Micro Talk on Physical-Statistical Modeling

All models are wrong, some are useful - George Box

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Bayesian Perspective

1. Data Model \[ Z \mid X, \theta \]

\[ Z = h(X) + e \]

\( X = \text{true/latent process (unobserved)} \)

\( e = \text{measurement error} \)

Examples:

- \( h = HX, H = \text{Incidence Matrix} \)
- Observations from different scales
Bayesian Perspective

2. **Process Model** \([ X \mid \theta ]\)
   - approximate science leads to uncertainty

   ● **CO2 Model** :
     \[ x_{i+1} = \phi(x_i) + G(u) + \epsilon_{i+1} \]

   ● **Ice Flow Velocity** (Paterson, 1994)
     \[ U = u_s + 2AH \frac{\tau^n}{n + 1} + \epsilon, \]

     where,

     \[ \tau = -\rho g H \frac{dS}{dx} \]

     and

     \[ H = S - B \]
Display of the Data Set

- Surface and Basal Topography (m)
- Distance (m)
- Velocity (m/yr)
Bayesian Perspective

3. Parameter Model [ $\theta$ ]

- Bayes’ Theorem:

\[ [X, \theta \mid Z] \propto [Z \mid X, \theta][X \mid \theta][\theta] \]

Comments

- Stages 1. vs 2.: Merging approximate scientific laws with Statistical uncertainty management.
- Non-identifiability of measurement error precision and process error precision
- Box, Hunter and Hunter (1978): Mechanistic Modeling
Thanks!!