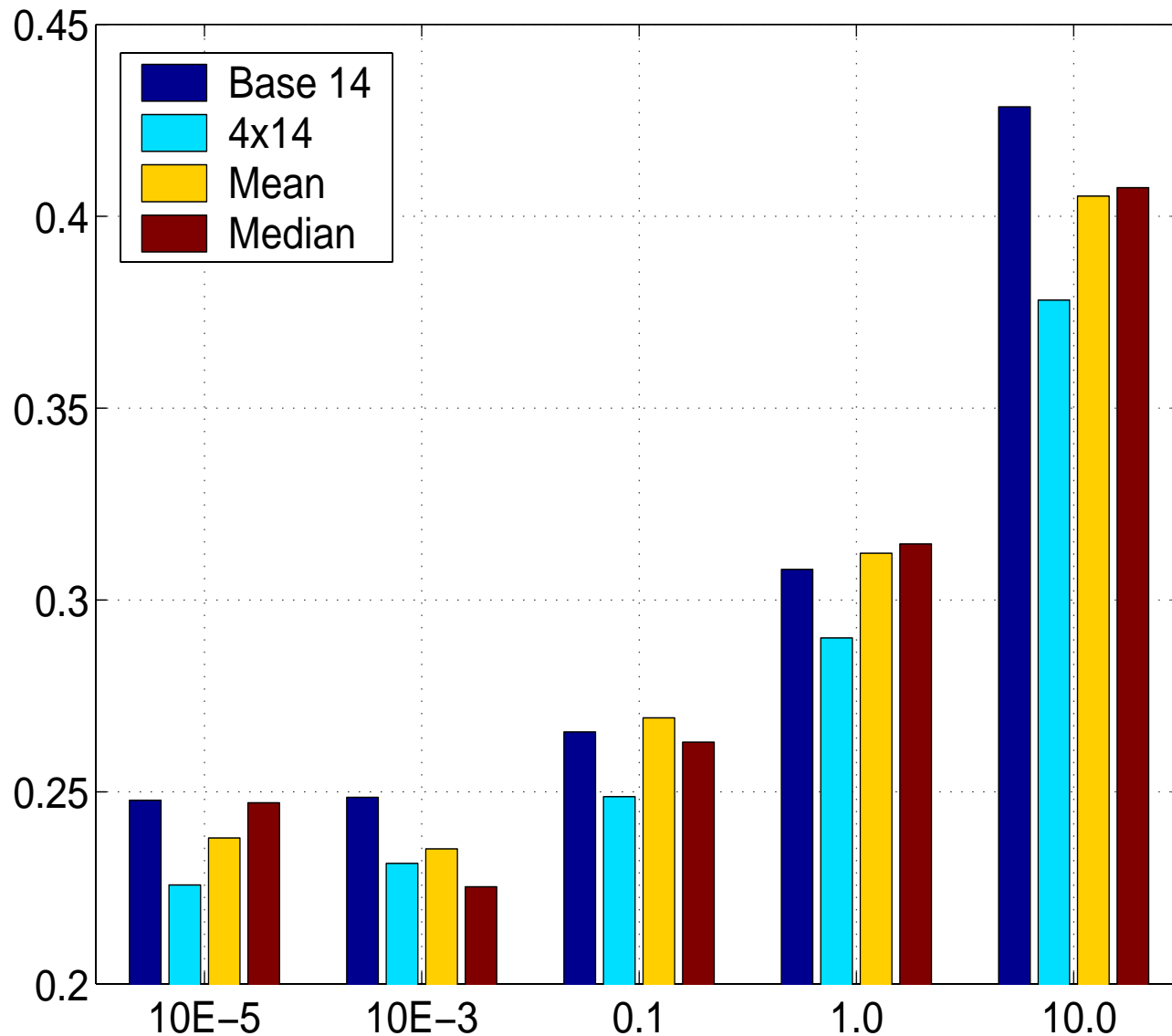
Single Gaspari Cohn half-width can't deal with this range of errors

## Time Median Envelopes: Varying Obs. Error Variance



**Climatological case is unique:** Looks like time mean coherence

# Time Median Envelopes: Varying Obs. Error Variance



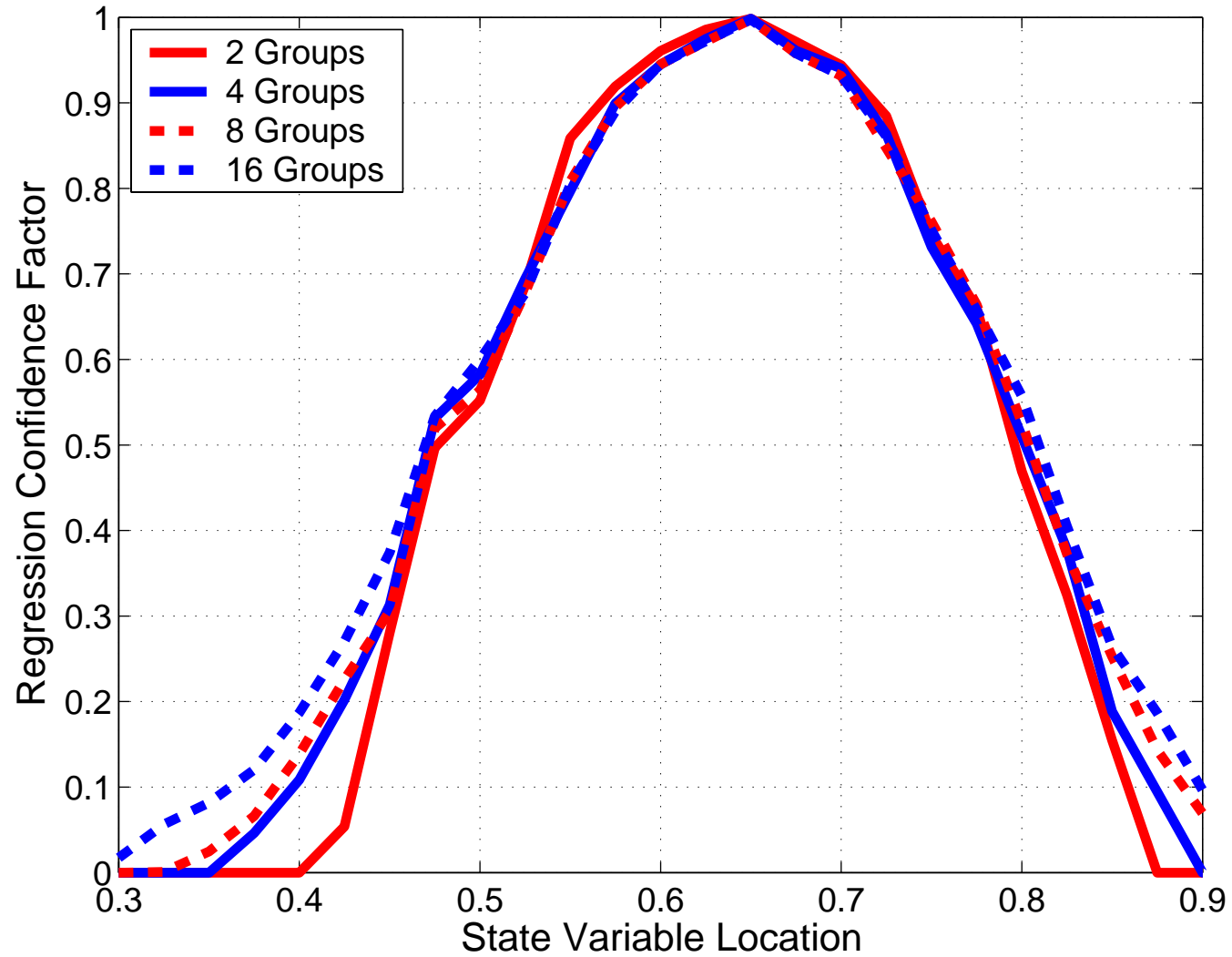
Error scaled by  
obs. error standard  
deviation

As error grows, fil-  
ter becomes less  
'efficient'

Things are more  
'nonlinear' for  
large errors

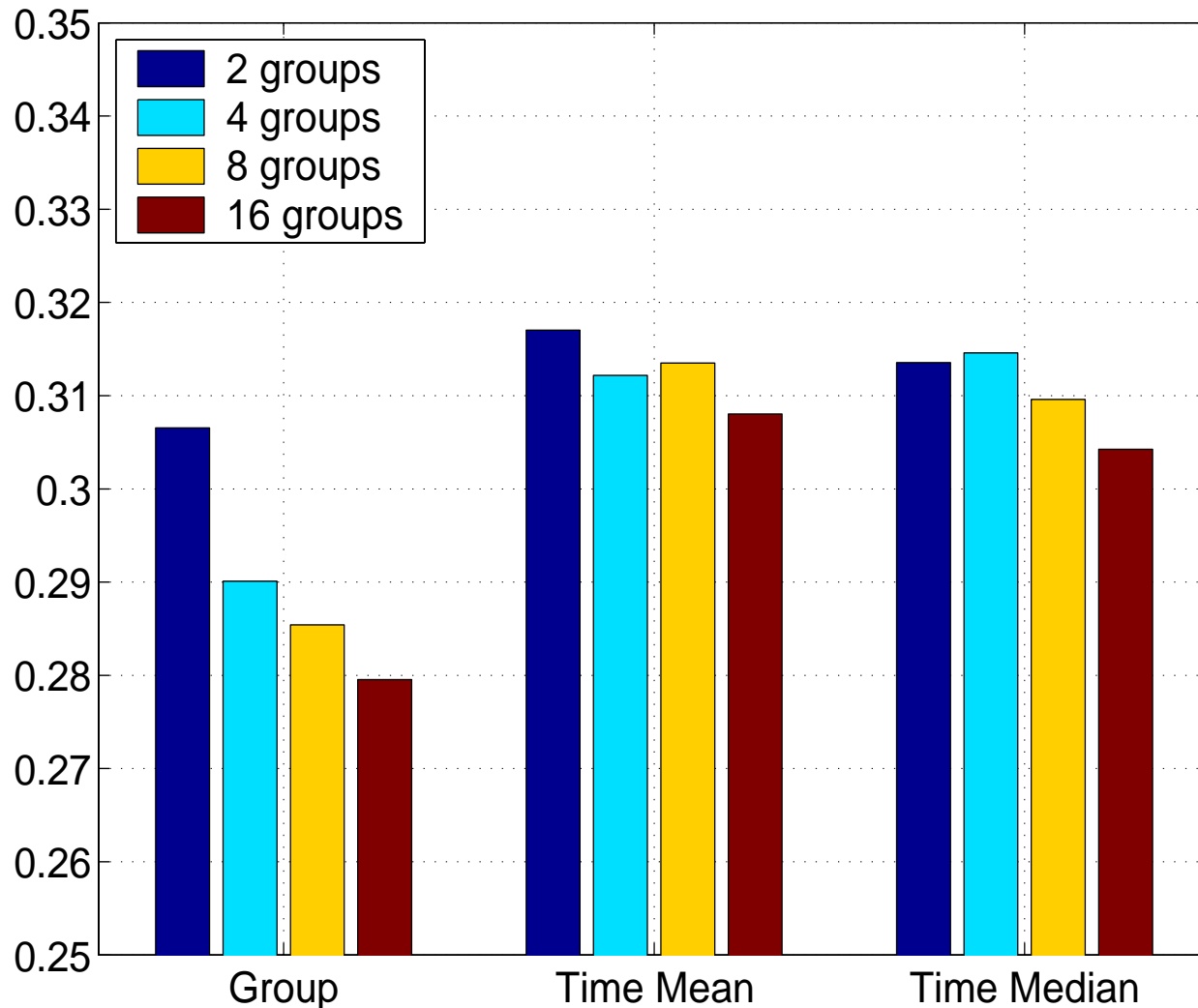
Group, mean and  
median compare  
favorably with  
tuned base for all  
cases!

## Sensitivity of results to group size



Obs. error variance 1.0, 14 member ensemble case

## Sensitivity of results to group size



Gradual improvement for increased group size

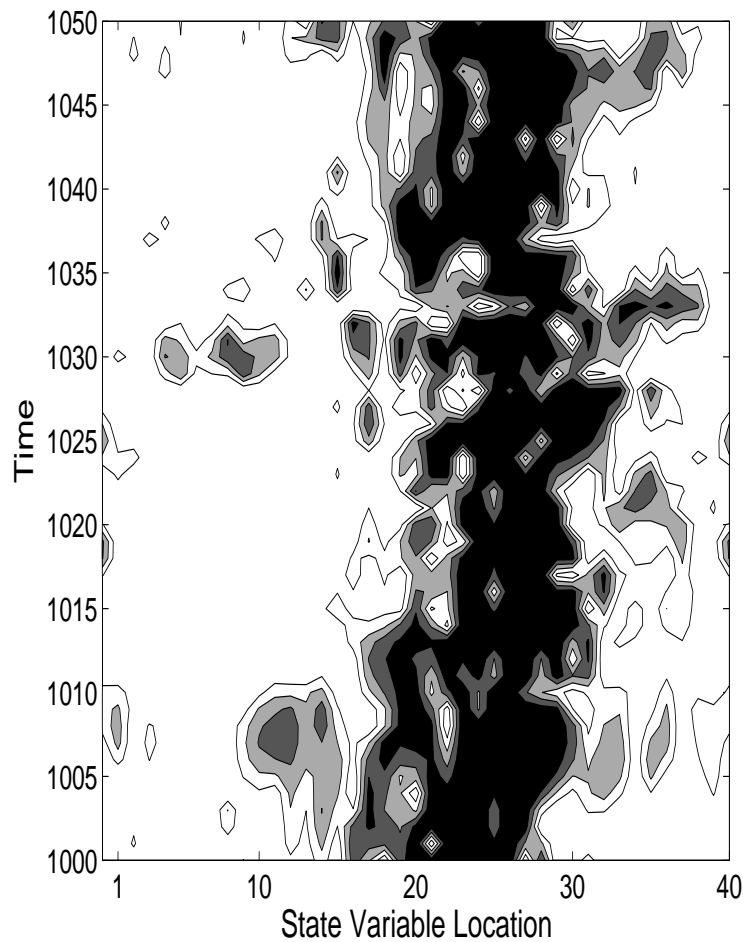
Time mean/median improve more slowly

Important that small groups give good results (for cost)

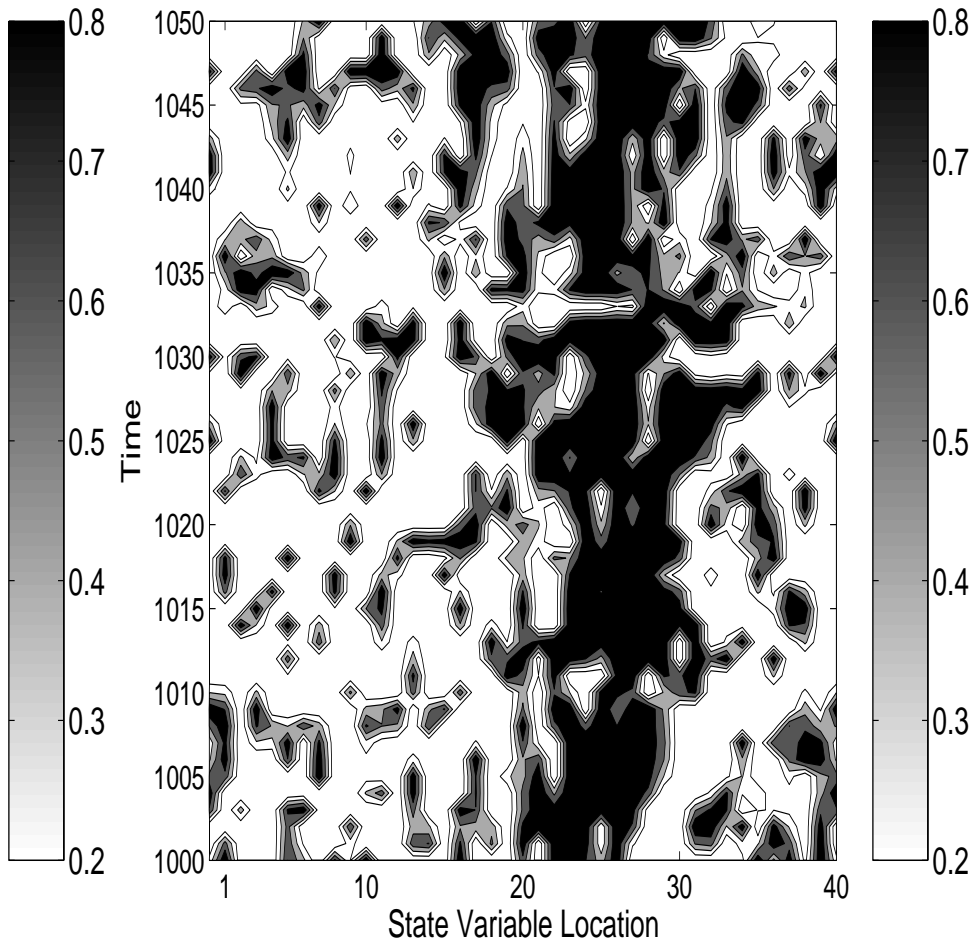
Group better than time mean implies some time varying information (time means should reduce noise)

Obs. error variance 1.0, 14 member ensemble case

# Time variation of regression confidence factor



16 groups

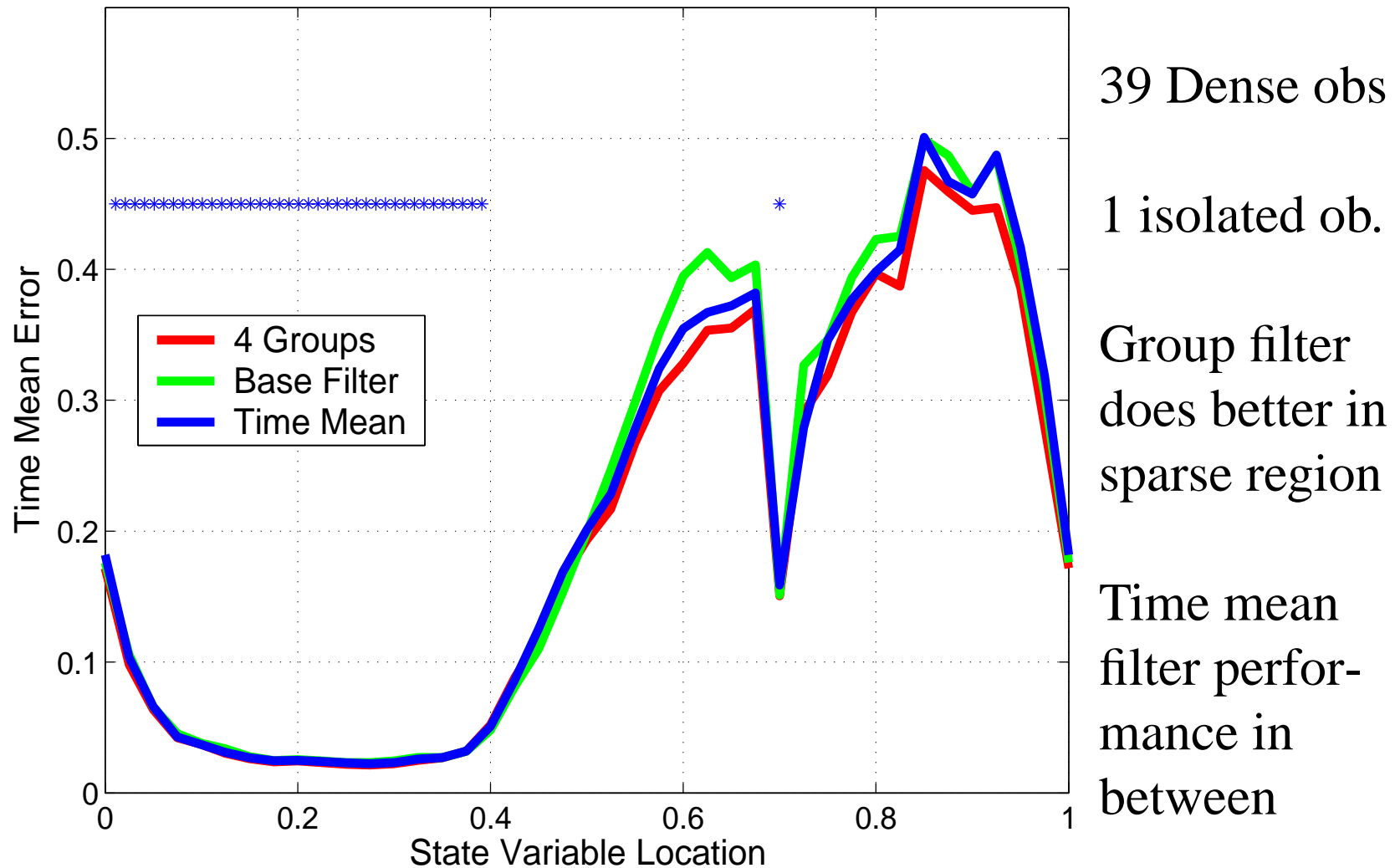


2 groups

Some consistent time variation; noise dominates far from obs.

Notice behavior around step 1033 for instance

# Challenging Traditional Localization: Varying spatial obs. density



39 Dense obs

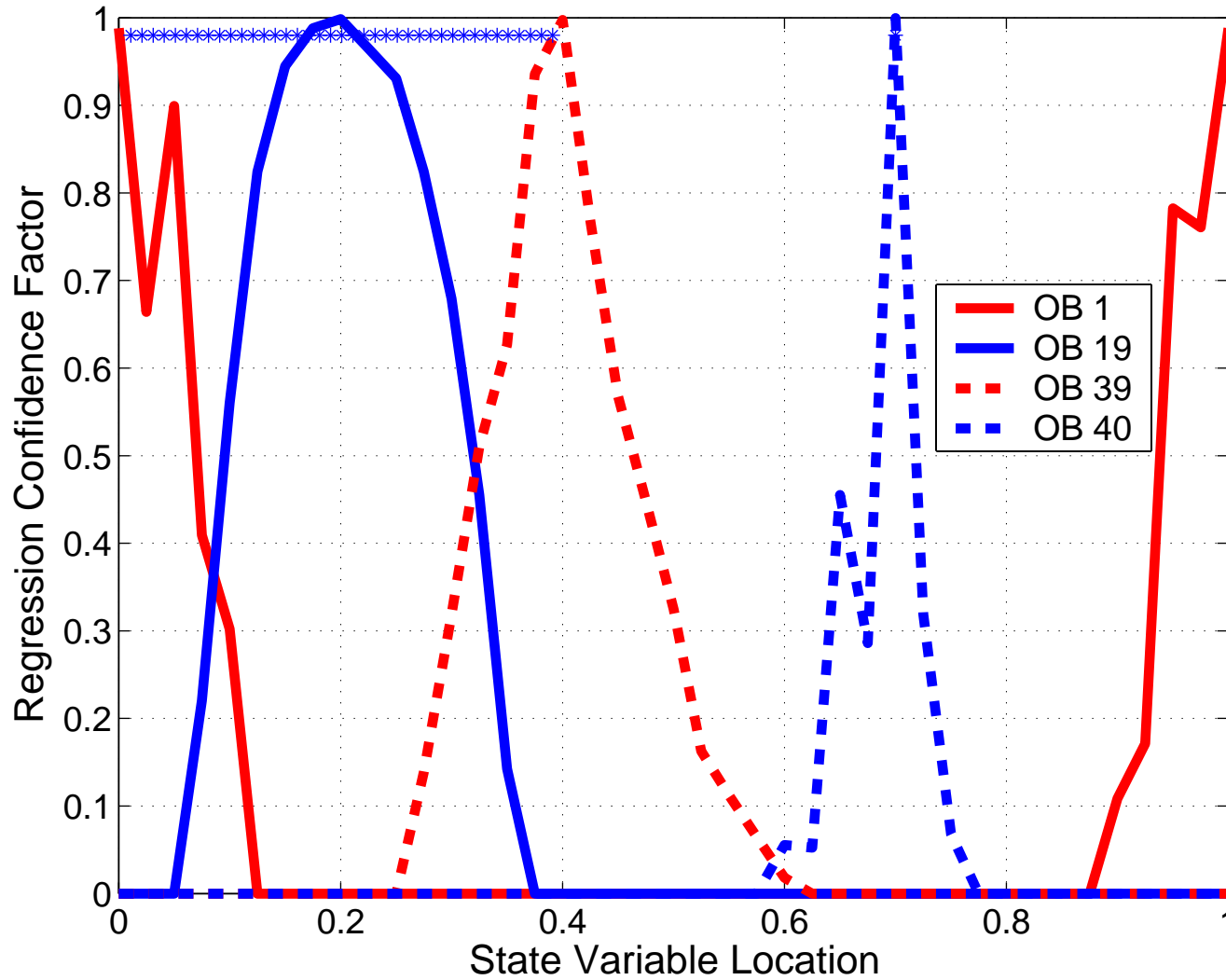
1 isolated ob.

Group filter  
does better in  
sparse region

Time mean  
filter perfor-  
mance in  
between

Time Mean Prior RMS Error as function of spatial location

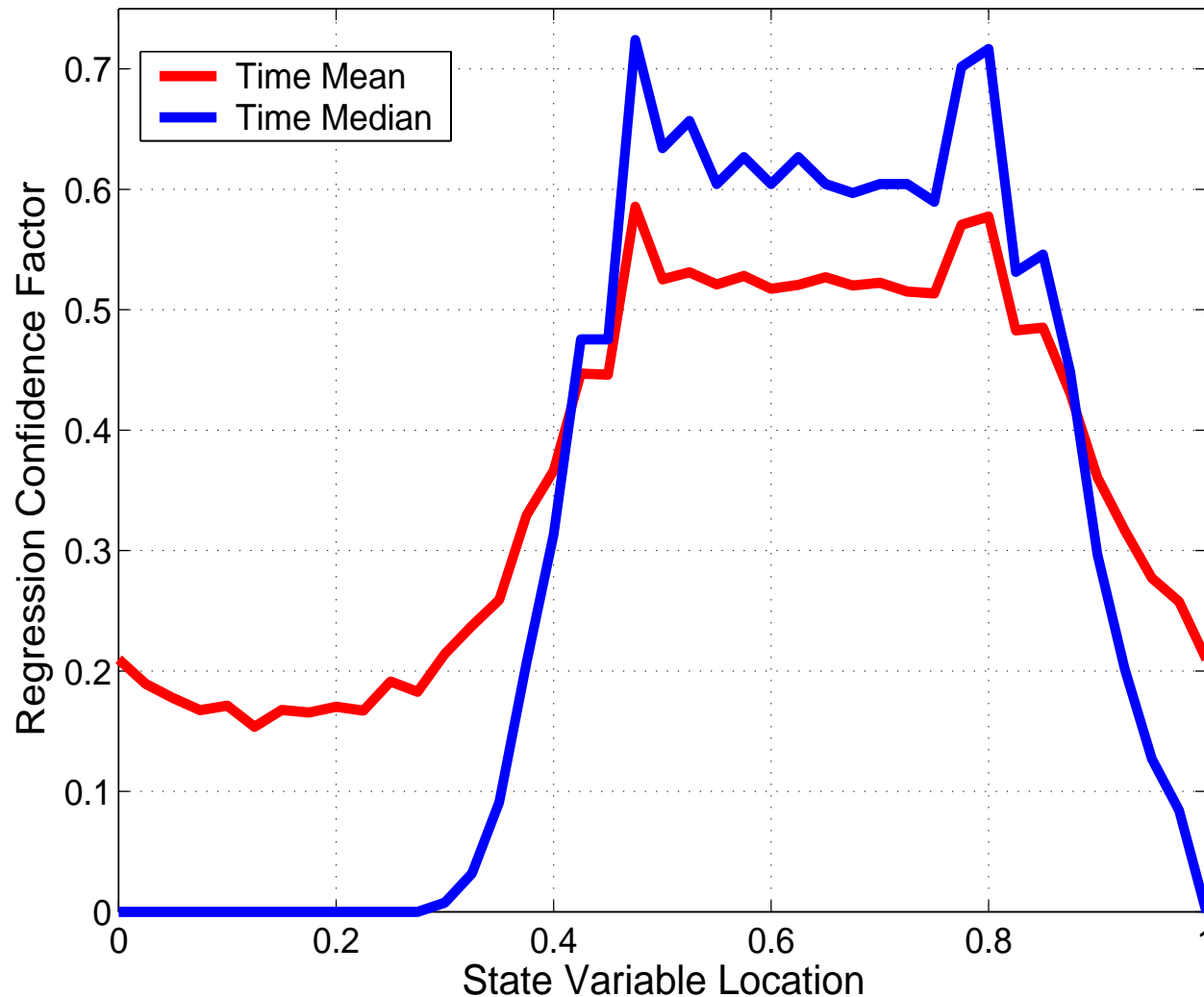
# Challenging Traditional Localization: Varying spatial obs. density



Time median envelopes vary lots with spatial location of obs.



## Challenging traditional localization: Spatial-mean obs.



Forward observation operator is fifteen grid point mean

Envelope has become less Gaussian

In one-d L96 domain, everything stays pretty Gaussian

**But..., group filter does eliminate need for tuning localization!**

## Assimilating observations at times different from state estimate

Ensemble smoothers: use future observations

Targeted observations: examine impact of obs. in past

Real-time assimilation: use of late arriving observations in forecast

Expect correlations to diminish as time separation increases

Need a 'localization' in time, too

Group filter can provide this

## Time 'localization': Experimental design

4 group, 14 ensemble member filter

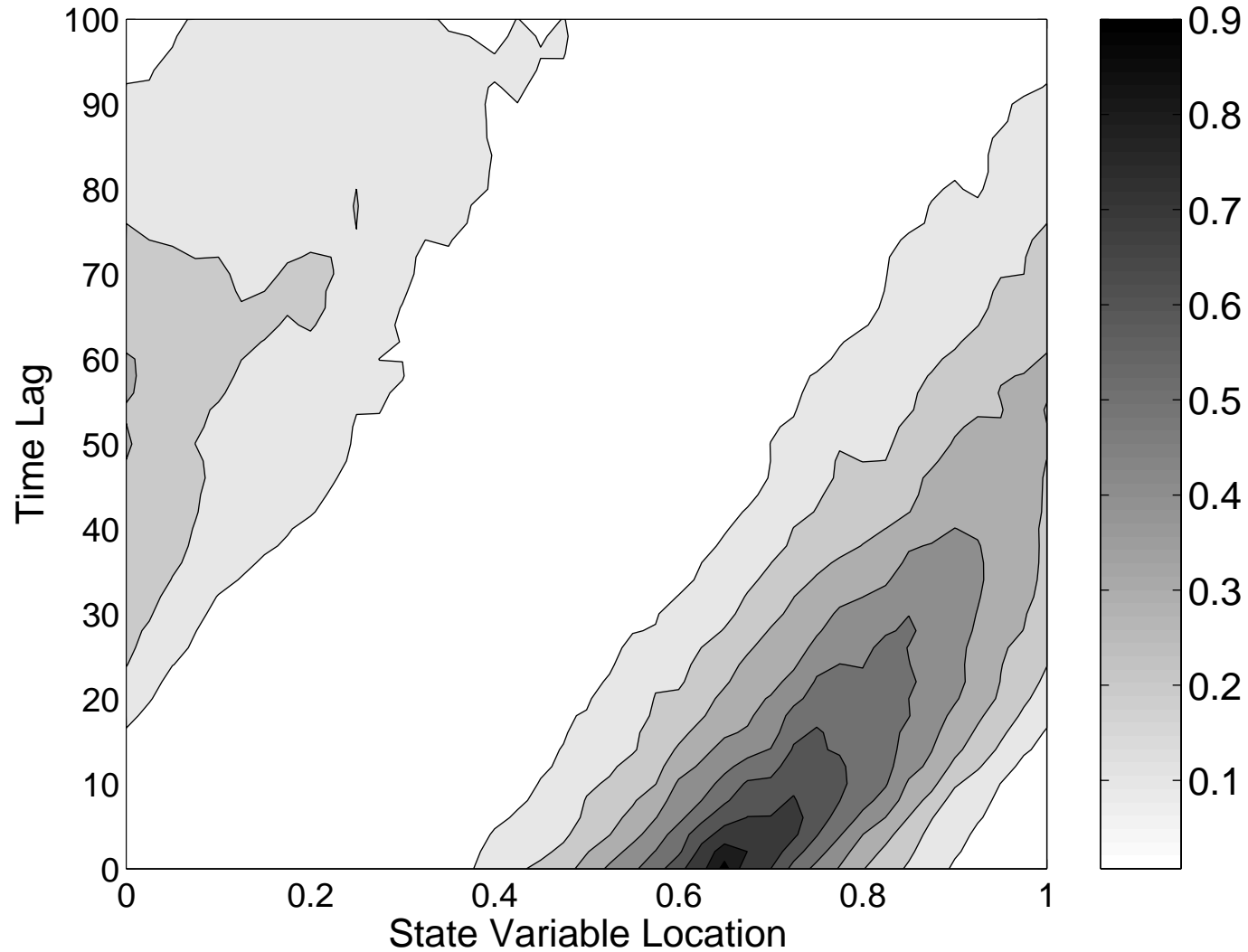
40 random obs. with 1.0 error variance

1 additional observation at location 0.642

The additional observation is from a prior time step

Time mean regression confidence envelope as function of time lag

# Regression confidence factor as function of obs. lag time

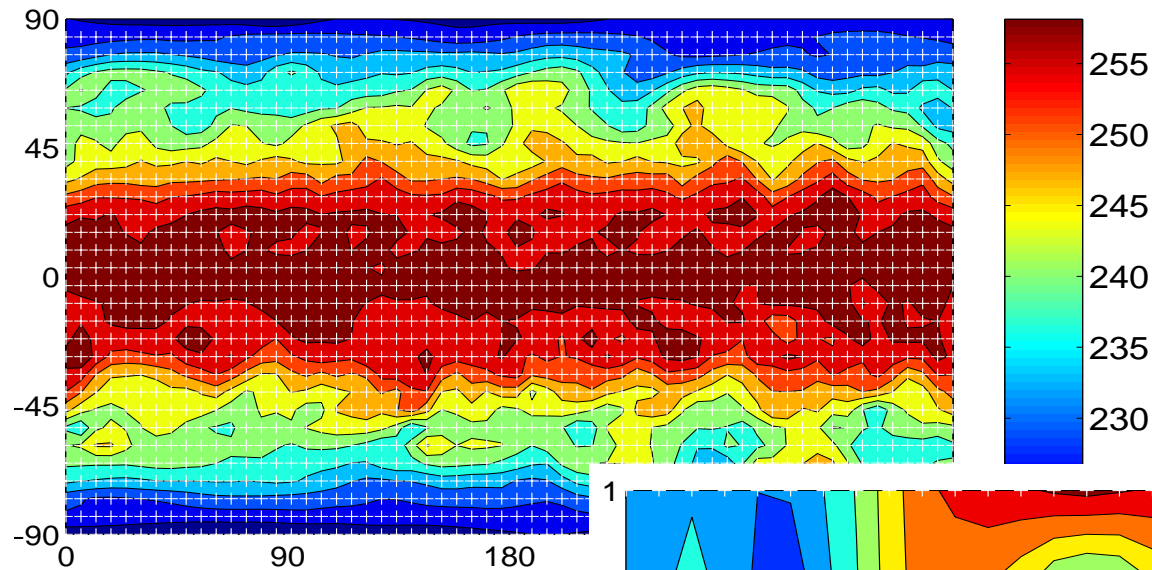


Moves with group velocity (approximately); dies off with lead

# Assimilation in Idealized AGCM: GFDL FMS B-Grid Dynamical Core (Havana)

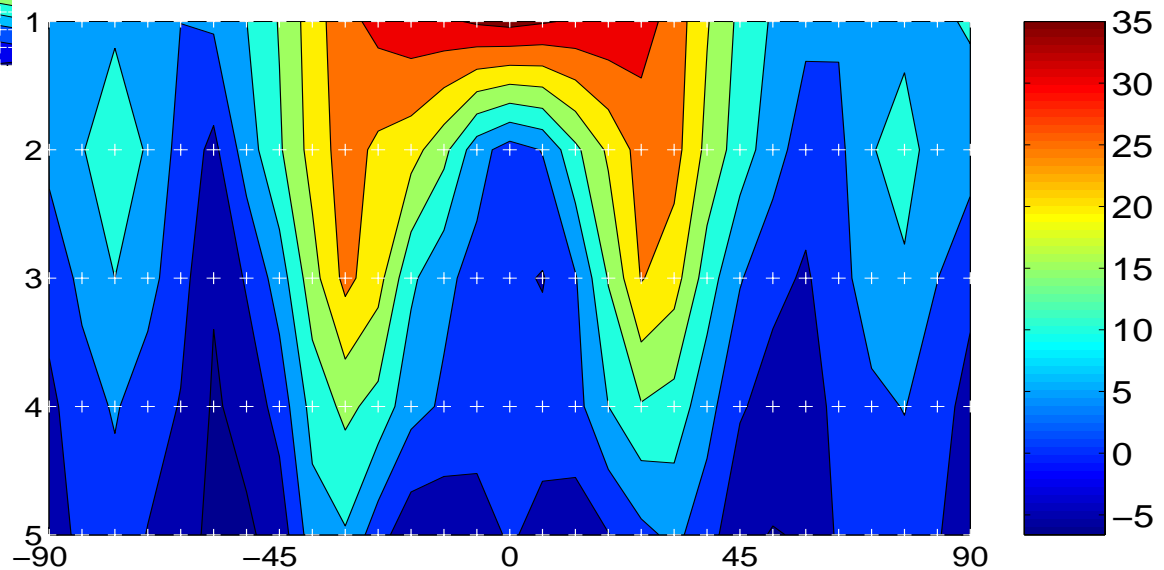
Held-Suarez Configuration (no zonal variation, fixed forcing)

Low-Resolution (60 lons, 30 lats, 5 levels); Timestep 1 hour

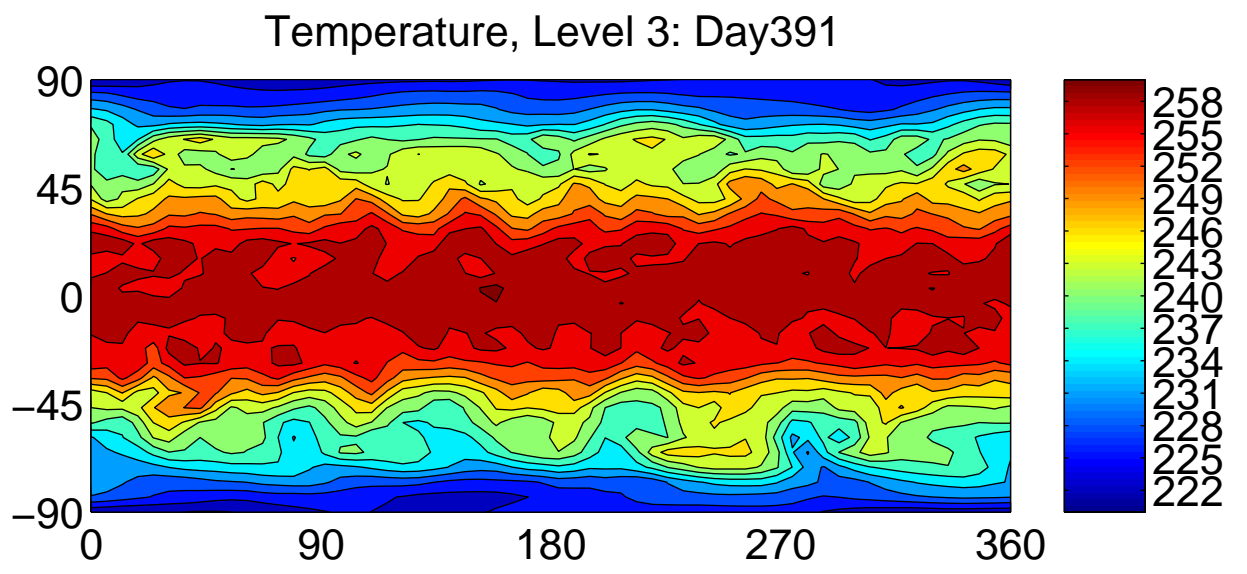
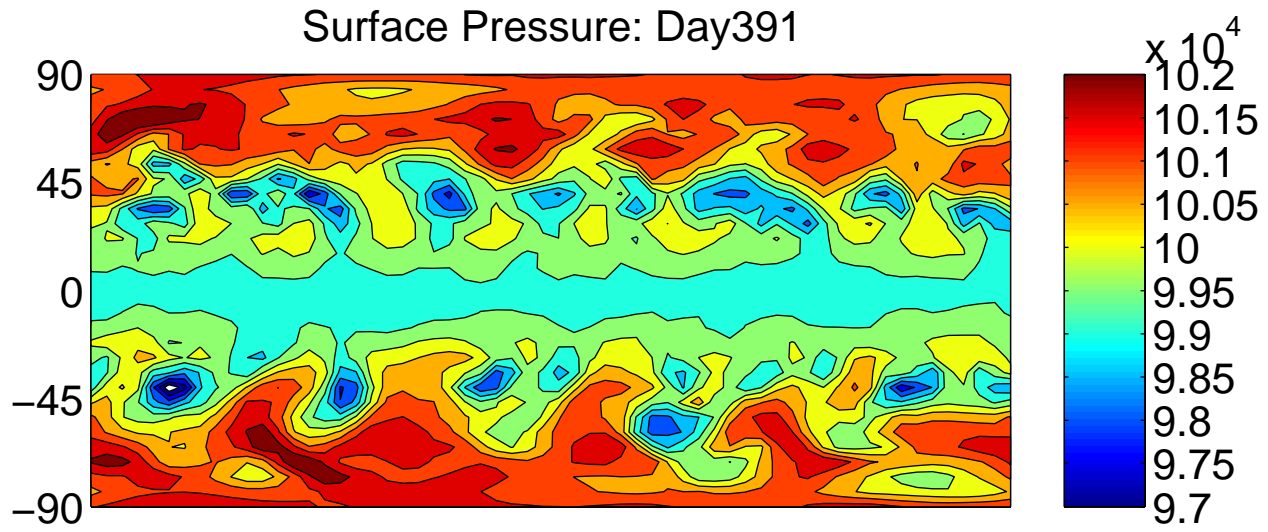


Cross Section of Zonal Mean U

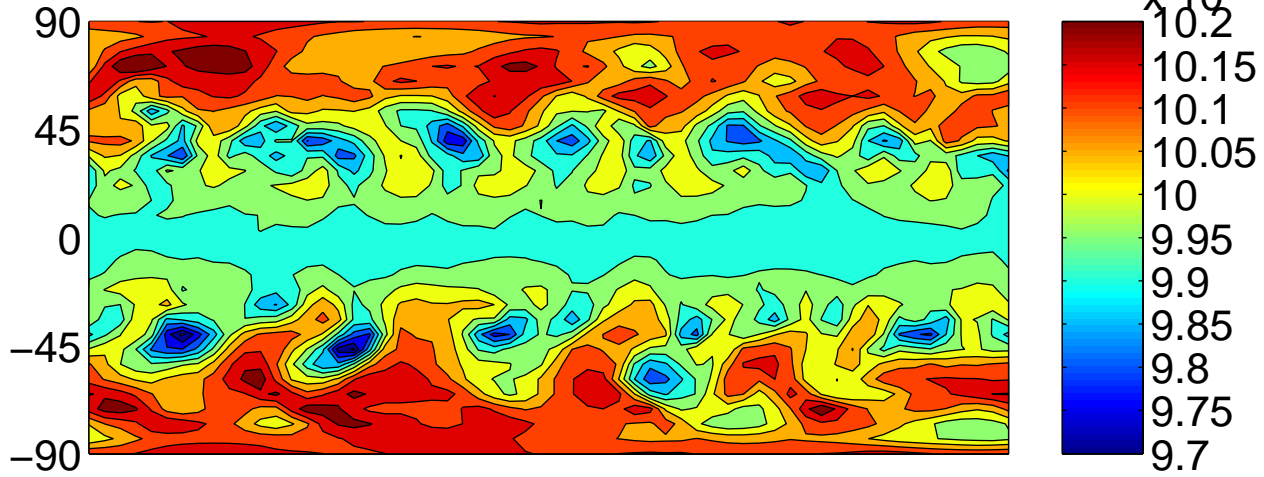
T at Level 3  
(Middle of Atmosphere)



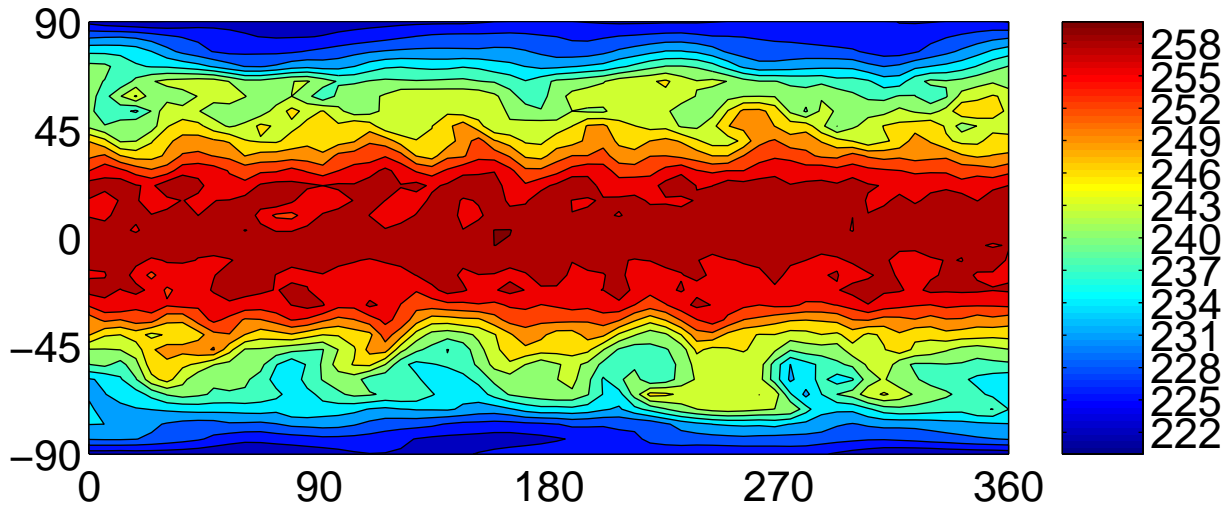
Has Baroclinic Instability



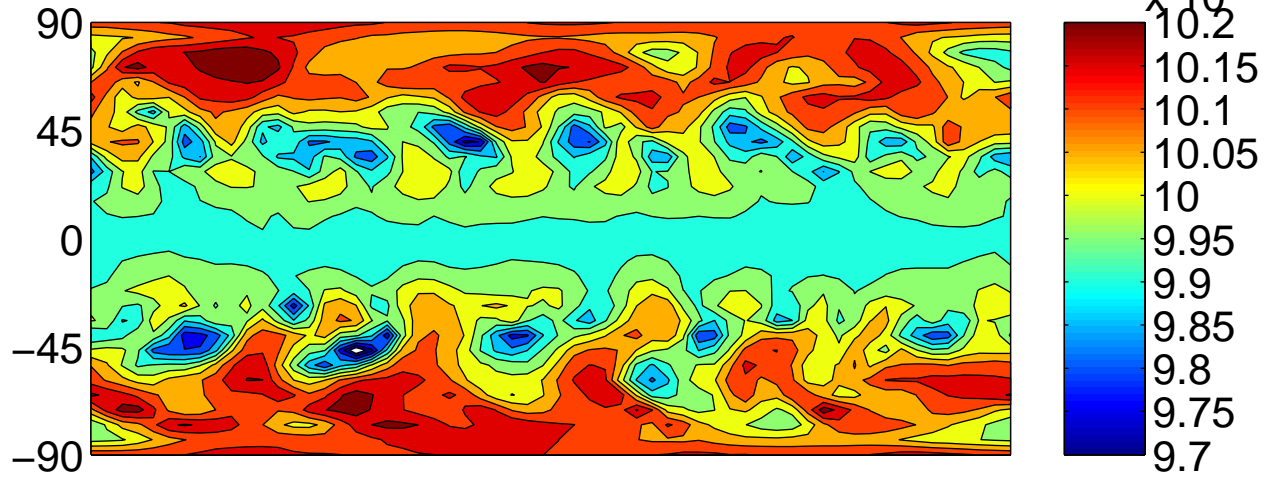
Surface Pressure: Day392



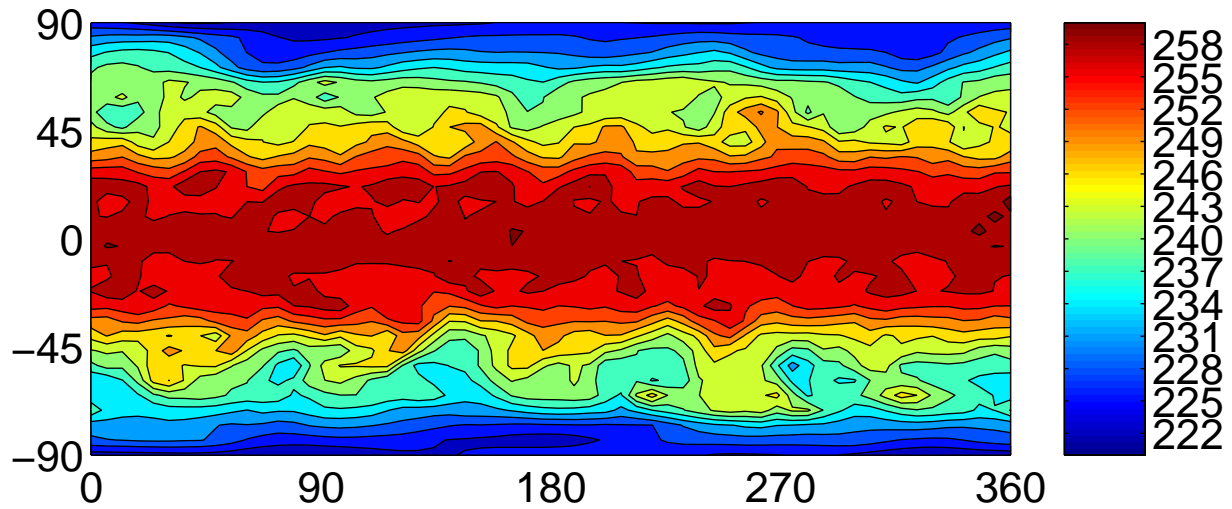
Temperature, Level 3: Day392



Surface Pressure: Day393

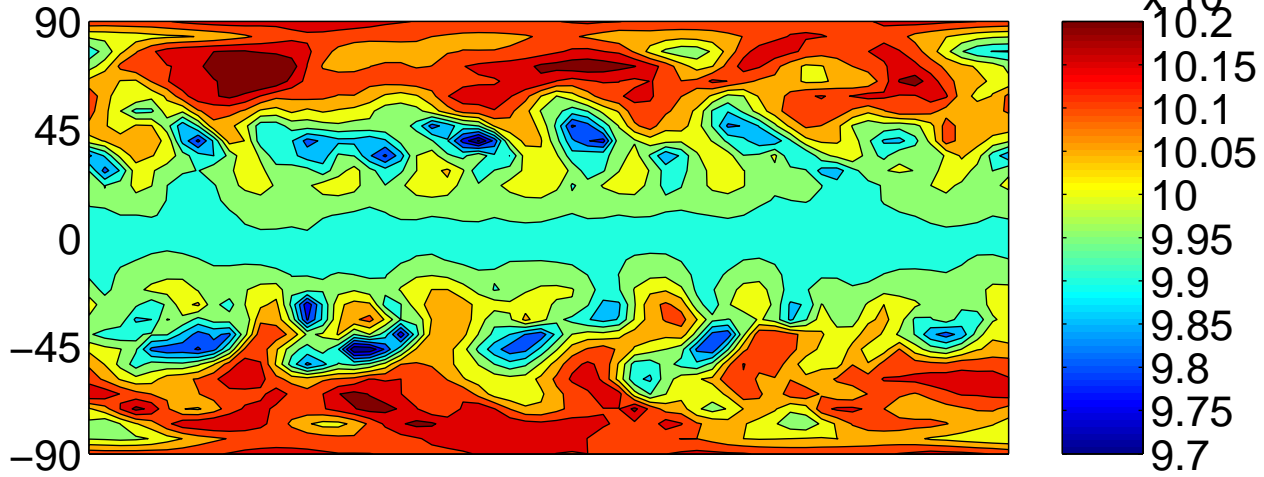


Temperature, Level 3: Day393

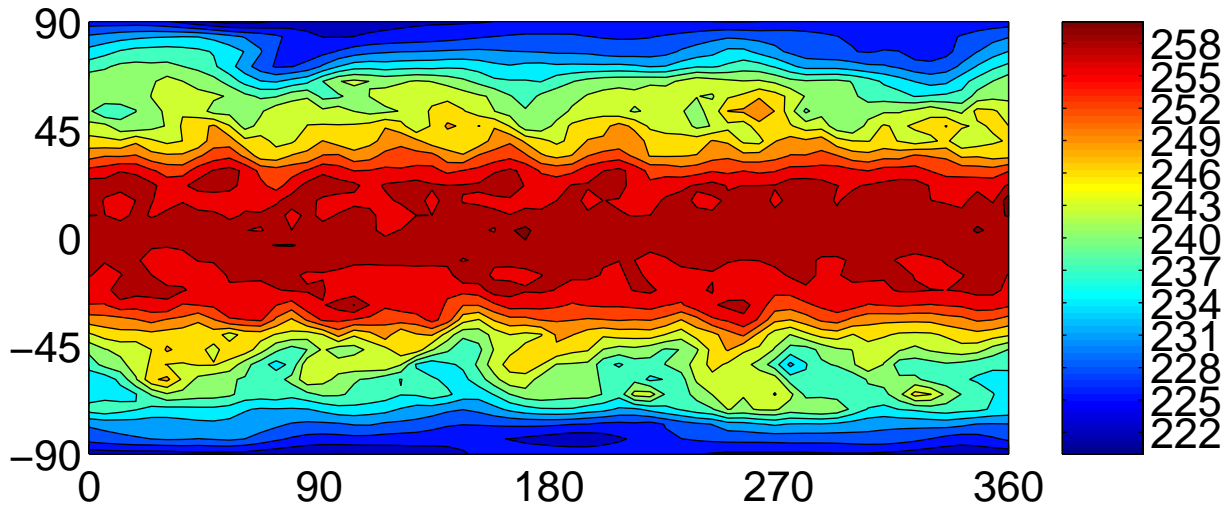




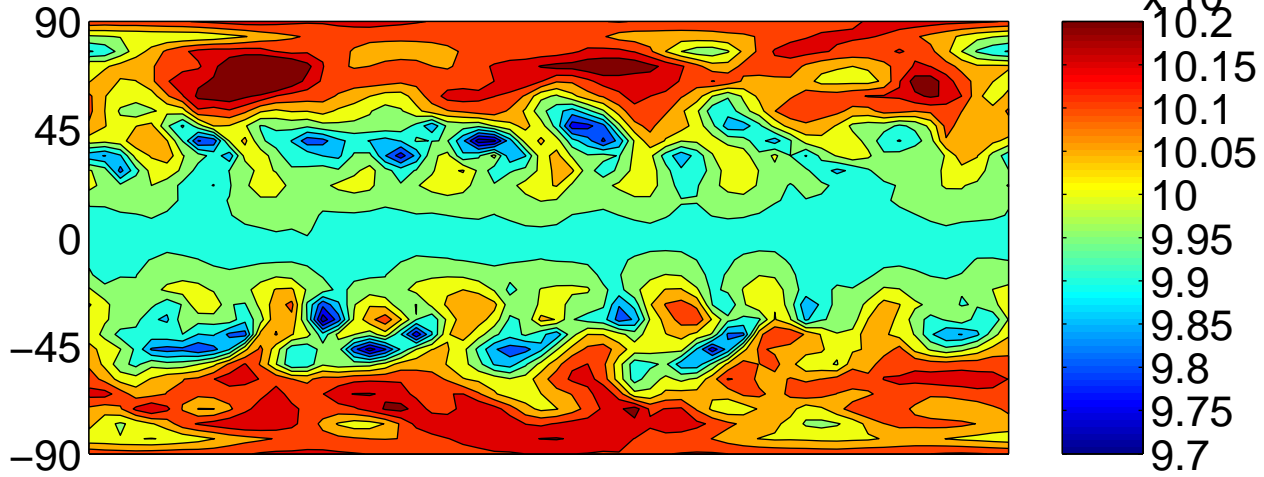
Surface Pressure: Day394



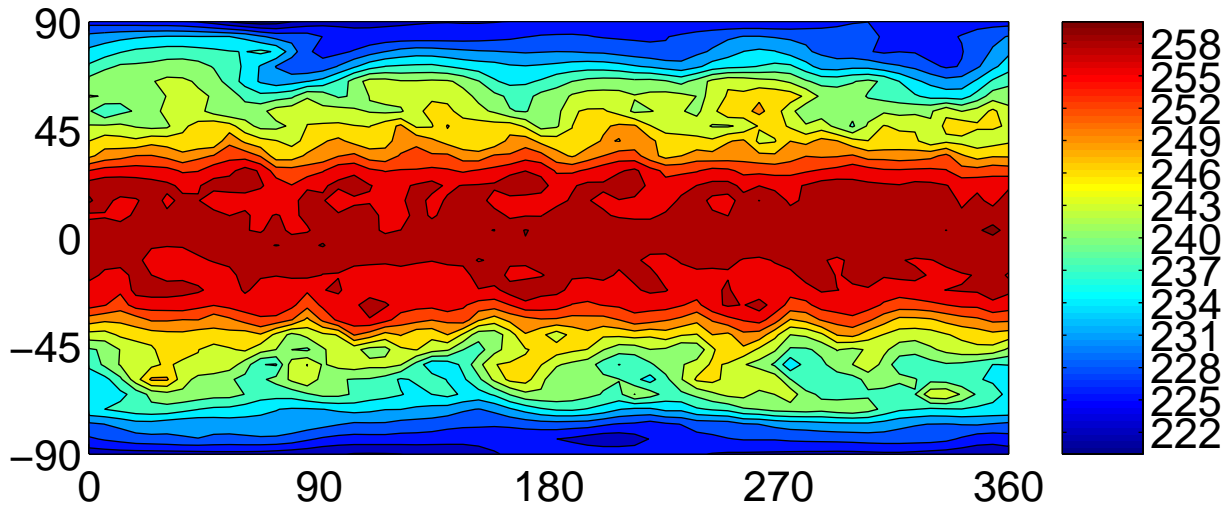
Temperature, Level 3: Day394



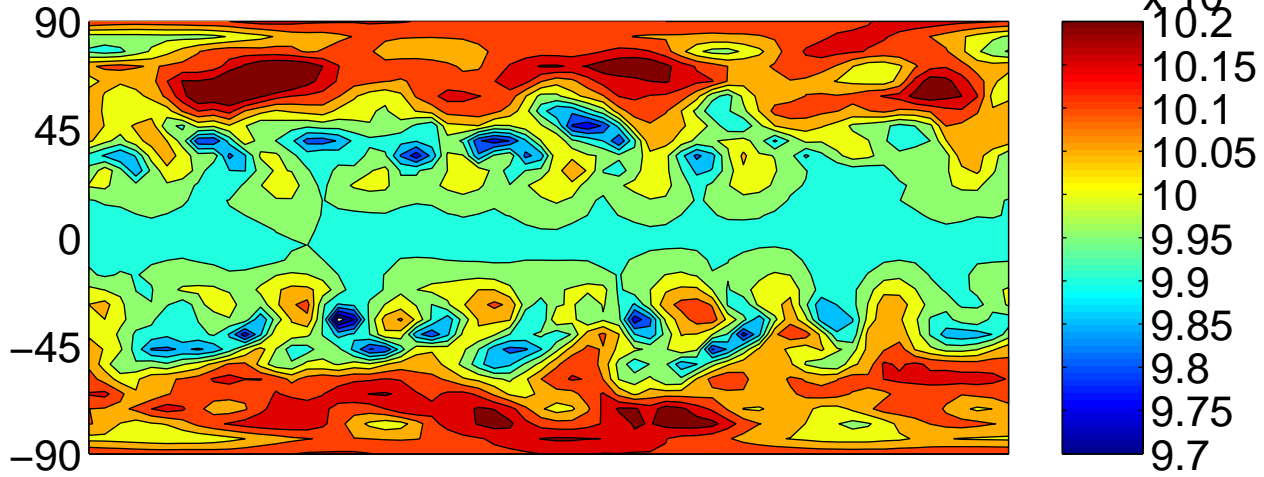
Surface Pressure: Day395



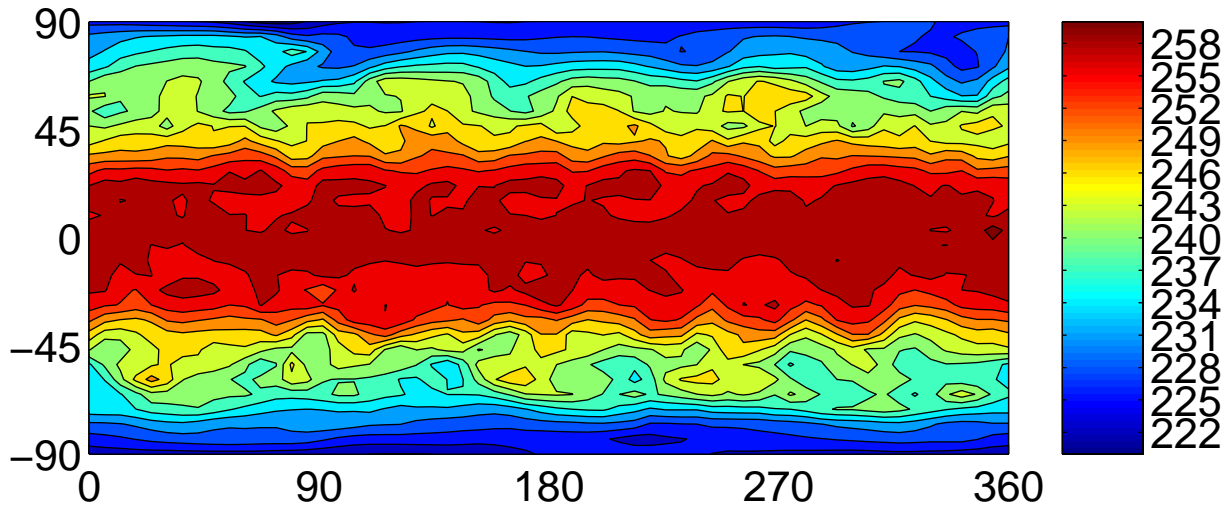
Temperature, Level 3: Day395



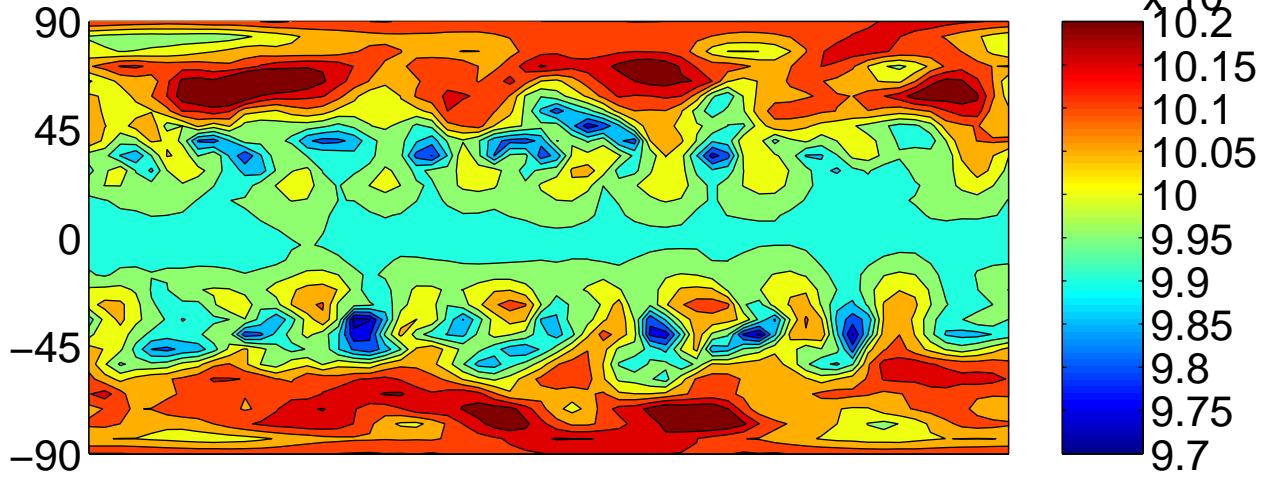
Surface Pressure: Day396



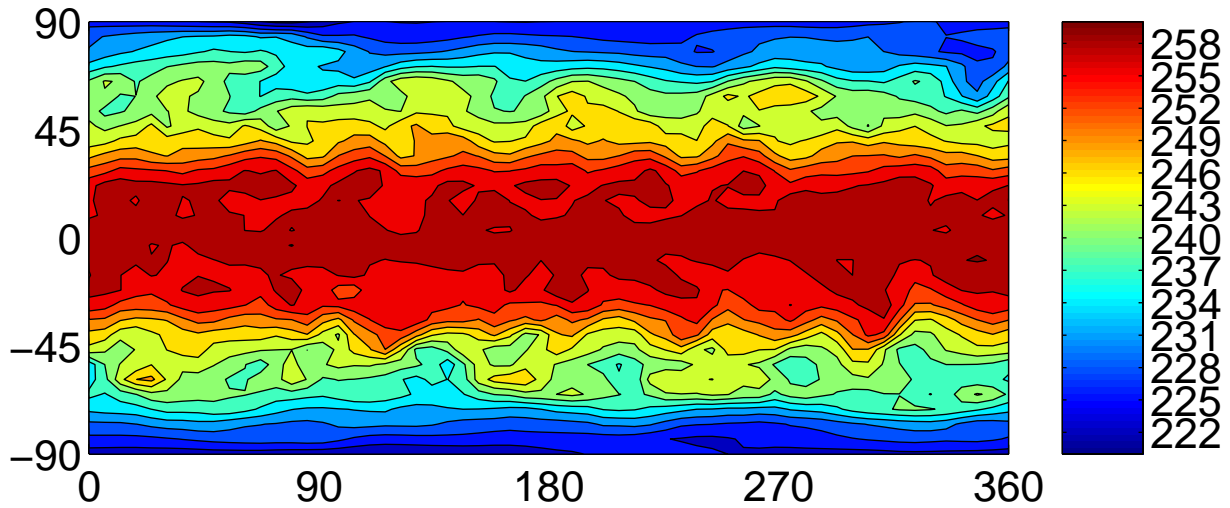
Temperature, Level 3: Day396



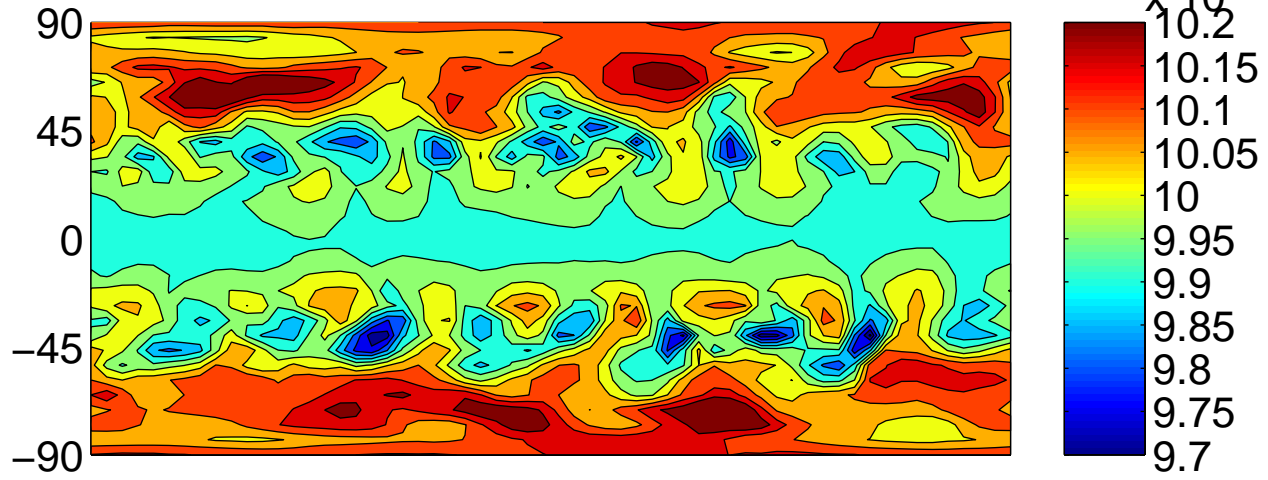
Surface Pressure: Day397



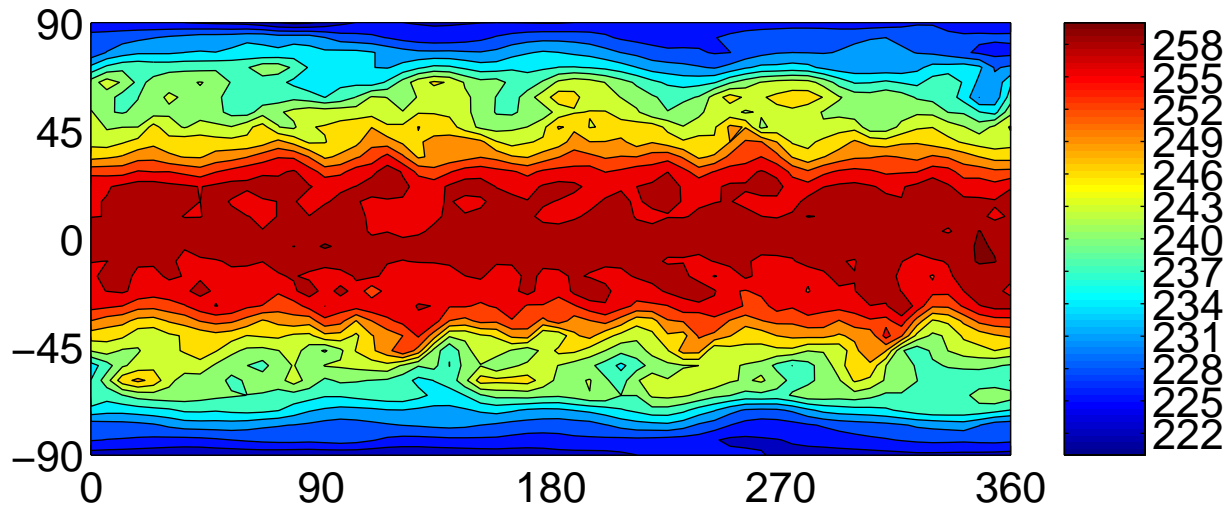
Temperature, Level 3: Day397



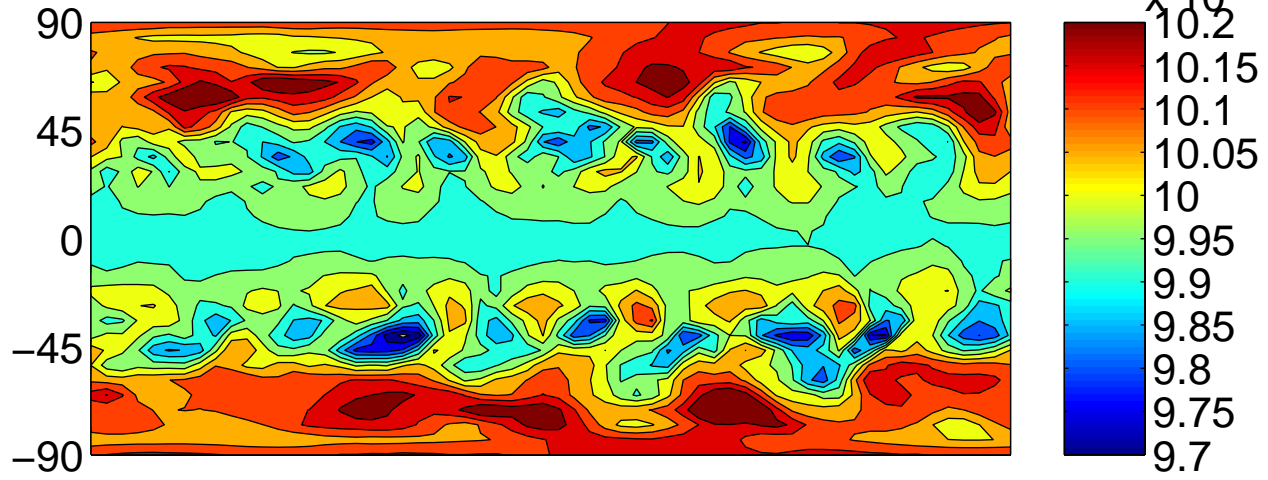
Surface Pressure: Day398



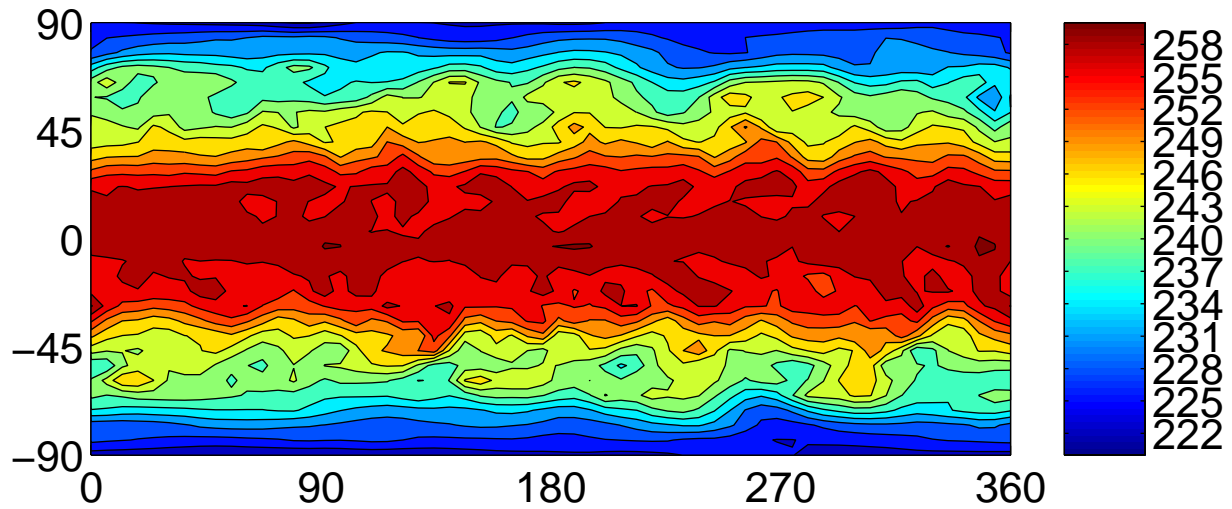
Temperature, Level 3: Day398



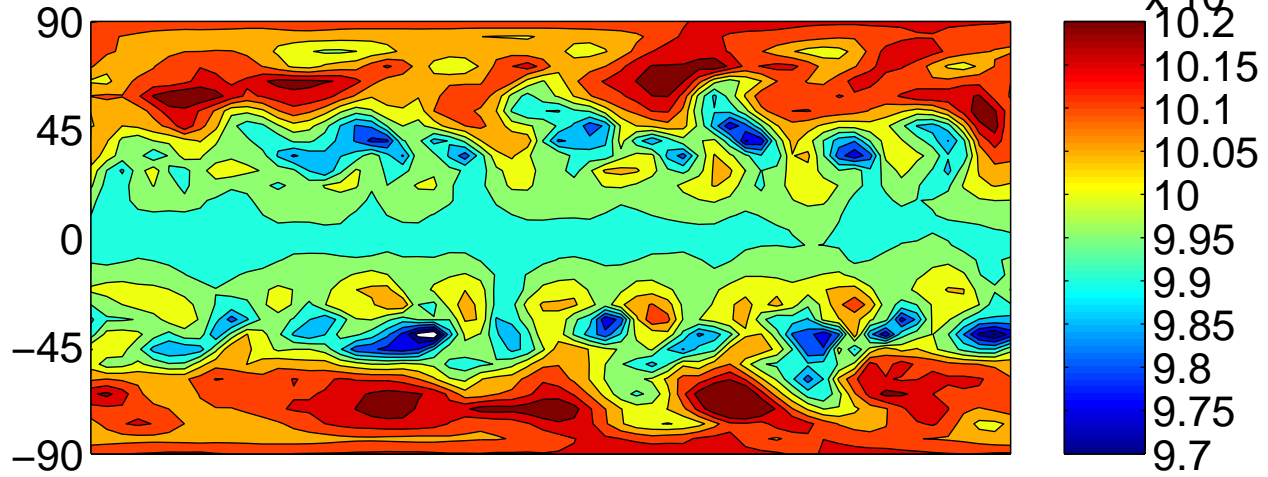
Surface Pressure: Day399



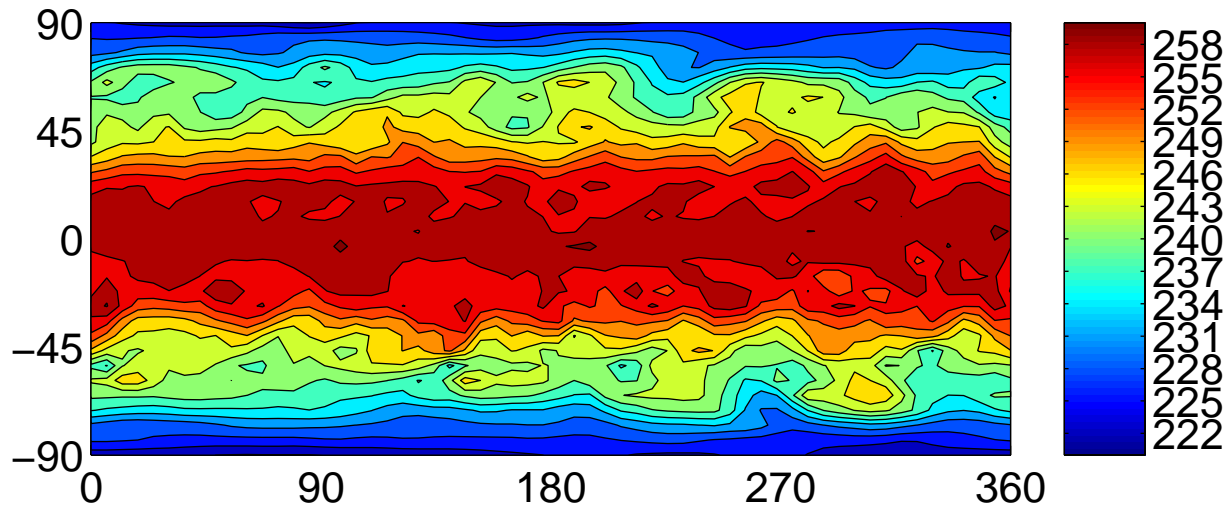
Temperature, Level 3: Day399



Surface Pressure: Day400



Temperature, Level 3: Day400



## Experimental Design Details: Bgrid AGCM

Results for 4x20 group filter

Assimilation for 400 days; starting from climatological distribution

Summary results are from last 200 days

No covariance inflation

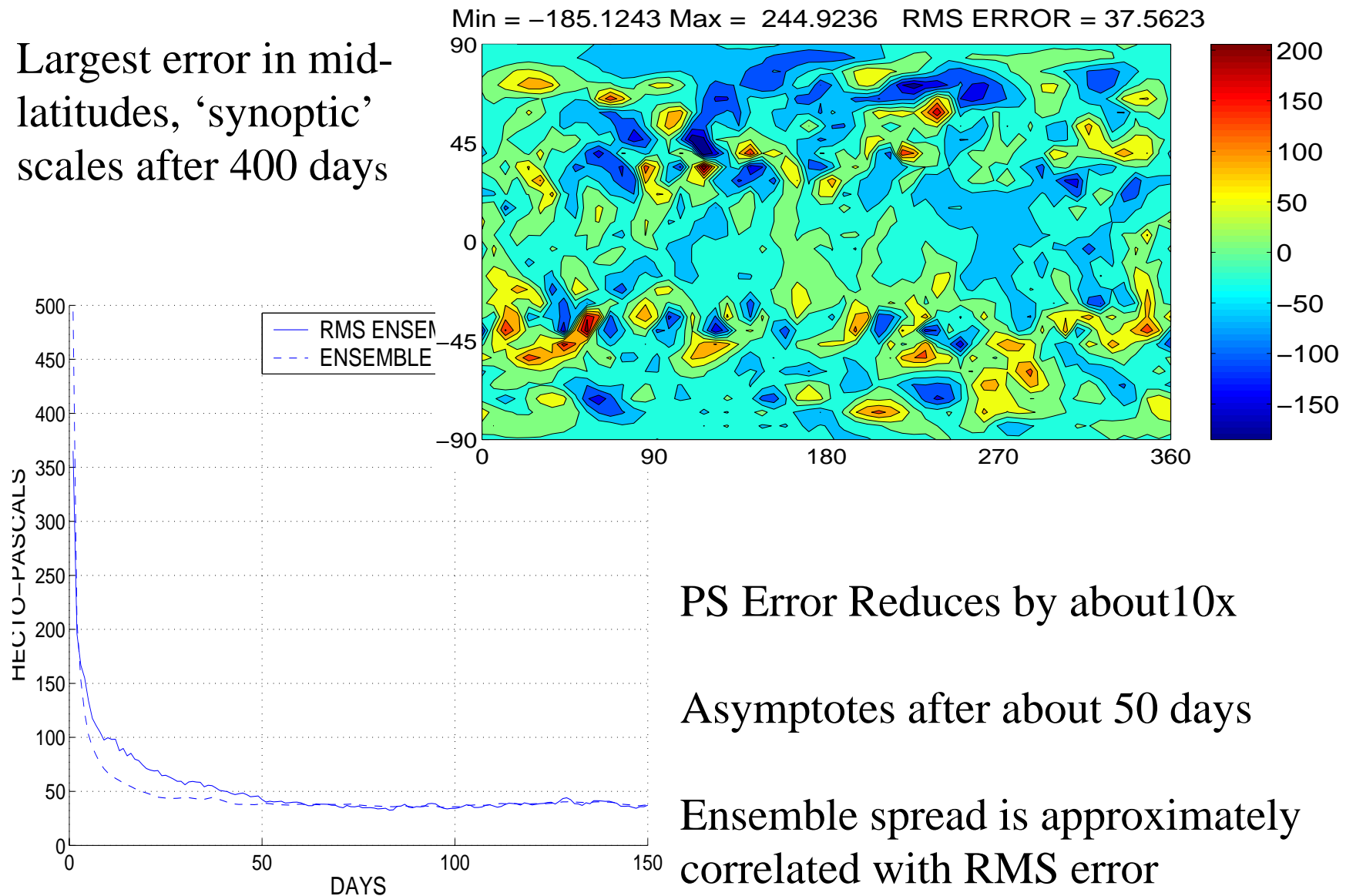
1800 randomly located surface pressure stations observe once every 24 hours

Observational error variance is 1 mb



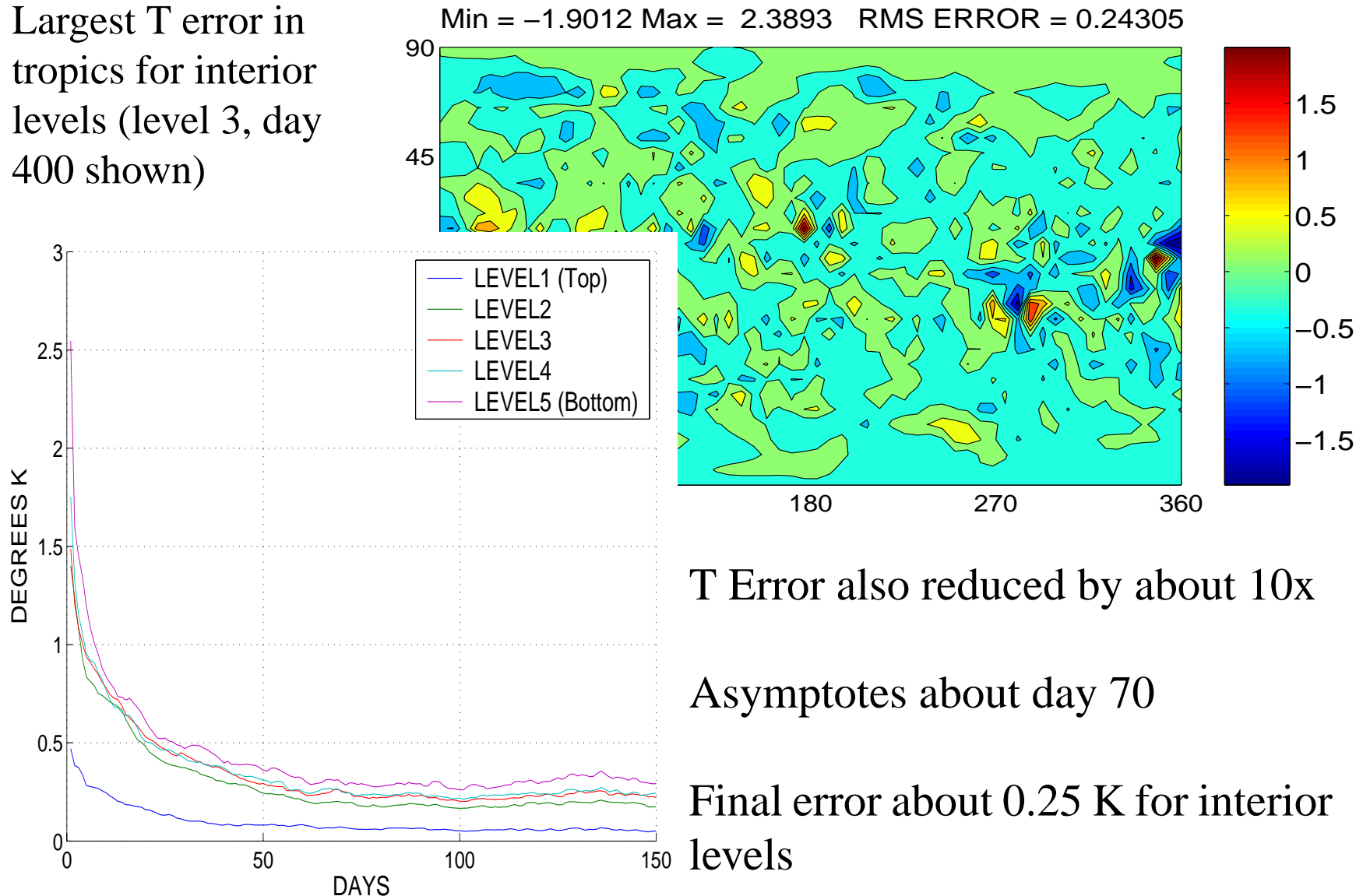
## Baseline Case: 1800 PS Obs every 24 hours

Largest error in mid-latitudes, 'synoptic' scales after 400 days

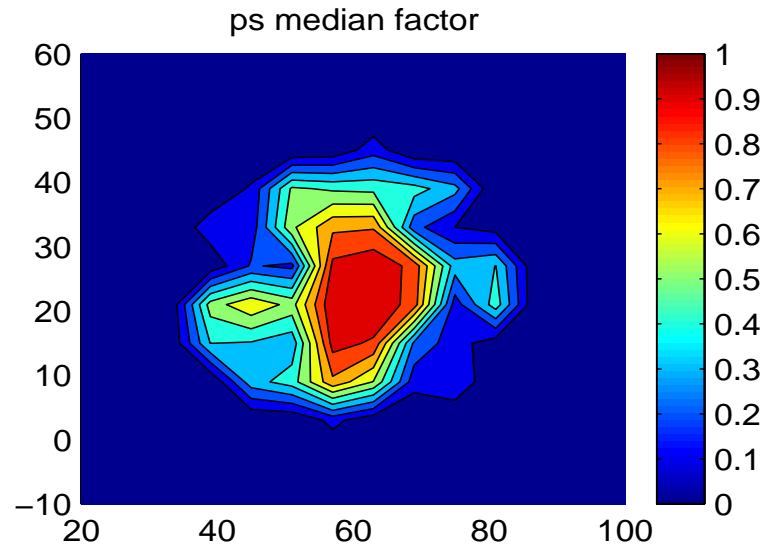
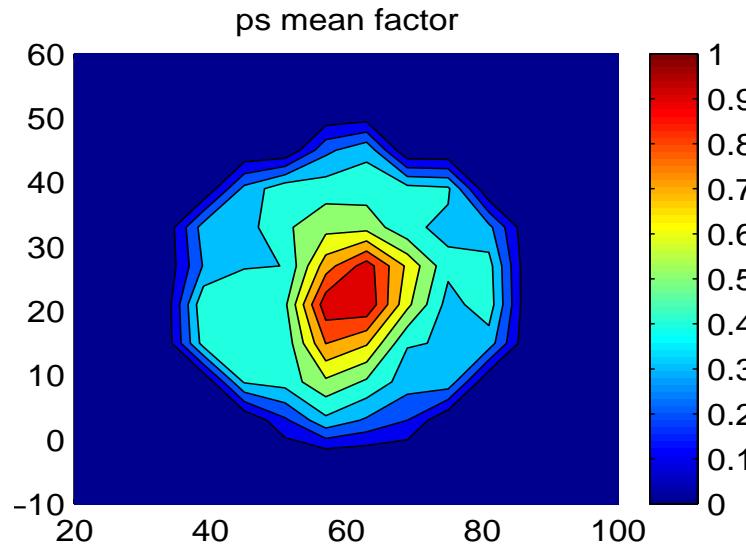


## Baseline Case: 1800 PS Obs every 24 hours

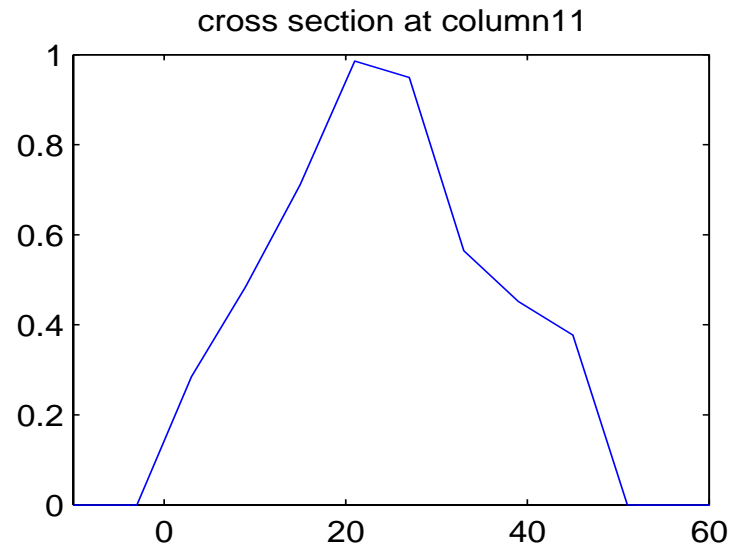
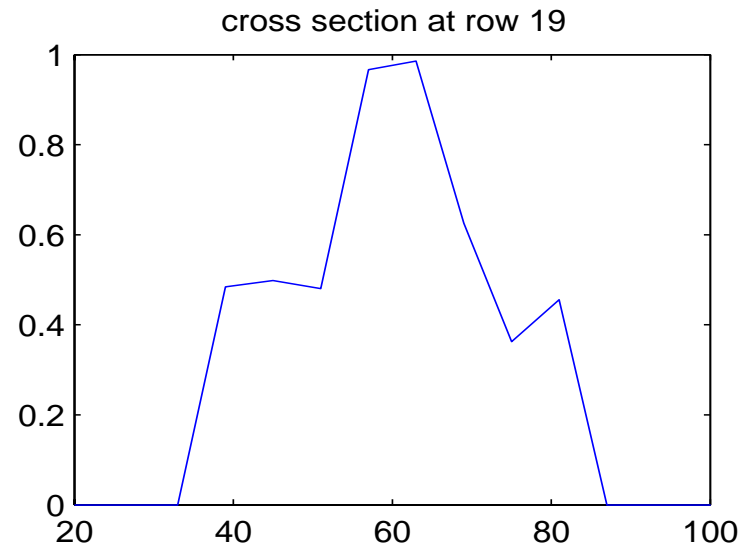
Largest T error in tropics for interior levels (level 3, day 400 shown)



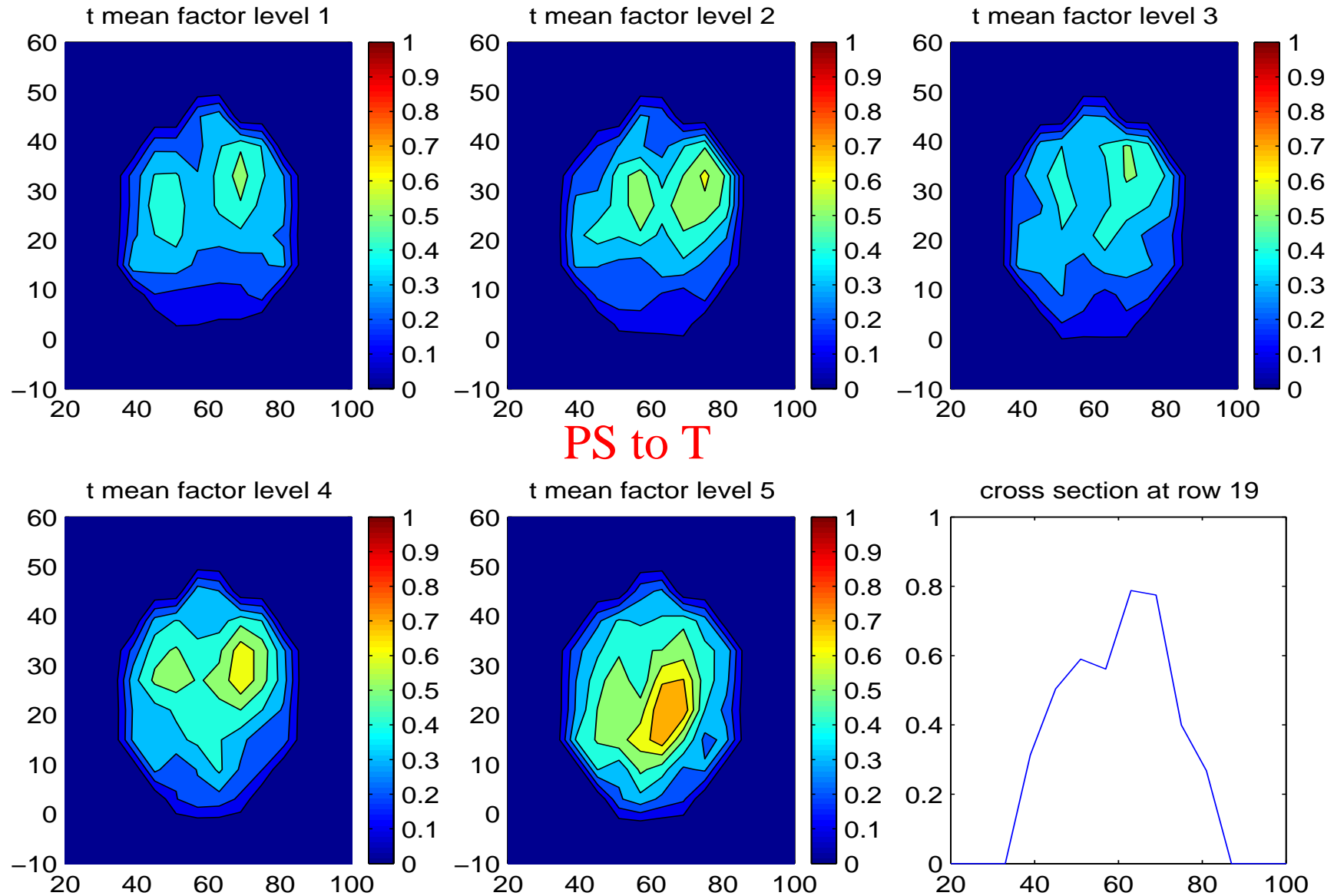
# Hierarchical Filter Regression Confidence Factors: PS Obs. at 20N, 60E



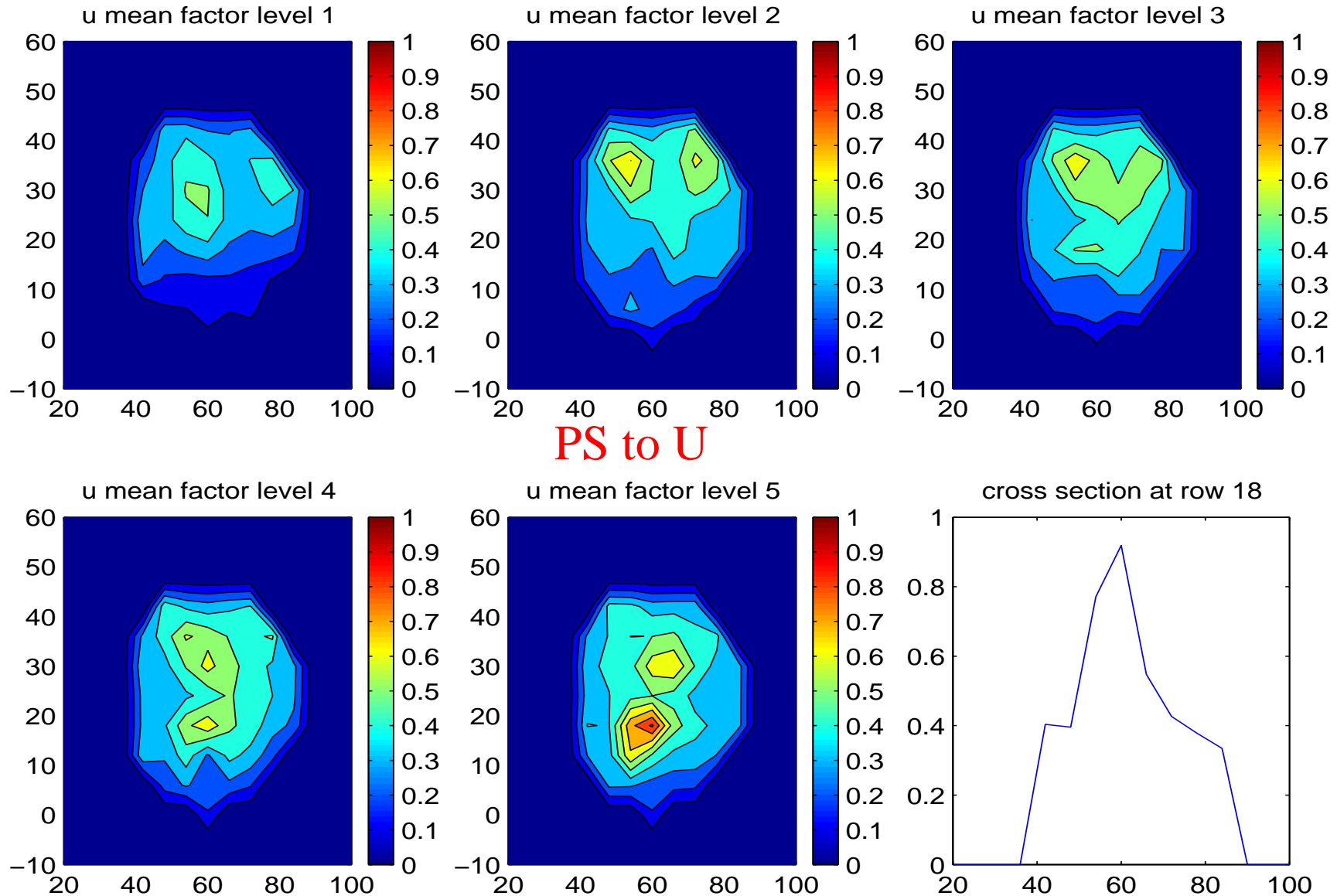
PS to PS



# Hierarchical Filter Regression Confidence Factors: PS Obs. at 20N, 60E



# Hierarchical Filter Regression Confidence Factors: PS Obs. at 20N, 60E



# Hierarchical Filter Regression Confidence Factors: PS Obs. at 20N, 60E

