Statistics for detecting climate change

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Supported by the Statistical and Applied Mathematical Sciences Institute
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Introduction

2006-2007 SAMSI climate and RM working group:

- reading group (EOF, detection & attribution,..)
- Undergraduate workshop
- Data portal: CMIP3 multi-model dataset at PCMDI (ex IPCC AR4 archive)
Search for evidence of climate change
Fingerprints, patterns of climate change

- Patterns are spatio-temporal responses of the climate to a certain forcing
- e.g. [only spatial] surface temperature change from increasing GHG or from anthropogenic sulfate aerosols
- Chosen a priori (physics or ensemble of runs of a model)
Detection and attribution


Detection of anthropogenetic signals

Decompose the observations $y$ into a linear combination of climate change “signals” $g_i$.

$g_i$: patterns (or building blocks for fingerprints) of responses to individual or combined forcings (solar, sulfate aerosol, GHG,..).
\[ y = \sum_{i=1}^{m} b_i g_i + \varepsilon \] 

(1)

**Note:** Linearity good assumption (Gillett et al., 2004)

Option: “orthogonalize” some signals prior to the regression (e.g. sulfate aerosol, greenhouse gas).
/home/guillas/climate/dataCM2/dataEOFdiffsJan/eof_ts--a1_ts_179W179E--89S89N_Jan_mon.Rdata ( Jan )
Similar results with more general approach:

Regression

- \( Y \): observations \( p \)-dimensional
- \( X \): patterns of climate change from model, known \( p \times m \) matrix of rank \( q \).
- each column of \( X \) is a vector representing a response-pattern spatially

Linear regression: (Mardia et al. 1979)

\[
Y = X\beta + u \tag{2}
\]

with \( u \) is the climate noise, \( Eu = 0, C(u) = \sigma^2 I \).
Ordinary Least Squares:

Let $\hat{\beta} = (X'X)^{-1} X'Y$.

Th. [Gauss-Markov Th]: $\hat{\beta}$ is the BLUE of $\beta$.

Generalized Least Squares:

When $C(u) \neq \sigma^2 I$, Consider the transformed model:

$$Z = C(u)^{-1/2} X \beta + v$$  \hspace{1cm} (3)

where $Z = C(u)^{-1/2} Y$, $v = C(u)^{-1/2} u$. 
"Pre-whitening" with any matrix $P$ such that:

$$
E(Puu'P') = PC(u)P' = I. \tag{4}
$$

$C(v) = I$, so we can apply G-M to this model:

Let

$$
\tilde{\beta} = \left( X'C(u)^{-1}X \right)^{-1} X'C(u)^{-1/2}z = \left( X'C(u)^{-1}X \right)^{-1} X'C(u)^{-1}y. \tag{5}
$$

$\tilde{\beta}$ is the BLUE of $\beta$, and $C(\tilde{\beta}) = (X'C(u)^{-1}X)^{-1}$. 
Possible to look at scalar diagnostics $\phi = w'Y$. (e.g. global mean, or focus on one mean for one grid cell)

Issue: noise in $X$ inflates the variance of $\tilde{\beta}$ by approximately $1 + 1/M$ ($M = \text{ens. size}$)

**Fingerprints**

The columns of $C(u)^{-1}X$ are the optimal fingerprints.
Climate noise

**Note:** $\mathcal{C}(u)$ is unknown, so:

- $\hat{\mathcal{C}}_n(u) = \frac{1}{n} Y_n Y'_n$ can be plugged in.
- $Y_n$ are “pseudo-observations” from a control run with features as close as possible to the observations (locations of missing data, ..)
Problem: $\hat{C}_{n}(u)$ not invertible, $(p > n)$, so:

1. use $k$ EOFs of control runs (or sometimes of forced runs)

2. Define $P_{(k)}$ as matrix of $k$ highest variance EOFs weighted by $\sqrt{\lambda_i}$

3. use the Moore-Penrose pseudo-inverse $P'_{(k)}P_{(k)}$ in place of $\hat{C}_{n}^{-1}$

So $P'_{(k)}P_{(k)} = I_k$

Issue: depends on $k$!
Tests and confidence regions

Under normality assumption for \( u \),

\[
(\tilde{\beta} - \beta) \left( X' C(u)^{-1} X \right)^{-1} (\tilde{\beta} - \beta) \sim \chi^2_m \quad (6)
\]

Using EOFs:

With an estimated Covariance matrix (often on another sample),

test becomes a \( T^2 \)-test, using \( F \)-distributions.

\[
(\tilde{\beta} - \beta) \left( X' C(u)^{-1} X \right)^{-1} (\tilde{\beta} - \beta) \sim T^2 \quad (7)
\]
Bayesian approach

Berliner et al. (2000)

True vector of temperatures $T_t$

Observations $Y_t$

$$Y_t | T_t, D_t \sim N (L_t T_t, D_t)$$  \hspace{1cm} (8)

with $L_t$ location matrix (only 0 except 1 for the location)

$$T_t | a, g, \Sigma \sim N (a . g, \Sigma^s)$$  \hspace{1cm} (9)

with $g$ spatial fingerprint, $\Sigma^s$ spatial covariance.
Assumption of space-time separability.

Prior on $a$: (actually collection of)

$$
\pi(a) = pn(0, \sigma^2) + (1 - p)n(\mu_A, \tau^2_A) \quad (10)
$$
Future work

Common EOFs
Benestad (2001) [downscaling studies]

- concatenate the fields (temp, pressure)
- Carry out the EOF decomposition alltogether.
- EOFs more representative of the patterns.

..not yet used for detection & attribution
Spatio-temporal EOFs
(North and Wu, 2001)

Issues:

• size of covariance matrices
• type of correlation
• truncation