Spatial Patterns of Probablistic Temperature Change Projections

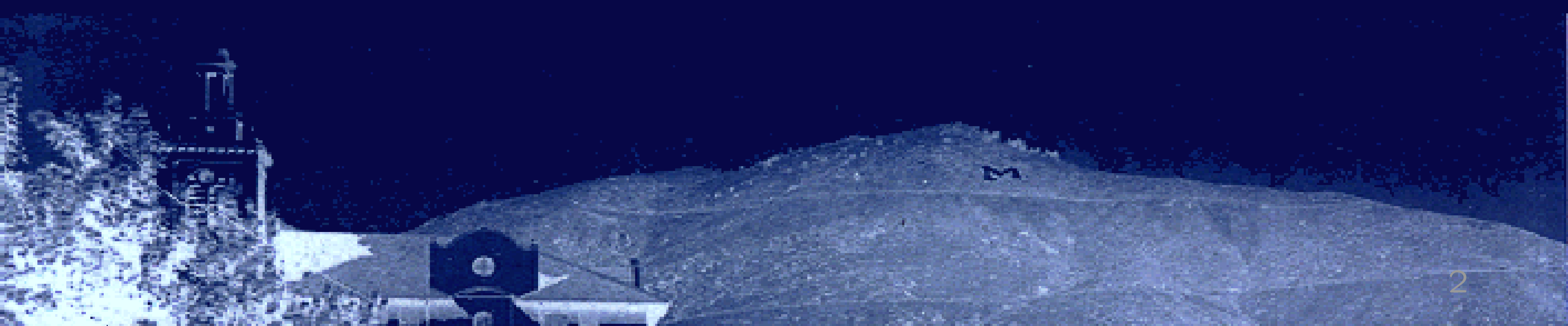
IMAGe ToYII, 05/07/07

Reinhard Furrer

We present probabilistic projections for spatial patterns of future temperature change using a hierarchical Bayesian model.

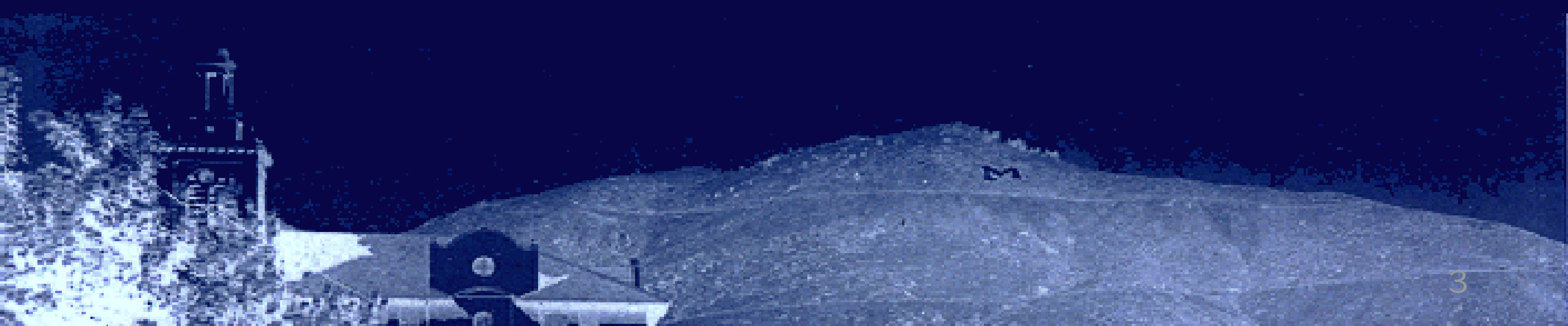
Collaboration with: Reto Knutti - ETHZ

Stephan Sain, Doug Nychka, Claudia Tebaldi,
Jerry Meehl, Linda Mearns, . . . - NCAR

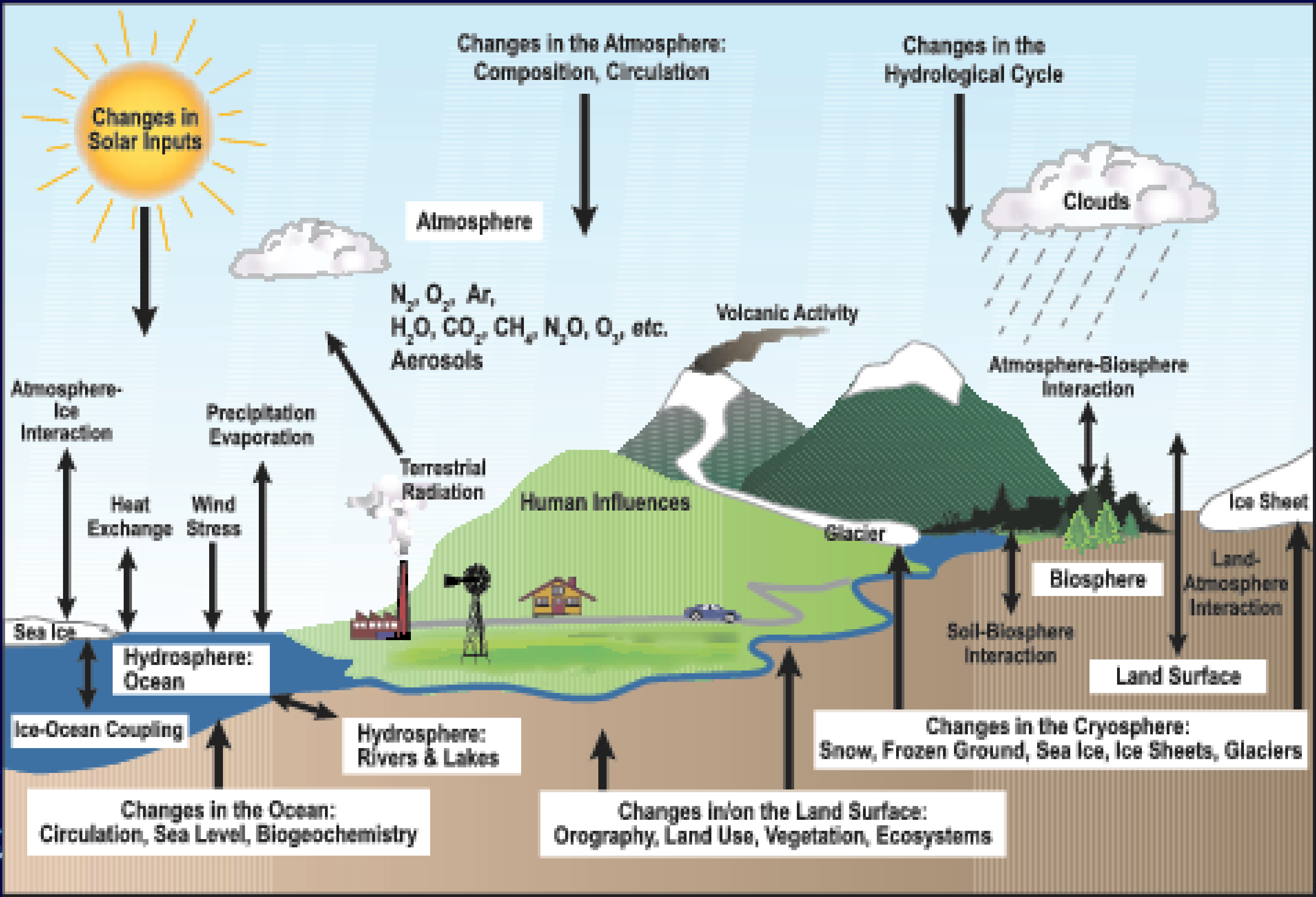


Outline of the Talk

- Climate projection data
- A simple hierarchical Bayesian model
- Presenting uncertainty results
- Model extensions
- Conclusion



Study Climate



Source: AR4, IPCC

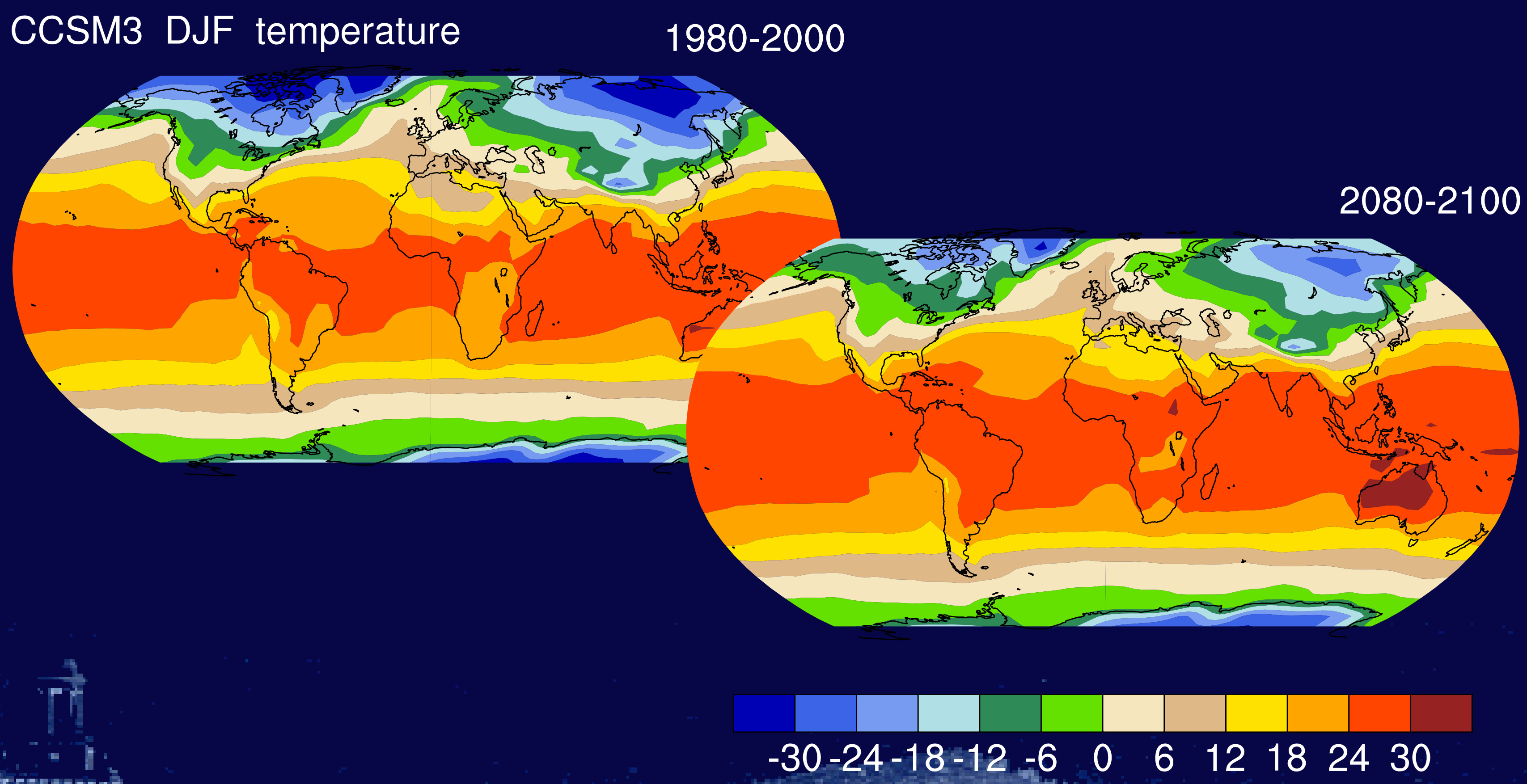
Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models

> Numerical models that calculate the detailed large-scale motions of the atmosphere or the ocean explicitly from hydrodynamical equations.

Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models



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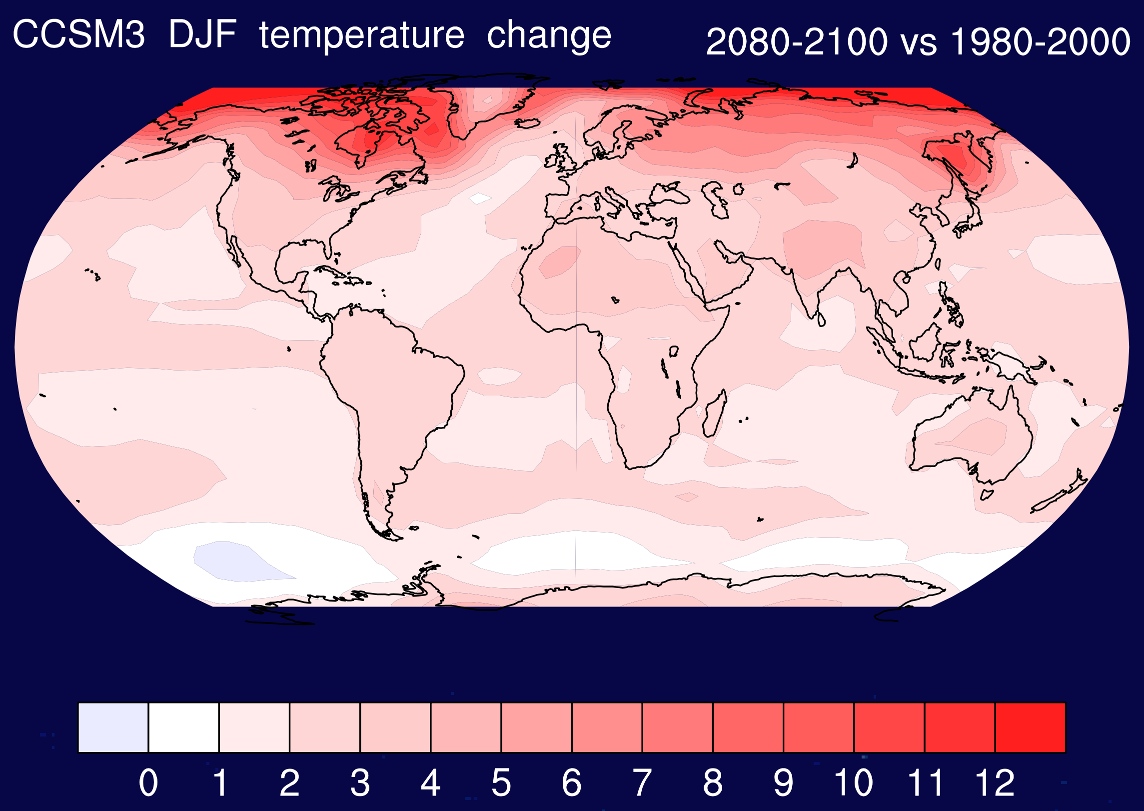


Example: Atmospheric Model

Input
- External forcings (radiation, volcanos, . . .)
- Anthropogenic forcings (GHG, aerosols, . . .)
- Initial conditions

$\downarrow$

- Flow dynamics, PDEs
- Discretization and simplifications
- Parametrization

$\downarrow$

Output
- Temperature and precipitation
- Pressure, wind, . . .

Models Do Not Agree

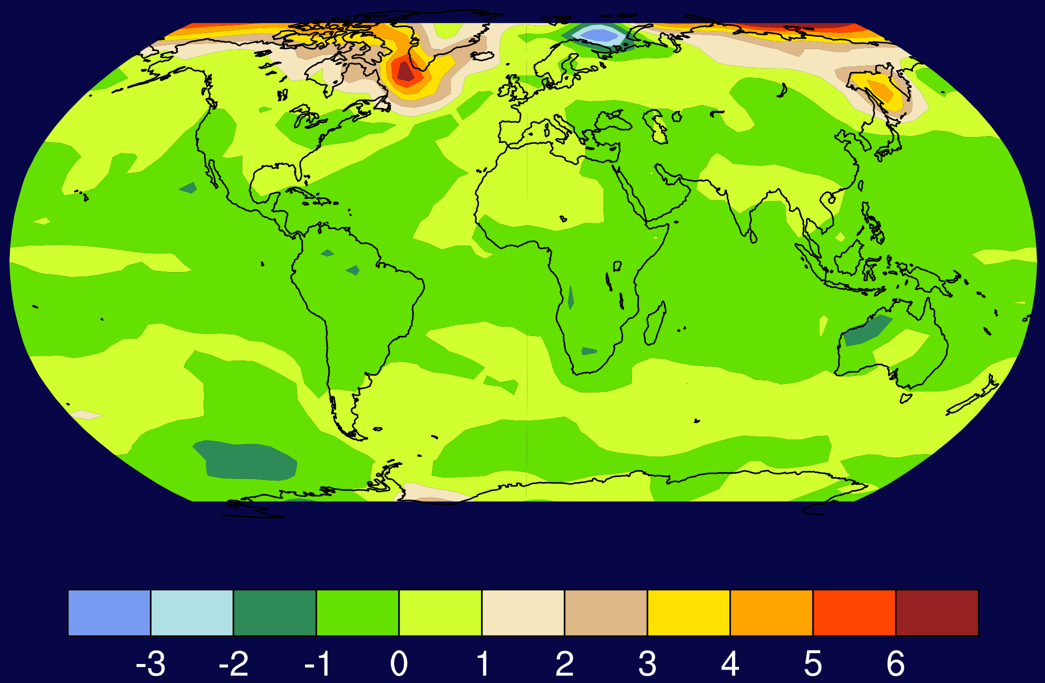
Model ID, Vintage	Sponsor(s), Country	Atmosphere Top Resolution[a] References	Ocean Resolution[b] Z Coord., Top BC References	Sea Ice Dynamics, Leads References	Coupling Flux Adjustments References	Land Soil, Plants, Routing References
1: BCC-CM1, 2005	Beijing Climate Center, China	top = 25 hPa T63 (1.9° x 1.9°) L16 Dong et al., 2000; CSMD, 2005; Xu et al., 2005	1.9° x 1.9° L30 depth, free surface Jin et al., 1999	no rheology or leads Xu et al., 2005	heat, momentum Yu and Zhang, 2000; CSMD, 2005	layers, canopy, routing CSMD, 2005
2: BCCR-BCM2.0, 2005	Bjerknes Centre for Climate Research, Norway	top = 10 hPa T63 (1.9° x 1.9°) L31 Déqué et al., 1994	0.5°–1.5° x 1.5° L35 density, free surface Bleck et al., 1992	rheology, leads Hibler, 1979; Harder, 1996	no adjustments Furevik et al., 2003	Layers, canopy, routing Mahfouf et al., 1995; Douville et al., 1995; Oki and Sud, 1998
3: CCSM3, 2005	National Center for Atmospheric Research, USA	top = 2.2 hPa T85 (1.4° x 1.4°) L26 Collins et al., 2004	0.3°–1° x 1° L40 depth, free surface Smith and Gent, 2002	rheology, leads Briegleb et al., 2004	no adjustments Collins et al., 2006	layers, canopy, routing Oleson et al., 2004; Branstetter, 2001
4: CGCM3.1(T47), 2005	Canadian Centre for Climate Modelling and Analysis, Canada	top = 1 hPa T47 (~2.8° x 2.8°) L31 McFarlane et al., 1992; Flato, 2005	1.9° x 1.9° L29 depth, rigid lid Pacanowski et al.,	rheology, leads Hibler, 1979; Flato and Hibler, 1992	heat, freshwater Flato, 2005	layers, canopy, routing Verseghy et al., 1993
5: CGCM3.1(T63), 2005		top = 1 hPa T63 (~1.9° x 1.9°) L31 McFarlane et al., 1992; Flato 2005				
6: CNRM-CM3, 2004	Météo-France/Centre National de Recherches Météorologiques, France	top = 0.05 hPa T63 (~1.9° x 1.9°) L45 Déqué et al., 1994				
7: CSIRO-MK3.0, 2001	Commonwealth Scientific and Industrial Research Organisation (CSIRO) Atmospheric Research, Australia	top = 4.5 hPa T63 (~1.9° x 1.9°) L18 Gordon et al., 2002				
8: ECHAM5/MPI-OM, 2005	Max Planck Institute for Meteorology, Germany	top = 10 hPa T63 (~1.9° x 1.9°) L31 Roeckner et al., 2003				
9: ECHO-G, 1999	Meteorological Institute of the University of Bonn, Meteorological Research Institute of the Korea Meteorological Administration (KMA), and Model and Data Group, Germany/Korea	top = 10 hPa T30 (~3.9° x 3.9°) L19 Roeckner et al., 1996				

Model	Forcing Agents													
	Greenhouse Gases						Aerosols							
	CO_2	CH_4	N_2O	Stratospheric Ozone	Tropospheric Ozone	CFCs	SO_4	Urban	Black carbon	Organic carbon	Nitrate	1st Indirect	2nd Indirect	Dust
BCC-CM1	Y	Y	Y	Y	C	4	4	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
BCCR-BCM2.0	1	1	1	C	C	1	2	C	n.a.	n.a.	n.a.	n.a.	n.a.	C
CCSM3	4	4	4	4	4	4	4	n.a.	4	4	n.a.	n.a.	n.a.	Y
CGCM3.1(T47)	Y	Y	Y	C	C	Y	2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	C
CGCM3.1(T63)	Y	Y	Y	C	C	Y	2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	C
CNRM-CM3	1	1	1	Y	Y	1	2	C	n.a.	n.a.	n.a.	n.a.	n.a.	C
CSIRO-MK3.0	Y	E	E	Y	Y	E	Y	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
ECHAM5/MPI-OM	1	1	1	Y	C	1	2	n.a.	n.a.	n.a.	n.a.	Y	n.a.	n.a.
ECHO-G	1	1	1	C	Y	1	6	n.a.	n.a.	n.a.	n.a.	Y	n.a.	n.a.
FGOALS-g1.0	4	4	4	C	C	4	4	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
GFDL-CM2.0	Y	Y	Y	Y	Y	Y	Y	n.a.	Y	Y	n.a.	n.a.	n.a.	C
GFDL-CM2.1	Y	Y	Y	Y	Y	Y	Y	n.a.	Y	Y	n.a.	n.a.	n.a.	C
GISS-AOM	5	5	5	C	C	5	2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
GISS-EH	Y	Y	Y	Y	Y	Y	Y	n.a.	Y	Y	Y	n.a.	Y	C

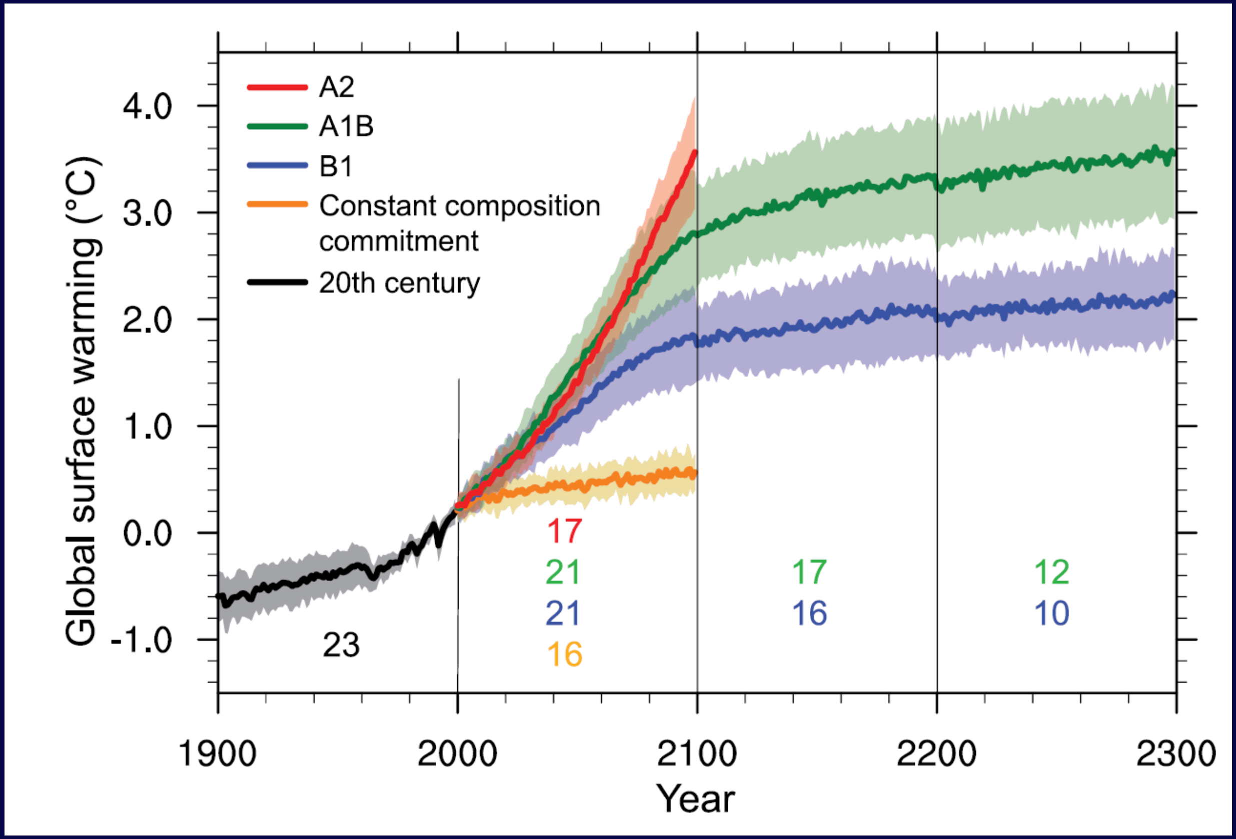
Source: AR4, IPCC

Models Do Not Agree

CCSM3 DJF temp change difference to sample mean (21 models)



Models Do Not Agree



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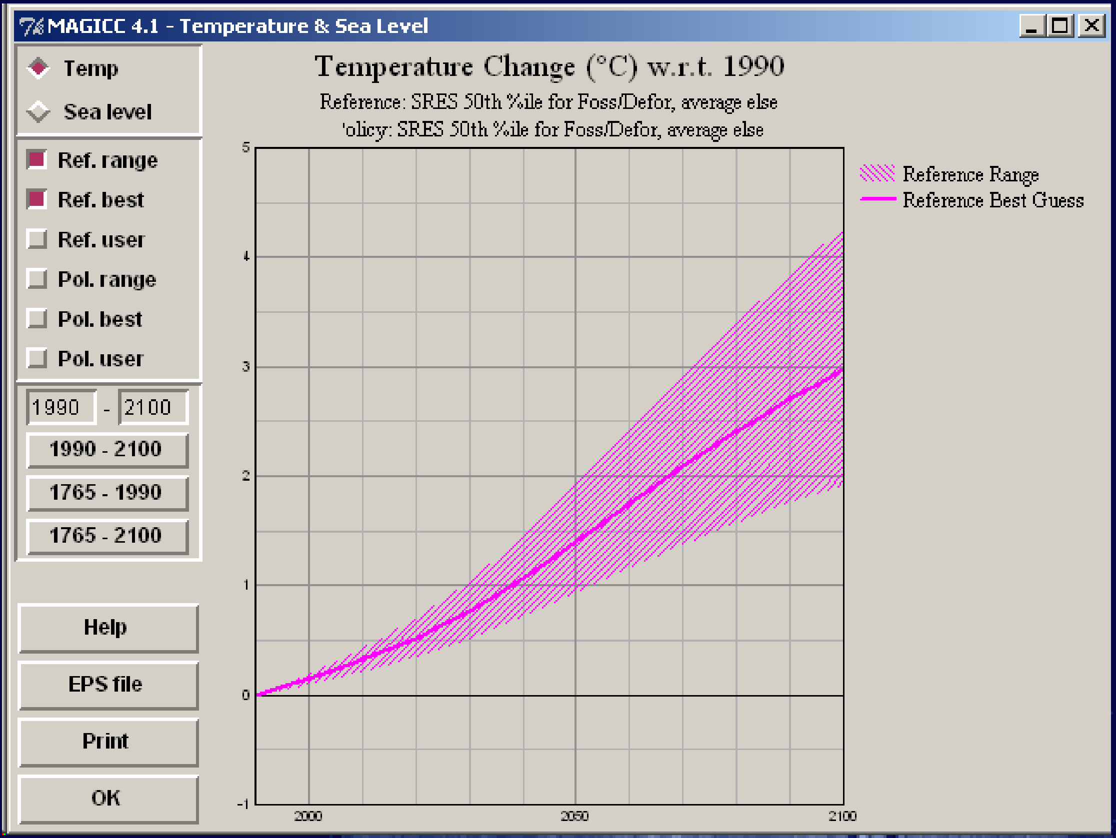
Quantifying Uncertainty

- Variability of global temperature increase across 16 models. MAGICC/SCENGEN program (Wigley, 2001, 2003).



Quantifying Uncertainty

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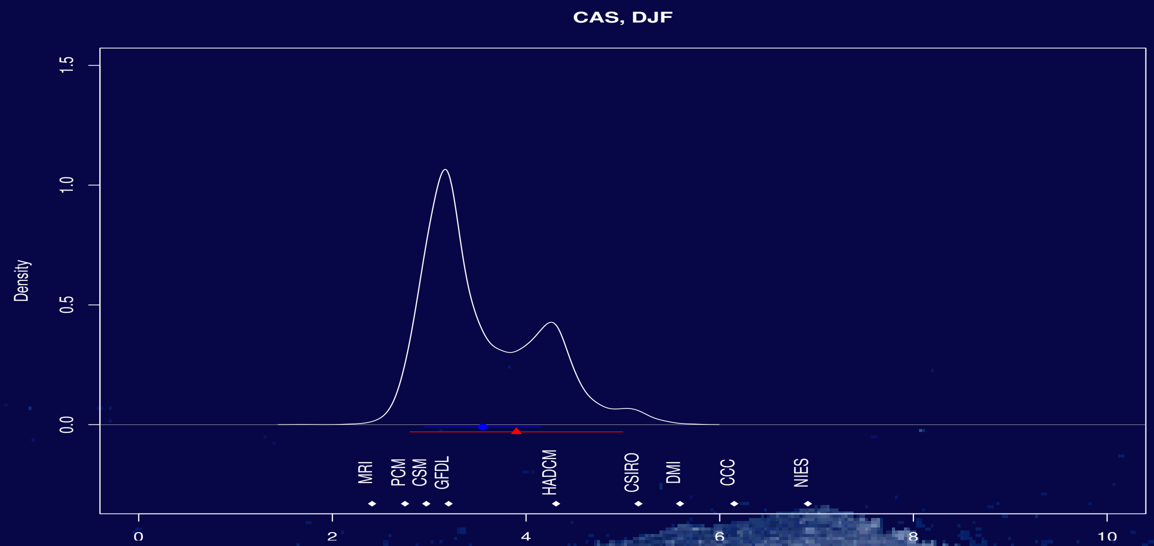
- Probabilistic description of regional climate changes. (Tebaldi et al. 2004, 2005).



Quantifying Uncertainty

- Variability of global temperature increase across 16 models. MAGICC/SCENGEN program (Wigley, 2001, 2003).

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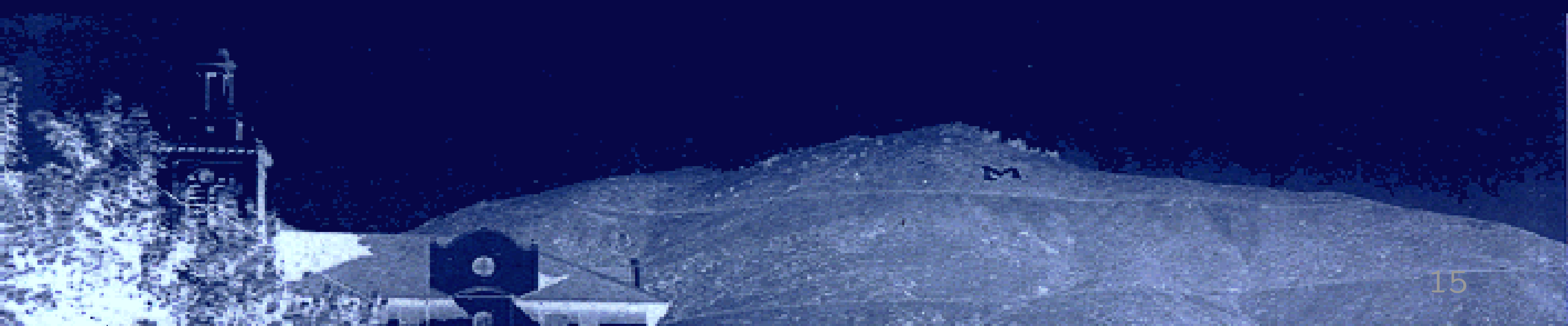
- Gridded, global, spatial approach . . .

Data

Data provided for the Fourth Assessment Report of IPCC:

- 21 models (CCSM, GFDL, HADCM, PCM, . . .)
- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42)
- Different scenarios (A2: "business as usual", A1B, B1)
- Temperature, precipitation, pressure, winds. . .



Data

Data provided for the Fourth Assessment Report of IPCC:

- 21 models (CCSM, GFDL, HADCM, PCM, ...)
- Around $2.8^\circ \times 2.8^\circ$ resolution (8192 data points, T42)
  aggregate to $5^\circ \times 5^\circ$ and omit the “poles” (3264 points).
- Different scenarios (A2: “business as usual”, A1B, B1)
- Temperature, precipitation, pressure, winds...
  seasonal averages over years 1980–1999 and 2080–2099

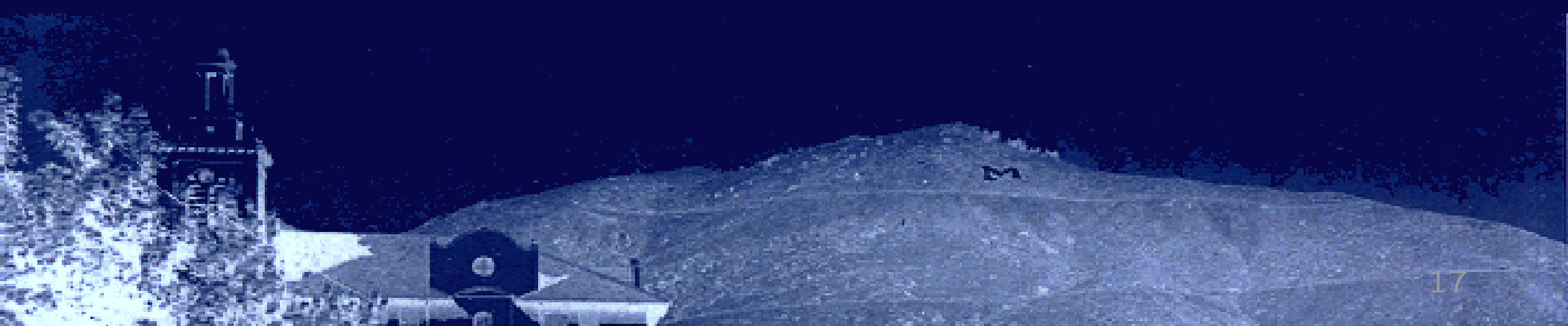
Statistical Model

For models $i = 1, \ldots, N$, stack the gridded output into vectors:

$$\mathbf{X}_i = \text{simulated present climate}_i$$
$$\mathbf{Y}_i = \text{simulated future climate}_i$$

Objective:

$$\boxed{\begin{array}{c}\text{Probabilistic description of modeled climate change} \\ \mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i\end{array}}$$



Statistical Model

Data level:

$$\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i = \text{simulated climate change}$$

Statistical Model

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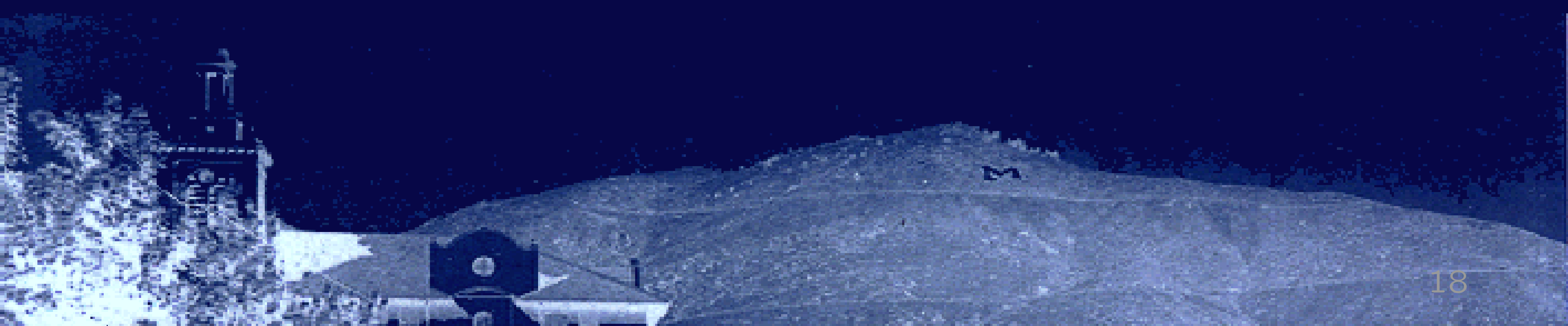
$$\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i = \text{simulated climate change}$$
$$= \text{large scale structure} + \text{small scale structure}$$



Statistical Model

Data level:

$$\begin{aligned}\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change}\\ &= \text{large scale structure} + \text{small scale structure}\\ &= \text{climate signal} + \text{model bias and internal variability}\end{aligned}$$

Statistical Model

Data level:

$$\begin{aligned}
\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change} \\
&= \text{large scale structure} + \text{small scale structure} \\
&= \text{climate signal} + \text{model bias and internal variability} \\
&= \qquad \boldsymbol{\mu}_i \qquad + \qquad \varepsilon_i
\end{aligned}$$

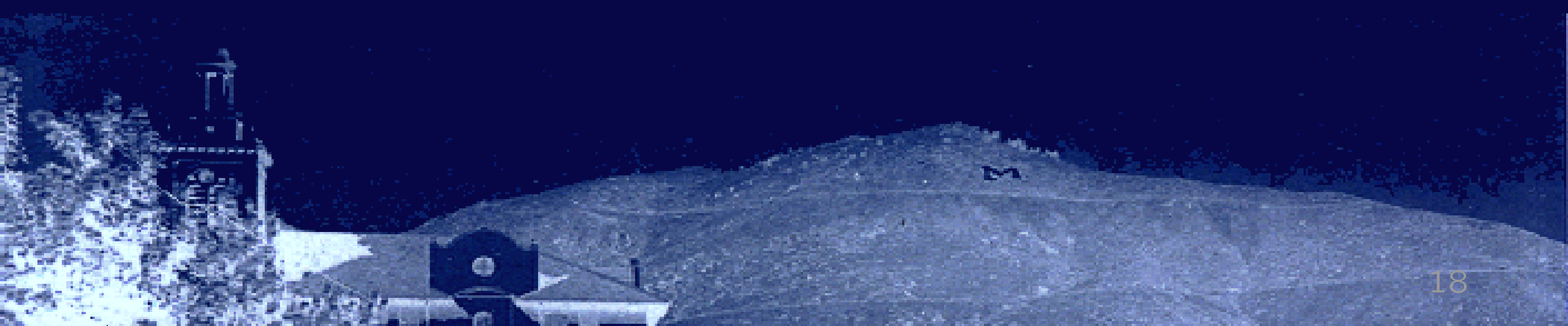
Statistical Model

Data level:

$$\begin{aligned}\mathbf{D}_i = \mathbf{Y}_i - \mathbf{X}_i &= \text{simulated climate change} \\ &= \text{large scale structure} + \text{small scale structure} \\ &= \text{climate signal} + \text{model bias and internal variability} \\ &= \quad \boldsymbol{\mu}_i \quad + \quad \boldsymbol{\varepsilon}_i\end{aligned}$$

$$\mathbf{D}_i \mid \boldsymbol{\mu}_i,\, \phi_i \overset{\text{iid}}{\sim} \mathcal{N}_n(\,\boldsymbol{\mu}_i,\, \phi_i \boldsymbol{\Sigma}\,) \qquad \phi_i > 0 \qquad i = 1, \ldots, N$$

for given $\boldsymbol{\Sigma}$

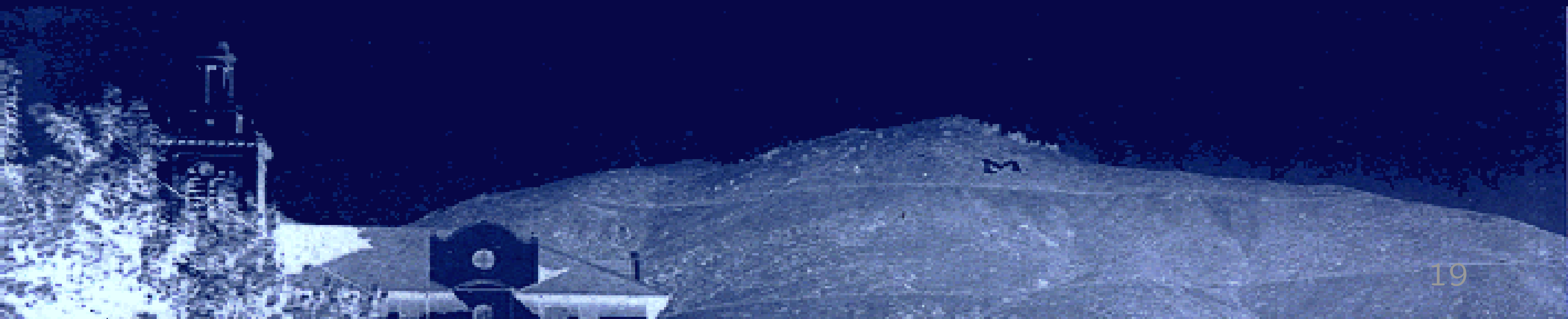


Statistical Model

Process level:

$$\boldsymbol{\mu}_i = \mathbf{M}\boldsymbol{\theta}_i$$

for given $\mathbf{M}$



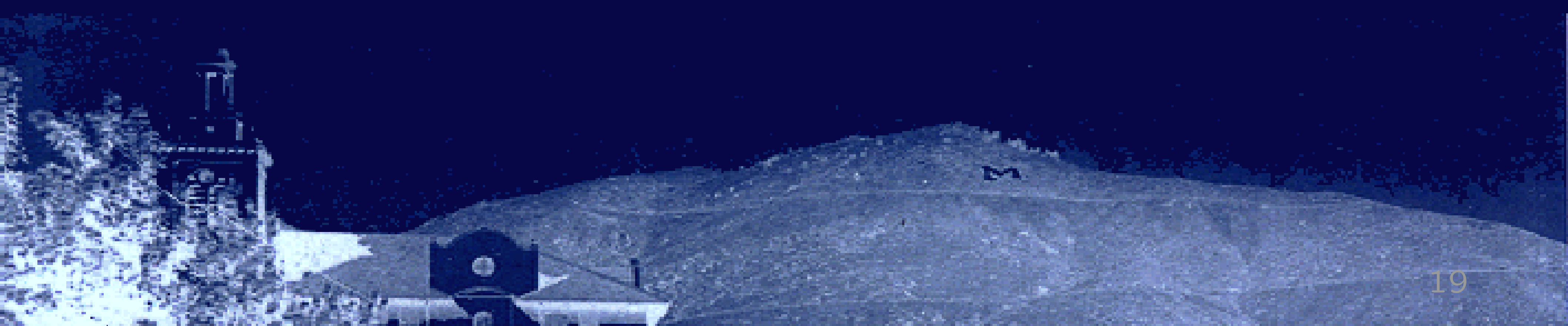
Statistical Model

Process level:

$$\boldsymbol{\mu}_i = \mathbf{M}\boldsymbol{\theta}_i$$

for given $\mathbf{M}$

$$\boldsymbol{\theta}_i \mid \boldsymbol{\vartheta},\ \psi_i \overset{\text{iid}}{\sim} \mathcal{N}_p(\,\boldsymbol{\vartheta},\ \psi_i\mathbf{I}\,) \qquad \psi_i > 0 \qquad i = 1, \ldots, N$$



Statistical Model

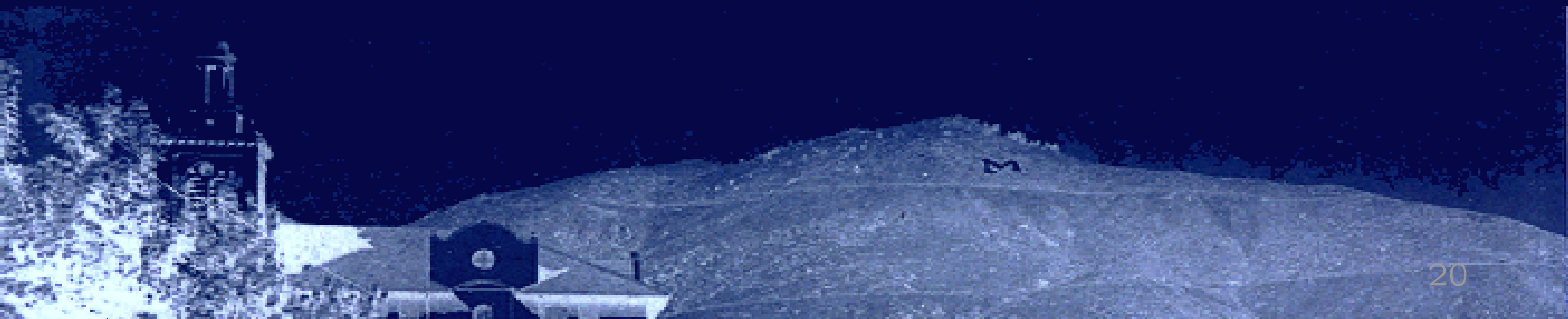
Prior level:

$$\phi_i \overset{\text{iid}}{\sim} \mathrm{I\Gamma}(\,\xi_1,\ \xi_2\,) \qquad \xi_1,\ \xi_2 > 0 \qquad i = 1, \ldots, N$$

$$\psi_i \overset{\text{iid}}{\sim} \mathrm{I\Gamma}(\,\xi_3,\ \xi_4\,) \qquad \xi_3,\ \xi_4 > 0 \qquad i = 1, \ldots, N$$

$$\boldsymbol{\vartheta} \sim \mathcal{N}_p(\,\mathbf{0},\ \xi_5 \mathbf{I}\,) \qquad \xi_5 > 0$$

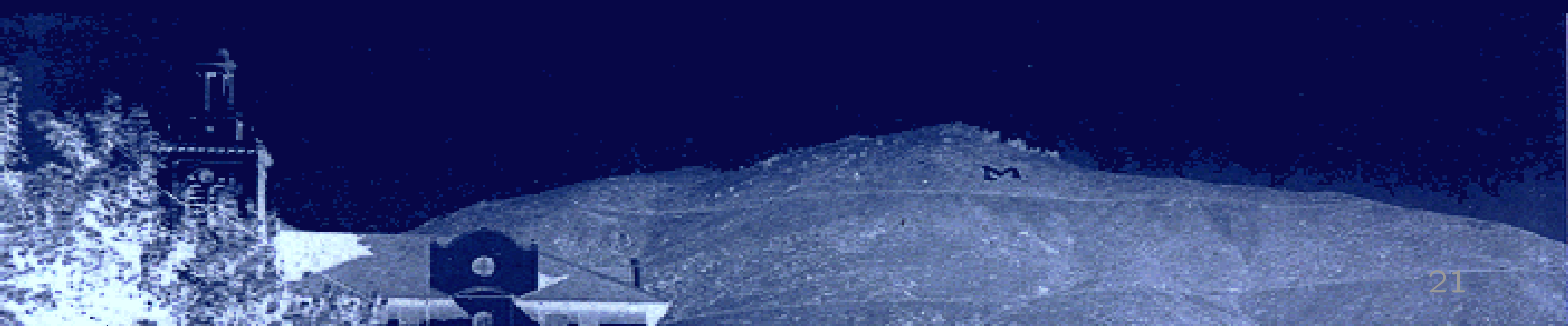
for given $\xi_1, \ldots, \xi_5$



Statistical Model

Need to specify:

- Covariance model for $\boldsymbol{\Sigma}$
- Basis functions used in $\mathbf{M}$
- Hyperparameters $\xi_1, \xi_2, \quad \xi_3, \xi_4, \quad \xi_5$



Covariance Model for $\mathbf{\Sigma}$

The covariance matrices $\phi_i\mathbf{\Sigma}$ are positive definite.

Examples of positive definite functions on the sphere:

1. representation with an infinite series of Legendre polynoms

$$c(h;\phi_i,\tau) = \phi_i\Big(1 - 2\tau\cos(h) + \tau^2\Big)^{-3/2}$$

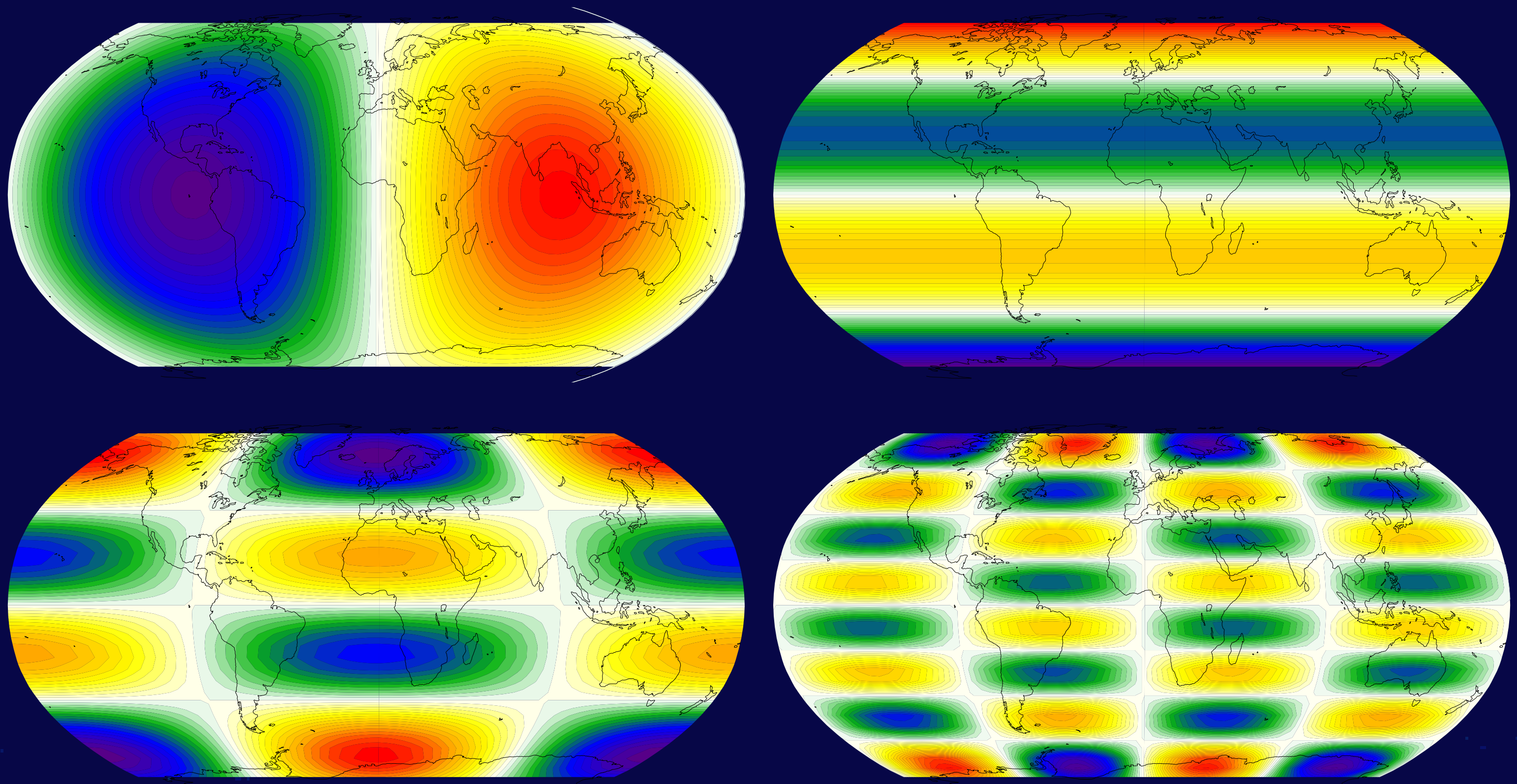
2. restriction of a positive definite function on $\mathbb{R}^3$ to the sphere

$$c(h;\phi_i,\tau) = \phi_i\exp\Big(-\tau\sin(h/2)\Big)$$

Range τ is choosen according to an "empirical Bayes" approach.

Basis Functions Used in M

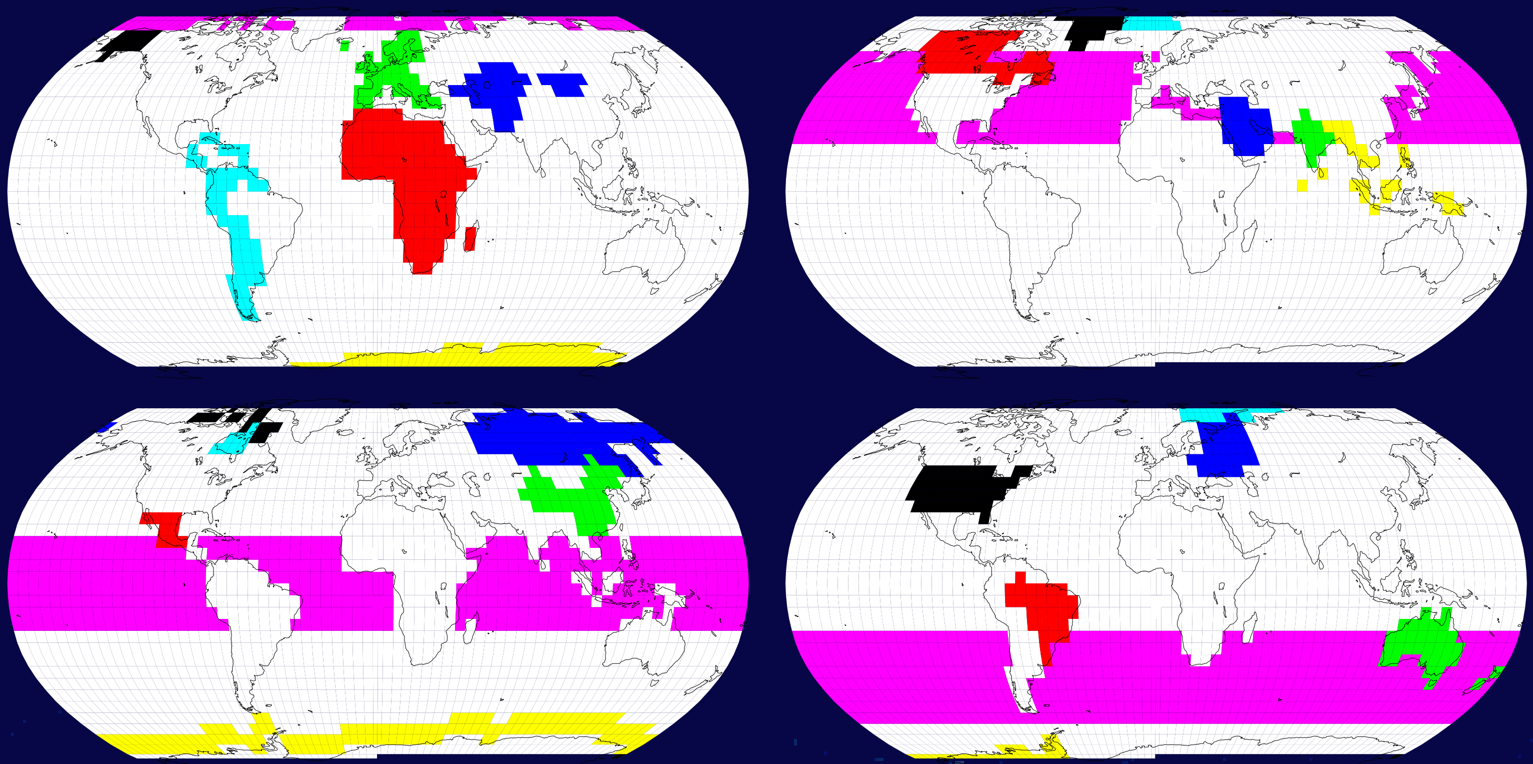
1. Spherical harmonics



Basis Functions Used in M

1. Spherical harmonics

2. Indicator functions



Hyperparameters $\xi_1, \ldots, \xi_5$

To make sure that variability around the truth is smaller than bias and internal variability

$$\phi_i > \psi_i$$

Choose ξ_1, ξ_2, ξ_3 small, $\xi_4 \in [\,1,\, 2.5\,]$, ξ_5 large.

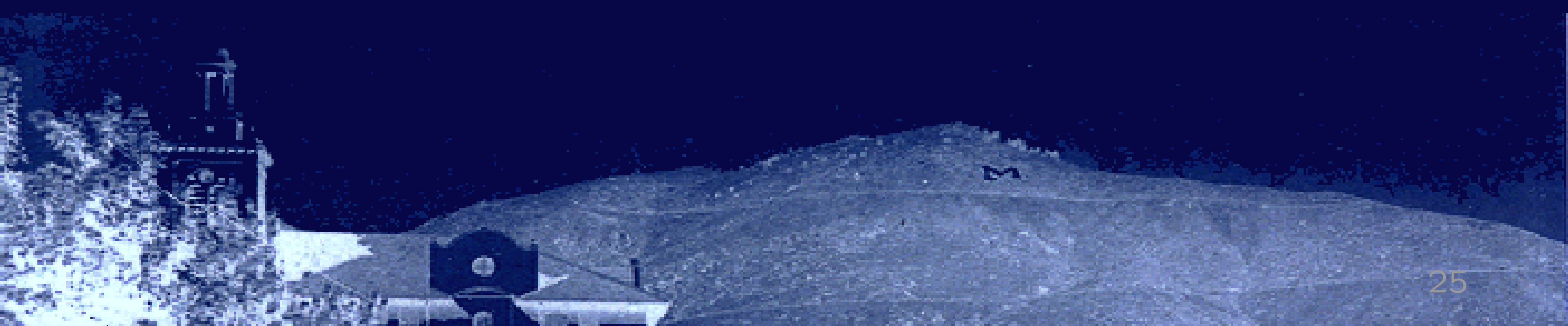
Hyperparameters $\xi_1, \ldots, \xi_5$

To make sure that variability around the truth
is smaller than bias and internal variability

$$\phi_i > \psi_i$$

Choose ξ_1, ξ_2, ξ_3 small, $\xi_4 \in [\,1,\, 2.5\,]$, ξ_5 large.

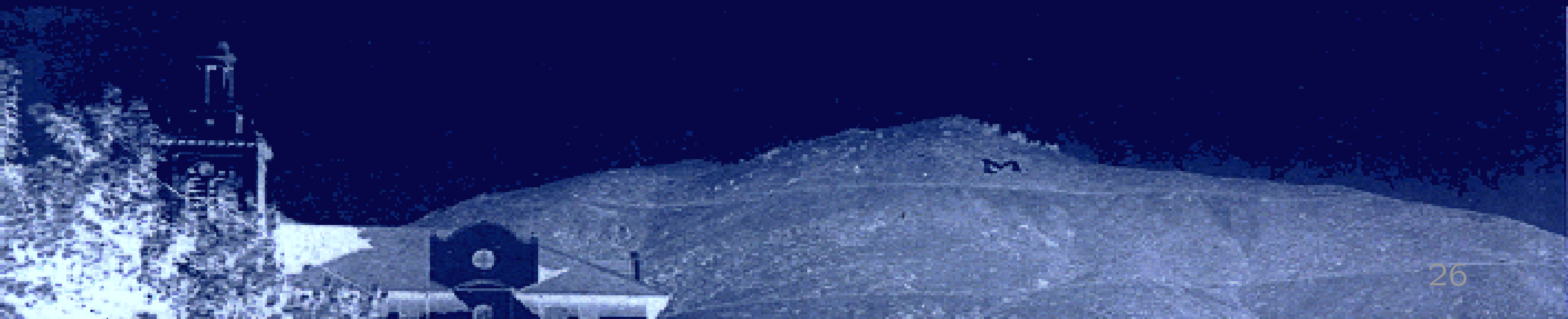
$\xi_4 \in [\,.5,\, 1.5\,]$ for 2020-2029 projections.



Posterior Distribution

The goal is the posterior distribution of $\mathbf{M}\vartheta$ given the data $\mathbf{D}_i$:

$$[\,\mathbf{M}\vartheta \mid \mathbf{D}_1, \ldots, \mathbf{D}_N, \ldots]$$



Posterior Distribution

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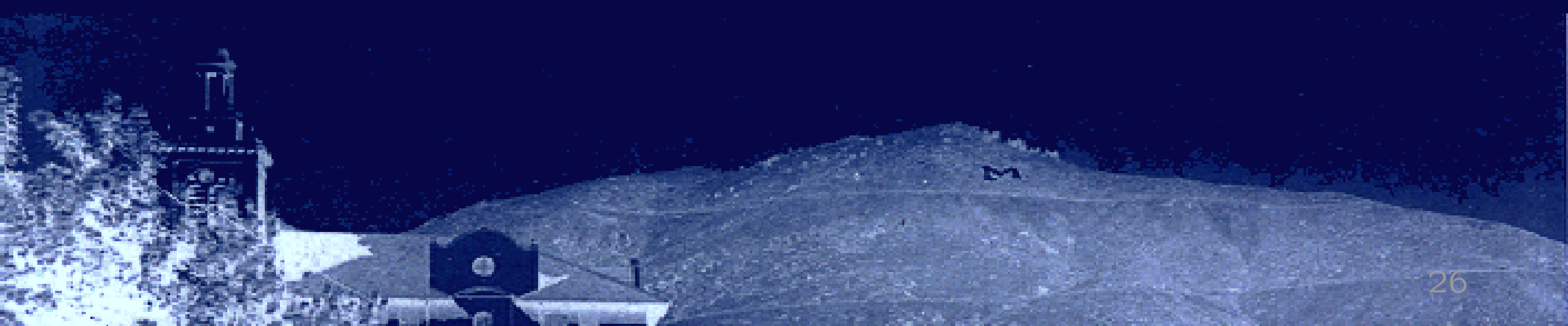
$$[\,\mathbf{M}\vartheta \mid \mathbf{D}_1, \ldots, \mathbf{D}_N, \ldots\,]$$

Via Bayes' theorem, the posterior density is

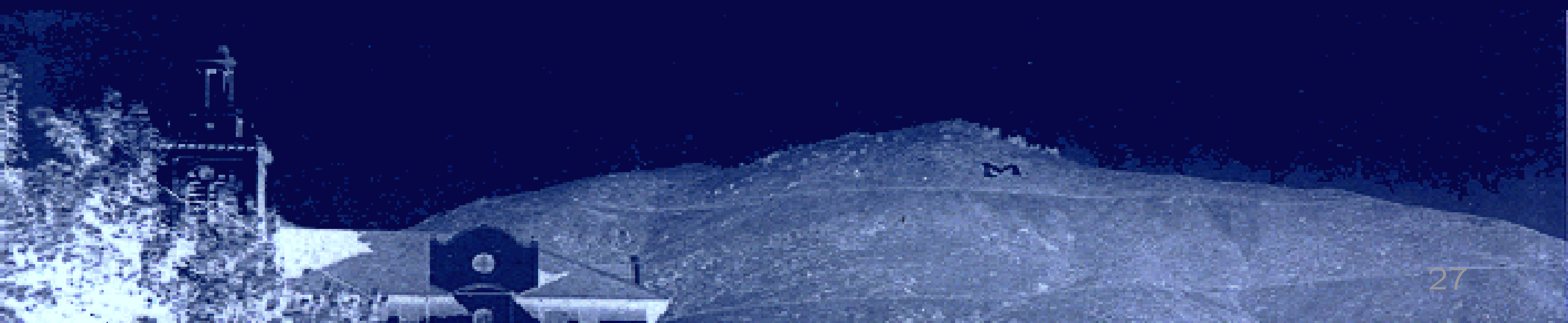
$$\begin{aligned}
&[\text{ process} \mid \text{data, parameters }] \\
&\qquad \propto [\text{ data} \mid \text{process, parameters }] \\
&\qquad\qquad \cdot [\text{ process} \mid \text{parameters }] \cdot [\text{ parameters }]
\end{aligned}$$



Posterior Distribution

No closed form of the posterior density.

Use computational approaches MCMC.



Posterior Distribution

No closed form of the posterior density.

Use computational approaches MCMC.

Gibbs sampler:

1. Express the distribution of the parameter conditional on everything else.

2. Cycle among the parameters by simulating a new value based on the full conditional distribution and the current values of the other parameters.

3. Repeat, . . .

Full Conditionals

Full conditionals for the parameters are available:

$$\boldsymbol{\vartheta} \mid \ldots \sim \mathcal{N}_p(\cdot,\cdot)$$

$$\boldsymbol{\theta}_i \mid \ldots \sim \mathcal{N}_p(\cdot,\cdot)$$

$$\phi_i \mid \ldots \sim \mathrm{I}\Gamma(\cdot,\cdot)$$

$$\psi_i \mid \ldots \sim \mathrm{I}\Gamma(\cdot,\cdot)$$

Full Conditionals

Full conditionals for the parameters are available:

$$\boldsymbol{\vartheta} \mid \ldots \sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b},\ \mathbf{A}^{-1})$$

$$\mathbf{A} = \frac{1}{\xi_5}\mathbf{I} + \sum_{i=1}^{N} \frac{1}{\psi_i}\mathbf{I} \qquad \mathbf{b} = \sum_{i=1}^{N} \frac{1}{\psi_i}\boldsymbol{\theta}_i$$

$$i = 1, \ldots, N: \ \boldsymbol{\theta}_i \mid \ldots \sim \mathcal{N}_p(\mathbf{A}^{-1}\mathbf{b},\ \mathbf{A}^{-1})$$

$$\mathbf{A} = \frac{1}{\psi_i}\mathbf{I} + \frac{1}{\phi_i}\mathbf{M}^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}\mathbf{M} \quad \mathbf{b} = \frac{1}{\psi_i}\boldsymbol{\vartheta} + \frac{1}{\phi_i}\mathbf{M}^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}\mathbf{D}_i$$

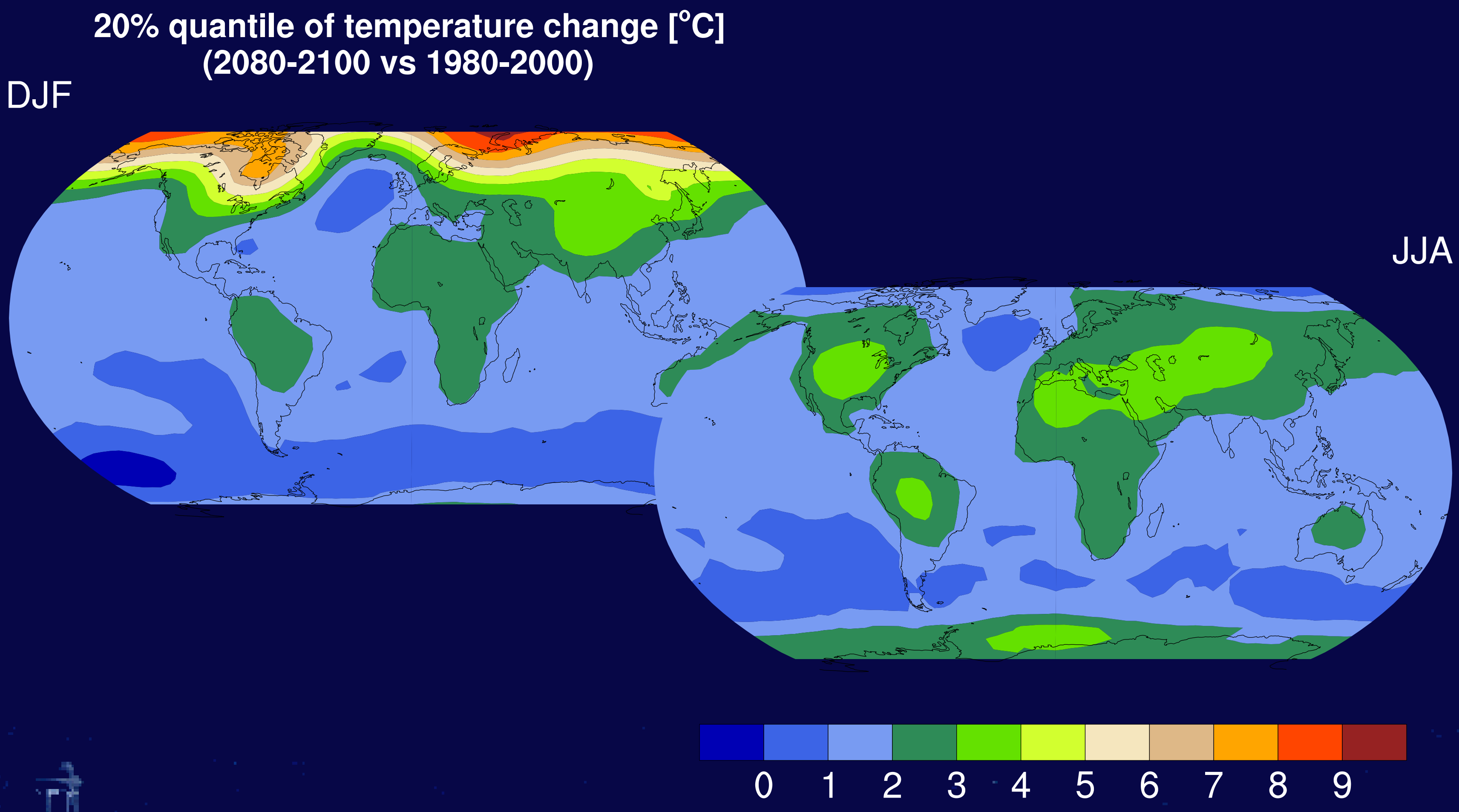
$$i = 1, \ldots, N: \ \phi_i \mid \ldots \sim \mathrm{I}\Gamma\left(\xi_1 + \frac{n}{2}, \xi_2 + \frac{1}{2}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{D}_i - \mathbf{M}\boldsymbol{\theta}_i)\right)$$

$$i = 1, \ldots, N: \ \psi_i \mid \ldots \sim \mathrm{I}\Gamma\left(\xi_3 + \frac{p}{2}, \xi_4 + \frac{1}{2}(\boldsymbol{\theta}_i - \boldsymbol{\vartheta})^{\mathsf{T}}(\boldsymbol{\theta}_i - \boldsymbol{\vartheta})\right)$$

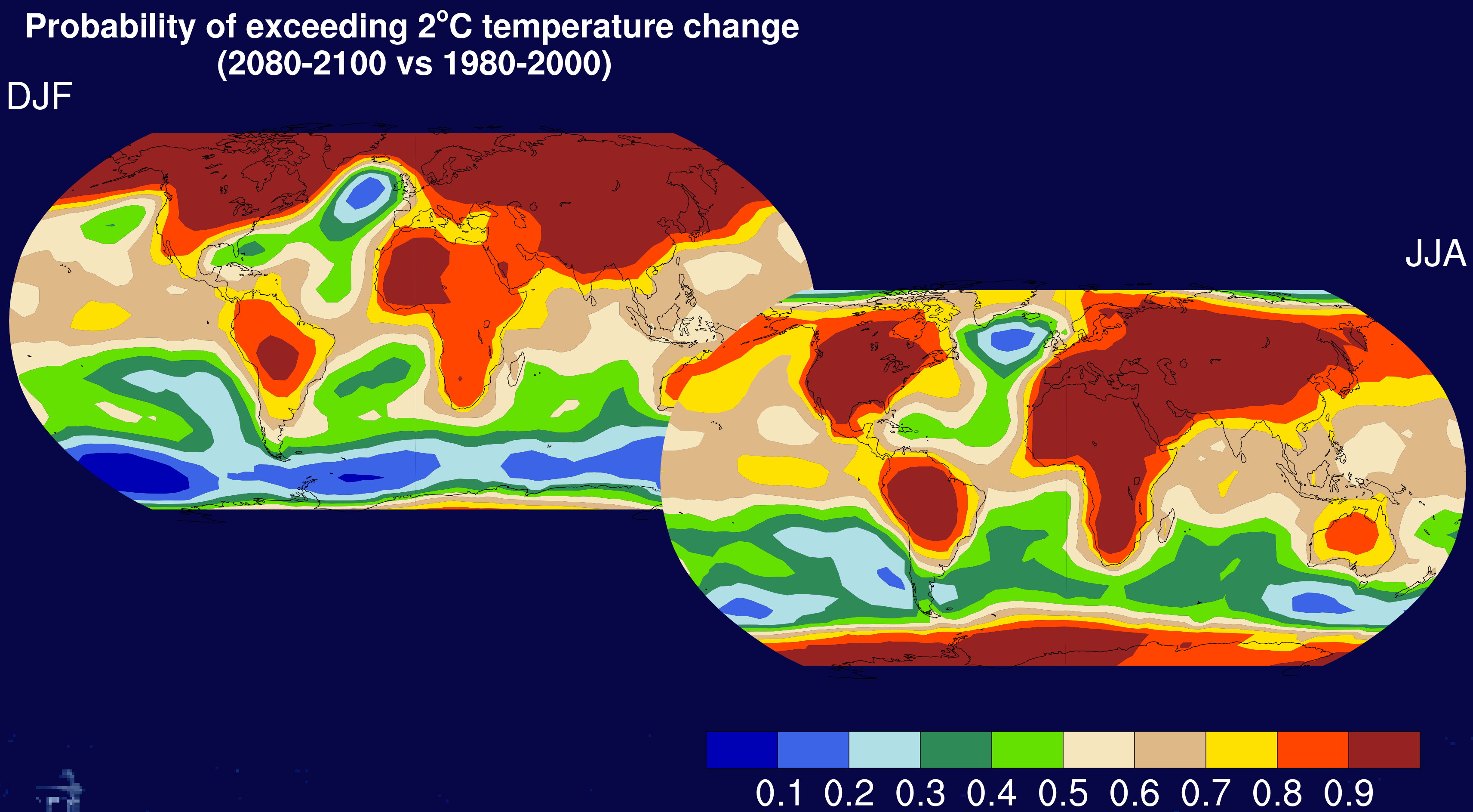
Gibbs Sampler

- Gibbs sampler programmed in R
- Run 20000 iterations
  (10000 burn-in, keep every 20th, takes few hours)
- Visual/primitive inspection of convergence

Temperature Change Quantiles



Exceedance Probabilities



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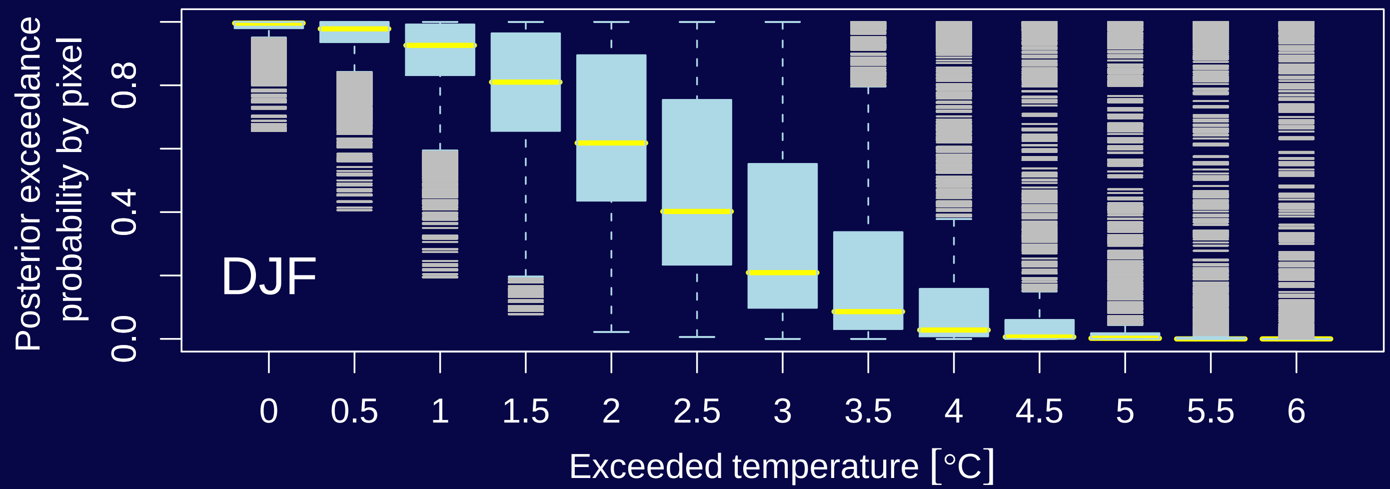
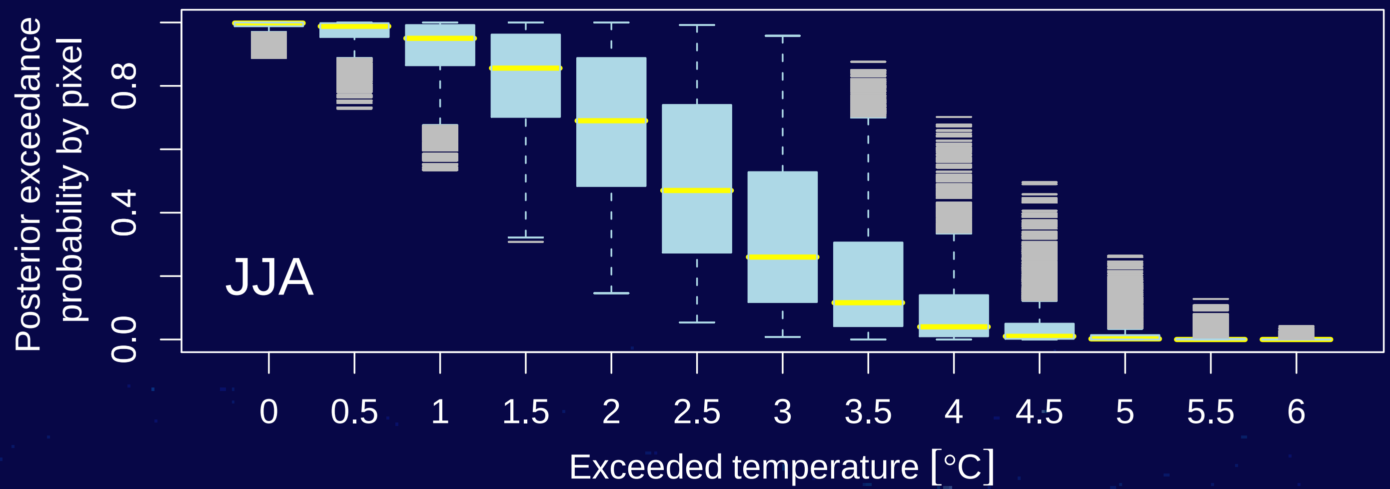
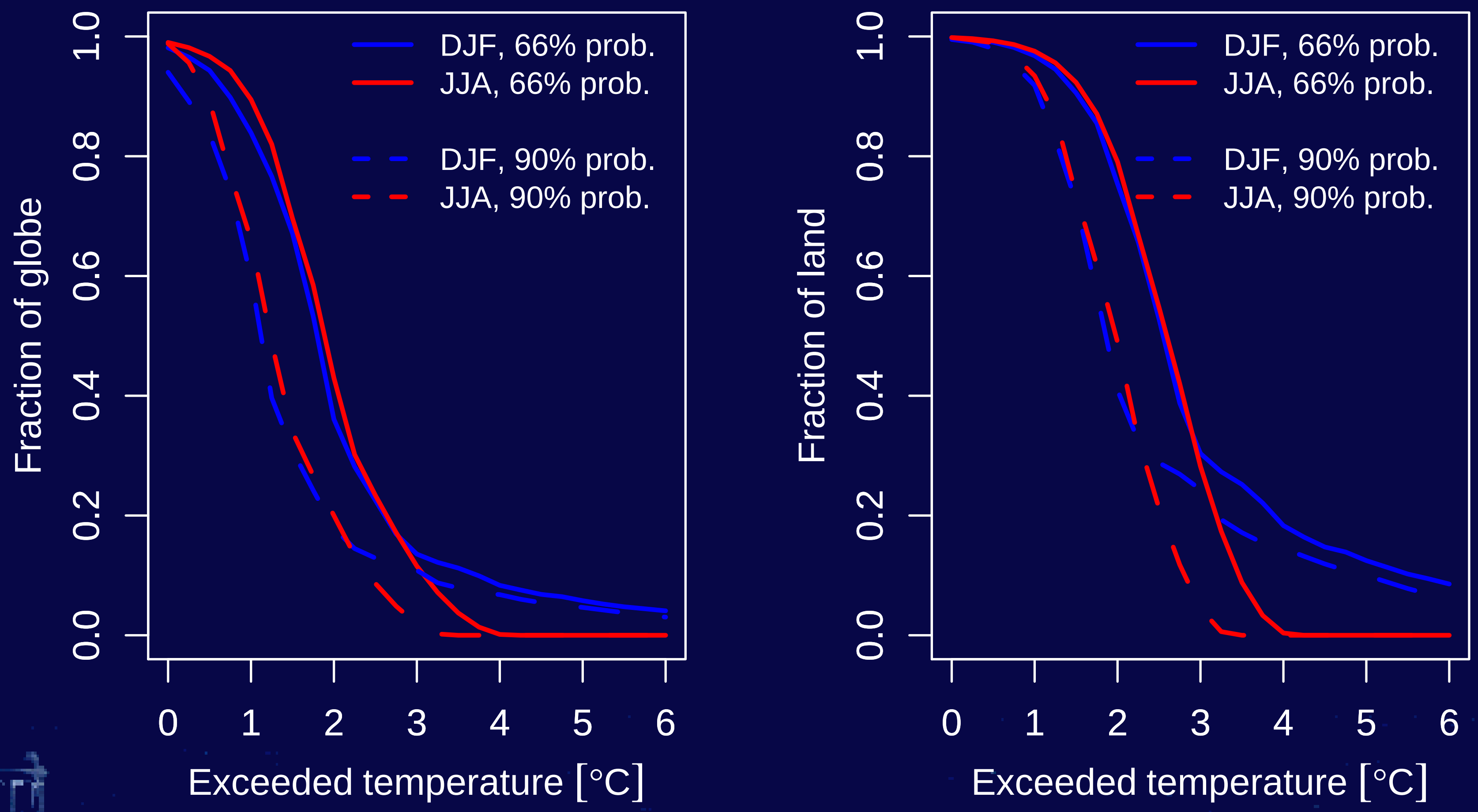
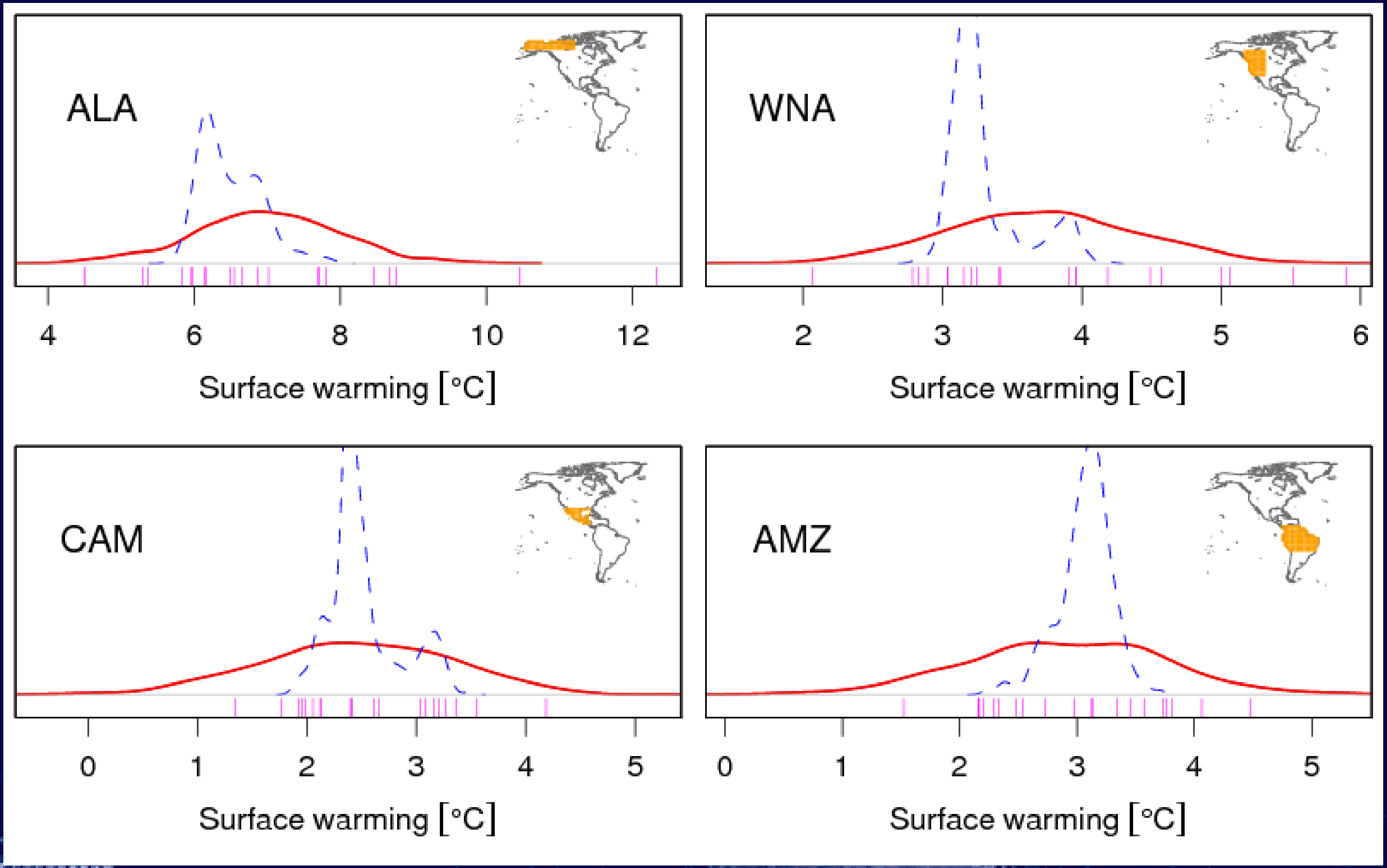




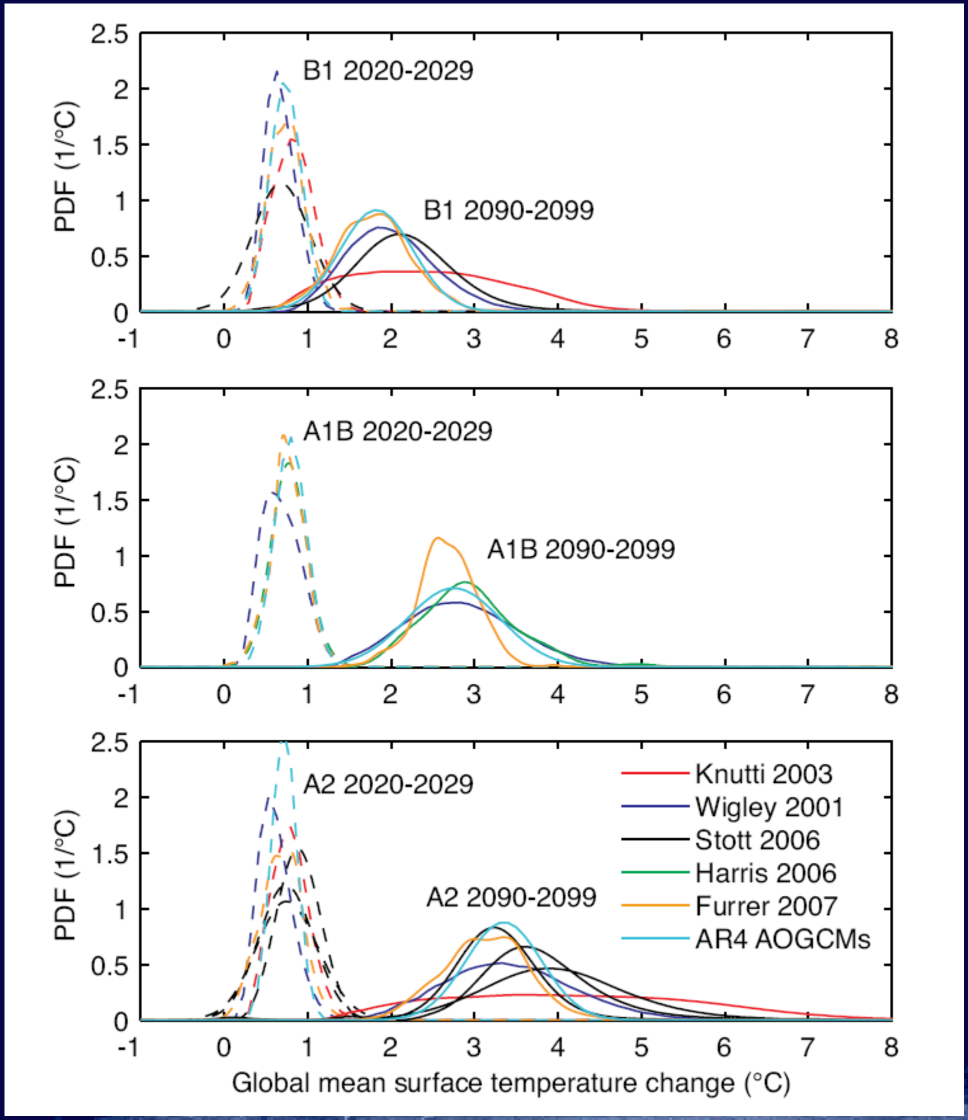
Exceedance Fractions



Regional Assessment



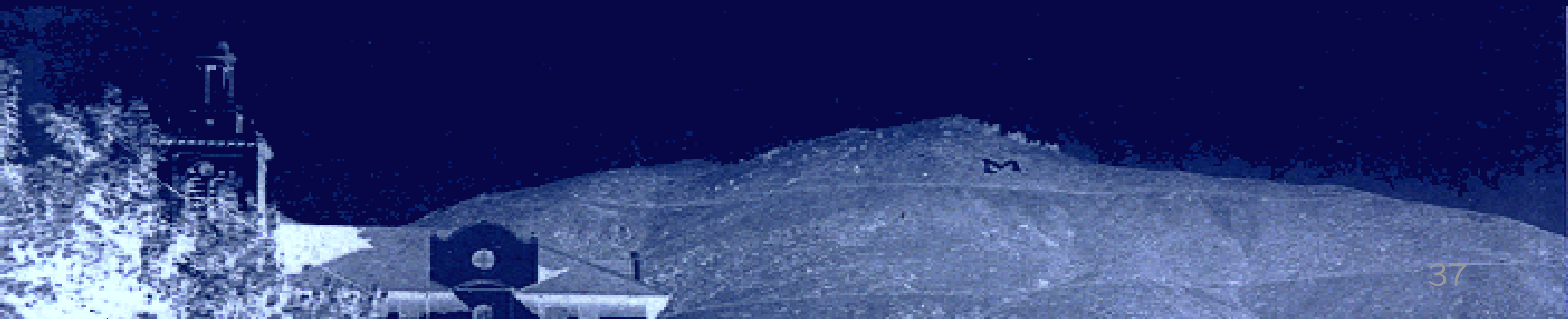
Global Assessment



Source: AR4, IPCC

Model Extensions

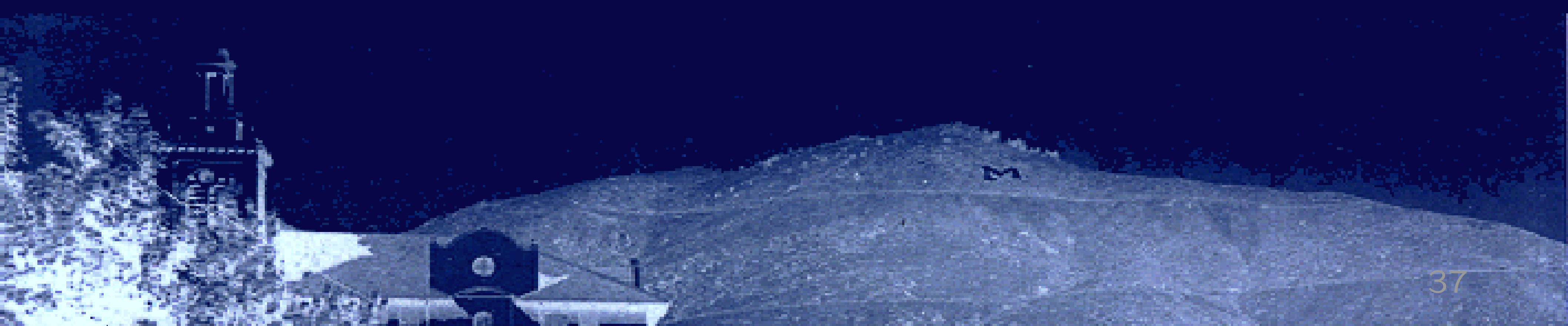
- Use “more” data
  - $\rightsquigarrow$ ensemble runs, model present and future individually, . . .



Model Extensions

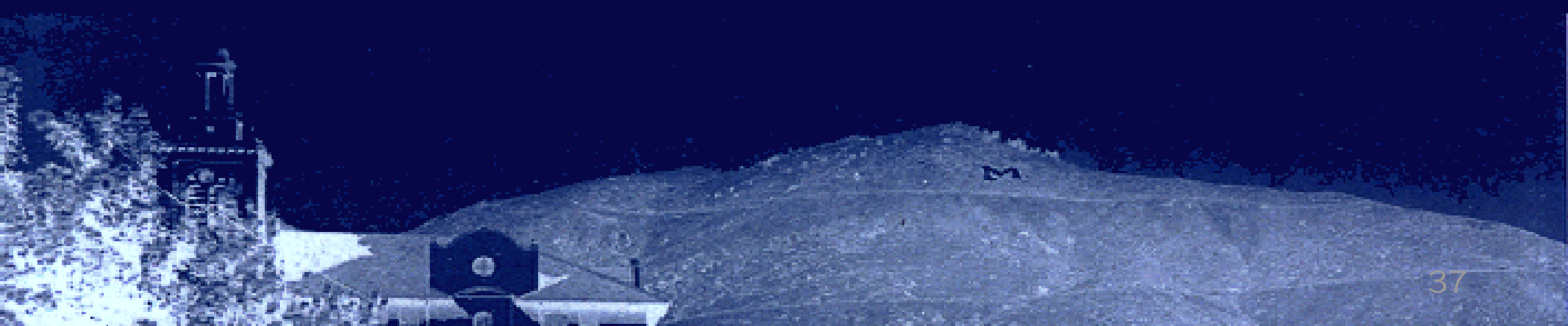
- Use "more" data
  - $\rightsquigarrow$ ensemble runs, model present and future individually, . . .

- Use AOGCM specific weighting
  - $\rightsquigarrow$ performance, "core" simililarities, . . .



Model Extensions

- Use “more” data
  - $\leadsto$ ensemble runs, model present and future individually, . . .
- Use AOGCM specific weighting
  - $\leadsto$ performance, “core” simililarities, . . .
- Parameterize covariance matrices
  - $\leadsto$ built in range, nonstationarity, . . .



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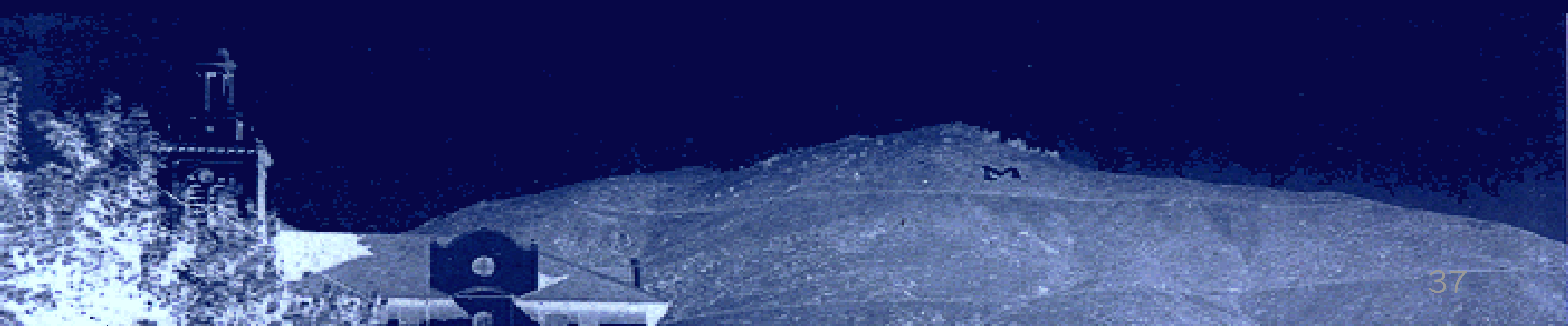
  - $\rightsquigarrow$ built in range, nonstationarity, . . .
- Building bi-/multivariate models
  - $\rightsquigarrow$ use temperature for precipitation prediction, . . .



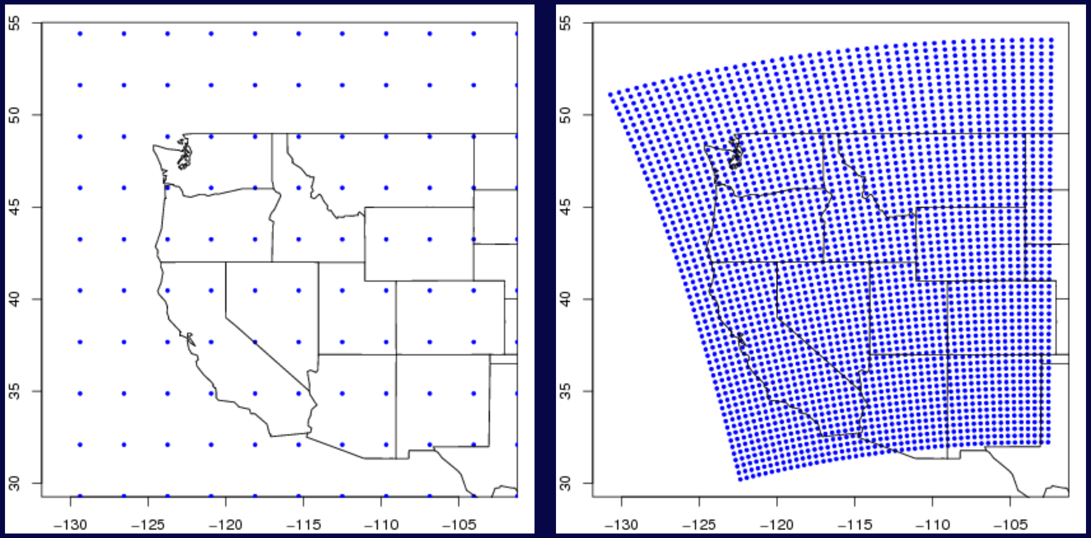
Model Extensions

- Use “more” data
  - $\leadsto$ ensemble runs, model present and future individually, . . .
- Use AOGCM specific weighting
  - $\leadsto$ performance, “core” simililarities, . . .
- Parameterize covariance matrices
  - $\leadsto$ built in range, nonstationarity, . . .
- Building bi-/multivariate models
  - $\leadsto$ use temperature for precipitation prediction, . . .
- Address computational complexity
  - $\leadsto$ sparsity, Metropolis-Hastings steps, . . .

Model Extensions

Is an geostatisitical approach adequat?

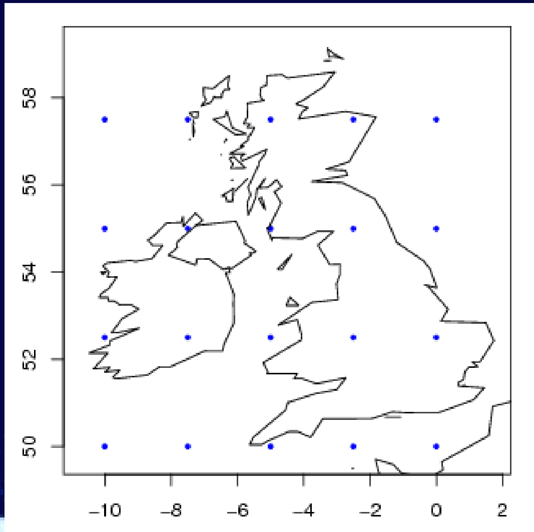
- NARCCAP: North American Regional Climate Change Assessment Program. `www.narccap.ucar.edu`

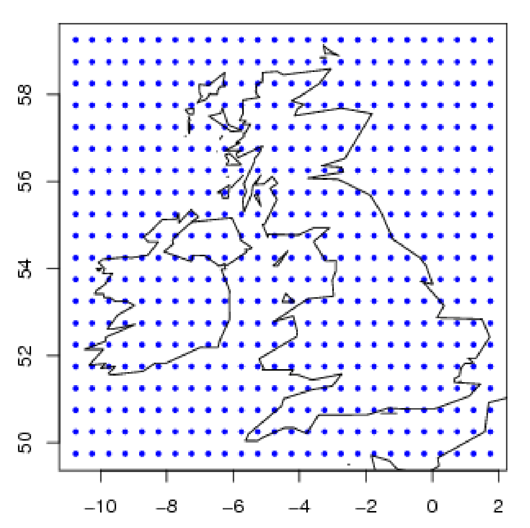


Model Extensions

Is an geostatisitical approach adequat?

- NARCCAP: North American Regional Climate Change Assessment Program. www.narccap.ucar.edu
- PRUDENCE: Prediction of Regional scenarios and Uncertainties for Defining EuropeaN Climate change risks and Effects. http://prudence.dmi.dk



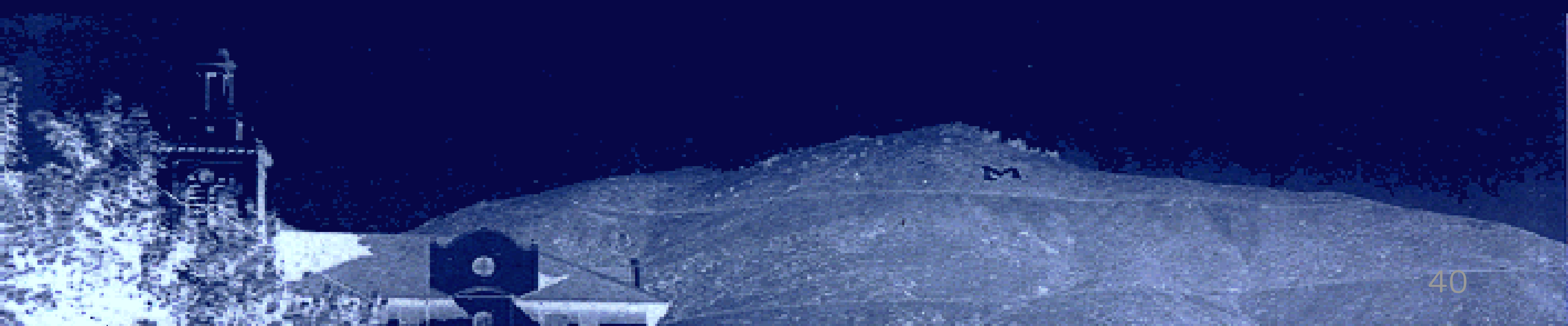


CAR and SAR Models

Spatial autoregressive models represent the data at a lattice site as a linear combination of neighboring locations.

1. Simultaneous autoregressive (SAR) models

2. Conditional autoregressive (CAR) models



CAR and SAR Models

Spatial autoregressive models represent the data at a lattice site as a linear combination of neighboring locations.

1. Simultaneous autoregressive (SAR) models

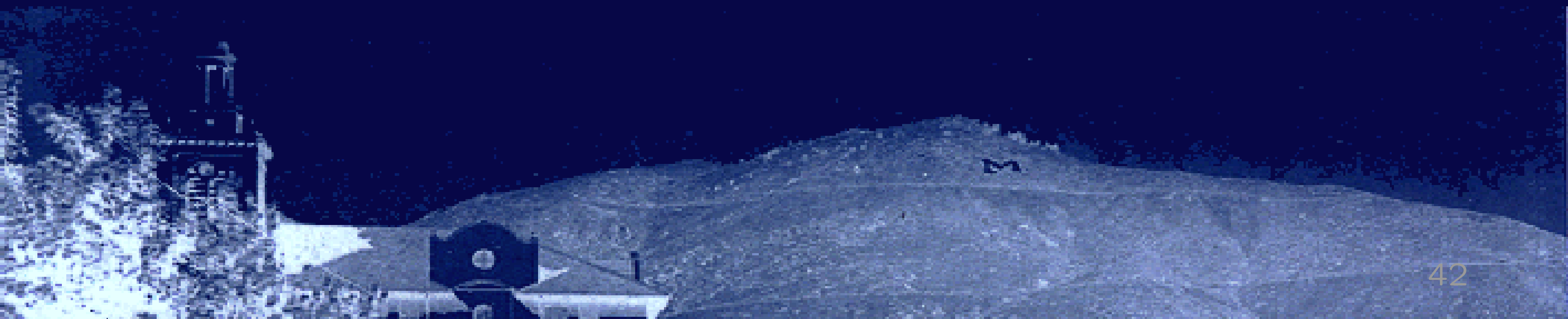
$$Y_i = \mu_i + \sum_j b_{ij}(Y_j - \mu_j) + \varepsilon_i$$

2. Conditional autoregressive (CAR) models

$f(Y_i \mid Y_{-i})$ with Y_{-i} all but Y_i

Discussion

- AOGCMs are not “equal”
- AOGCMs are not “unbiased”
- AOGCMs are not “independent”



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