Stochastic Model for Clouds Uncoupled to the Boundary Layer

Howard Bondell

bondell@stat.ncsu.edu

NC STATE UNIVERSITY

Joint with: Montse Fuentes, Josh Hacker, Brian Reich, Justin Shows, Robert Tardif

Outline

- Uncoupled Clouds
  - Focus on Cloud Base Height
- Stochastic Model for Uncoupled Cloud Height
  - Time Series Model
  - Mixture Model
- Future Directions

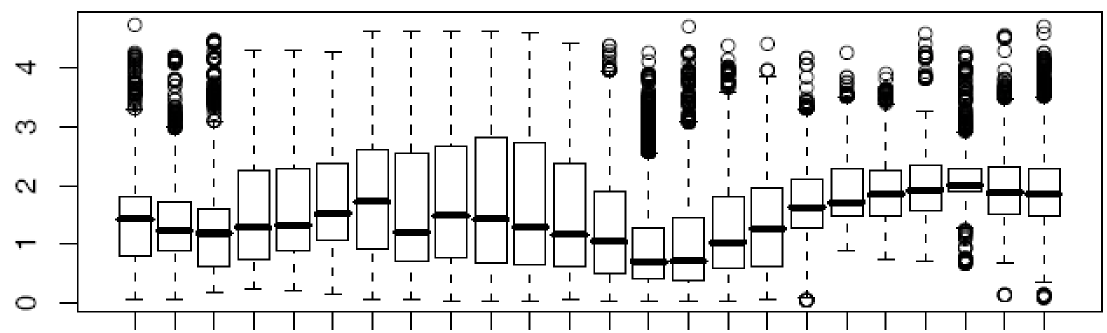
Uncoupled Clouds

- Data:
  - Cloud Base Height and Liquid Water Path
  - 3 1/2 years at 20 second time steps from single column
  - Only under fair weather conditions - Lots of missings!
  - >1 million observations
  - About 7% have uncoupled cloud, rest do not
- Goal:
  - Stochastic model for Cloud Height and LWP
  - Examine effects of using random generations of these processes as inputs to model for boundary layer conditions
  - Short term (1 or 2 days)

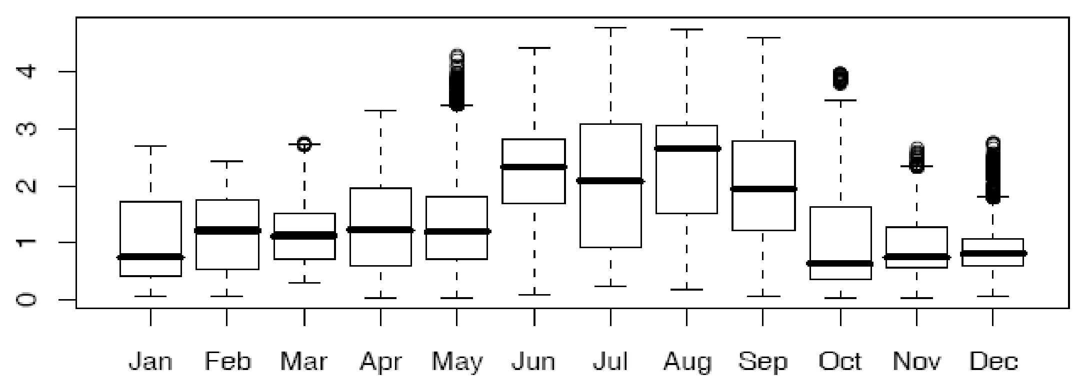
Uncoupled Clouds

- Cloud Height: Diurnal and Seasonal Trends

Cloud Base Height by Hour

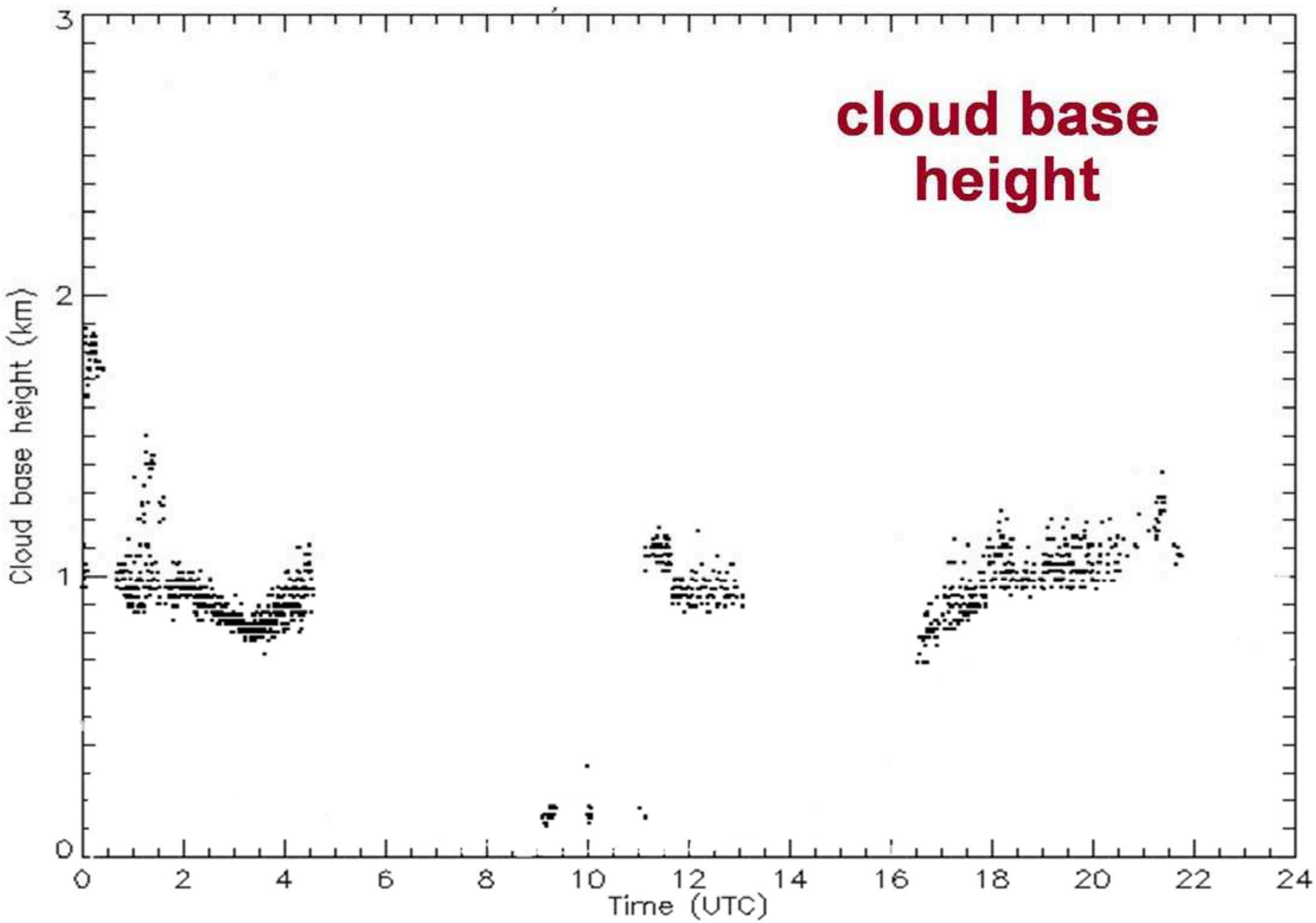


Cloud Base Height by Month



Uncoupled Clouds

- A Day in the Life of Cloud Height



Model for Uncoupled Clouds

- Purely stochastic model
  - Not affected by boundary layer processes
- Conditional on presence of uncoupled cloud

$$Y_t = \mu_t + \epsilon_t$$

Y_t represents Cloud Height (or log Water Path) at time t
μ_t is mean trend (depends on month and time of day)
ϵ_t is error at time t

- Data in 20 second time steps
- Errors clearly not independent across time!
  - Same cloud over multiple time points

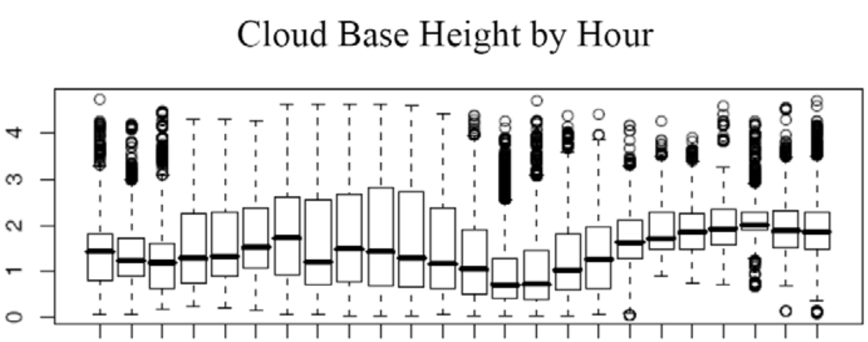
Model for Cloud Height

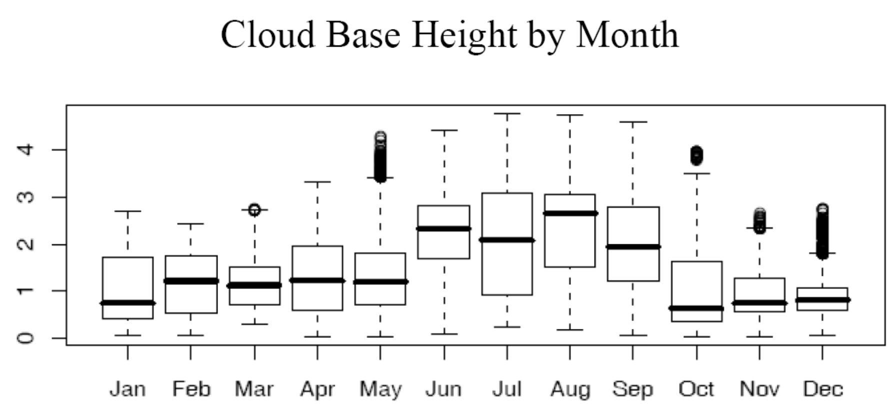
- Recall: $Y_t = \mu_t + \epsilon_t$
- Model trend with seasonal and daily components

$$\mu_t = \alpha + \beta_1 \sin\left(\frac{2\pi M}{12}\right) + \beta_2 \cos\left(\frac{2\pi M}{12}\right) + \beta_3 \sin\left(\frac{2\pi H}{12}\right) + \beta_4 \cos\left(\frac{2\pi H}{12}\right)$$

M is time in months and H is time in hours

- Simple trigonometric model gives good fit to overall trend





NC STATE UNIVERSITY

Time Series Model for Errors

- Recall: $Y_t = \mu_t + \epsilon_t$
- Autoregressive Moving Average (ARMA) Model

$$\epsilon_t = \rho_1 \epsilon_{t-1} + ... + \rho_k \epsilon_{t-k} + \phi_1 w_{t-1} + ... + \phi_h w_{t-h} + w_t$$

  $\{w_t\}$ is a White Noise sequence with variance σ^2

- ARMA (k, h)
  k is order of the autoregressive part
  h is order of the moving average part
  - AR propogates current value to future
  - MA propogates random shock to future

Model for Cloud Height Errors

- Recall: $Y_t = \mu_t + \epsilon_t$
- Error terms given by ARMA (1,2)
  - Estimated coefficients differ by month
  - Interest in short term effects, may differ by season
- $\epsilon_t = \rho\epsilon_{t-1} + \phi_1 w_{t-1} + \phi_2 w_{t-2} + w_t$
  - AR term $\rho \approx 1 \rightarrow$ simplification: differences follow MA (2)
  - $\epsilon_t - \epsilon_{t-1} = \phi_1 w_{t-1} + \phi_2 w_{t-2} + w_t$
  - MA components = 0 $\rightarrow$ Simple Random Walk
  - Negative here, so less drift than SRW

Model for Presence of Cloud

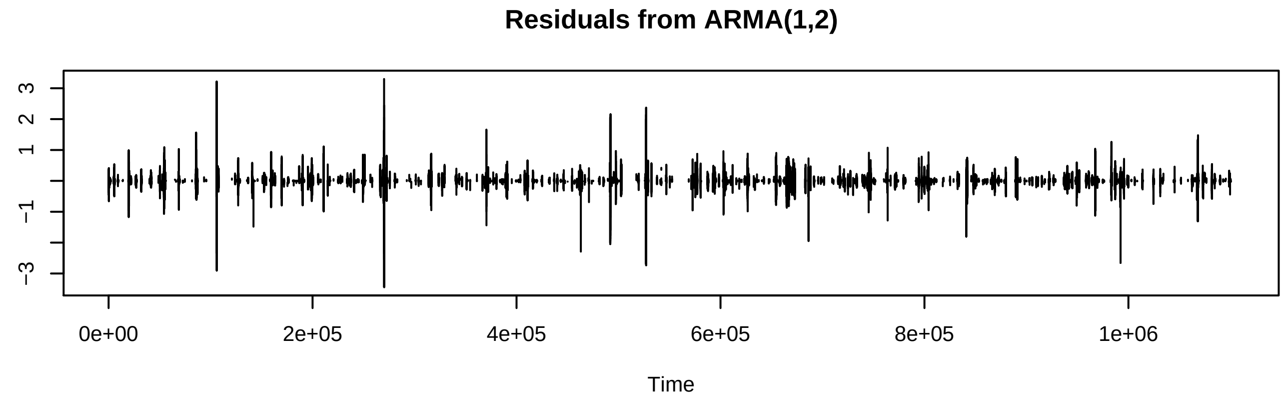
- Overall 6.9% of time points have uncoupled cloud
- Logistic Regression Model
  - Let $Z_t = 1$ if cloud, 0 otherwise
  - Let $\tilde{Z}_t$ represent the history up to time t
  - $Pr[Z_t = 1|\tilde{Z}_t] = \frac{\exp(\alpha+\beta\{Z_{t-1}+Z_{t-2}+Z_{t-3}\})}{1+\exp(\alpha+\beta\{Z_{t-1}+Z_{t-2}+Z_{t-3}\})}$
  - Current cloud status depends on proportion of cloud cover in last minute
- Can add other covariates such as time of day, humidity, etc.

Algorithm to Simulate Clouds

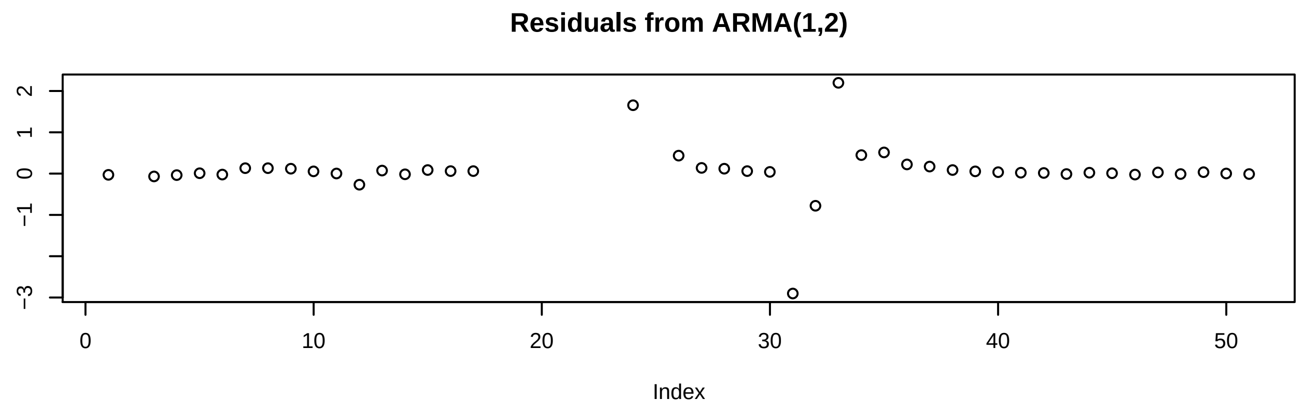
- Generate presence/absence of cloud from logistic model
- Once new cloud is present, initial height is generated from $N(\mu_t, \sigma_t^2)$
  - μ_t given by seasonal and diurnal trend
  - σ_t^2 is variance that is estimated monthly
- Generate heights from appropriate ARMA process until cloud disappears
- Restart after new cloud appears
- Good fit to data, MOSTLY!

BUT ...

- Residuals from ARMA process fit
  - Overall

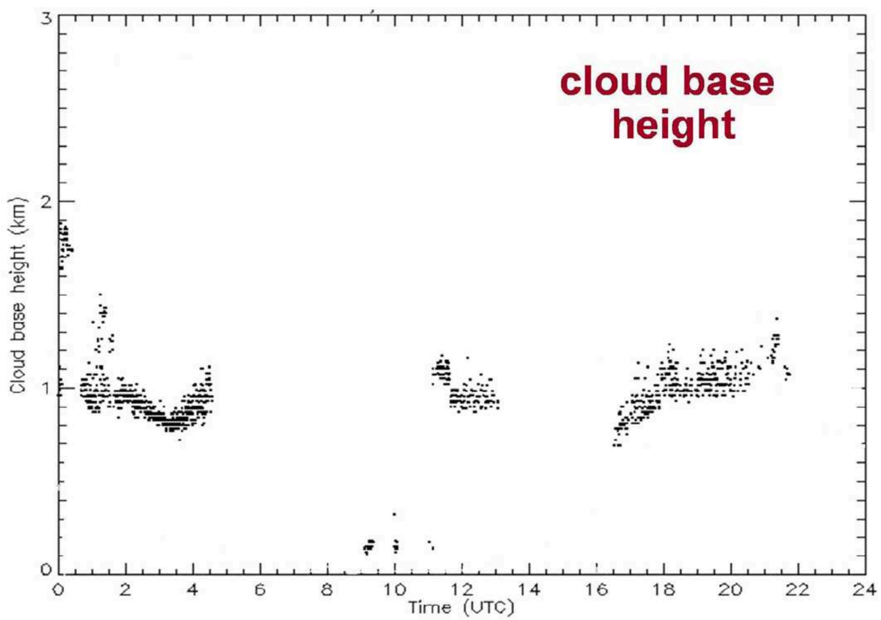


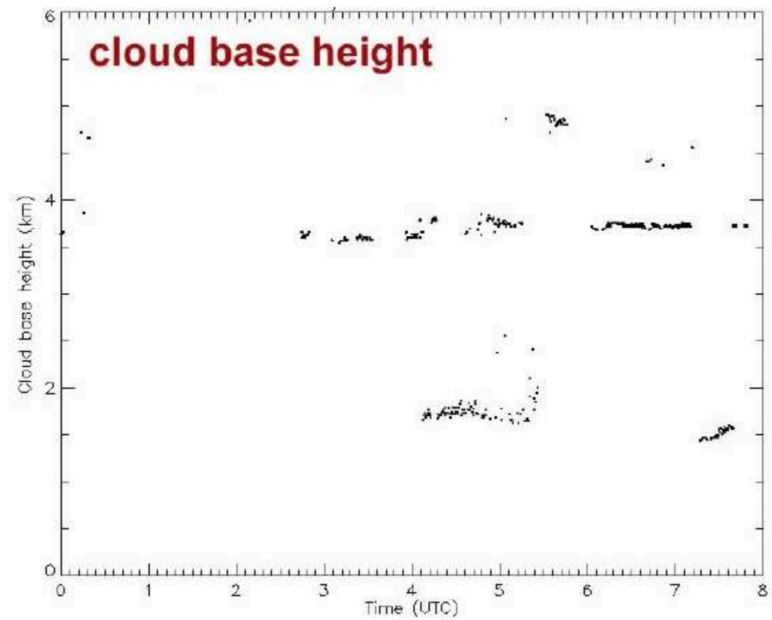
  - Zoomed in on 1/2 hour



Drawback

- Cloud height measurement based on lowest detected cloud



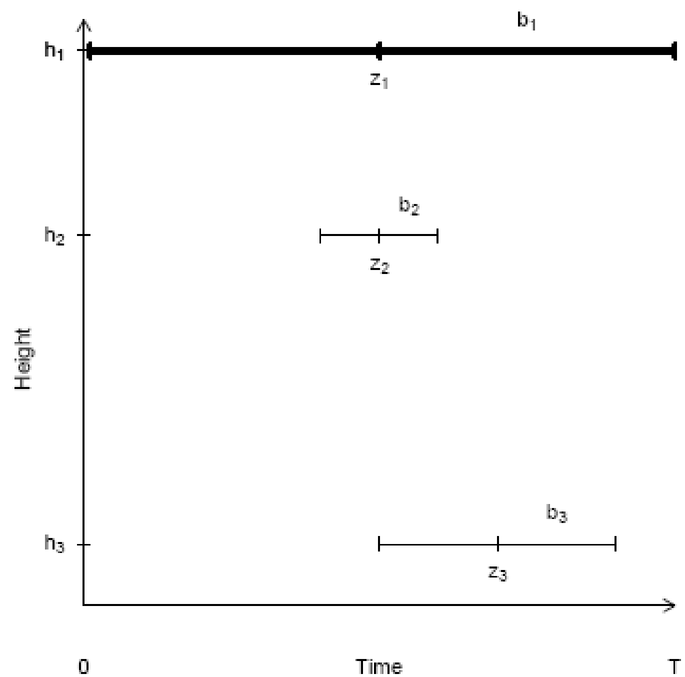


- How to allow for jumps from upper to lower cloud layers?
- Is the simple model sufficient?

NC STATE UNIVERSITY

Mixture Model

- Let N = Total # of clouds above boundary layer in $[0, T]$
- Knot z_j = time that center of cloud j is overhead
- Bandwidth b_j = 1/2 length of time cloud j is overhead
- Height h_j = average base height of cloud j



Placing the Clouds in the Window

- Number of clouds $N \sim$ Poisson(λT)
- Knots z_j drawn from distribution on $[0, T]$
- Bandwidths b_j drawn from distribution on $[0, T/2]$
- Heights h_j drawn from Normal

- Simple case:
  - Knots and Bandwidths are uniform
- In general:
  - Distributions can depend on covariates such as time, etc.

- But what data do we actually have?

Cloud Base Height Model

- Observed cloud base height at time t

$$Y_t \overset{\mathcal{D}}{=} \sum_{j\in M_t} p_j(t)\delta\left(h_j\right) + \varepsilon_t$$

  - $M_t = \{j | z_j - b_j \leq t \leq z_j + b_j\}$ is subset of clouds overhead at time t
  - $p_j(t)$ is probability of picking cloud j
  - $\delta\left(h_j\right)$ is point mass at height h_j
  - $\varepsilon_t \overset{iid}{\sim}$ N(0,σ_e^2) is variation in height

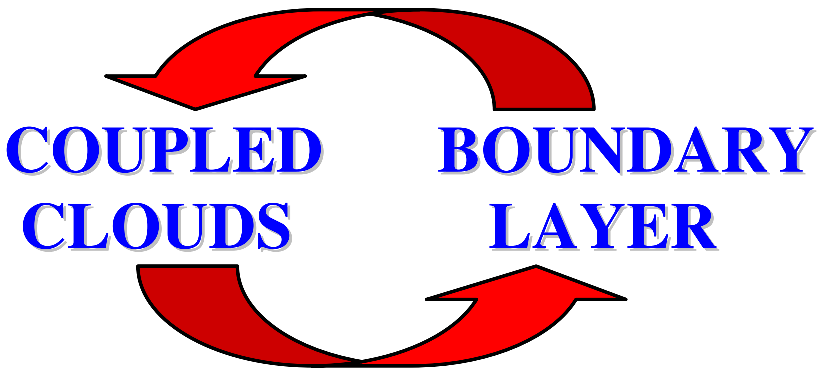
- How to determine probabilities?

Which Cloud is Observed?

- For each cloud in the window
  - Cloud Density, d_j = conditional prob. of detection given no lower detections
  - Densities $d_j \overset{iid}{\sim}$ Beta (a, b)
- For time t, order possible clouds from lowest to highest
  - Shoot beam upward and see what is detected
  - $Pr[$ pick cloud $j]$, $p_j(t) = d_j \prod_{\substack{k \in M(t) \\ h_k < h_j}} (1 - d_k)$
- Parameters in assumed distributions can be estimated from observed data
  - Then can generate cloud process from this model

Further Work: Coupled Clouds

- Uncoupled clouds can affect, but are not affected by Boundary Layer conditions
- Coupled clouds feedback into the boundary layer



- Stochastic parameterizations for coupled clouds need to allow for this interplay
- Need to combine stochastic model for uncoupled clouds with more physically based models for coupled clouds