Spatial Patterns of Probabilistic Temperature Change Projections

IMAGe ToYII, 05/07/07

Reinhard Furrer
We present probabilistic projections for spatial patterns of future temperature change using a hierarchical Bayesian model.

Collaboration with: Reto Knutti - ETHZ
Stephan Sain, Doug Nychka, Claudia Tebaldi, Jerry Meehl, Linda Mearns, . . . - NCAR
Outline of the Talk

- Climate projection data
- A simple hierarchical Bayesian model
- Presenting uncertainty results
- Model extensions
- Conclusion
Study Climate

Source: AR4, IPCC
Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models

Numerical models that calculate the detailed large-scale motions of the atmosphere or the ocean explicitly from hydrodynamical equations.
Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models

CCSM3 DJF temperature

1980-2000

2080-2100

-30 -24 -18 -12 -6 0 6 12 18 24 30
Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models

CCSM3 DJF temperature change  2080-2100 vs 1980-2000
Example: Atmospheric Model

Input

- External forcings (radiation, volcanos, ...)
- Anthropogenic forcings (GHG, aerosols, ...)
- Initial conditions

↓

- Flow dynamics, PDEs
- Discretization and simplifications
- Parametrization

Output

- Temperature and precipitation
- Pressure, wind, ...
### Models Do Not Agree

Source: AR4, IPCC

<table>
<thead>
<tr>
<th>Model ID, Vintage</th>
<th>Sponsor(s), Country</th>
<th>Atmosphere</th>
<th>Ocean</th>
<th>Sea Ice</th>
<th>Coupling</th>
<th>Land</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Atmosphere Top Resolution</strong></td>
<td><strong>Z Coord., Top BC References</strong></td>
<td><strong>Dynamics, Leads References</strong></td>
<td><strong>Flux Adjustments References</strong></td>
<td><strong>Soil, Plants, Routing References</strong></td>
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<tr>
<td>BCC-CM1, 2005</td>
<td>Beijing Climate Center, China</td>
<td>top = 25 hPa</td>
<td>T85 (1.9° x 1.9°)</td>
<td>L16 Dong et al., 2000; CSMD, 2005; Xu et al., 2005</td>
<td>1.9° x 1.9° L30 depth, free surface Jin et al., 1999</td>
<td>heat, momentum Yu and Zhang, 2000; CSMD, 2005</td>
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<tr>
<td>BCCR-BCM2.0, 2005</td>
<td>Bjerknes Centre for Climate Research, Norway</td>
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<td>T85 (1.9° x 1.9°)</td>
<td>L31 Déqué et al., 1994</td>
<td>0.5°-1.5° x 1.5° L35 density, free surface Biek et al., 1992</td>
<td>no adjustments Furutake et al., 2003</td>
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<td>National Center for Atmospheric Research, USA</td>
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<td>T85 (2.5° x 1.4°) L26 Collins et al., 2004</td>
<td>0.3°-1° x 1° L40 depth, free surface Smith and Gent, 2002</td>
<td>no adjustments Collins et al., 2006</td>
<td>layers, canopy, routing Olesen et al., 2004; Branstetter, 2001</td>
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<td>CGCM3.1(T47), 2005</td>
<td>Canadian Centre for Climate Modelling and Analysis, Canada</td>
<td>top = 1 hPa</td>
<td>T47 (2.5° x 2.5°)</td>
<td>L31 McFarlane et al., 1992; Flato, 2005</td>
<td>1.9° x 1.9° L29 depth, rigid lid Rasoloson et al., 1997</td>
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<td>CGCM3.1(T85), 2005</td>
<td>Canadian Centre for Climate Modelling and Analysis, Canada</td>
<td>top = 0.55 hPa</td>
<td>T85 (1.9° x 1.9°)</td>
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<td>CNRM-CM3, 2004</td>
<td>Météo-France/Centre National de Recherches Météorologiques, France</td>
<td>top = 0.5 hPa</td>
<td>T85 (1.9° x 1.9°)</td>
<td>L31 McFarlane et al., 1992; Flato, 2005</td>
<td>1.9° x 1.9° L29 depth, rigid lid Rasoloson et al., 1997</td>
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<td>CSIRO-MK3.0, 2001</td>
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<td>ECMAM/MPI-CM, 2005</td>
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<td>ECHO-G, 1999</td>
<td>Meteorological Institute of the University of Bonn, Meteorological Research Institute of the Korea Meteorological Administration (KMA), and Model and Data Group, Germany/Korea</td>
<td>top = 10 hPa</td>
<td>T85 (1.9° x 1.9°)</td>
<td>L19 Roeckner et al., 1998</td>
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<thead>
<tr>
<th>Model</th>
<th>Greenhouse Gases</th>
<th>Forcing Agents</th>
<th>Aerosols</th>
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<tbody>
<tr>
<td>BCC-CM1</td>
<td>CO₂, CH₄, N₂O</td>
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<td>SO₂, Black carbon</td>
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<td>GISS-CHE</td>
<td>Y, Y, Y, Y, Y, Y</td>
<td></td>
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</tbody>
</table>

Source: AR4, IPCC
Models Do Not Agree

CCSM3 DJF temp change difference to sample mean (21 models)
Models Do Not Agree

Source: AR4, IPCC
Quantifying Uncertainty

Quantifying Uncertainty


Quantifying Uncertainty


- Gridded, global, spatial approach ...
Data

Data provided for the Fourth Assessment Report of IPCC:

- 21 models (CCSM, GFDL, HADCM, PCM, ...)
- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42)
- Different scenarios (A2: “business as usual”, A1B, B1)
- Temperature, precipitation, pressure, winds...
Data

Data provided for the Fourth Assessment Report of IPCC:

- 21 models (CCSM, GFDL, HADCM, PCM, ...)

- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42) aggregate to $5^\circ \times 5^\circ$ and omit the “poles” (3264 points).

- Different scenarios (A2: “business as usual”, A1B, B1)

- Temperature, precipitation, pressure, winds... seasonal averages over years 1980–1999 and 2080–2099
Statistical Model

For models $i = 1, \ldots, N$, stack the gridded output into vectors:

- $X_i = \text{simulated present climate}_i$
- $Y_i = \text{simulated future climate}_i$

Objective:

**Probabilistic description of modeled climate change**

$$D_i = Y_i - X_i$$
Statistical Model

Data level:

\( D_i = Y_i - X_i = \text{simulated climate change} \)
Statistical Model

Data level:

\[ D_i = Y_i - X_i = \text{simulated climate change} \]
\[ = \text{large scale structure} + \text{small scale structure} \]
**Statistical Model**

Data level:

\[ D_i = Y_i - X_i = \text{simulated climate change} \]

\[ = \text{large scale structure} + \text{small scale structure} \]

\[ = \text{climate signal} + \text{model bias and internal variability} \]
Statistical Model

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\[ D_i = Y_i - X_i = \text{simulated climate change} \]
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\[ = \mu_i + \epsilon_i \]
Statistical Model

Data level:

\[ D_i = Y_i - X_i = \text{simulated climate change} \]
\[ = \text{large scale structure} + \text{small scale structure} \]
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\[ = \mu_i + \varepsilon_i \]

\[ D_i \mid \mu_i, \phi_i \overset{iid}{\sim} N_n(\mu_i, \phi_i \Sigma) \]
\[ \phi_i > 0 \quad i = 1, \ldots, N \]
for given \( \Sigma \)
Statistical Model

Process level:

\[ \mu_i = M \theta_i \]

for given \( M \)
Statistical Model

Process level:

\[ \mu_i = \mathbf{M} \theta_i \]
for given \( \mathbf{M} \)

\[ \theta_i \mid \vartheta, \psi_i \overset{\text{iid}}{\sim} N_p(\vartheta, \psi_i \mathbf{I}) \quad \psi_i > 0 \quad i = 1, \ldots, N \]
Statistical Model

Prior level:

\[ \phi_i \overset{\text{iid}}{\sim} \text{IG}(\xi_1, \xi_2) \quad \xi_1, \xi_2 > 0 \quad i = 1, \ldots, N \]

\[ \psi_i \overset{\text{iid}}{\sim} \text{IG}(\xi_3, \xi_4) \quad \xi_3, \xi_4 > 0 \quad i = 1, \ldots, N \]

\[ \vartheta \sim \mathcal{N}_p(0, \xi_5 I) \quad \xi_5 > 0 \]

for given \( \xi_1, \ldots, \xi_5 \)
Statistical Model

Need to specify:

- Covariance model for $\Sigma$
- Basis functions used in $\mathbf{M}$
- Hyperparameters $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$
Covariance Model for $\Sigma$

The covariance matrices $\phi_i \Sigma$ are positive definite.

Examples of positive definite functions on the sphere:

1. representation with an infinite series of Legendre polynomials

$$c(h; \phi_i, \tau) = \phi_i \left(1 - 2\tau \cos(h) + \tau^2\right)^{-3/2}$$

2. restriction of a positive definite function on $\mathbb{R}^3$ to the sphere

$$c(h; \phi_i, \tau) = \phi_i \exp\left(-\tau \sin(h/2)\right)$$

Range $\tau$ is choosen according to an “empirical Bayes” approach.
Basis Functions Used in M

1. Spherical harmonics
Basis Functions Used in M

1. Spherical harmonics

2. Indicator functions
Hyperparameters $\xi_1, \ldots, \xi_5$

To make sure that variability around the truth is smaller than bias and internal variability

$$\phi_i > \psi_i$$

Choose $\xi_1, \xi_2, \xi_3$ small, $\xi_4 \in [1, 2.5]$, $\xi_5$ large.
Hyperparameters $\xi_1, \ldots, \xi_5$

To make sure that variability around the truth is smaller than bias and internal variability

$$\phi_i > \psi_i$$

Choose $\xi_1, \xi_2, \xi_3$ small, $\xi_4 \in [1, 2.5]$, $\xi_5$ large.

$\xi_4 \in [.5, 1.5]$ for 2020-2029 projections.
Posterior Distribution

The goal is the posterior distribution of $M \vartheta$ given the data $D_i$:

$$[M\vartheta \mid D_1, \ldots, D_N, \ldots]$$
The goal is the posterior distribution of $M\theta$ given the data $D_i$:

$$[M\theta \mid D_1, \ldots, D_N, \ldots]$$

Via Bayes' theorem, the posterior density is

$$[\text{process} \mid \text{data, parameters}] \propto [\text{data} \mid \text{process, parameters}] \cdot [\text{process} \mid \text{parameters}] \cdot [\text{parameters}]$$
Posterior Distribution

No closed form of the posterior density.

Use computational approaches MCMC.
Posterior Distribution

No closed form of the posterior density.

Use computational approaches MCMC.

Gibbs sampler:

1. Express the distribution of the parameter conditional on everything else.

2. Cycle among the parameters by simulating a new value based on the full conditional distribution and the current values of the other parameters.

3. Repeat, ...
Full Conditionals

Full conditionals for the parameters are available:

\[ \vartheta | \ldots \sim \mathcal{N}_p(\cdot, \cdot) \]

\[ \theta_i | \ldots \sim \mathcal{N}_p(\cdot, \cdot) \]

\[ \phi_i | \ldots \sim \mathcal{I}_\Gamma(\cdot, \cdot) \]

\[ \psi_i | \ldots \sim \mathcal{I}_\Gamma(\cdot, \cdot) \]
Full Conditionals

Full conditionals for the parameters are available:

\[ \vartheta | \ldots \sim \mathcal{N}_p(A^{-1}b, A^{-1}) \]
\[ A = \frac{1}{\xi_5} I + \frac{1}{\psi_i} \sum_{i=1}^{N} \frac{1}{\psi_i} I \]
\[ b = \sum_{i=1}^{N} \frac{1}{\psi_i} \theta_i \]
\[ i = 1, \ldots, N : \theta_i | \ldots \sim \mathcal{N}_p(A^{-1}b, A^{-1}) \]
\[ A = \frac{1}{\psi_i} I + \frac{1}{\phi_i} \mathbf{M}^T \Sigma^{-1} \mathbf{M} \]
\[ b = \frac{1}{\psi_i} \vartheta + \frac{1}{\phi_i} \mathbf{M}^T \Sigma^{-1} \mathbf{D}_i \]
\[ i = 1, \ldots, N : \phi_i | \ldots \sim \Gamma \left( \xi_1 + \frac{n}{2}, \xi_2 + \frac{1}{2} (\mathbf{D}_i - \mathbf{M} \theta_i)^T \Sigma^{-1} (\mathbf{D}_i - \mathbf{M} \theta_i) \right) \]
\[ i = 1, \ldots, N : \psi_i | \ldots \sim \Gamma \left( \xi_3 + \frac{p}{2}, \xi_4 + \frac{1}{2} (\theta_i - \vartheta)^T (\theta_i - \vartheta) \right) \]
Gibbs Sampler

- Gibbs sampler programmed in R
- Run 20000 iterations (10000 burn-in, keep every 20th, takes few hours)
- Visual/primitive inspection of convergence
Temperature Change Quantiles

20% quantile of temperature change [°C]
(2080-2100 vs 1980-2000)

DJF

JJA
Exceedance Probabilities

Probability of exceeding 2°C temperature change
(2080-2100 vs 1980-2000)

DJF

JJA
Exceedance Fractions

Fraction of globe

Fraction of land

Exceeded temperature [°C]
Regional Assessment

ALAD

WNA

CAM

AMZ

Surface warming [°C]

Surface warming [°C]
Global Assessment

Source: AR4, IPCC
Model Extensions

- Use "more" data
  \[\Rightarrow\] ensemble runs, model present and future individually, . . .
Model Extensions

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  \[\rightarrow\] ensemble runs, model present and future individually, . . .

- Use AOGCM specific weighting
  \[\rightarrow\] performance, "core" similarities, . . .
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- Parameterize covariance matrices
  - built in range, nonstationarity, . . .
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  ⟷ use temperature for precipitation prediction, . . .
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- Address computational complexity
  - sparsity, Metropolis-Hastings steps, . . .
Model Extensions

Is an geostatisitical approach adequat?

- NARCCAP: North American Regional Climate Change Assessment Program. [www.narccap.ucar.edu](http://www.narccap.ucar.edu)
Model Extensions

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CAR and SAR Models

Spatial autoregressive models represent the data at a lattice site as a linear combination of neighboring locations.

1. Simultaneous autoregressive (SAR) models

2. Conditional autoregressive (CAR) models
CAR and SAR Models

Spatial autoregressive models represent the data at a lattice site as a linear combination of neighboring locations.

1. Simultaneous autoregressive (SAR) models
\[ Y_i = \mu_i + \sum_j b_{ij} (Y_j - \mu_j) + \varepsilon_i \]

2. Conditional autoregressive (CAR) models
\[ f(Y_i \mid Y_{-i}) \text{ with } Y_{-i} \text{ all but } Y_i \]
Discussion

- AOGCMs are not “equal”
- AOGCMs are not “unbiased”
- AOGCMs are not “independent”

