A Generalized Linear Modeling Approach to Stochastic Weather Generators

and

The Kriging Estimator as a Local Smoother

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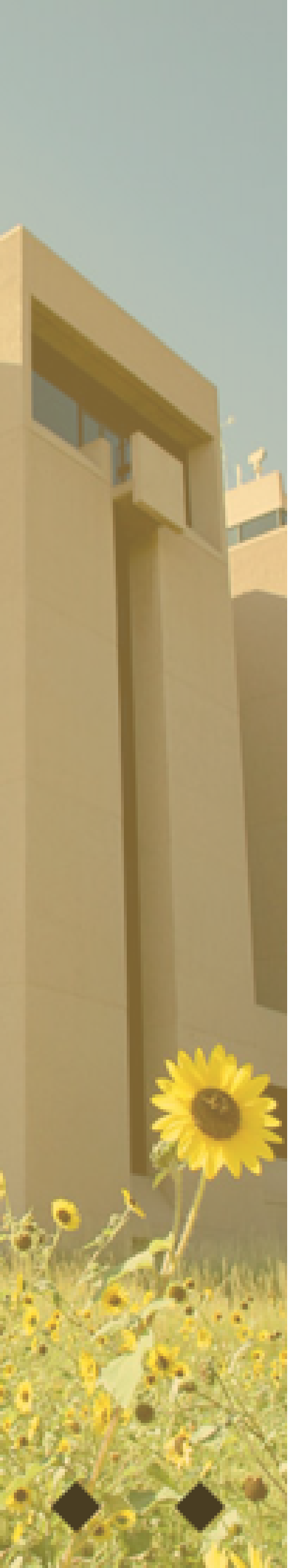
Advisory Panel Meeting, April 27 2006



Personal Background

- Dissertation at the Swiss Federal Institute of Technology in Lausanne under the supervision of Prof. Stephan Morgenthaler (Oct 2001):

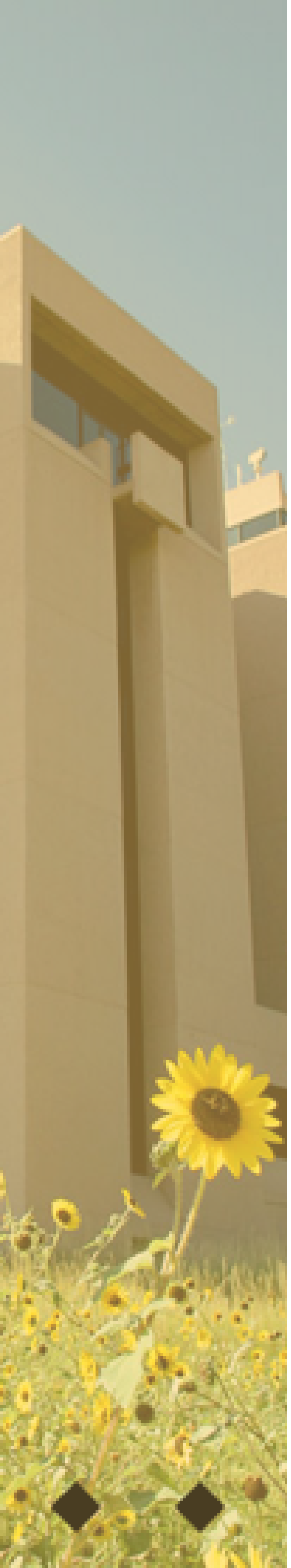
  'Estimating Cumulative Distributions by Spline Functions'

- Consulting statistician (for food scientists, chemists, biologists, consumer experts etc.) at the Nestlé Research Center in Lausanne (Nov 2001 – July 2003):

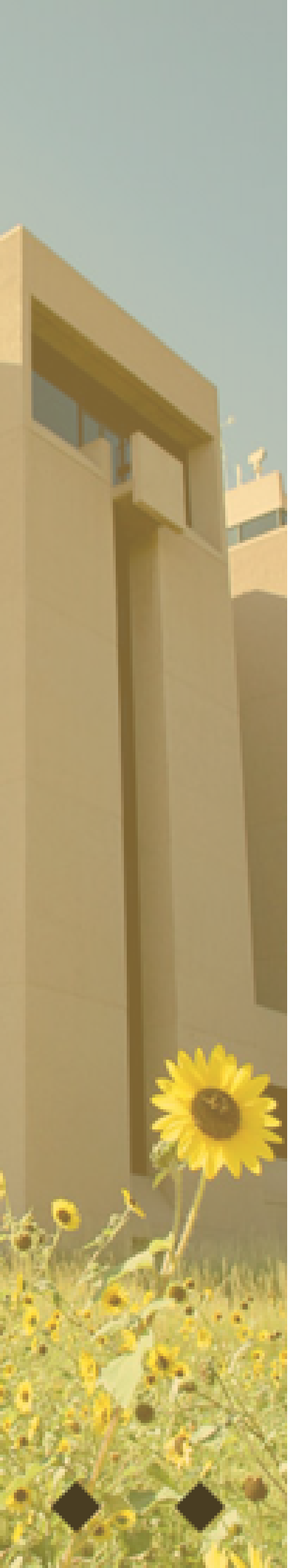
  Experimental designs & analysis of results, visualization, multivariate techniques common to consumer science

- Postdoctoral researcher, chair of statistical hydrology, National Scientific Research Institute, Water, Earth and Environment, Québec (Sep 2003 – March 2005):

  Nonstationarity in regional frequency analysis, short term discharge prediction using a hidden Markov model



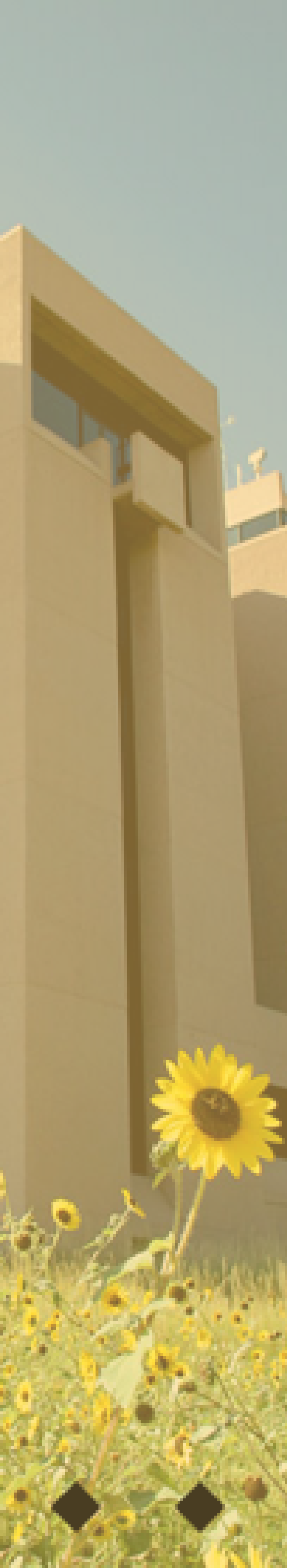
A Generalized Linear Modeling Approach to Stochastic Weather Generators



NSF Project

Understanding and Modeling the Scope for Adaptive Management in Agroecosystems in the Argentine Pampas in Response to Inter-annual and Decadal Climate Variability and Other Risk Factors

Biocomplexity in the Environment / Dynamics of Coupled Natural and Human Systems



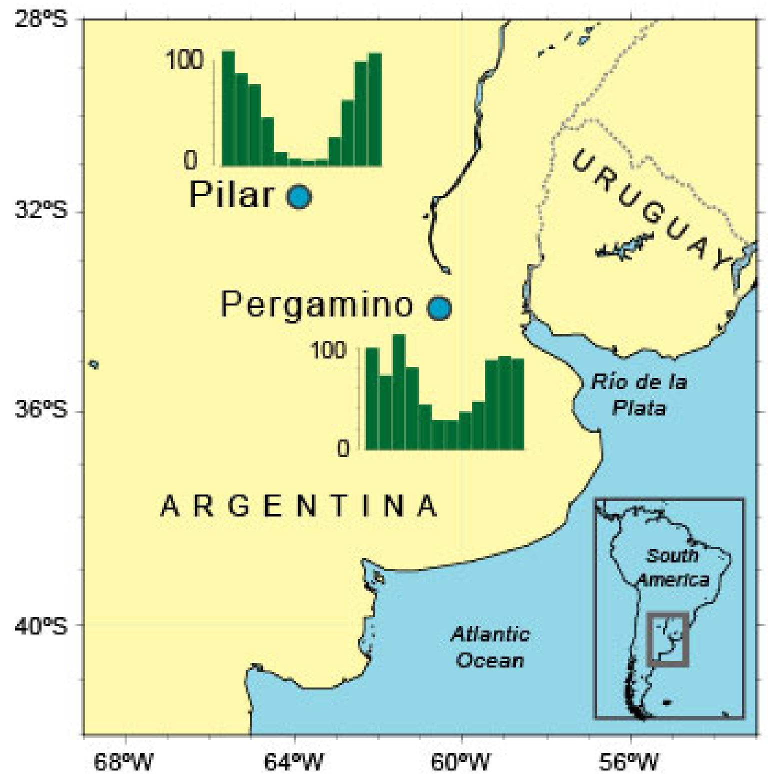
(Selected) Project Objectives

1. Build plausible scenarios of inter-annual and inter-decadal climate variability.
   $\Rightarrow$ weather generator

2. Explore best practices for the characterization of uncertainty, and the design and communication of climate information.
   $\Rightarrow$ probabilistic treatment & ?

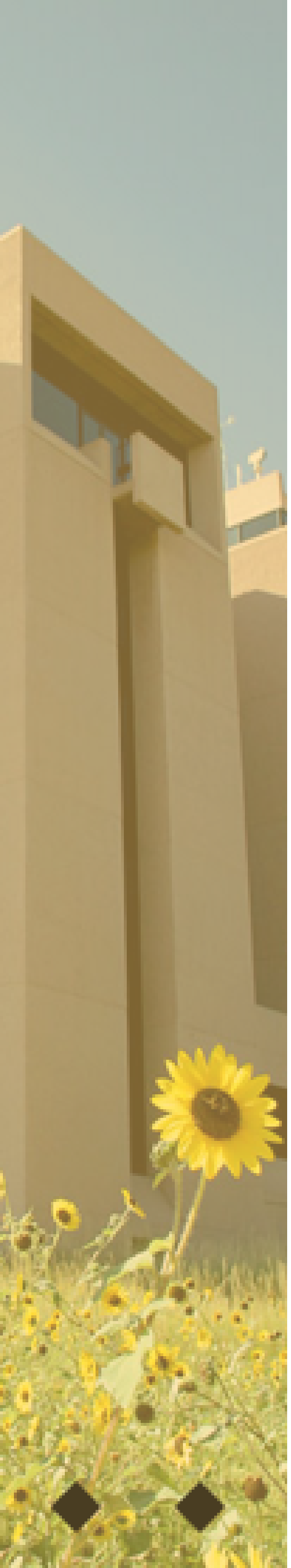
Location: Argentine Pampas

Marked interannual (ENSO) and inter-decadal (increase in precipitation since 1970) climate signals.



Pergamino:
near-optimal conditions

Pilar:
marginal conditions (semi-arid)



Classical Weather Generator

Generates consistent daily weather series based on data from the target location by:

- Two-state (wet and dry), first order Markov chain for precipitation occurrence.

- Gamma distribution for precipitation intensity: conditional on occurrence, independent of previous occurrence or intensity.

- 1) Standardization of minimum and maximum temperature series conditional on precipitation occurrence.

  2) Bivariate AR(1) model for the standardized series, normality assumption satisfied at Pergamino.

Precipitation: Occurrence

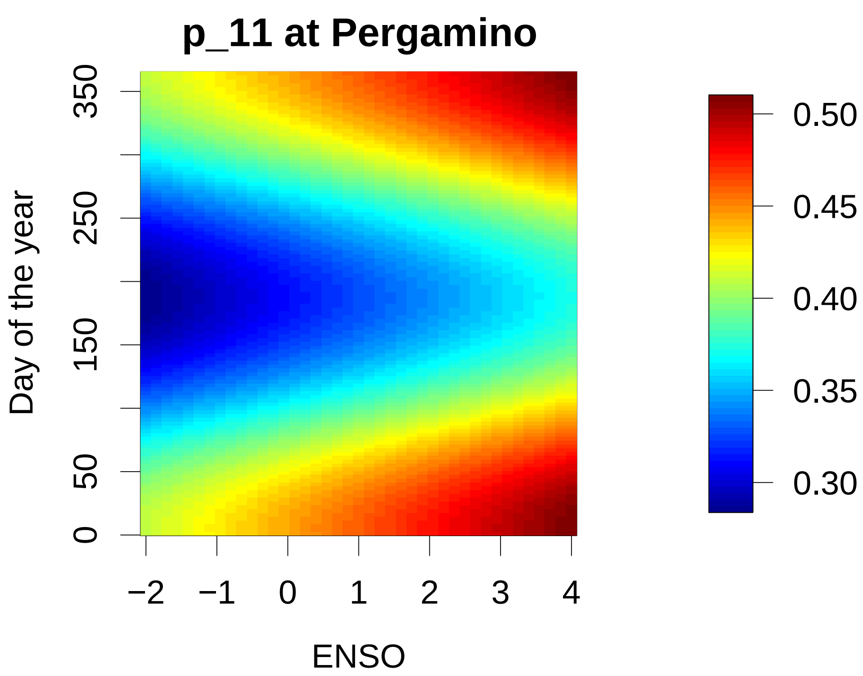
Model: binomial with log link

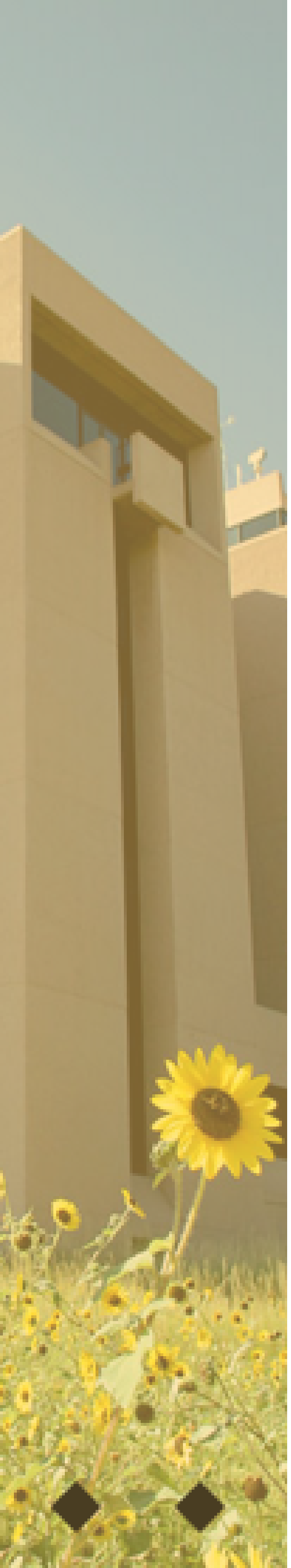
$$\log\left(\frac{p_k}{1-p_k}\right) = \mathbf{x}_k^T \boldsymbol{\beta}$$

where p_k probability of rain for the kth record

$\mathbf{x}_k$ covariates (previous occurrence, season, ENSO)

$\boldsymbol{\beta}$ parameter vector





Precipitation: Intensity

Model: Gamma with log link

$$\log\left(\mu_k\right) = \boldsymbol{\xi}_k^T \gamma$$

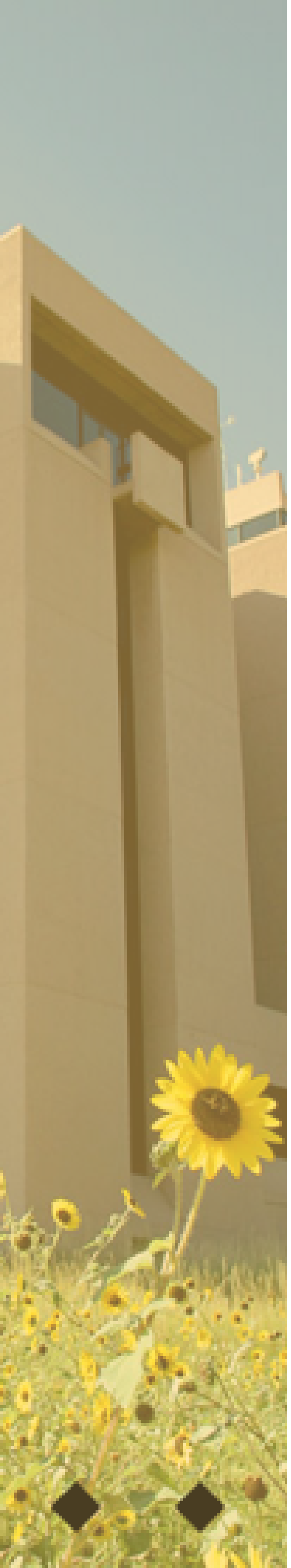
where μ_k mean precipitation intensity for the kth record (if wet)

$\boldsymbol{\xi}_k$ covariates (season, ENSO)

γ parameter vector

$\Rightarrow$ constant coefficient of variation

Only seasonal cycle is significant.



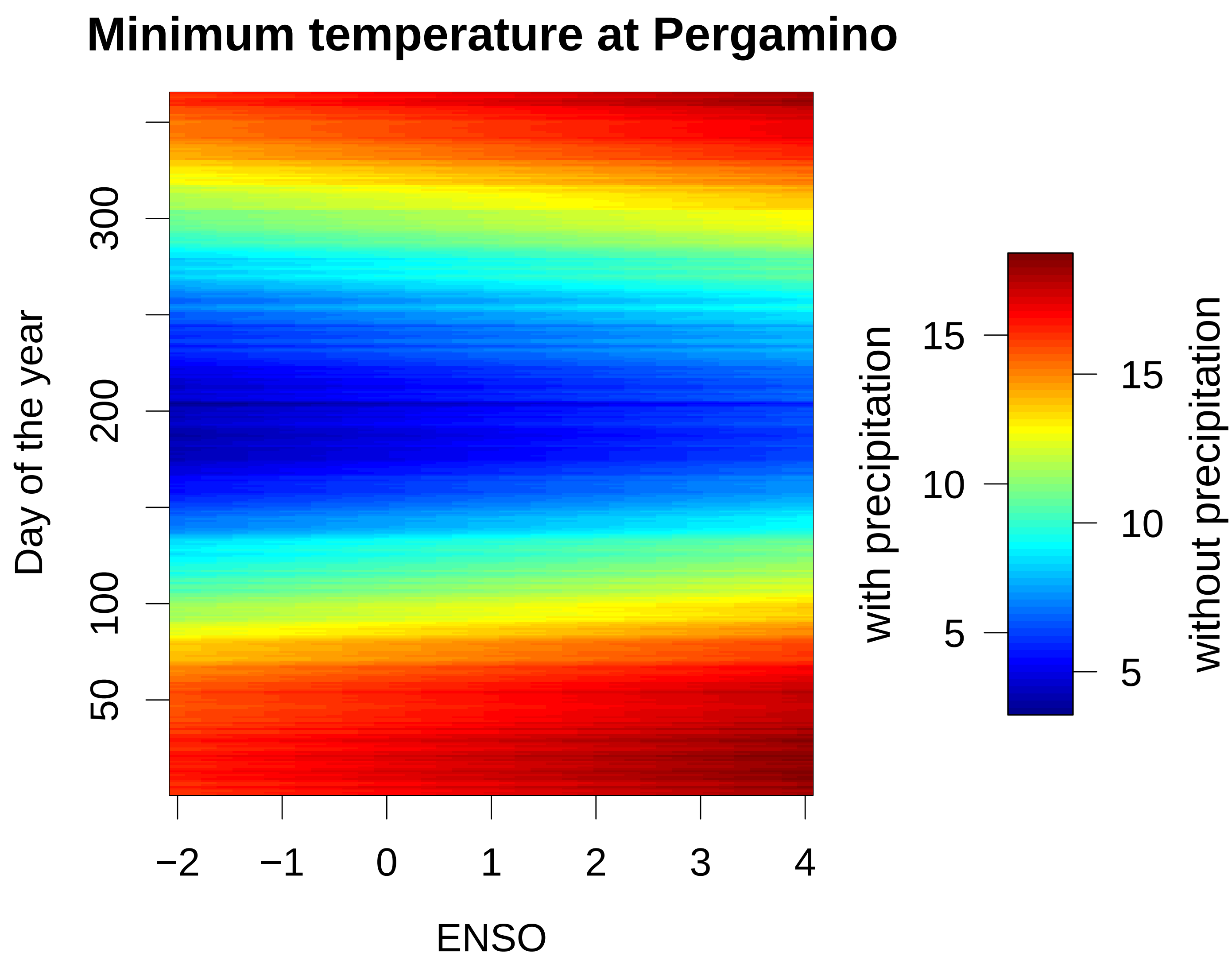
Temperature: Model

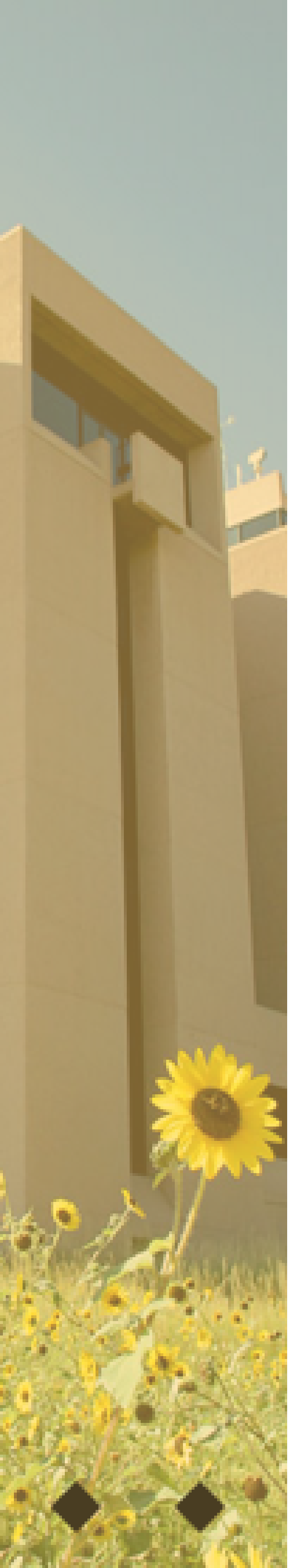
Model:
$$X_t = \mu_{X,0} + \mu_{X,1} J_t + \psi_X Y_{t-1} + \boldsymbol{\xi}_k^T \boldsymbol{\gamma}_X + \varepsilon_{X,t}$$
$$\varepsilon_{X,t} = \phi_X \varepsilon_{X,t-1} + u_{X,t}$$

$$Y_t = \mu_{Y,0} + \mu_{Y,1} J_t + \psi_Y X_t + \boldsymbol{\xi}_k^T \boldsymbol{\gamma}_Y + \varepsilon_{Y,t}$$
$$\varepsilon_{Y,t} = \phi_Y \varepsilon_{Y,t-1} + u_{Y,t}$$

where	X_t, Y_t	daily minimum and maximum temperature
	J_t	precipitation occurrence, 1=wet, 0=dry
	$\varepsilon_{X,t}$, $\varepsilon_{Y,t}$	AR(1)
	$u_{X,t}$, $u_{Y,t}$	$\mathcal{N}(0, \sigma_X^2)$ resp. $\mathcal{N}(0, \sigma_Y^2)$, uncorrelated
	$\boldsymbol{\xi}_k$	covariates (season, ENSO)

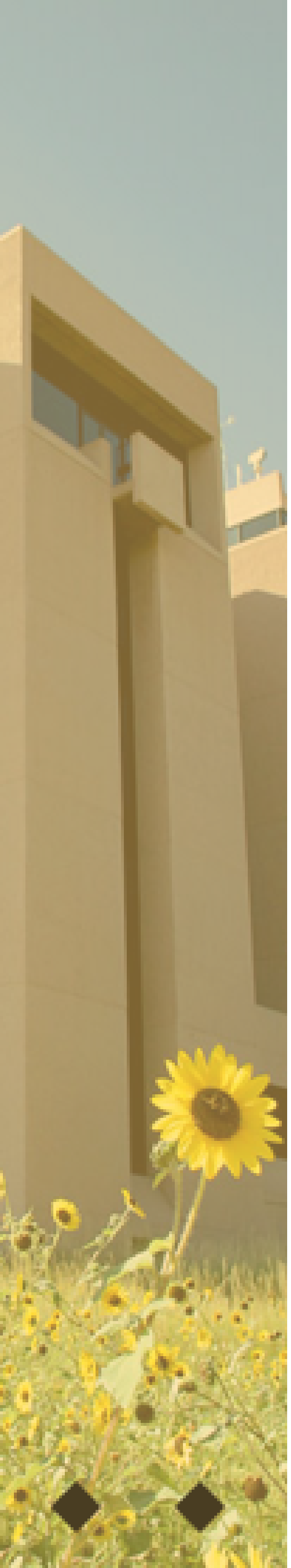
Temperature: Illustration



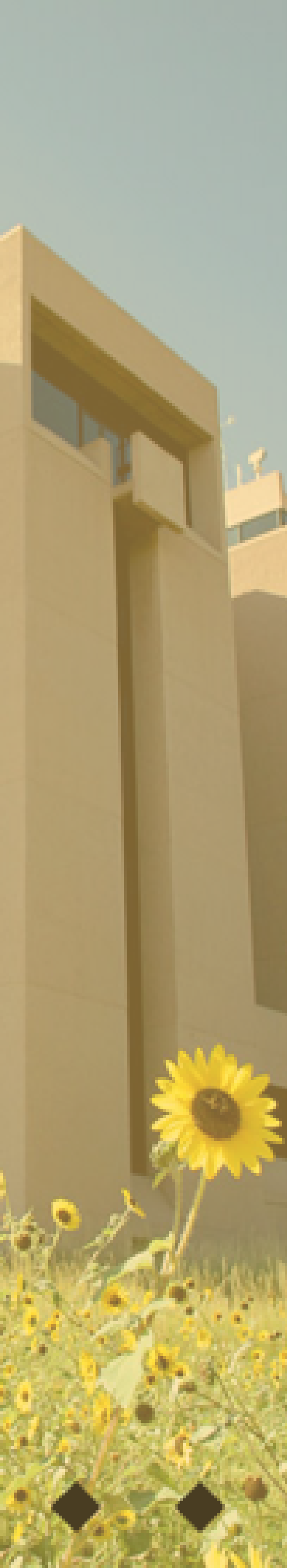


To Do

- Generate weather series from the generalized weather generator.
- Provide “usable” programming of the generalized weather generator, for example as an R-package.
- Take into account parameter uncertainty in generating climate scenarios.
- Improve treatment of precipitation/temperature extremes.
- Model the dependence of the variability of the temperature variables on the season (or on covariates).



The Kriging Estimator as a Local Smoother



Splines as Local Smoothers

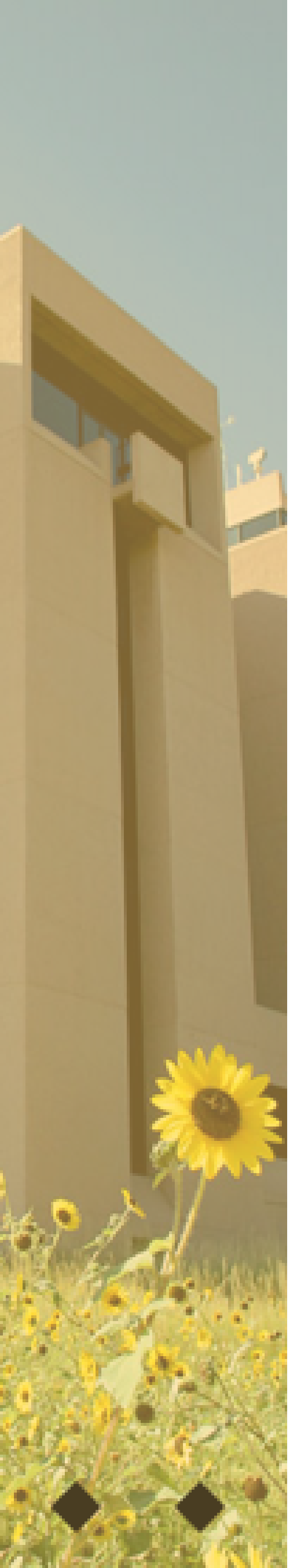
Model: $y_i = g(x_i) + \varepsilon_i, \qquad \varepsilon_i$ iid $\mathcal{N}(0, \sigma^2)$

Estimation: $\min\limits_{f \in \mathcal{H}} \left[\frac{1}{n} \sum\limits_{i=1}^{n} \big(y_i - f(x_i)\big)^2 + \lambda \int_0^1 f''(x)^2 \mathrm{d}x \right]$

Solution: $\widehat{g}(x) = \frac{1}{n} \sum\limits_{i=1}^{n} \omega(x, x_i) y_i$ (spline weight fcn)

Approximation: $\omega(x, x_i) \approx G_\lambda(x, x_i)$ reproducing kernel
under conditions on $x_1, \ldots, x_n$

Asymptotics: Exponential envelope condition for G_λ
$\Rightarrow$ asymptotic bias and variance of $\widehat{g}$



Kriging in "Spline Form"

Model: $y_i = g(\mathbf{x}_i) + \varepsilon_i,$

$g \sim \mathcal{MN}(\mathbf{0}, \rho\mathbf{K}) \perp \varepsilon_i \sim \mathcal{MN}(\mathbf{0}, \sigma^2\mathbf{I}),$

$\mathrm{Cov}(g(\mathbf{x}), g(\mathbf{x}')) = \rho k(\mathbf{x}, \mathbf{x}')$

Estimation: $$\min_{f \in \mathcal{H}} \left[\frac{1}{n} \sum_{i=1}^{n} \Big(y_i - f(\mathbf{x}_i) \Big)^2 + \lambda \langle f, f \rangle_{\mathrm{P}} \right]$$

Solution: $$\widehat{g}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \omega(\mathbf{x}, \mathbf{x}_i) y_i$$ (kriging weight fcn)

Approximation: $\omega(\mathbf{x}, \mathbf{x}_i) \approx G_\lambda(\mathbf{x}, \mathbf{x}_i)$ reproducing kernel

under conditions on $\mathbf{x}_1, \ldots, \mathbf{x}_n$

Asymptotics: Exponential envelope condition for G_λ

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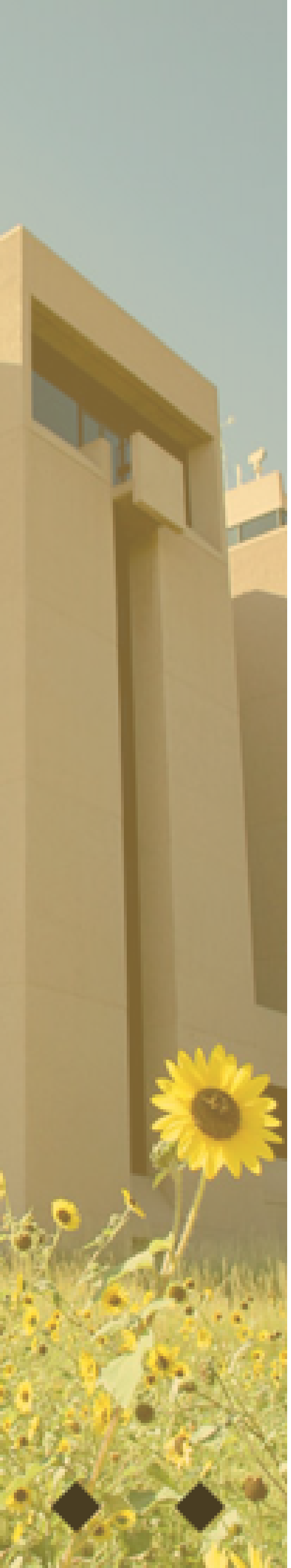


Obstacle

Exponential envelope condition for G_λ?

Problem: G_λ known through its Fourier transform

$$\widehat{G}_\lambda(\omega) = \left(1 + \lambda \frac{1}{\widehat{k}(\omega)}\right)^{-1}$$

Approach: Matern class of covariance fcns

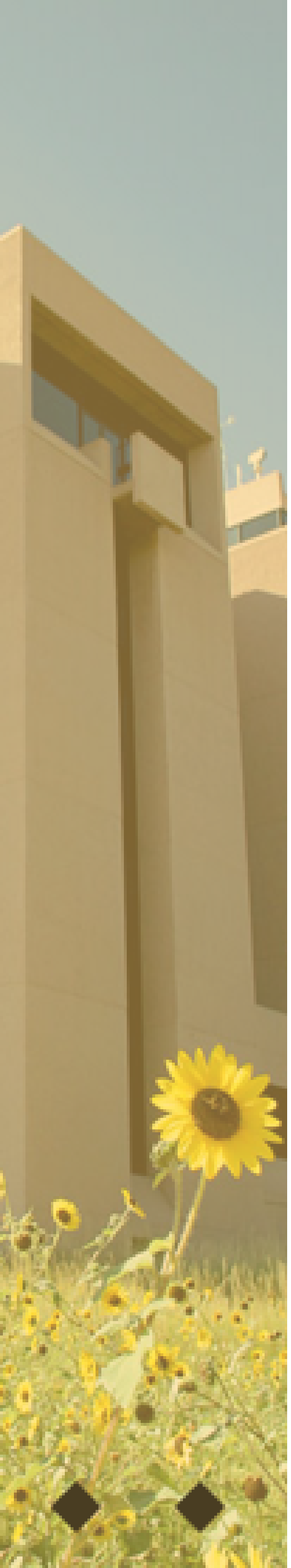
$$\widehat{k}(\omega) = \frac{1}{(\alpha^2+\omega^2)^{\nu+1}}$$

Next step: Approximate inverse Fourier transform and prove the condition

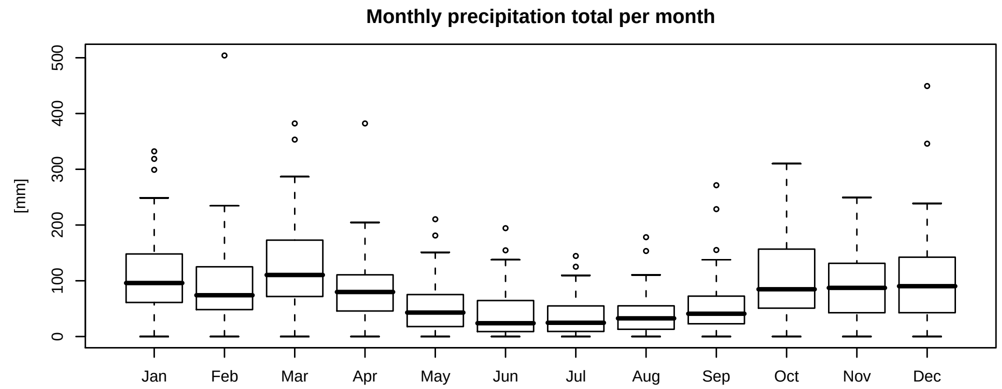
or

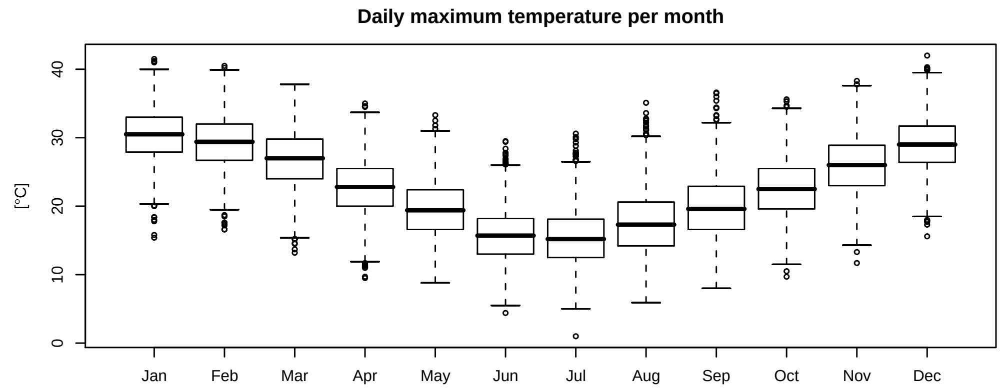
Use connections with Laplace transform without the functional form of G_λ

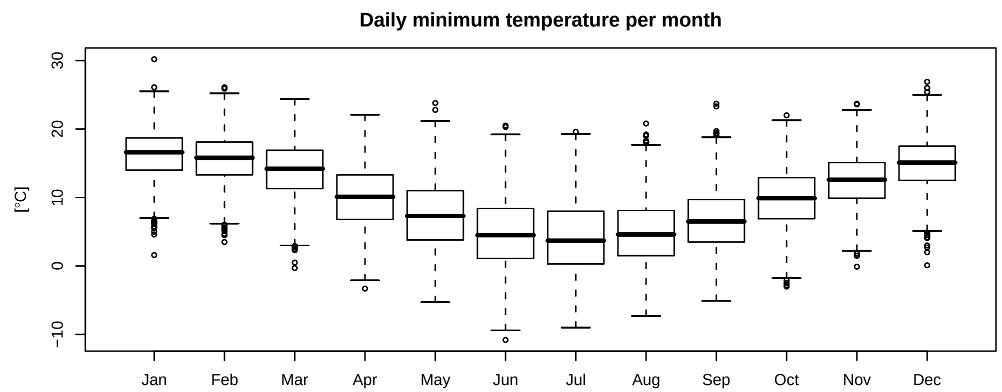
Additional Slides



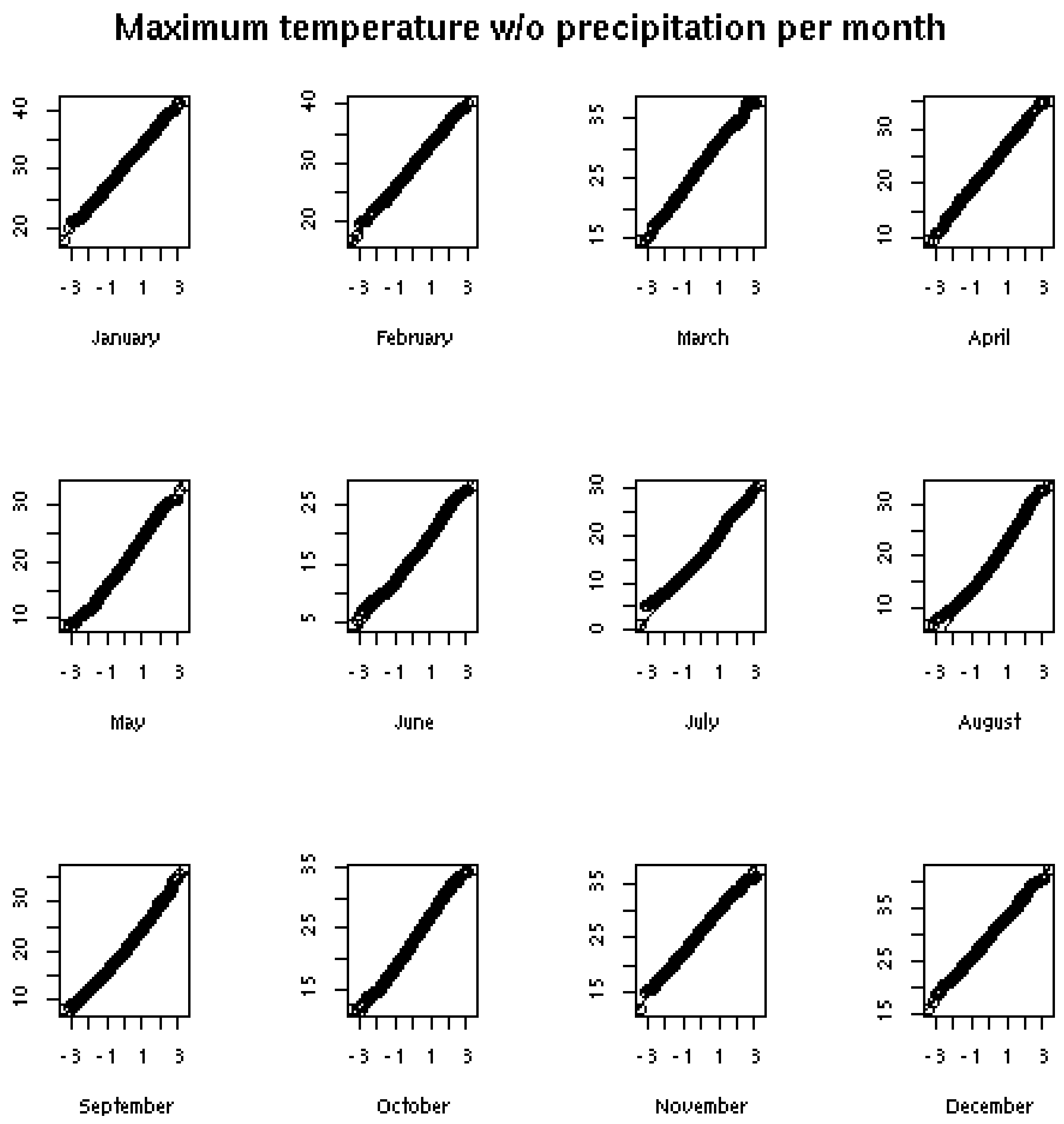
Data at Pergamino



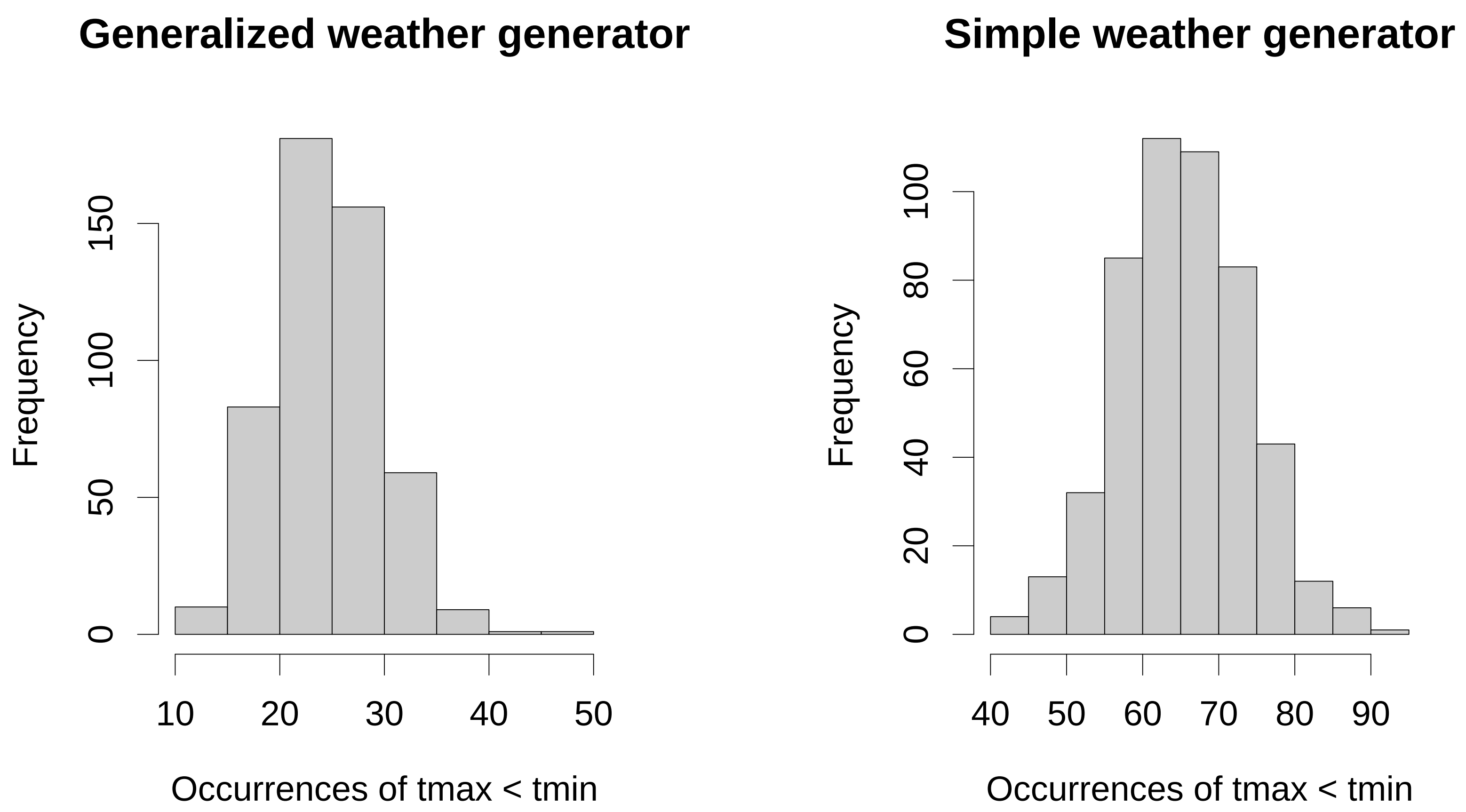




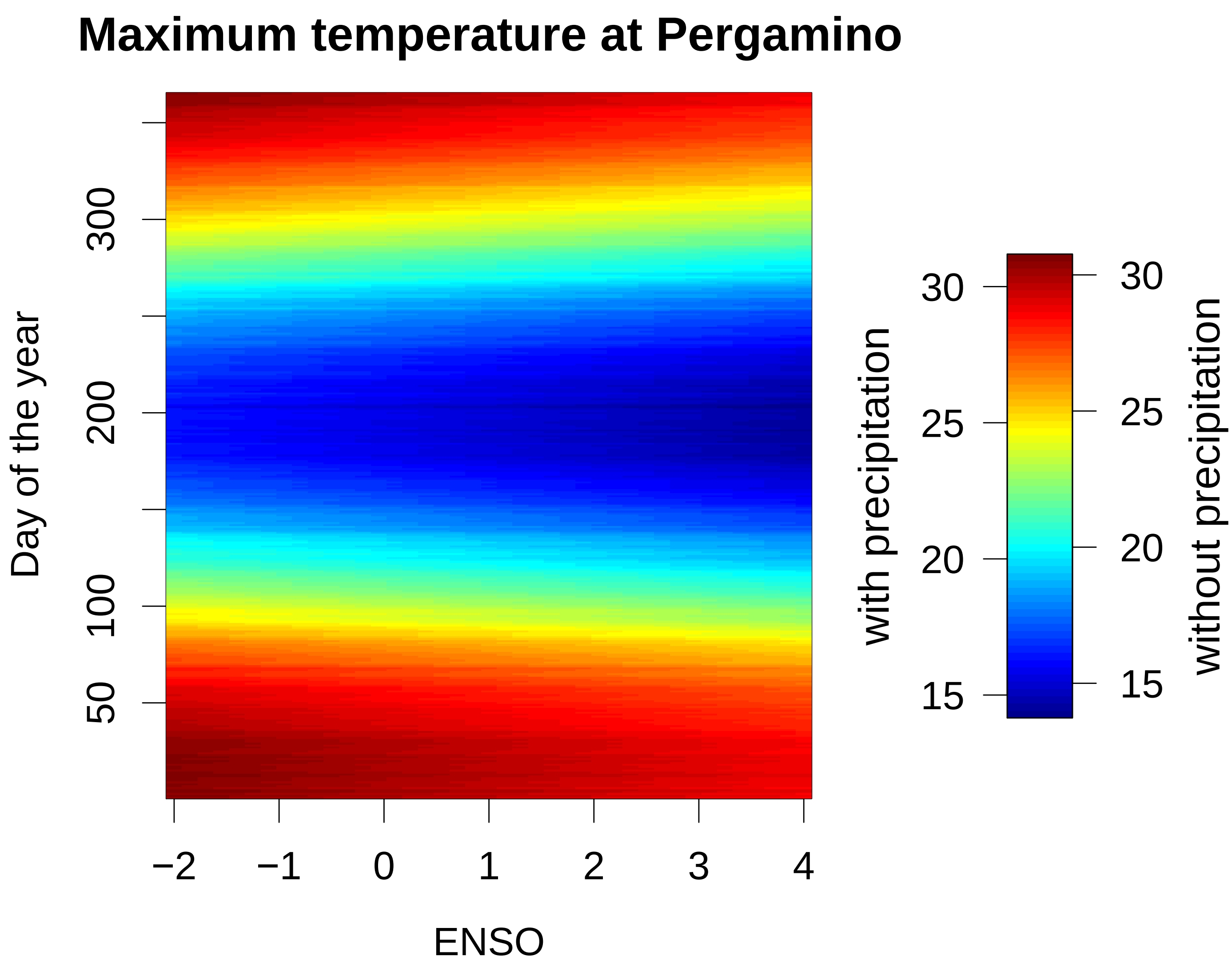
Normality


Maximum temperature w/o precipitation per month
January
February
March
April
May
June
July
August
September
October
November
December

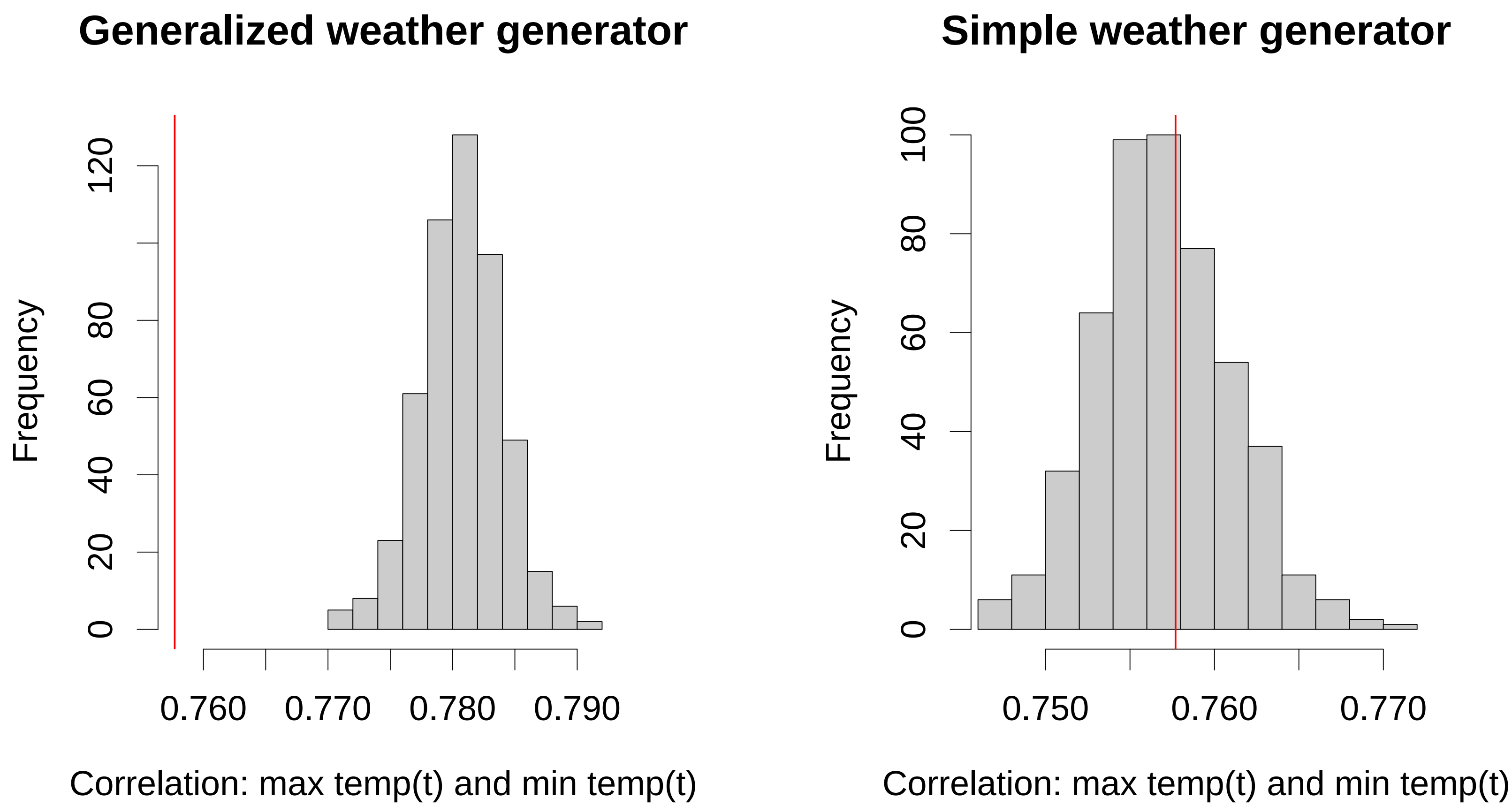
Defects

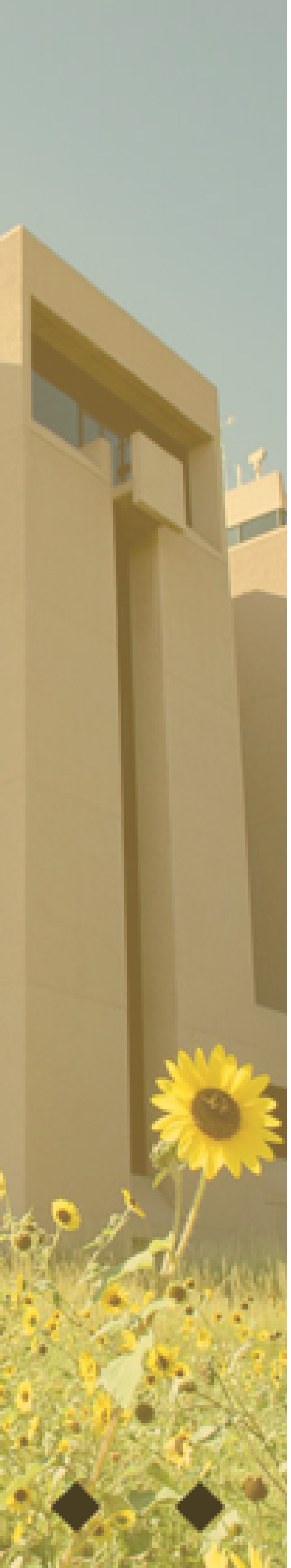

Generalized weather generator
Frequency
0
50
100
150
10
20
30
40
50
Occurrences of tmax < tmin
Simple weather generator
Frequency
0
20
40
60
80
100
40
50
60
70
80
90
Occurrences of tmax < tmin

Temperature: Illustration



Temperature: Crosscorrelation





Temperature: Lag 1 Crosscorr.