Micro Talk on Physical-Statistical Modeling

All models are wrong, some are useful **- George Box**

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Bayesian Perspective

1. Data Model $[\mathbf{Z} \mid \mathbf{X}, \boldsymbol{\theta}]$

$$Z = h(X) + e$$

X = **true/latent process (unobserved)**
e = **measurement error**

Examples :

- **h = HX, H = Incidence Matrix**
- **Observations from different scales**

Bayesian Perspective

2. Process Model [X | $\boldsymbol{\theta}$]

- approximate science leads to uncertainty

- **CO2 Model :**

$$x_{i+1} = \phi(x_i) + G(u) + \epsilon_{i+1}$$

- **Ice Flow Velocity (Paterson, 1994)**

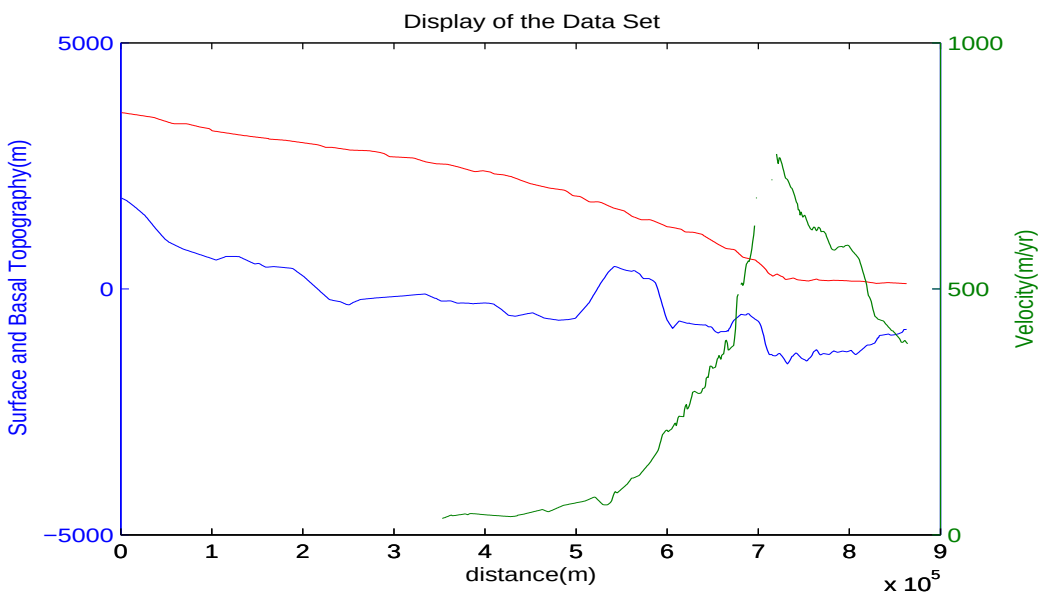
$$U = u_s + 2AH\frac{\tau^n}{n+1} + \epsilon,$$

where,

$$\tau = -\rho g H \frac{dS}{dx}$$

and

$$H = S - B$$


Display of the Data Set
5000
0
−5000
1000
500
0
Surface and Basal Topography(m)
Velocity(m/yr)
0
1
2
3
4
5
6
7
8
9
distance(m)
x 10^5

Bayesian Perspective

3. Parameter Model [$\boldsymbol{\theta}$]

- **Bayes' Theorem:**

$$[X, \boldsymbol{\theta}|Z] \propto [Z|X, \boldsymbol{\theta}][X|\boldsymbol{\theta}][\boldsymbol{\theta}]$$

Comments

- **Stages 1. vs 2.: Merging approximate scientific laws with Statistical uncertainty management.**
- **Non-identifiability of measurement error precision and process error precision**
- **Box, Hunter and Hunter(1978): Mechanistic Modeling**
- **Berliner(2003): Physical-Statistical Modeling**

Thanks !!