Introduce Myself - Elizabeth Shamseldin

- PhD Candidate under Richard Smith at UNC-Chapel Hill
- Working with Bayesian methods in spatial statistics
- Opportunity to work on extreme value project at NCAR

Extreme Precipitation: An Application Modeling N-Year Return Levels at the Station Level

Presented by: Elizabeth Shamseldin
Joint work with: Richard Smith, Doug Nychka, Steve Sain, Dan Cooley

Statistics Group, IMAGe

National Center for Atmospheric Research

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Motivating Question

Extreme precipitation under various climate scenarios

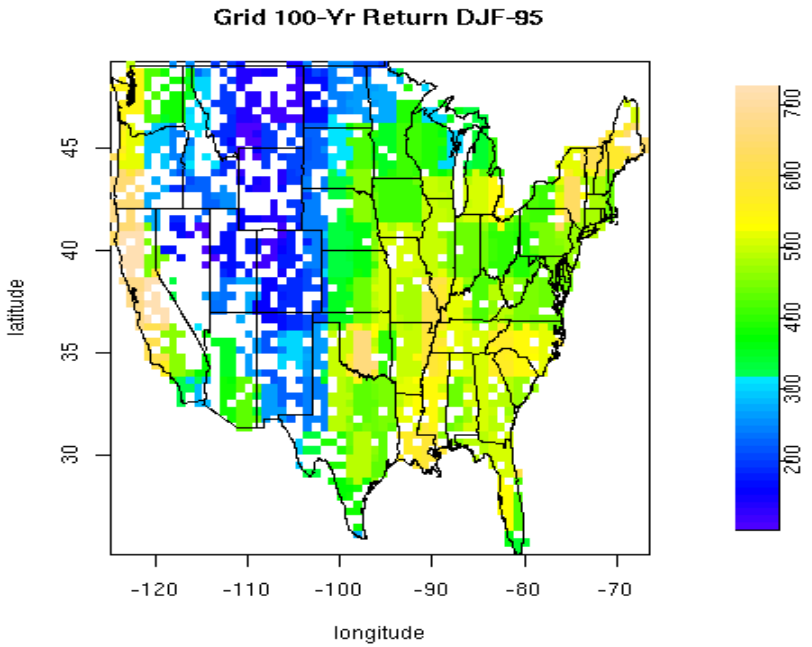
- Can regional climate model return level estimations be used to obtain return level predictions at the station level?
- Relationship between extremes of grid-cell data (re-analysis NCEP) and weather station data (NCDC)
- Leads to relationship between climate model data and point source data

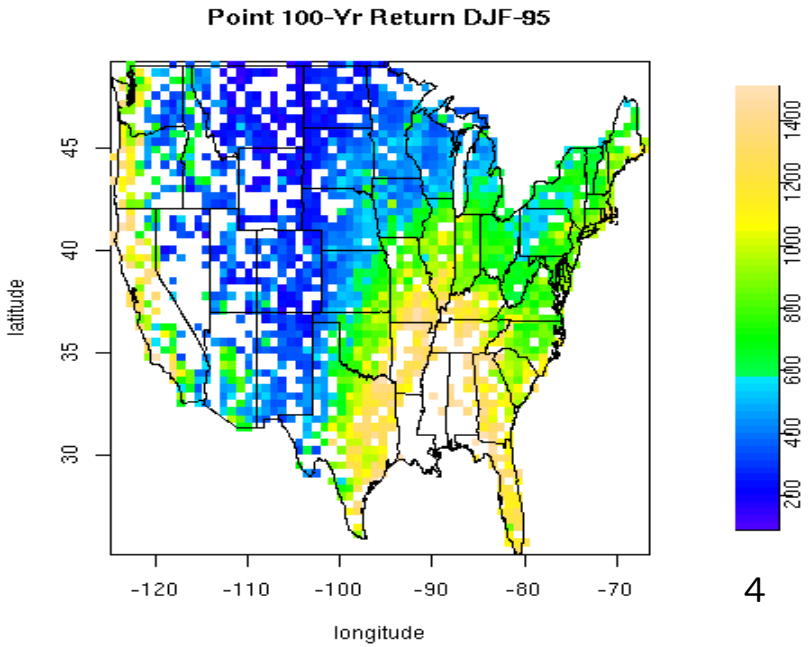
Data

- Point-source observational data from NCDC, originally obtained from Dr. Pavel Groisman
- Covers period 1950-1999 over 5873 stations.
- The data are daily rainfall values; units are tenths of a millimeter.
- Grid-cell data are from NCEP
- Covering period 1948–2003 with no missing data on 288 2.5^{o} grid cells, converted to the same units as the NCDC data.
- Rainfall values are considered over the four seasons
- Threshold values are determined by the 95^{th} and 97^{th} percentiles
- Clustering method is used to define peaks

Outline of Techniques

- Relationship of grid cell n-year return levels to the n-year return levels at point locations is explored
- Patterns are similar, scales differ
- Need model to translate between grid level and point level returns





Outline of Techniques

Tail of the Generalized Extreme Value distribution (GEV) is fit to the grid cell data above a given threshold

$$\Pr\{Y \leq y\} \;=\; \exp\left\{-\left(1+\xi\frac{y-\mu}{\psi}\right)_{+}^{-1/\xi}\right\} \qquad (1)$$

where Y is a random variable, μ is a location parameter, ψ a scale parameter and ξ is the extreme-value shape parameter; μ and ξ can take any value in $(-\infty, \infty)$ but ψ has to be > 0.

Technique

- Peaks Over Threshold model: considers $\Pr\{X \leq u + x \mid X > u\}$ for a given threshold u; the parameters are directly tied to the threshold value

- Point process approach is similar to the peaks over threshold method: all observations over a given threshold are considered. Tail of GEV is estimated.

- However, in the PP approach, the parameters are not tied to the threshold value.

Theory **Provided the model fits the data:**

- PP approach should produce equivalent parameter values as the POT approach

- Parameters are independent of the threshold (adjusting for estimation error)

- Ideal threshold determined by considering where the parameter values stabilize

Theory

Choose to model n-year return due to interpretability. *Event so extreme it is expected to happen once every n years:* ($\frac{1}{n}$ probability/yr)

The n-year return values can be directly obtained using the estimated GEV parameters. Define y_n by the equation:

$$\left(1+\xi\frac{y_n-\mu}{\psi}\right)^{-1/\xi} = \frac{1}{n}$$

which leads to the formula:

$$y_n = \begin{cases}\mu+\psi\frac{n^{\xi}-1}{\xi} & \text{if } \xi\neq 0,\\ \mu+\psi\log n & \text{if } \xi = 0.\end{cases} \tag{2}$$

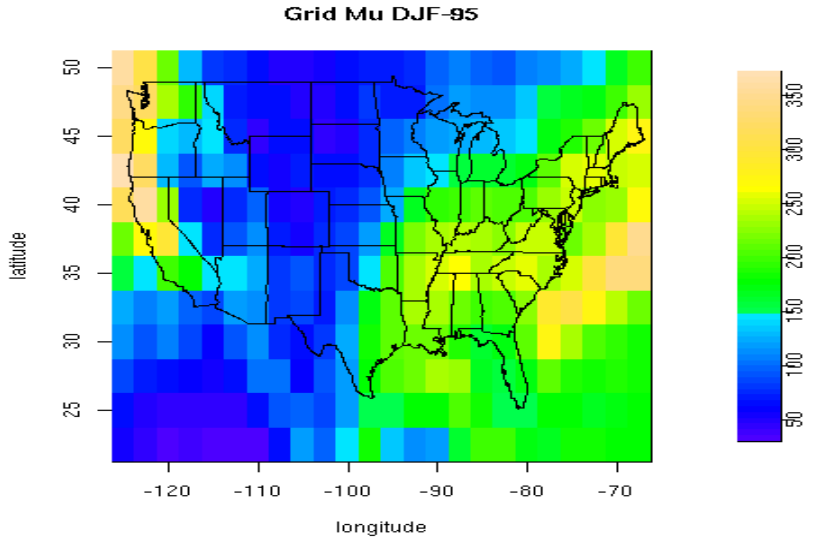
Recap

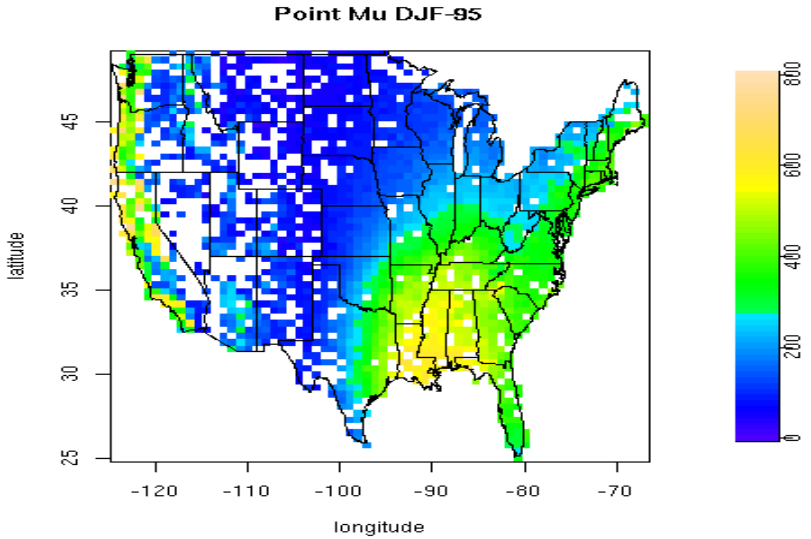
- GEV parameter estimates are obtained and checked through various diagnostics

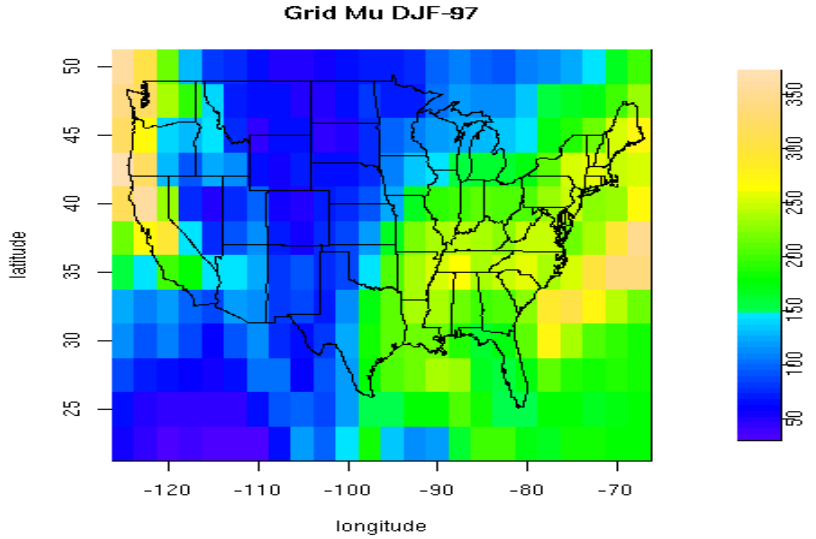
- GEV parameter estimates are used to generate n-year return levels at grid and station levels

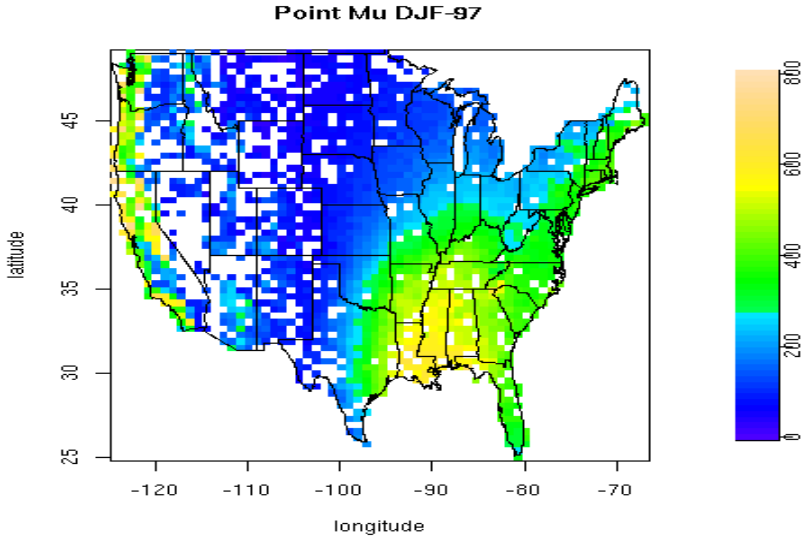
- Various models are explored to predict point location n-year return from grid cell n-year return

GEV Model Fit - Mu Parameter: 95 Grid - Point vs 97 Grid - Point

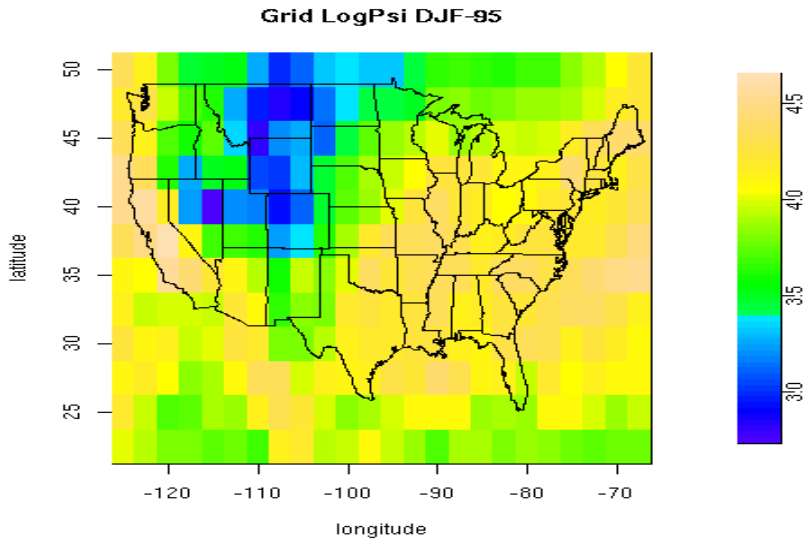


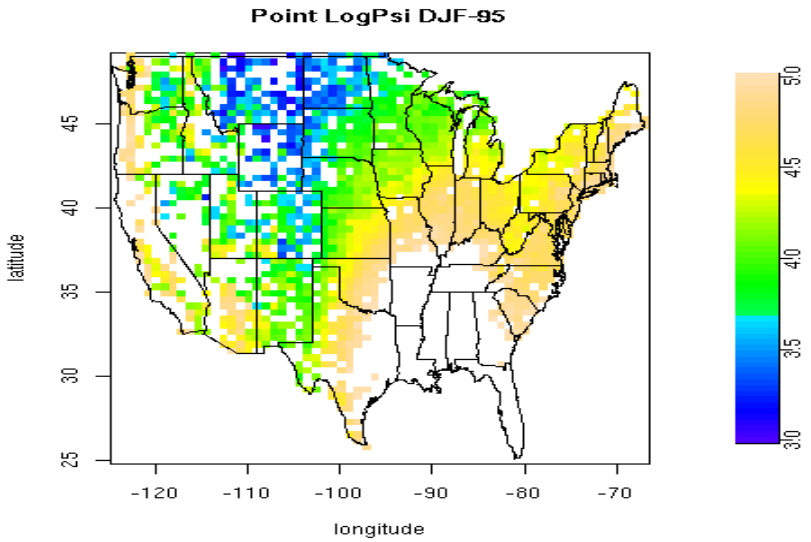


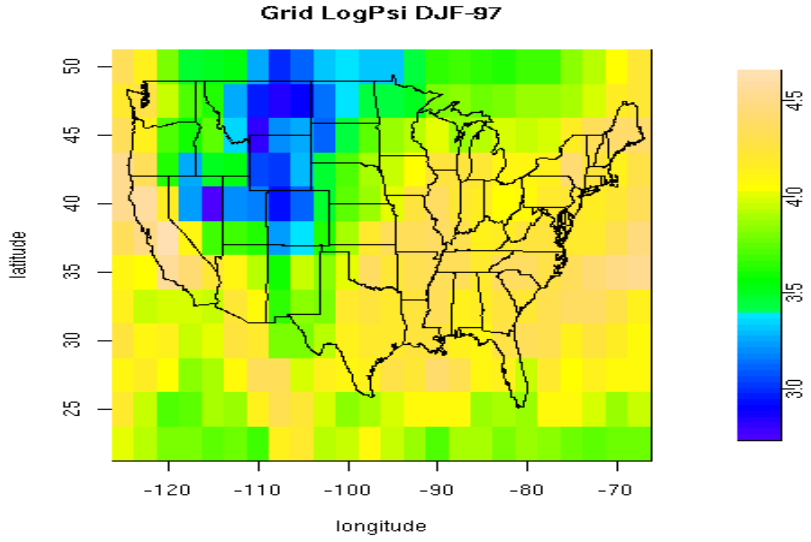


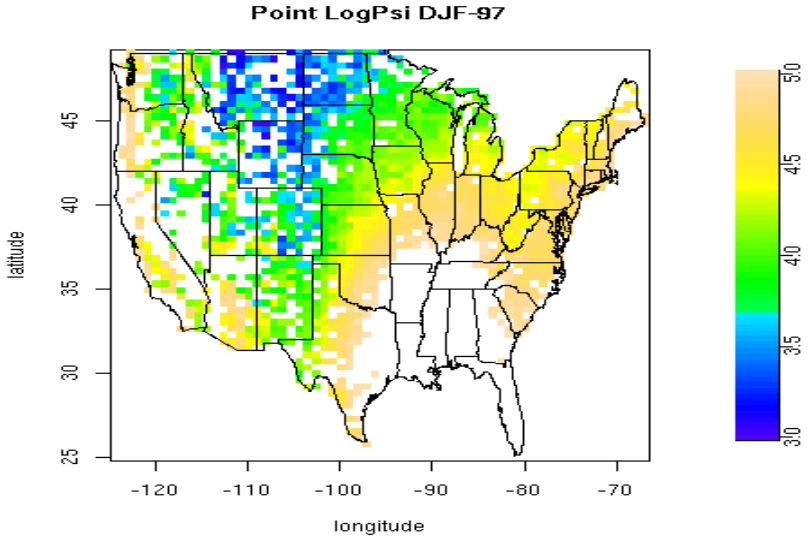


GEV Model Fit - LogPsi Parameter: 95 Grid - Point vs 97 Grid - Point

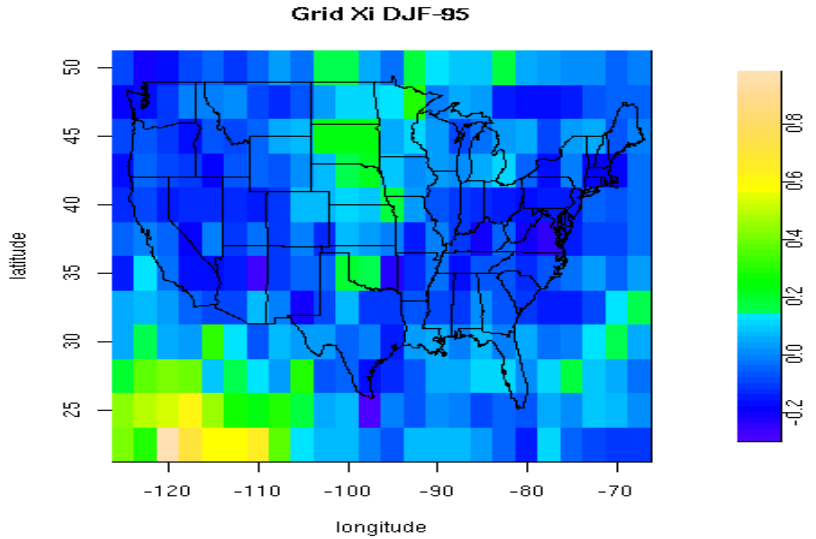


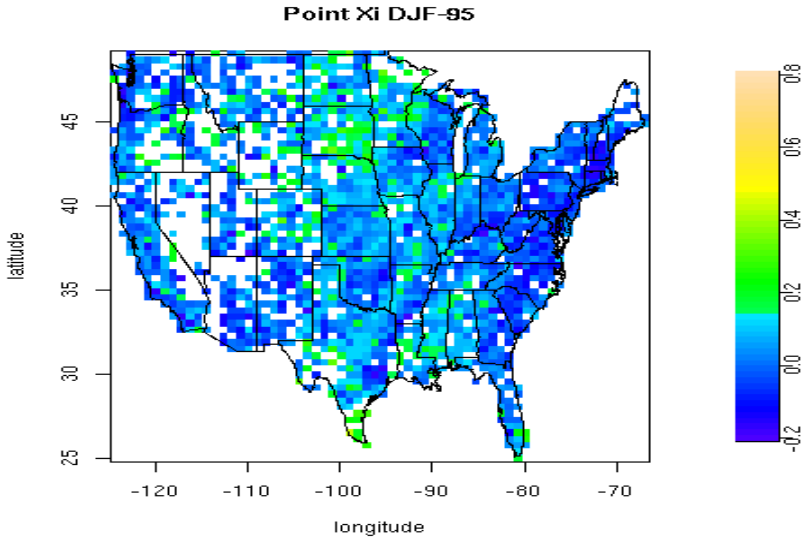


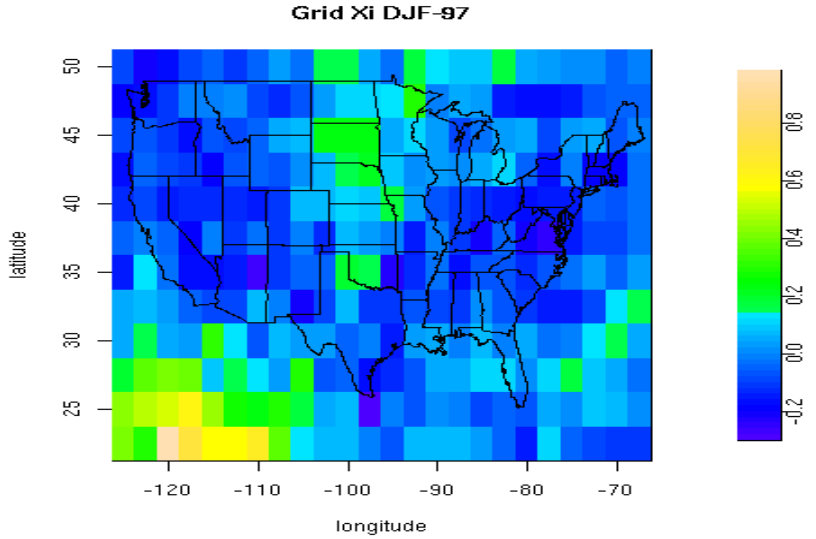


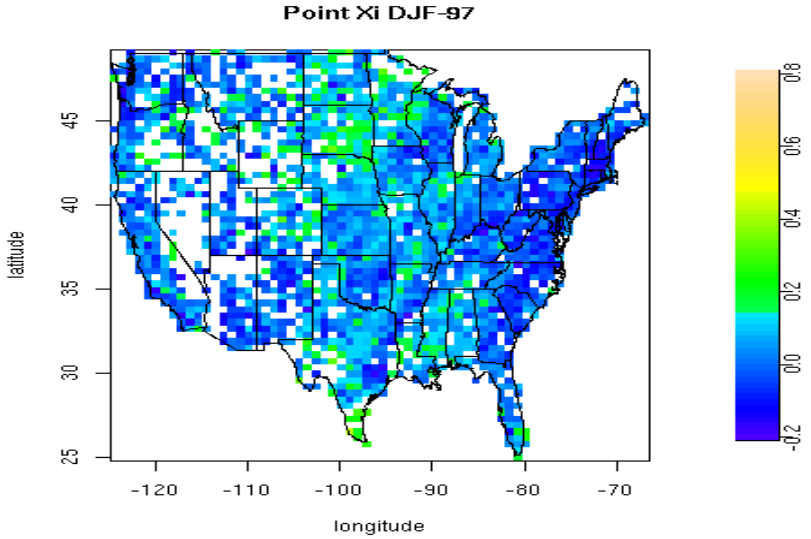


GEV Model Fit - Xi Parameter: 95 Grid - Point vs 97 Grid - Point

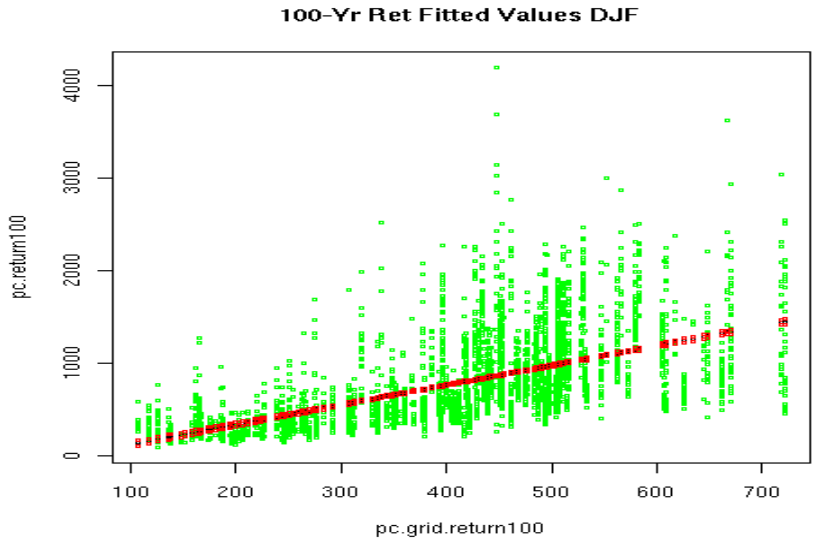


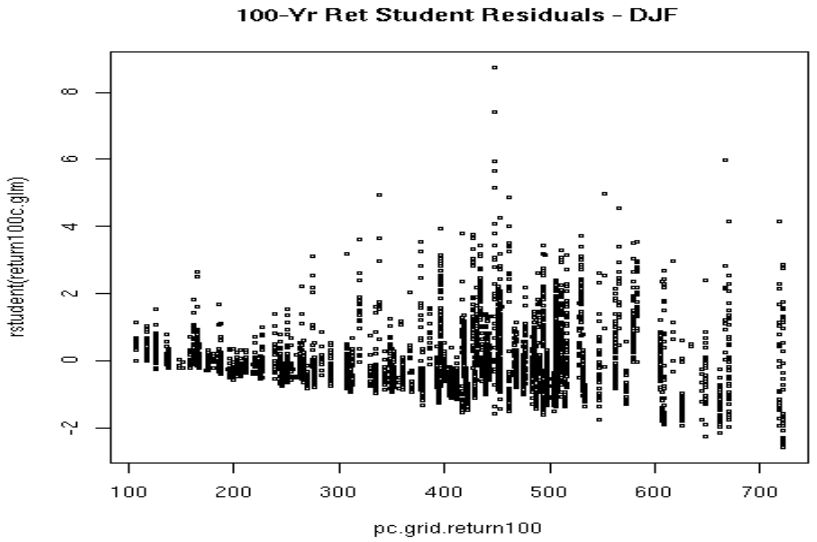


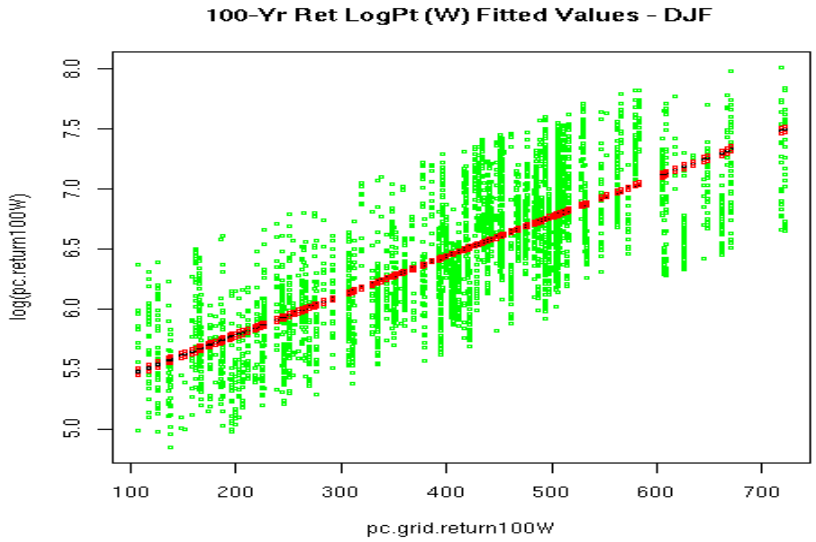


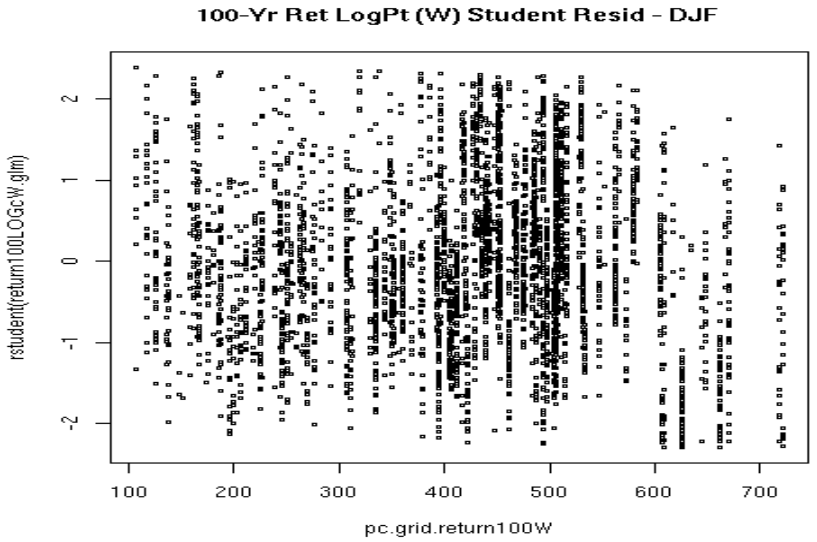


100-Year Return: Point vs Grid - LogPoint vs Grid

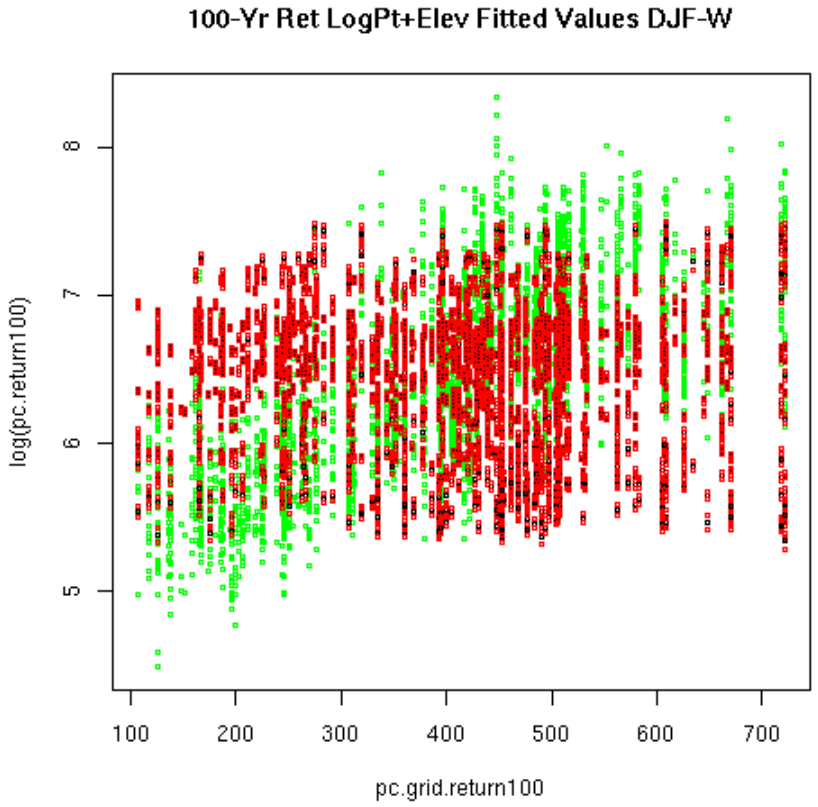


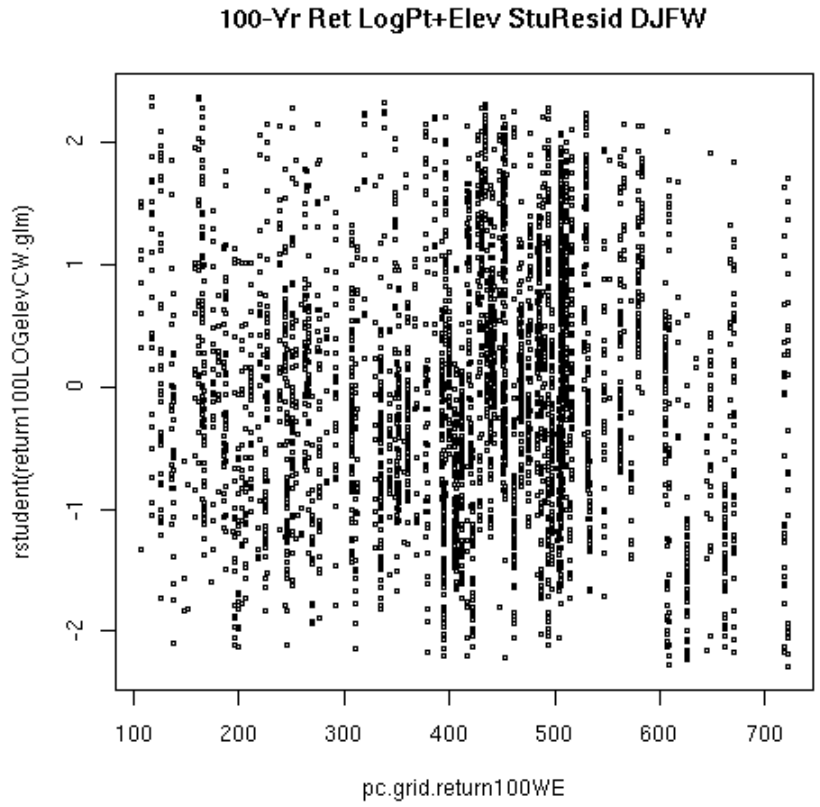






100-Year Return: LogPoint + Elevation vs Grid



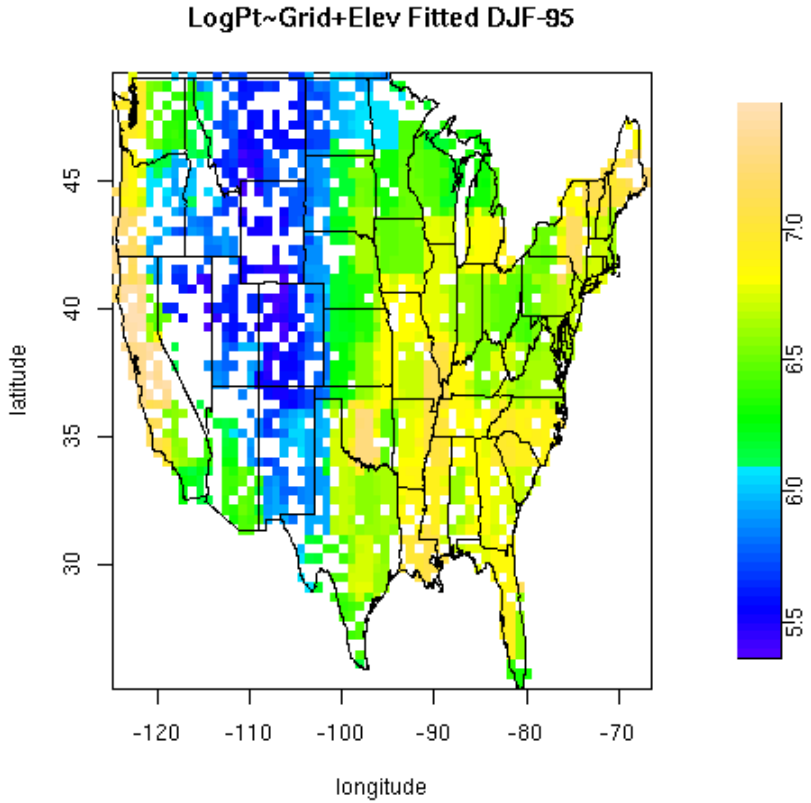


Model Results and Comparison Univariate Regression Predicting Point Locations 100-Year Return Level Using Grid Cell 100-Year Return

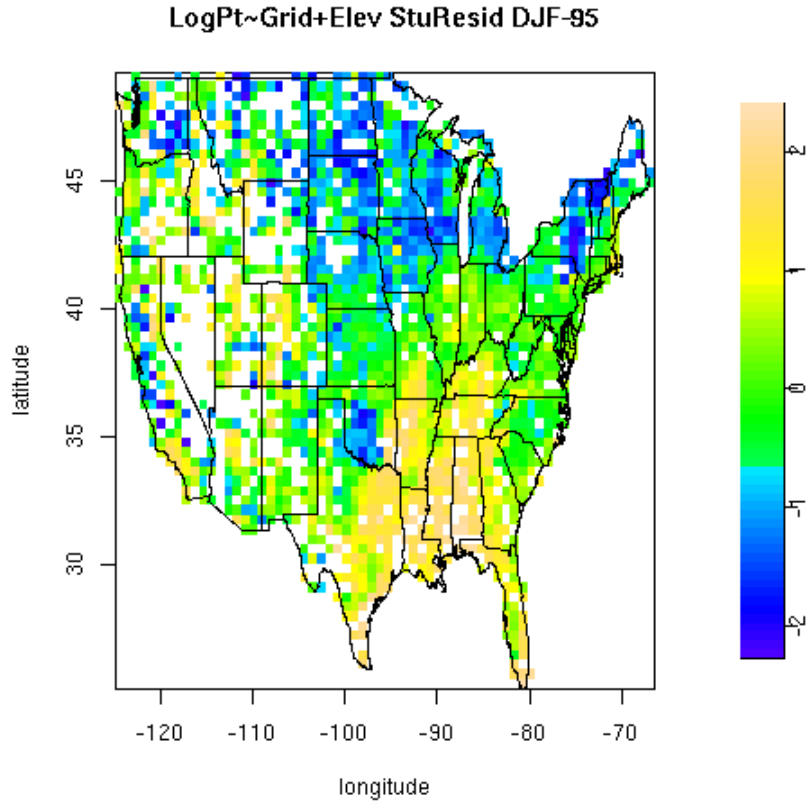
WINTER - 100	OBS	AIC	Intercept	X1	X2
log(Pt) Grid+Elev	4025	3425	5.3186	0.0030	-0.000124
97 log(Pt) Grid+Elev	4013	3356	5.3074	0.0030	-0.000113
SPRING - 100	OBS	AIC	Intercept	X1	X2
log(Pt) Grid+Elev	4138	1924	5.8960	0.0023	-0.000258
97 log(Pt) Grid+Elev	4145	2282	5.9736	0.0021	-0.000281

100-Year Return: LogPoint ~ Grid + Elevation Spatial Trend

Fitted 100-Yr Return Values

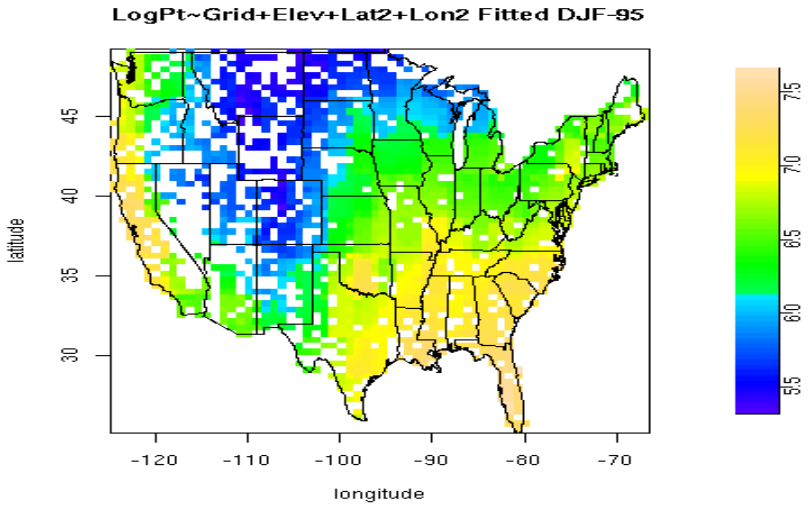


Residuals

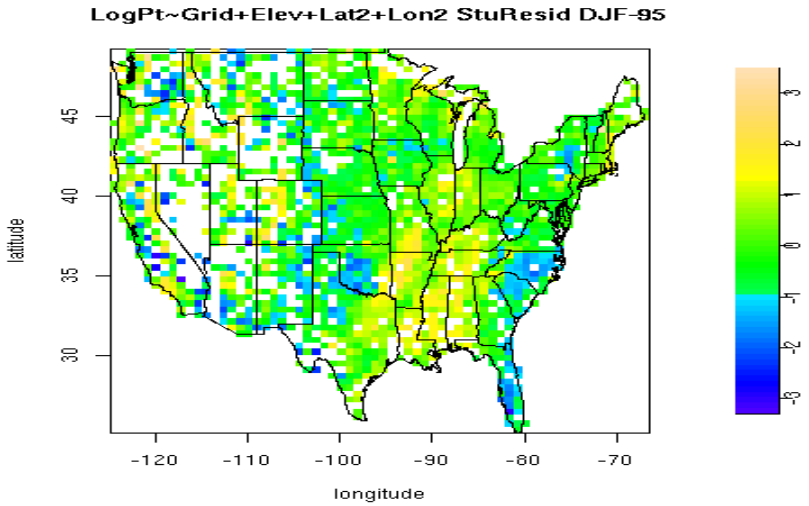


100-Year Return: LogPt $\sim$ Grid + Elevation Including Lat and Lon

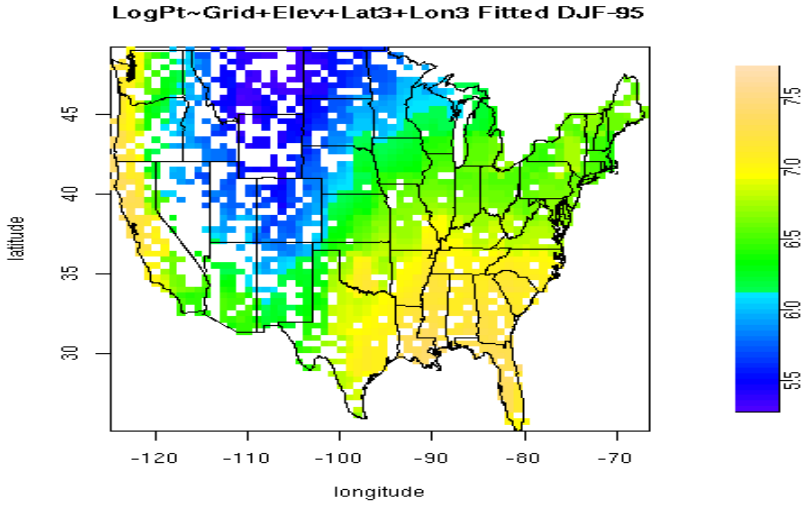
Quadratic Model



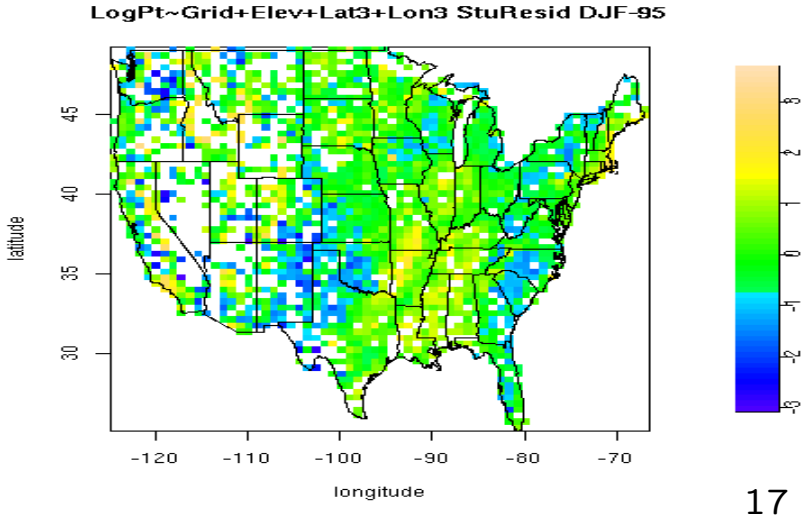
Residuals



Cubic Model

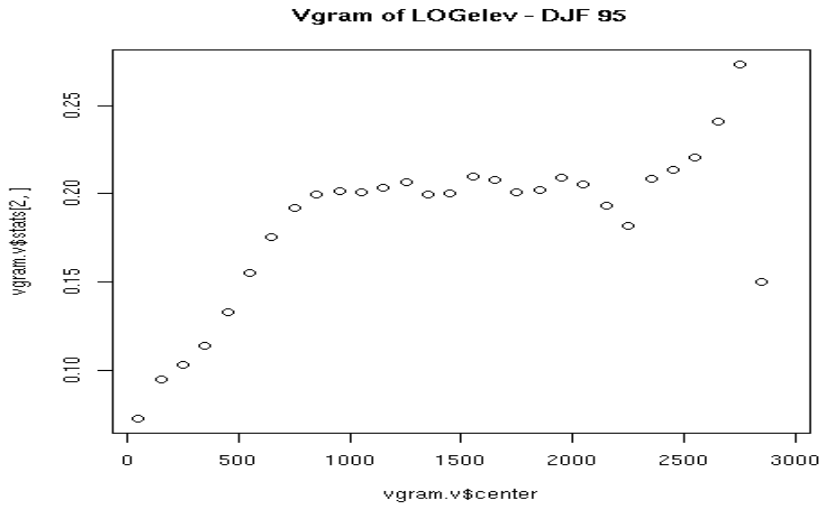


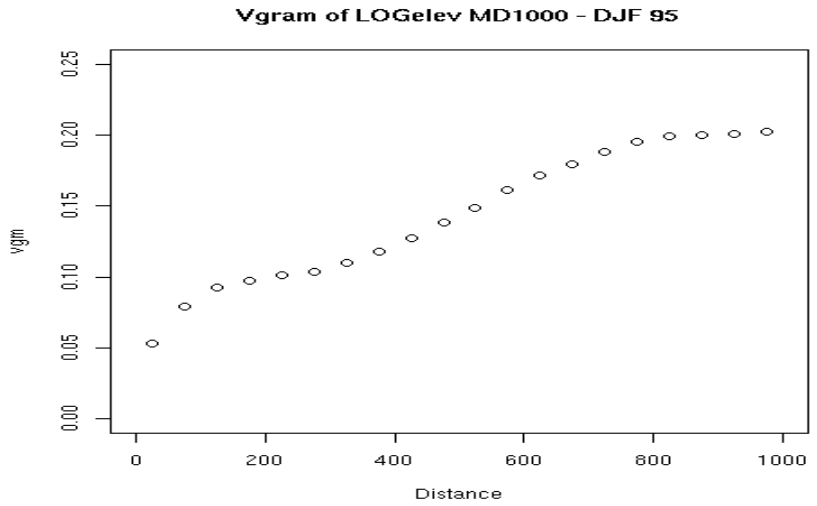
Residuals



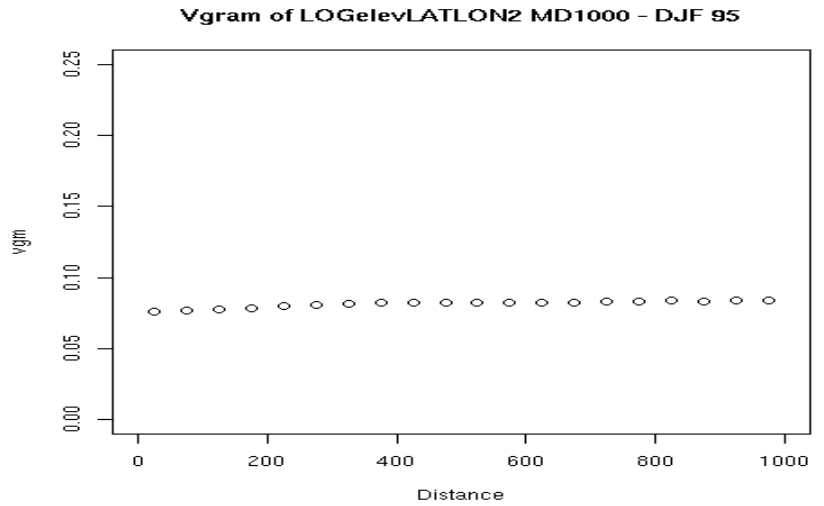
100-Year Return: LogPt ∼ Grid + Elevation Including Lat and Lon

Variograms of Log(Pt)+Elevation

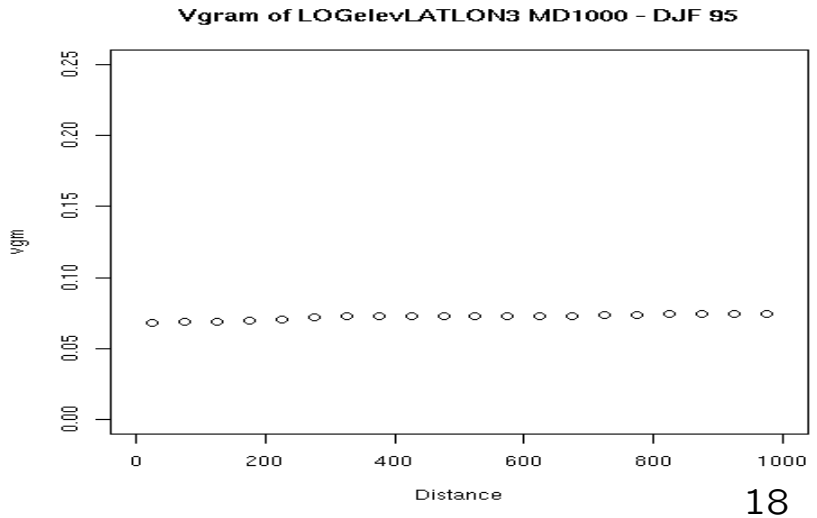




Quadratic Variogram



Cubic Variogram



Other Models Considered

Cubic vs Quartic

- Cubic and quartic models show little difference in the remaining spatial trend
- Cubic model: all terms and interaction terms are significant in the model
- Quartic model: some terms are no longer significant, including elevation
- AIC vs BIC

Elevation

- Quadratic models in elevation: did not resolve the spatial correlation among station return levels as well as the higher order models in latitude and longitude

Conclusion and Future Work

- Modeling the tail of the GEV Distribution appears to produce stable GEV parameter estimates and model coefficients within seasons and across 95% and 97% thresholds

- 100 year return levels are successfully modeled by season at the point (station) level using grid-level return values, station elevation, and station latitude and longitude coordinates

- The regression relationship between climate model predictions and future modeled extremes is best expressed through a cubic model in latitude and longitude to account for the spatial correlation, with $R^2 = 0.77$ and standard errors within 1.008 and 1.056 tenths of a millimeter.

- Future work includes plans to test grid-point models on RCM and CCSM data

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Theory

To take account of these deficiencies, an alternative class of methods has been developed, often known as the *Peaks Over Thresholds* (POT) approach. For the distribution of the excess values, a common family of probability density functions is the *Generalized Pareto Distribution* (GPD), introduced by Pickands (1975), and given by

$$\Pr\{X \leq u + x \mid X > u\} \;=\; 1 - \left(1 + \xi\frac{x}{\sigma}\right)_{+}^{-1/\xi}. \qquad (3)$$

One drawback in the POT model is that the parameters are directly tied to the threshold value, u.

A third approach, the *point process approach* (Smith 1989, 2003, Coles 2001), although operationally very similar to the POT approach, uses a representation of the probability distribution that leads directly to the GEV parameters (μ, ψ, ξ).

Theory

The Point Process method considers N peaks, $Y_1 ... Y_N$, observed at times $T_1 ... T_N$. Pairs are viewed as points in the space $[0, T] \times (u, \infty)$ (u=threshold), which form a nonhomogeneous Poisson process with intensity measure:

$$\lambda(t, y) = \frac{1}{\psi}\left(1 + \xi\frac{(y-\mu)}{\psi}\right)_+^{\frac{-1}{\xi}-1}$$

Theory

By standard formulae for a Poisson Process, the likelihood is of the form:

$$\begin{aligned} L(\mu,\psi,\xi) &= \prod_{i=1}^{N} \lambda(T_i,Y_i)\cdot \exp\left\{-\int_0^T\int_u^\infty \lambda(t,y)dtdy\right\} \\ &= \prod_{i=1}^{N} \lambda(T_i,X_i)\cdot \exp\left\{-T\left(1+\xi\frac{u-\mu}{\psi}\right)_+^{-1/\xi}\right\}. \end{aligned} \quad (4)$$

Theory

In practice we work with the negative log likelihood, $\ell = -\log L$, which leads to

$$\begin{aligned} \ell(\mu, \psi, \xi) &= N \log \psi + \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{N} \log \left(1 + \xi \frac{Y_i - \mu}{\psi}\right)_+ \\ &+ T \left(1 + \xi \frac{u - \mu}{\psi}\right)_+^{-1/\xi} \end{aligned} \tag{5}$$

where T is the length of the observation period in years and the $(...)_+$ symbols essentially mean that the expression are only evaluated if $1 + \xi \frac{u-\mu}{\psi} > 0$ and $1 + \xi \frac{Y_i - \mu}{\psi} > 0$ for each i (if these constraints are violated, L is automatically set to 0).

Theory

- The basic method of estimation is therefore to choose the parameters (μ, ψ, ξ) to maximize (**??**) or equivalently to minimize (5).

  This is performed by numerical nonlinear optimization.

- In practice it is convenient to replace (μ, ψ, ξ) by $(\theta_1, \theta_2, \theta_3)$ where $\theta_1 = \mu,\ \theta_2 = \log \psi,\ \theta_3 = \xi$ (defining θ_2 to be $\log \psi$ rather than ψ itself makes the algorithm more numerically stable, and has the advantage that we don't have to build the constraint $\psi > 0$ explicitly into the optimization procedure).