Carbon Cycle:
An Inverse Problem

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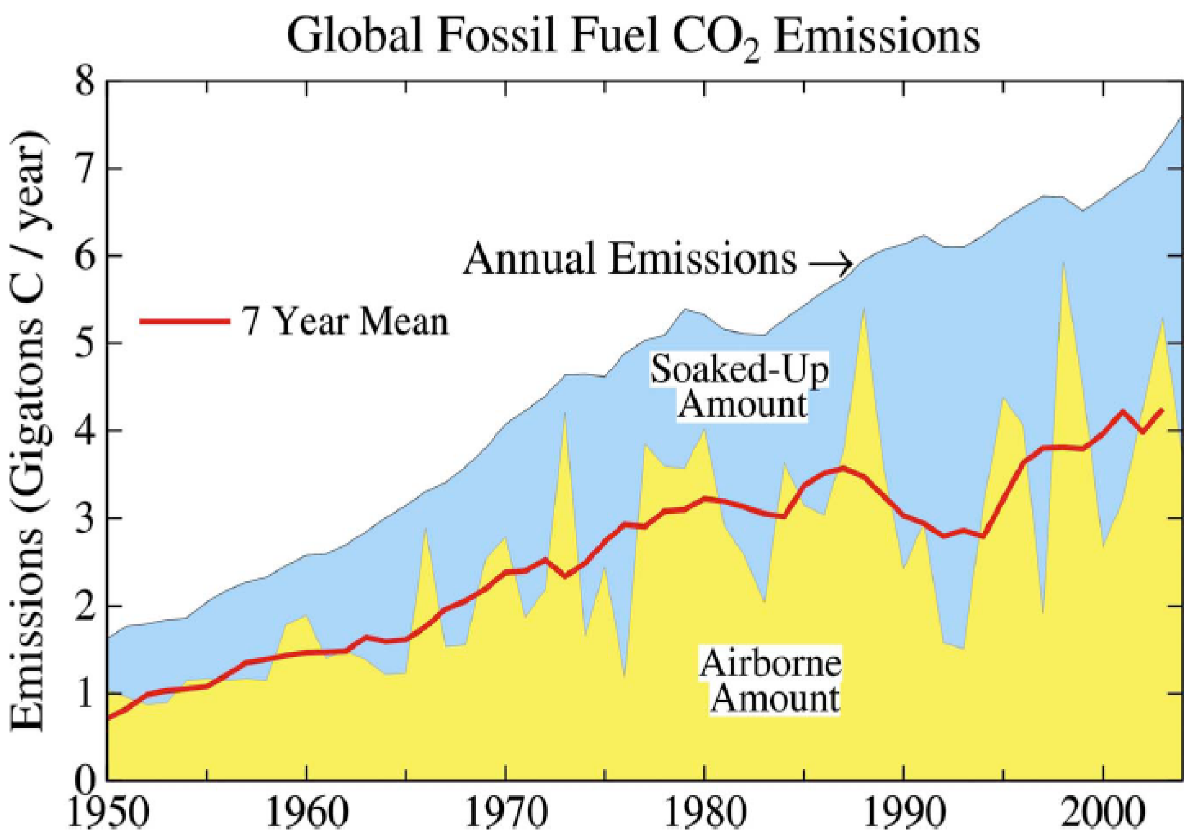
Outstanding Questions

- Only half of the CO_2 produced by human activities is remaining in the atmosphere
- Where are the *sinks* that are absorbing over 40% of the CO_2 that we emit?
  - Land or ocean?
  - Eurasia/North America?
- Why does CO_2 buildup vary dramatically with nearly uniform emissions?
- How will CO_2 sinks respond to climate change?



Global fossil fuel CO_2 emissions with division into portions that remain airborne or are soaked up by the ocean and land.

Source: Hansen and Sato, *PNAS*, **101**, 16109, 2004.

Atm Carbon Models

$$\frac{\partial C}{\partial t} + \underbrace{\Im}_{Atm\_transport+mixing}(C) = \underbrace{S|_{z=0}}_{SourcesSinks}$$

Kalnay

$$x_b(t_{i+1}) = M(x_a(t_i))$$

X=conc, fluxes, parameters

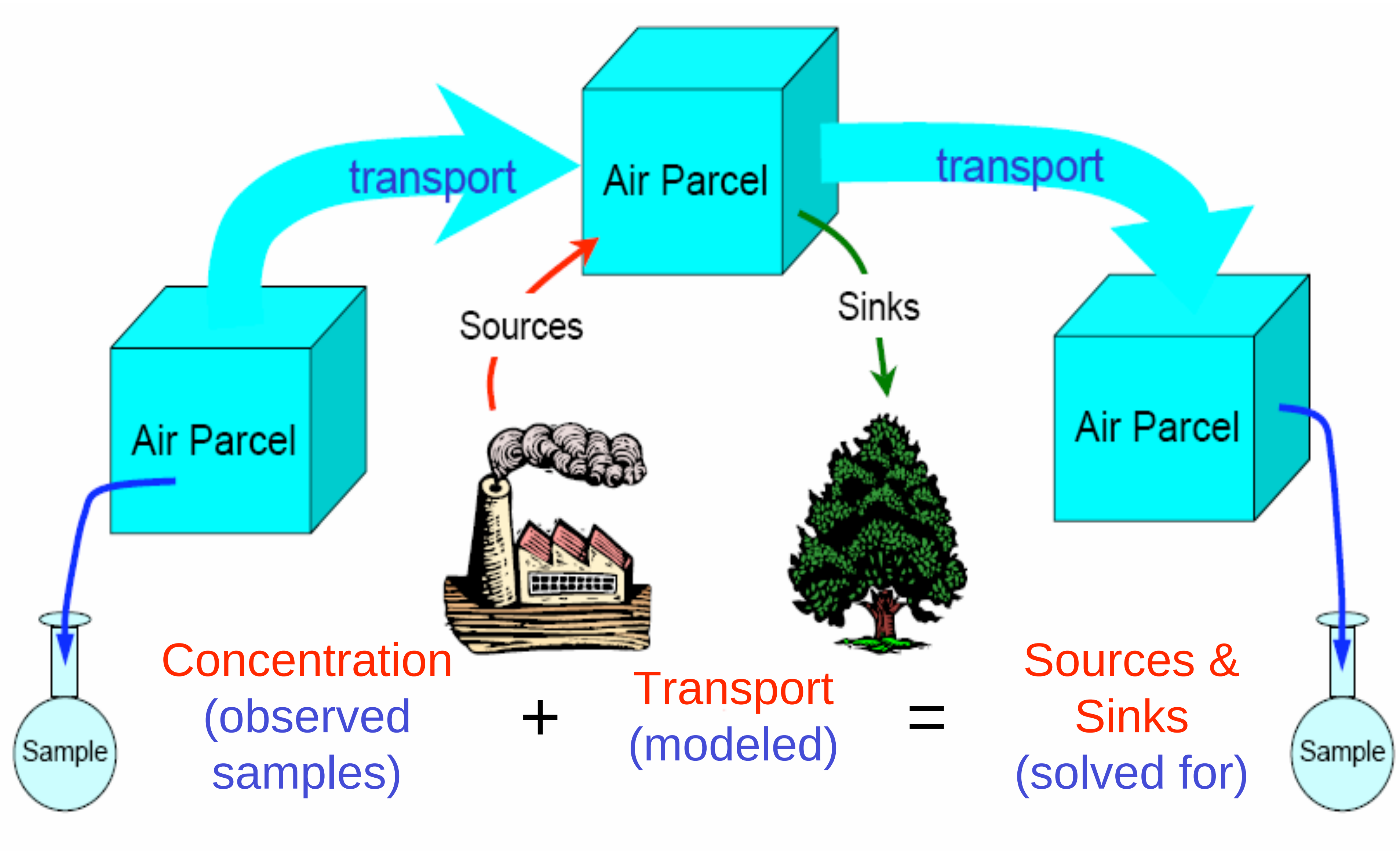
Nychka

$$x_{i+1} = \Phi(x_i) + G(u)$$

X = conc

u = fluxes

Atmospheric Inverse Modeling of CO_2



An Atm Carbon Cycle Model

$$\frac{\partial C}{\partial t} + \underbrace{\Im(C)}_{Atm\_transport+mixing} = \underbrace{S|_{z=0}}_{SourcesSinks}$$

$$S = FF + LandUse + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})$$

What we've got:

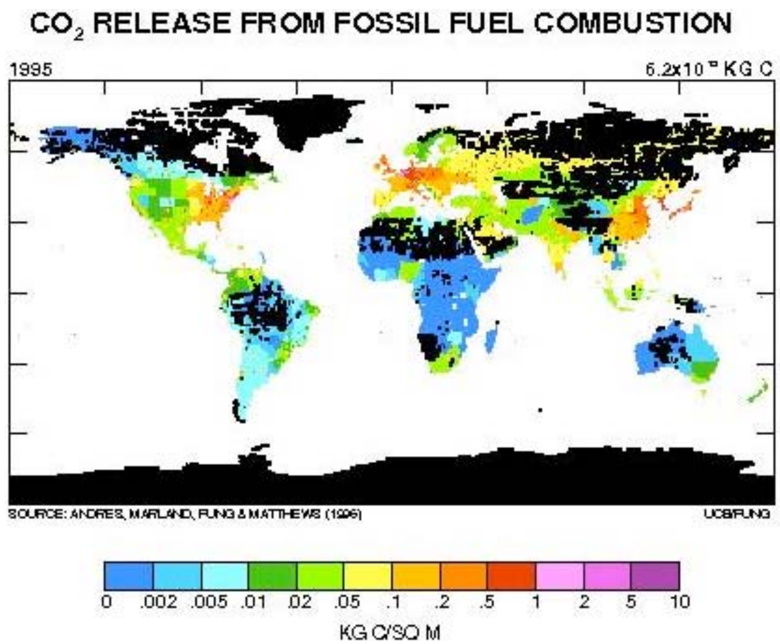
- **Sources/Sinks S known approximately or not well constrained**
- **C_{obs} (actually mixing ratios X_{obs}) biweekly, at ~100 stations near the surface**
- **"Decent" transport model (winds, turbulent mixing)**

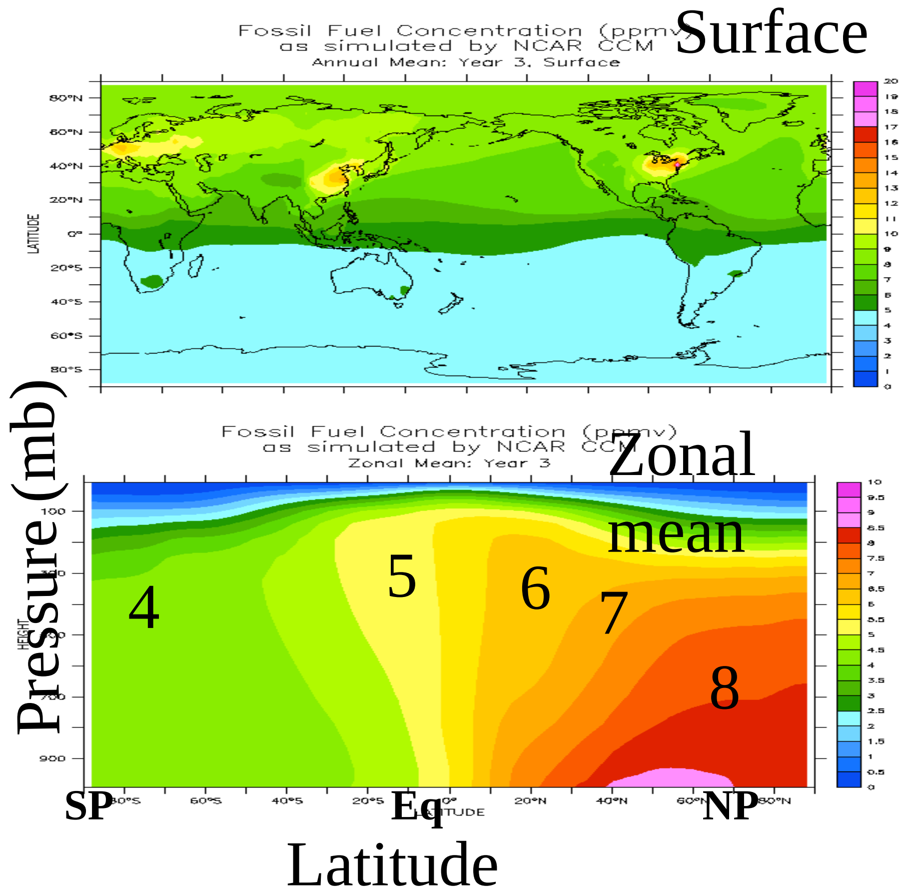
What we want:

- **where has the fossil fuel CO2 gone?** {Better estimates of the magnitude and distribution of S (e.g. land exchange)}
- **How did the fossil fuel CO2 get there? {improved understanding and representation of processes, e.g.**
  - **F_{ab}=LUE*AvailableLight; F_{ba}=exp(αT);**

What we've got: (1) The Model: NCAR climate model

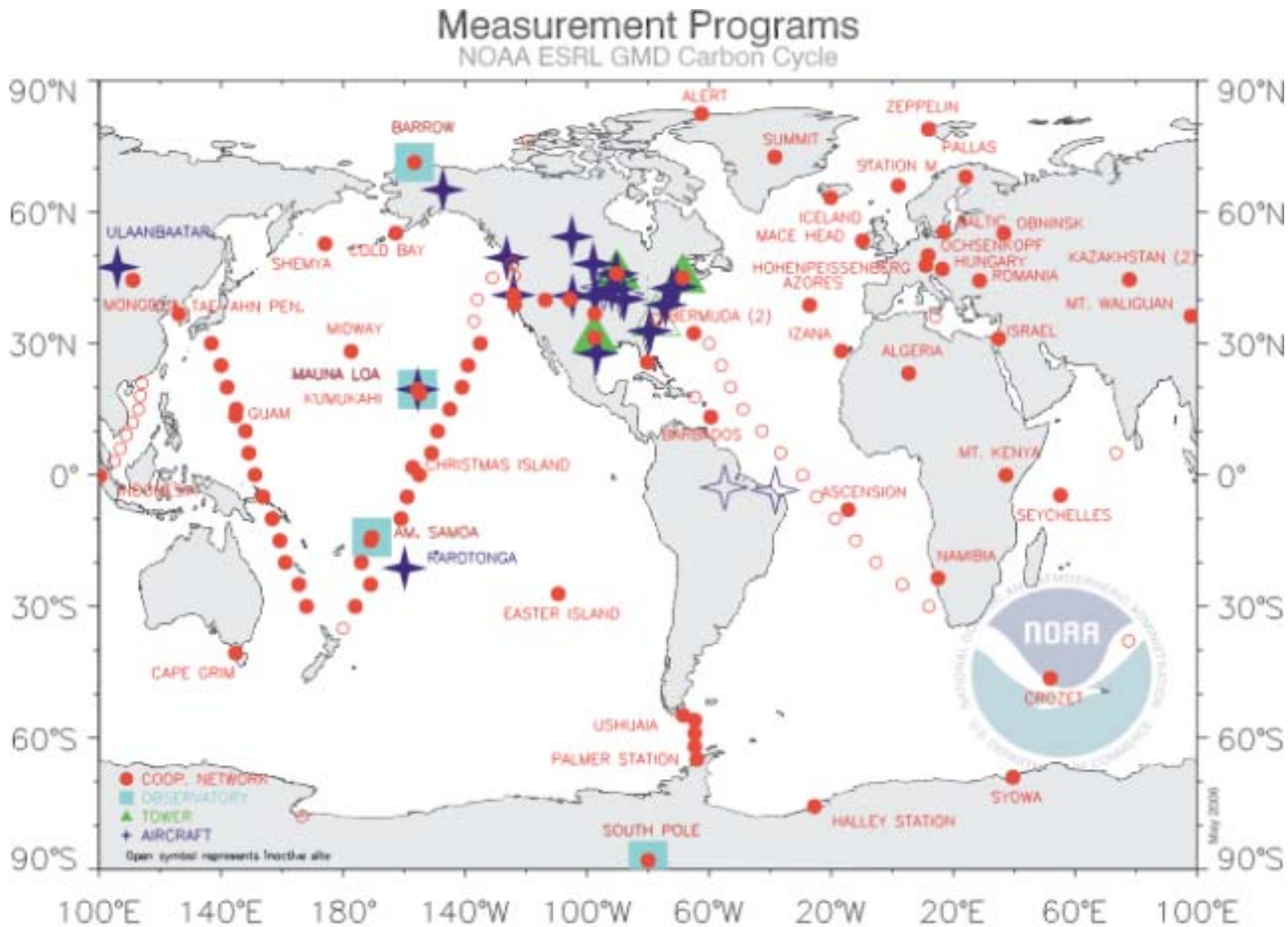
Source: Fossil fuel combustion (6 PgC/y) → ***C(x,y,z)* at steady state**





What We've got: The data: Atm CO2 (for now)

- Discrete surface flasks (~weekly)
- Continuous surface (hourly) observatories
- Tall towers continuous (hourly)
- Aircraft profiles (~weekly)

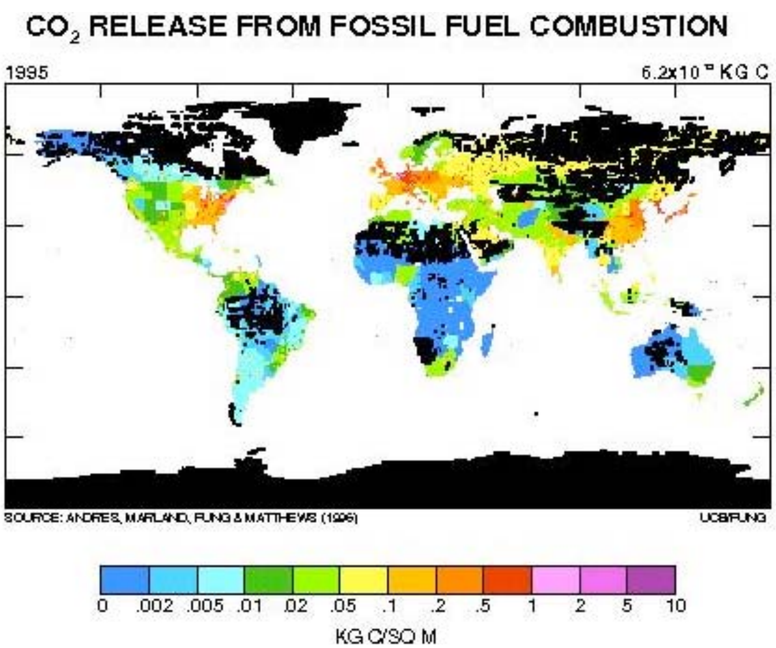


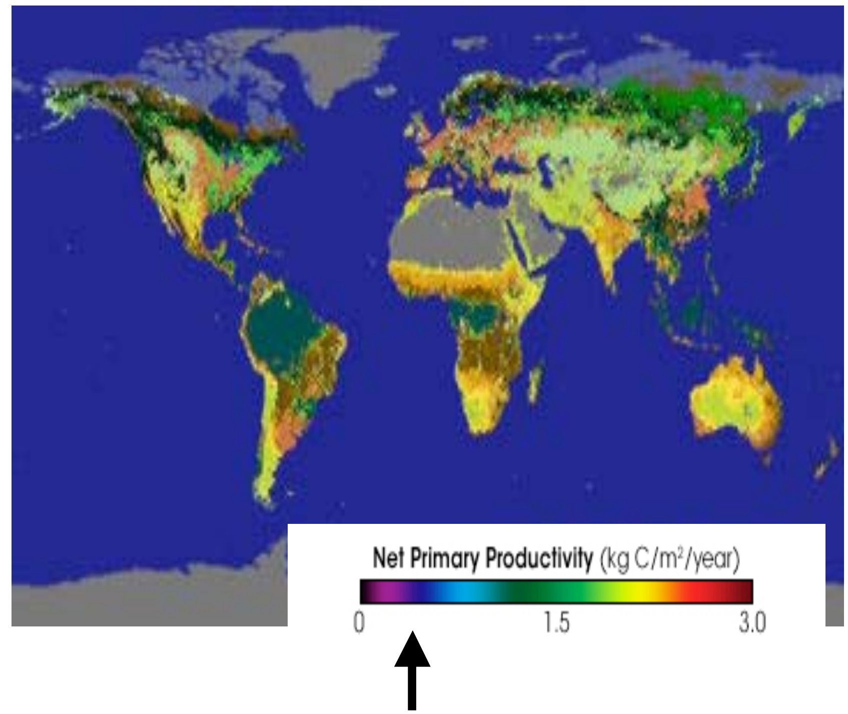
GMD Carbon Cycle operates 4 measurement programs. Semi-continuous measurements are made at 4 GMD baseline observatories and from tall towers. Discrete samples from the cooperative air sampling network and aircraft are measured at GMD. Presently, atmospheric carbon dioxide, methane, carbon monoxide, hydrogen, nitrous oxide, sulfur hexafluoride, and the stable isotopes of carbon dioxide and methane are measured. Contact: Dr. Pieter Tans, NOAA ESRL GMD Carbon Cycle, Boulder, Colorado, (303) 497-6678 (pieter.tans@noaa.gov, http://www.cmdl.noaa.gov/ccgg).

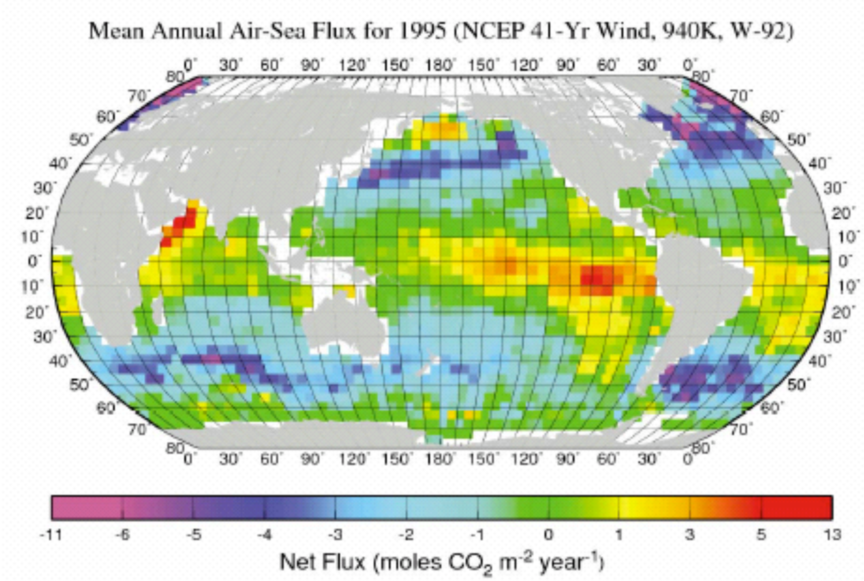
What We've Got: (3) The Flux Priors

$$\frac{\partial C}{\partial t} + \underbrace{\Im(C)}_{Atm\_transport+mixing} = \underbrace{S\big|_{z=0}}_{SourcesSinks}$$

$$S = \underbrace{FF}_{"well-known"} + LandUse + \underbrace{(F_{oa} - F_{ao})}_{extrapolation\ of\ sparse\ obs} + (F_{ba} - F_{ab})$$

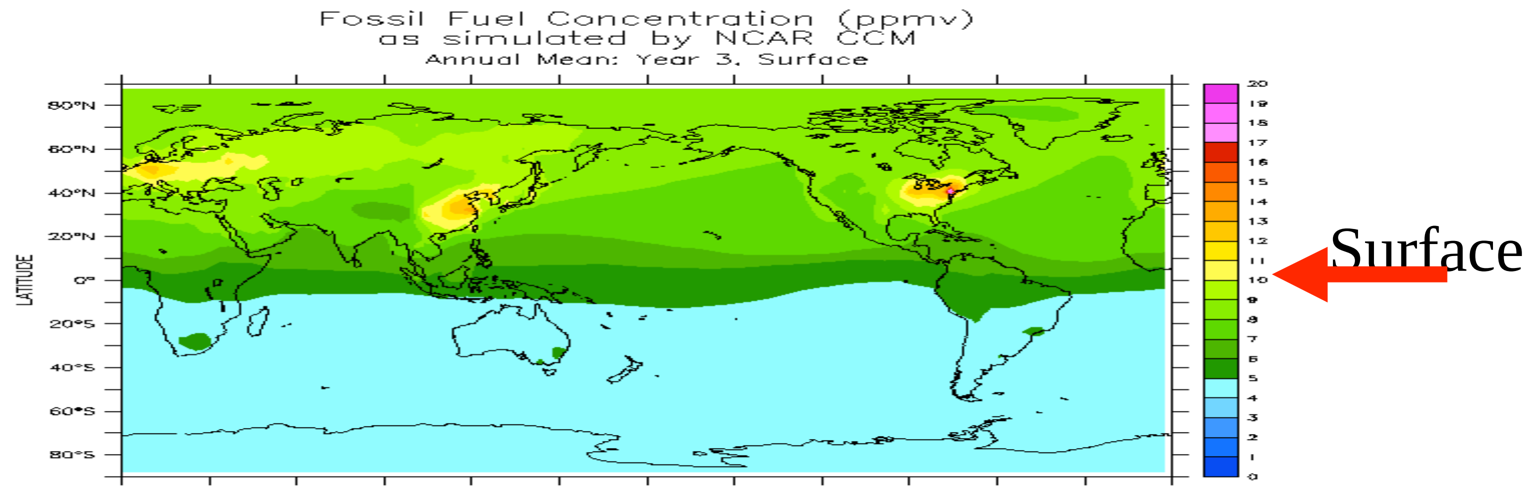


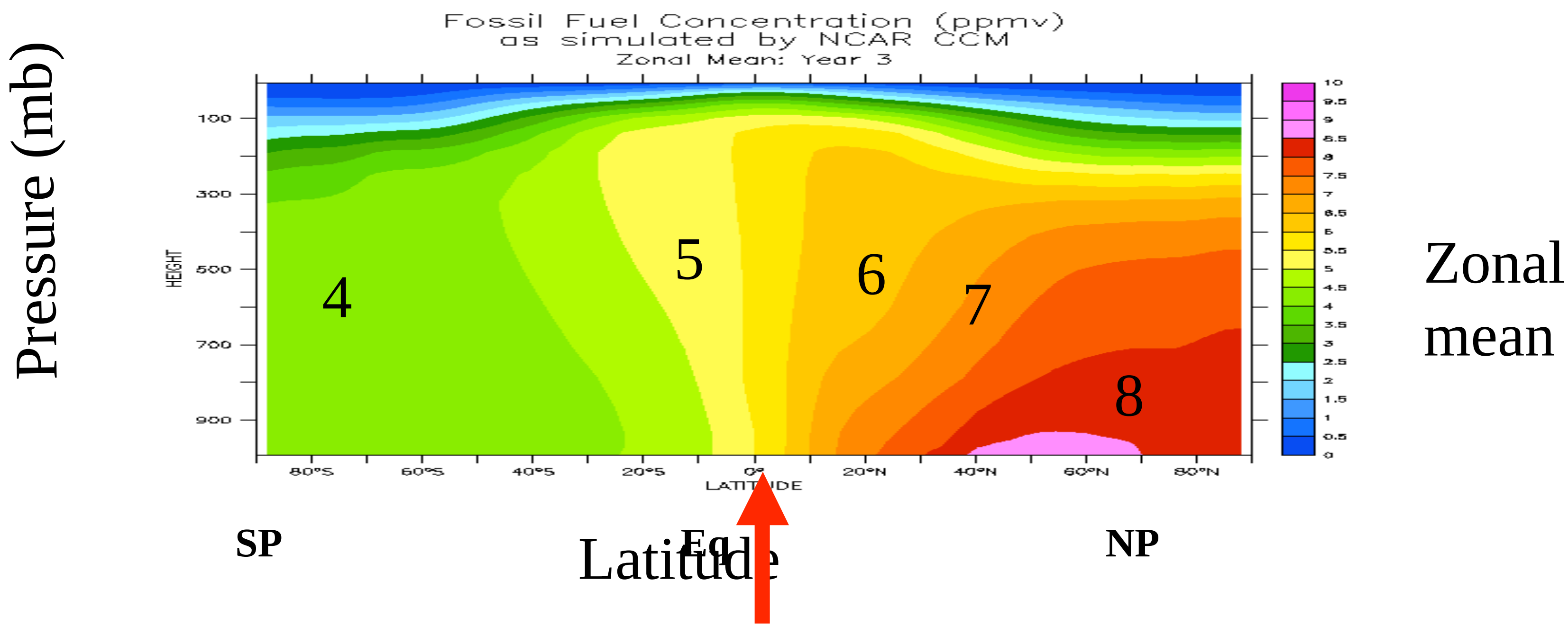




should net land flux (F_{ba}-F_{ab}) be prop to F_{ab}?

Example I: A Simpler Model - reduce 3D atm to 2 hemisphere

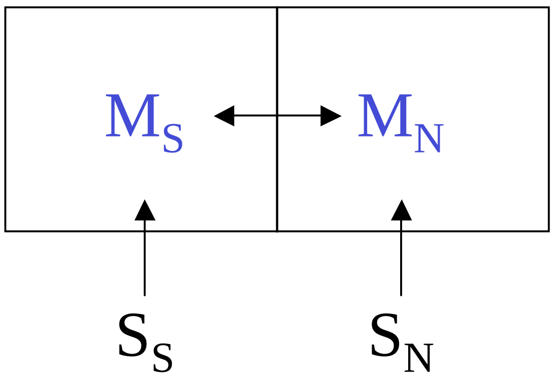




Example I: Interhemispheric Mixing: Two-Box Model, everything is perfect.

$$\frac{\partial M_N}{\partial t} = -\frac{M_N - M_S}{\tau} + S_N$$

$$\frac{\partial M_S}{\partial t} = +\frac{M_N - M_S}{\tau} + S_S$$



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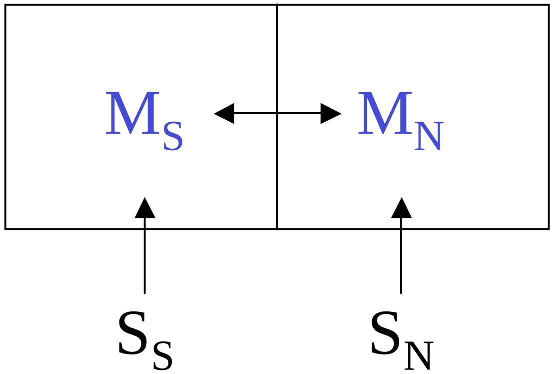


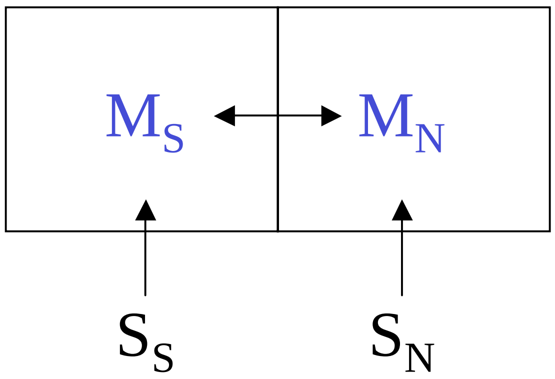
$$\frac{\partial (M_N - M_S)}{\partial t} = -2\frac{M_N - M_S}{\tau} + (S_N - S_S) = 0 \ @ \ Steady State$$

$$\tau = 2\frac{M_N - M_S}{S_N - S_S}$$

Interhemispheric exchange time τ determined from inert tracers (e.g. CFC, with S_s=0): ~1-2 years

Ex I: 2-Box Model Applied to the Carbon Cycle

$$M_N - M_S = \frac{\tau}{2}(S_N - S_S)$$



Consider the case $S_N = 6\ PgC/yr;\ S_s = 0$

$\tau = 1\ yr$

$\rightarrow M_N - M_S = 3\ PgC$

Recall 1 PgC → 0.5 ppmv if mixed in entire atm.

1 PgC → 1 ppmv if mixed in a hemisphere.

$\rightarrow X_N{}^{column} - X_S{}^{column} = 3\ ppmv$

Guess (3D model) surface gradient 1.5x column mean gradient

$\rightarrow X_N{}^{sfc} - X_S{}^{sfc} = 4.5\ ppmv$

Britt Stephens: new obs of vertical profile

Ex I: 2-Box Model Applied to the Carbon Cycle

Forward problem: If 100% FF CO_2 remained in atm

$$M_N - M_S = \frac{\tau}{2}(S_N - S_S)$$

$$S_N = 6 \; PgC/yr; \; S_s = 0$$

$$\tau = 1 \; yr$$

$$\rightarrow M_N - M_S = 3 \; PgC$$

$$\rightarrow X_N^{sfc} - X_S^{sfc} = 4.5 \; ppmv$$

$$But \; (X_N^{sfc} - X_S^{sfc})_{obs} = 2.5 \; ppmv$$

Obs → only 50% of FF CO_2 remains in atm

$$\frac{\partial(M_N + M_S)}{\partial t} = S_N + S_S = sources - sinks$$

$$\left.\frac{\partial(M_N + M_S)}{\partial t}\right|_{obs} = 3 \; PgC/yr$$

$$sources = 6 \; PgC/yr$$

$$\rightarrow Sinks_N + Sinks_S = 3 \; PgC/yr$$

Ex I: 2-Box Model Applied to the Carbon Cycle

Inverse problem

Obs operator

X=H(M)

$$\textit{Model:}\ M_N - M_S = \frac{\tau}{2}(S_N - S_S)$$

$$\textit{Given:}\ (X_N^{sfc} - X_S^{sfc})_{obs} = 2.5\ ppmv$$

$$\rightarrow (X_N^{column} - X_S^{column})_{obs} = 1.7\ ppmv$$

$$\rightarrow M_N - M_S = 1.7\ PgC$$

$$\textit{Invert model} \rightarrow S_N - S_S = 2\frac{M_N - M_S}{\tau} = 3.4\ PgC/yr$$

$$(sources_N - sinks_N) - (sources_S - sinks_S) = 3.4\ PgC/yr$$

$$(6\ PgC/yr - sinks_N) - (0 - sinks_S) = 3.4\ PgC/yr$$

$$\rightarrow sinks_N - sinks_S = 2.6\ PgC/yr$$

Obs Carbon Budget

$$Sinks_N + Sinks_S = 3\ PgC/yr$$

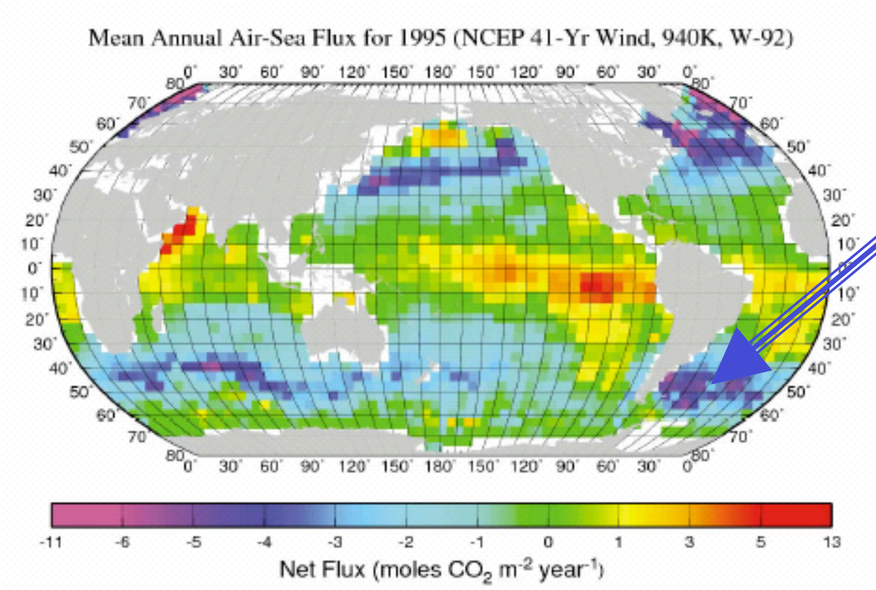
Where are the Carbon Sinks?

$$\textit{Budget} \qquad sinks_N + sinks_S = +3\ PgC/yr$$

$$\textit{Gradient} \qquad sinks_N - sinks_S = 2.6\ PgC/yr$$

$$\rightarrow sinks_N = 2.8\,PgC/yr;\quad sinks_S = 0.2\ PgC/yr$$

Northern sinks > Southern Sinks !!!!!!!



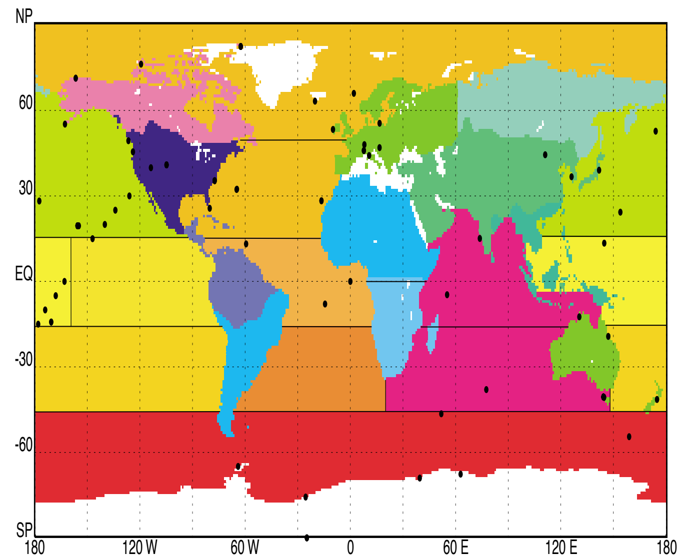
"Data/Obs": Huge C sink in the large expanse of southern ocean; but large uncertainty in obs

N ocn "better observed" → large Northern land sink!!!

Example II: Perfect 3D atm circulation model. Steady state

(1) Forward Step

- Premise: Atm CO_2 = linear combination of response to each source or sink
- Divide surface into "basis regions"
- Specify unitary source (e.g. 1 PgC/year) each year from each region
- Simulate atm CO_2 "basis" response with atm general circulation model
- Reconstruct fluxes and concentrations: unknown μ_k



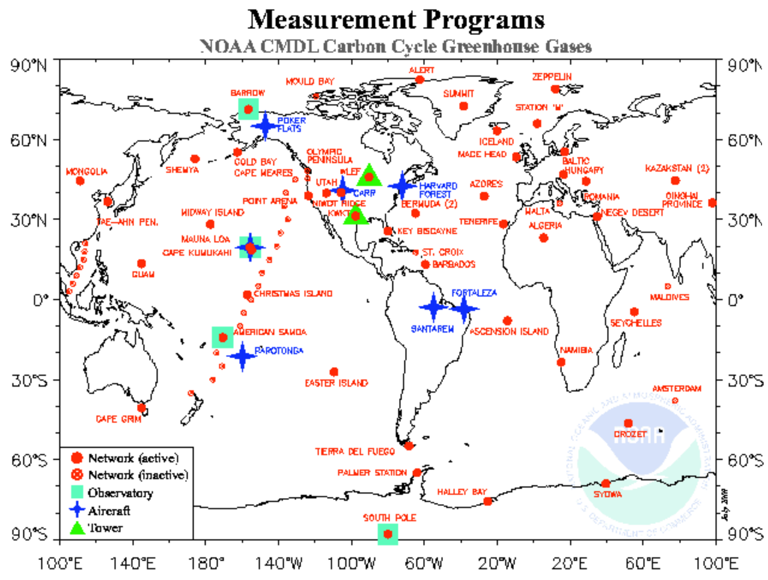
$$\hat{s}_k(x,y)$$

$$\hat{s}_k(x,y) \rightarrow \hat{c}_k(x,y,z,t)$$

$$S = \sum_{k-regions} \mu_k \times \hat{s}_k(x,y)$$

$$c(x,y,z) = \sum_{k-regions} \mu_k \times \hat{c}_k(x,y,z)$$

Ex II: (Step 2) Bayesian Inversion: perfect circulation model



The NOAA CMDL Carbon Cycle Greenhouse Gases group operates 4 measurement programs. In situ measurements are made at the CMDL baseline observatories: Barrow, Alaska; Mauna Loa, Hawaii; Tutuila, American Samoa; and South Pole, Antarctica. The cooperative air sampling network includes samples from fixed sites and commercial ships. Measurements from tall towers and aircraft began in 1992. Presently, atmospheric carbon dioxide, methane, carbon monoxide, hydrogen, nitrous oxide, sulfur hexaflouride, and the stable isotopes of carbon dioxide and methane are measured. Dr. Pieter Tans, Carbon Cycle Greenhouse Gases, Boulder, Colorado, (303) 497-6678. ptans@cmdl.noaa.gov.

Inversion: Seek the optimal source/sink combination $\{\mu_k\}$ to match atmospheric CO_2 data: *minimize*

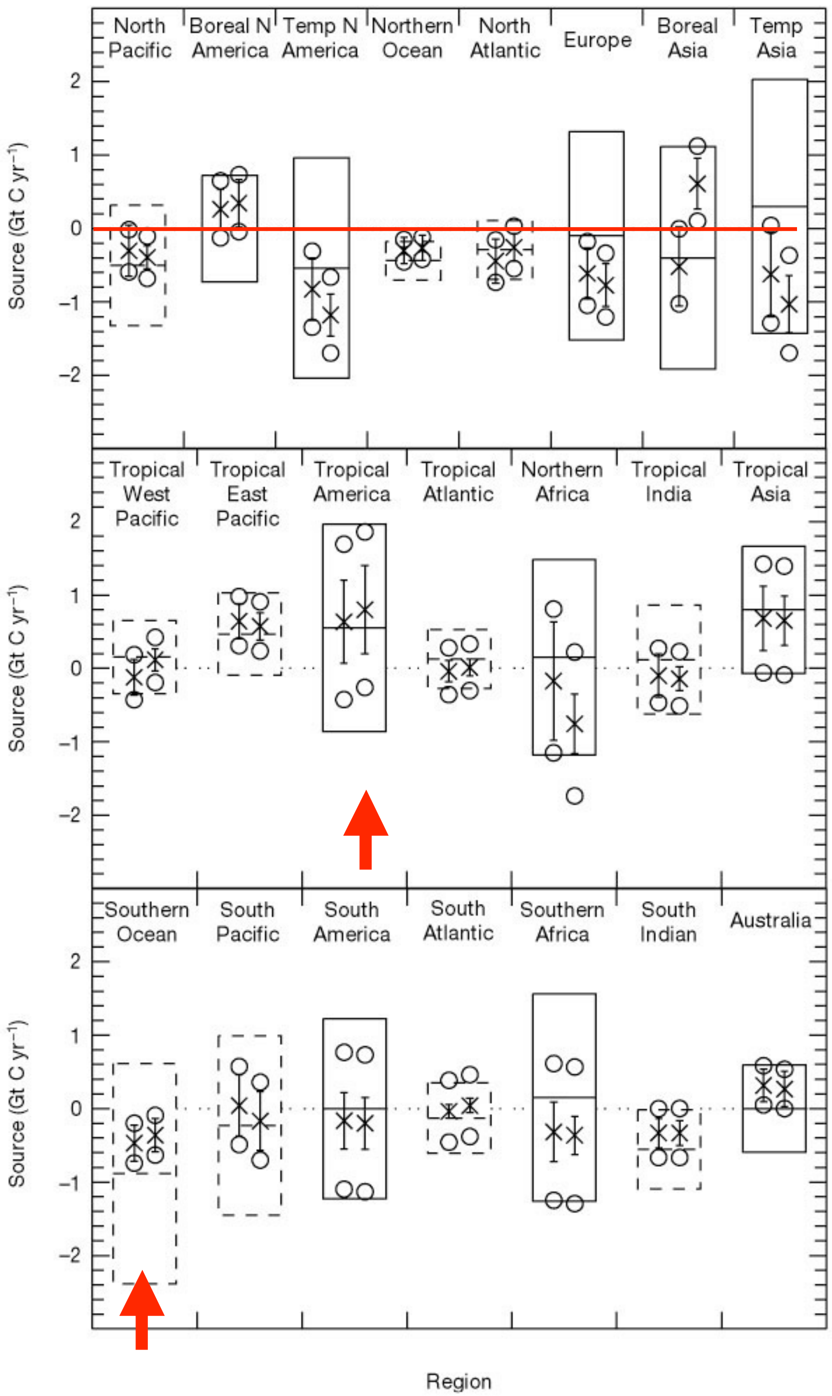
$$J = \sum_{stn} \frac{[C_{obs}(stn) - \sum_{k-regions} \mu_k \times \hat{c}_k(stn)]^2}{\sigma_{stn}^{\ 2}}$$

$$+ \sum_{k-regions} \frac{[\mu_k - \mu_k^{prior}]^2}{[\sigma_k^{prior}]^2}$$

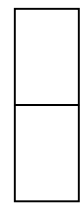
- **Obs. Network –**
  - **mainly remote marine locations**

Trying to infer information over land

Undetermined; non-unique solutions; prior estimates of source/sinks as additional constraints



Ex IIa: Posterior from many "perfect" circulation models

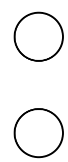


$\mu_k^{prior} \pm \sigma_k^{prior}$

Model m:

$\{\mu_{mk}^{posterior} \pm \sigma_{mk}^{posterior}\}$

Mean,std_dev ($\mu_{mk}^{posterior}$)



Mean ($\sigma_{mk}^{posterior}$)

Little innovation in tropics, Africa

Great innovation in S. Ocean

Gurney et al. Nature 2005

What next? Anticipating satellite data

Separating transport, initial conditions & surface fluxes

Kalnay $x_b^{i+1} = M(x_a^i)$ Analysis at time i => forecast at time i+1

Nychka $x_b^{i+1} = \underbrace{\Phi(x^i)}_{\text{transport}} + \underbrace{G(u^i)}_{\text{Fluxes, parameters}}$

transport Fluxes, parameters

$\underline{x^0} \neq x^0_{prior}$

initial conditions

4D Variational methods: adjust initial conditions to better match future data

$$J(x) = \frac{1}{2}\Big\{(x^0 - x^0_{prior})^T B^{-1}(x^0 - x^0_{prior}) + \left[y_o - H(x)\right]^T R^{-1}\left[y_o - H(x)\right]$$

Deviation of initial conditions from "prior"

Deviation of x from "observations"

$$+(u - u_{prior})^T P^{-1}(u - u_{prior})\Big\}$$

Deviation of fluxes from "prior"