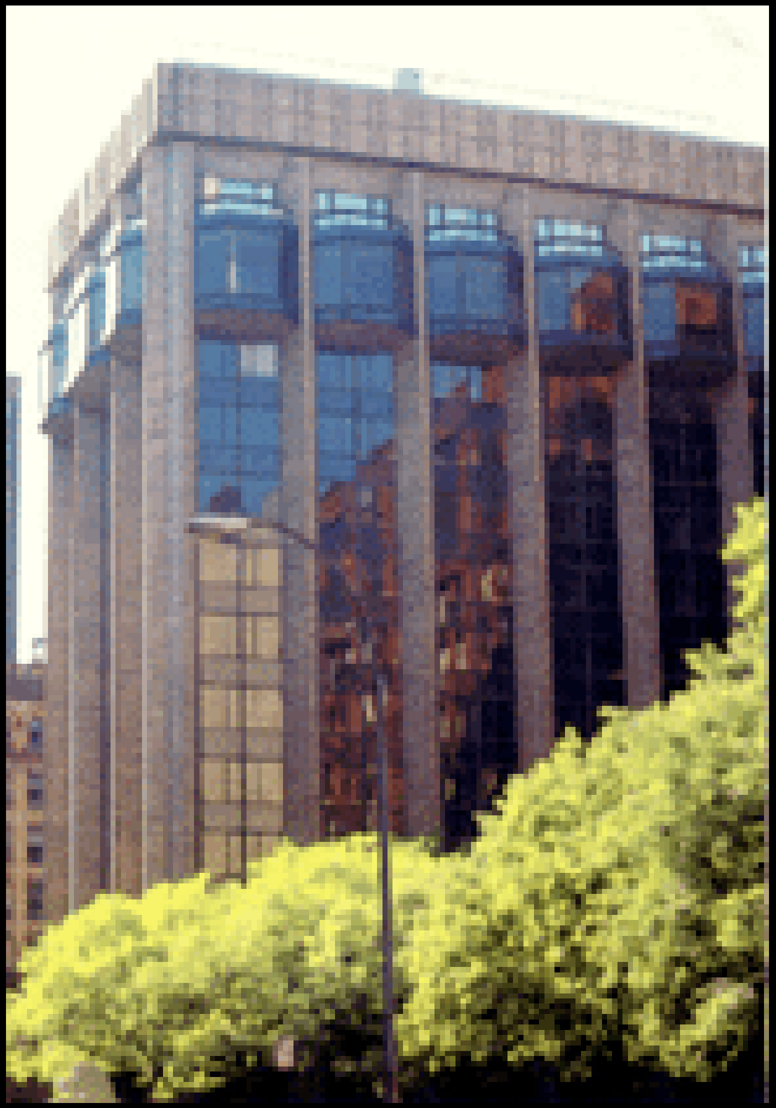
Gravity-wave refraction by three-dimensional winds: new effects and a new parametrization scheme



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New York University

CAOS

Gravity waves

Gravity waves
evolve locally

but they affect the
circulation globally

Think locally act globally

Current theory paper in JFM 2005

J. Fluid Mech. (2005), *vol.* 534, *pp.* 67–95.
doi:10.1017/S0022112005004374 Printed in the United Kingdom

Wave capture and wave–vortex duality

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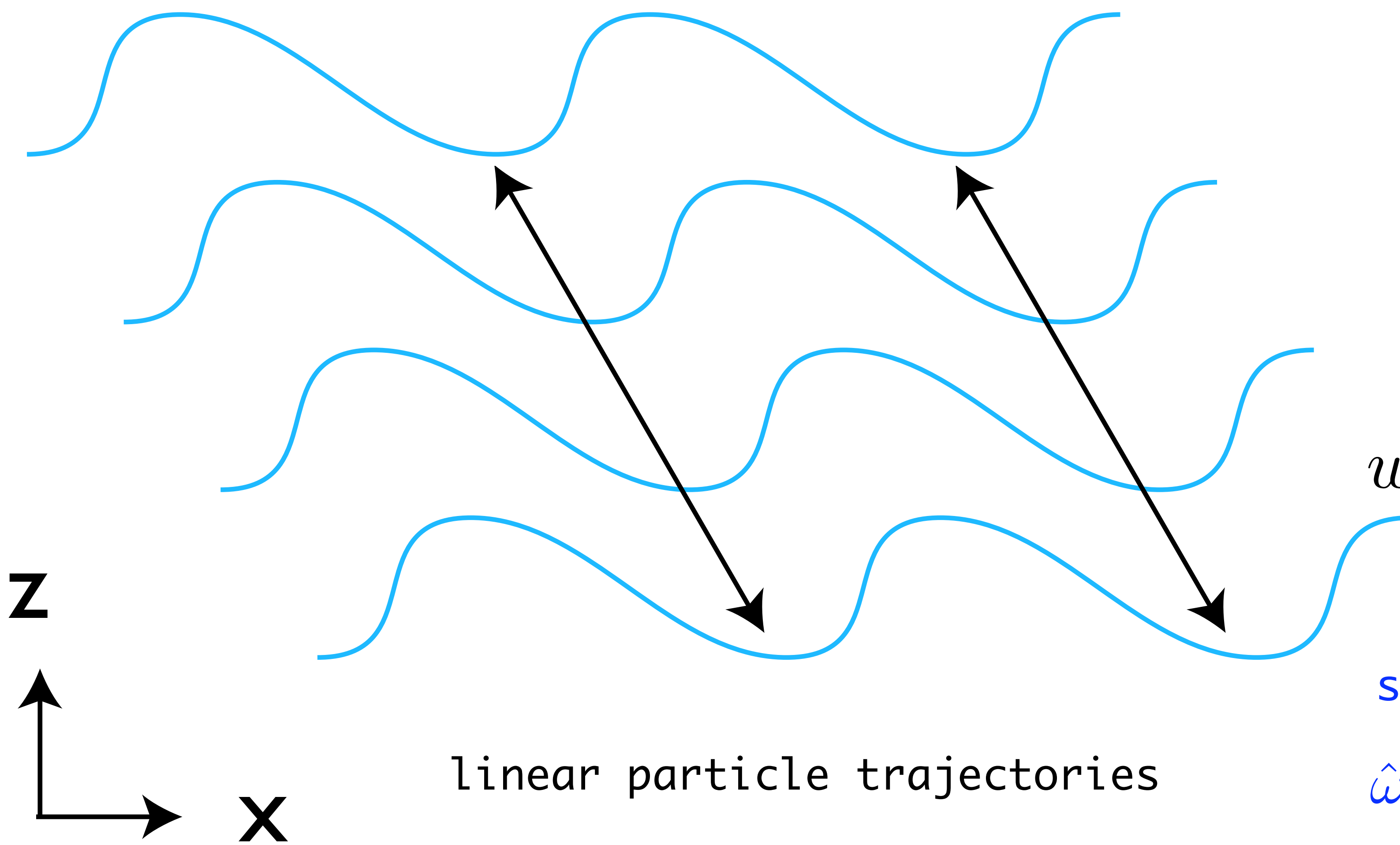
Inertia-gravity waves



undulating material stratification surfaces

(isentropes/isopycnals)

surfaces are flat at rest

$$w' \propto \exp(i[kx + mz - \hat{\omega}t])$$

scale-free dispersion relation

$$\hat{\omega}^2 = (N^2 - f^2)\frac{k^2}{k^2 + m^2} + f^2$$

$$f^2 \leq \hat{\omega}^2 \leq N^2$$

Momentum flux $\overline{u'w'} < 0$ *(unlike surface waves)*

Internal waves make a significant contribution to atmospheric angular-momentum fluxes... convergence yields wave drag

Oliver Bühler, Courant Institute, New York University, 2005

Wave breaking

Large-amplitude waves overturn and break nonlinearly, leading to 3d turbulence, irreversible mixing, and convergence of wave momentum flux

before after

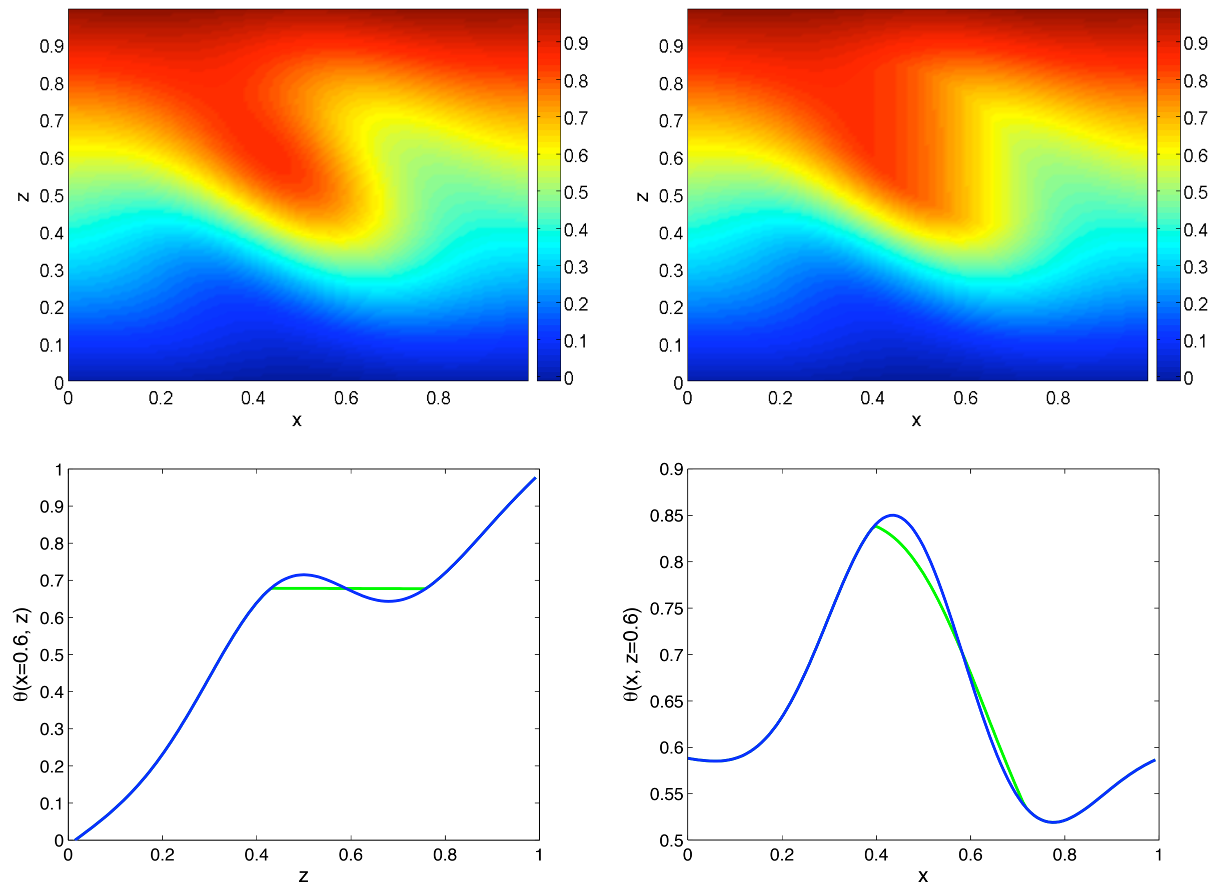
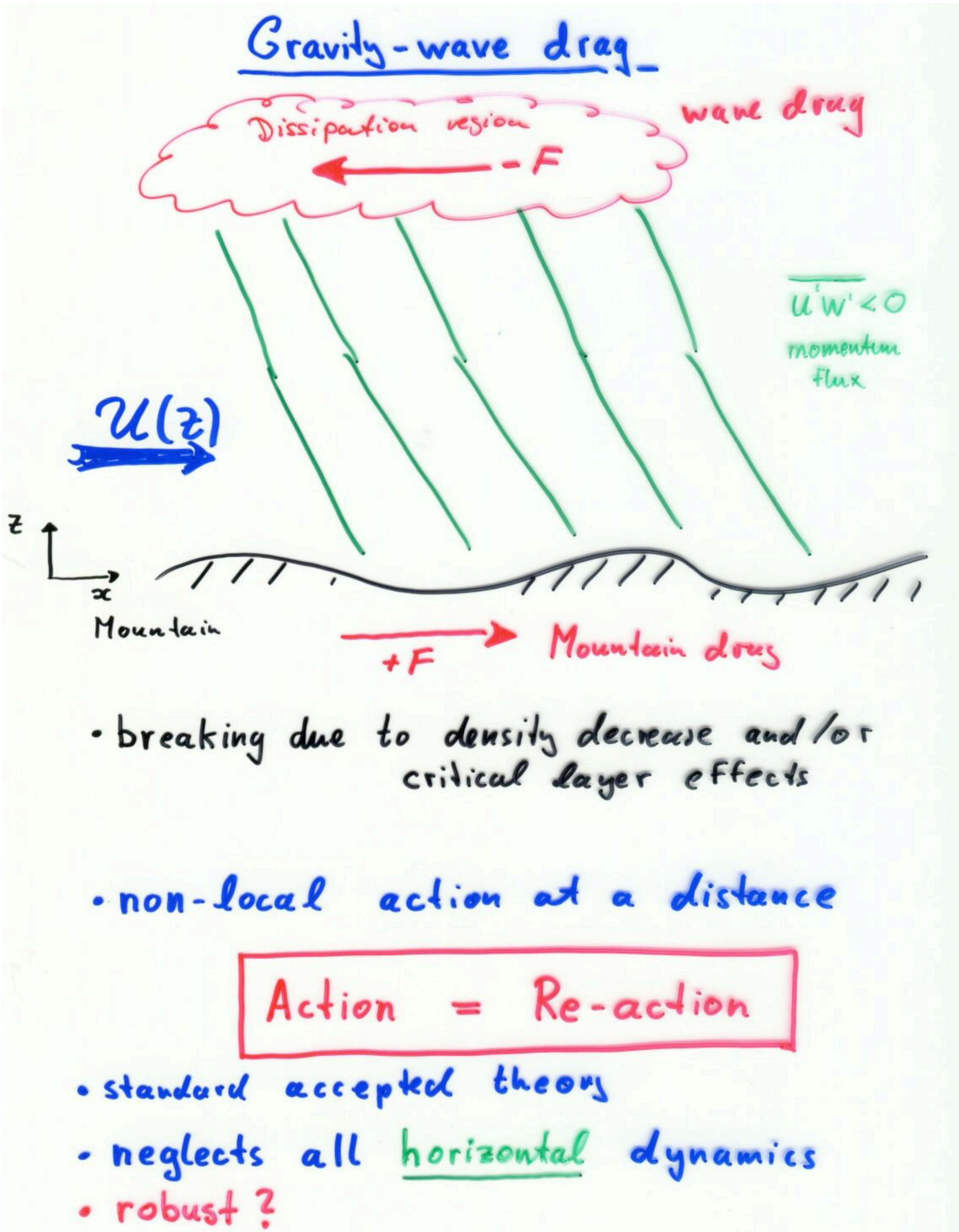


Figure 3: Diffusive scheme applied to the unstable buoyancy profile: $b(x,z) = 0.3\exp[-((x-0.5)^2+(z-0.5)^2)/0.09]\cos[2\pi(x+z)]$. (a) (b) are surface plots of the potential temperature distribution before and after mixing. (c) and (d) give potential temperature profiles on the transects $x = 0.6$ and $z = 0.6$ respectively. The blue curve is the profile before mixing, and the green curve is the profile after mixing.

Marcus Roper,
GFD report 2005

The classical gravity wave drag picture



Ray-tracing theory for steady wavetrain as function of altitude z

Horizontal mean-flow inhomogeneity is ignored ("co-dimension 1")

Vertical flux of horizontal pseudomomentum equals vertical flux of horizontal mean momentum

Vertical flux of horizontal pseudomomentum is constant unless waves are dissipating or breaking

This results in the classical force balance: there is an equal-and-opposite action at a distance between mountain drag and wave drag

Wave drag on a rotating planet
"gyroscopic pumping"

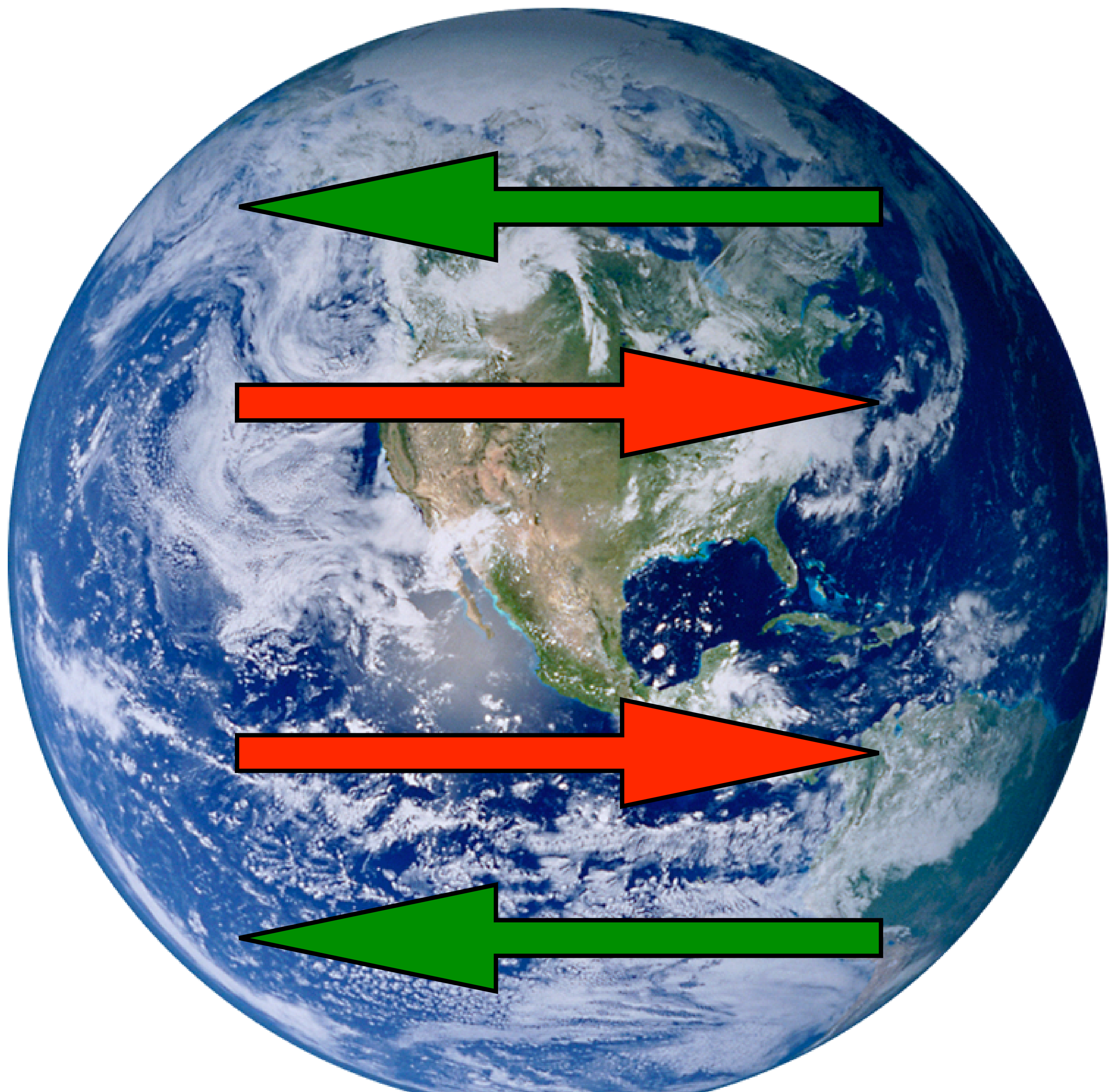
Retrograde force if $k/\hat{\omega} < 0$

Convergence of angular-momentum flux acts as effective force on zonal-mean flow

Prograde force if $k/\hat{\omega} > 0$

Retrograde force -> poleward motion

All Rossby waves
Winter gravity waves



Prograde force -> equatorward motion

Summer gravity waves

Oliver Bühler, Courant Institute, New York University, 2005

Vertical propagation of topographic Rossby and gravity waves

GW $f^2 \leq \hat{\omega}^2 \leq N^2$ RW $\hat{\omega} \leq 0$ Absolute frequency: $\omega = \hat{\omega} + Uk$

Topographic waves have zero absolute frequency: $\omega = 0$

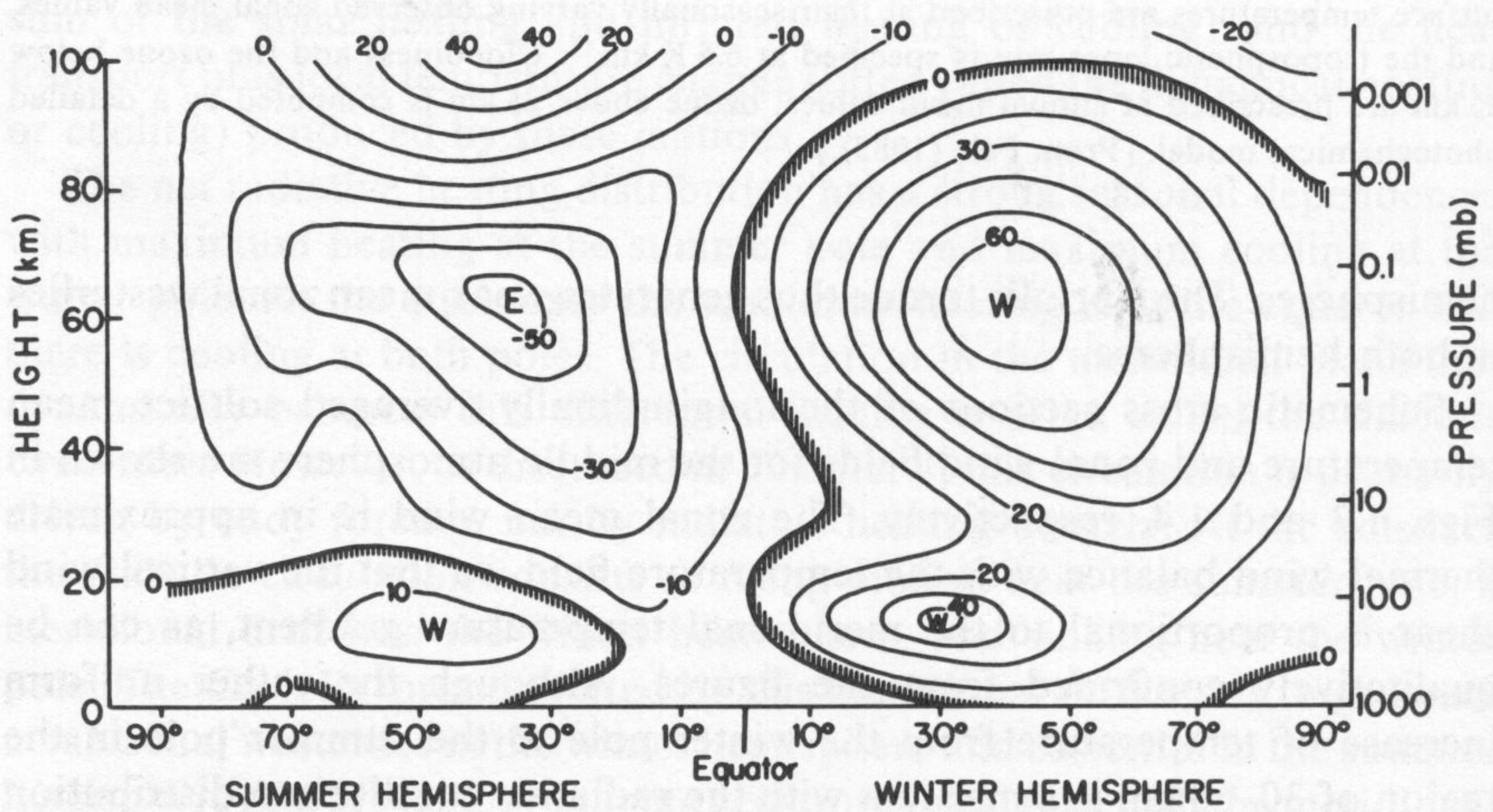


Fig. 1.4. Schematic latitude–height section of zonal mean zonal wind (m s^{-1}) for solstice conditions; W and E designate centers of westerly (from the west) and easterly (from the east) winds, respectively. (Courtesy of R. J. Reed.)

Topographic waves are retrograde if U>0 near surface

Vertical critical layer occurs where shear pushes intrinsic frequency towards lower limit

Summer zero-wind surface acts to filter topographic waves

Wave-driven global circulation

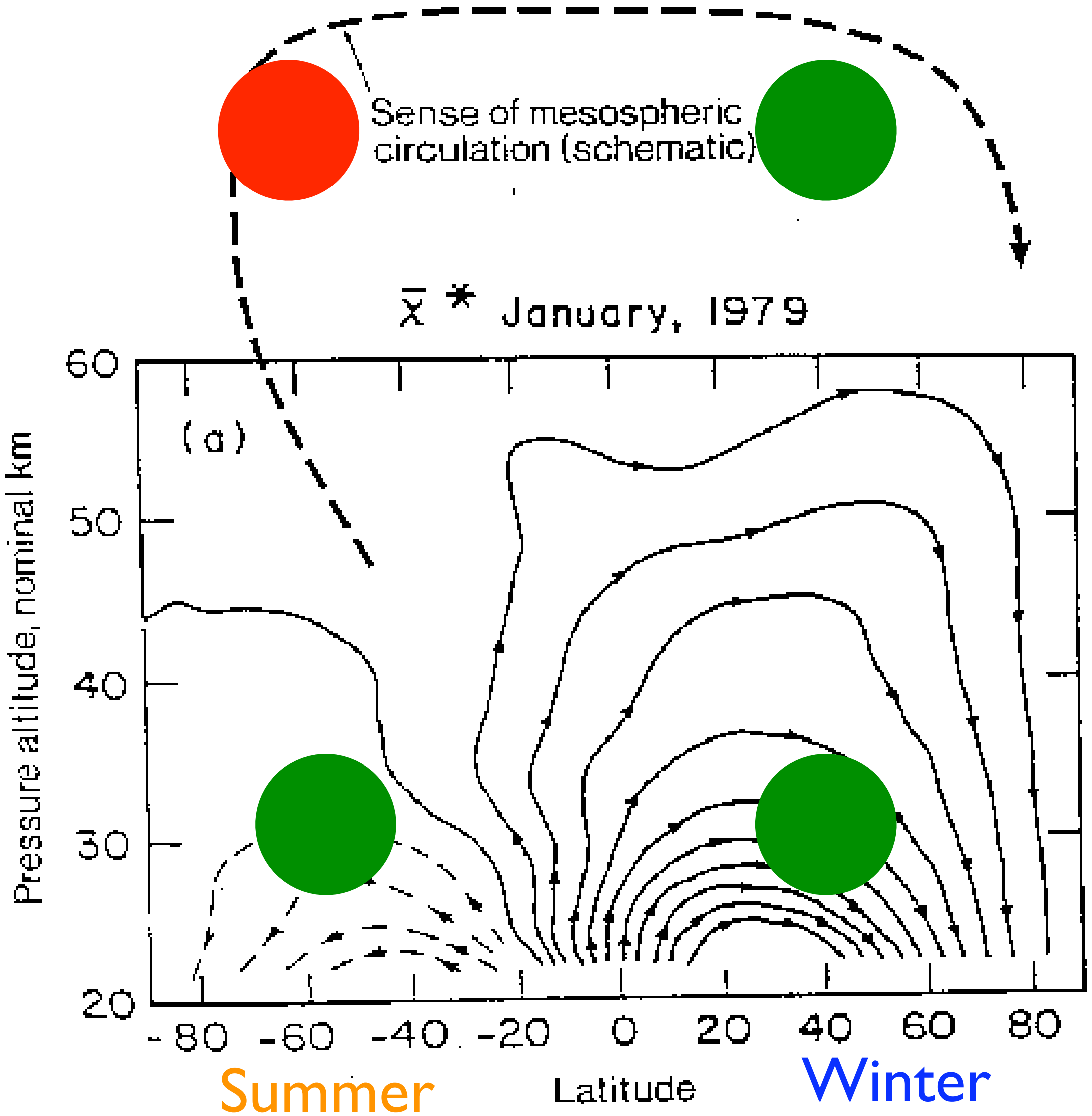


Fig. 3. Mass transport streamlines of the global-scale mean circulation for January 1979 (light curves), from reference [36], estimated using satellite data. The picture gives the typical latitude and height dependence of the longitude and time averaged mass circulation of the stratosphere, defined in a quasi-Lagrangian sense giving a simplified, but roughly correct, indication of the vertical advective transport of chemical constituents (see text, sect. 4). The heavy dashed streamline (schematic only) indicates the qualitative sense of the mesospheric "Murgatroyd-Singleton circulation", deduced from other observational and theoretical evidence (*e.g.*, [18,28]) and not from ref. [36]. The left-hand scale is pressure altitude $z = H_p \ln(p_0/p)$ in nominal kilometres, assuming a pressure e-folding scale height $H_p = 7$ km. Thus the vertical domain shown corresponds roughly to the middle third of fig. 1 (multiply the right-hand scale in fig. 1 by 7 km). The upward mean velocities are a smallish fraction of a millimetre per second, of the order of 0.2 mm s^{-1} [10,37,39,40], albeit somewhat larger in the case of January 1979 when the wintertime stratosphere was dynamically very active (sect. 9). The northward mean velocities at top right (not counting the heavy dashed streamline) are of the order of two or three ms^{-1}. The time for a marked fluid element to rise from the tropopause to, say, 40 km is generally of the order of two years [39]. The lower pair of circulation cells is often collectively referred to as the "Brewer–Dobson circulation"; for historical reviews see [27,41].

Prograde and retrograde mesospheric gravity and Rossby-wave drag:
Murgatroyd--Singleton circulation from summer to winter pole

Retrograde stratospheric Rossby-wave drag:
Brewer--Dobson circulation from equator to pole

Summer polar mesosphere is sunniest place on Earth

Also the coldest (-163^{o} Celsius)

Fermi review,
ME McIntyre 1993

New York University, 2005

Rough guide to gravity wave scales

To resolve a scale need 10 grid points across it

Gravity wave scales (excludes breaking details)		Model resolution (grid size * 10)
Horizontal scale	10 - 1000 km	1000 km
Vertical scale	0.1 - 10 km	10 km
Time scale	10 mins - 1 day	200 mins

GWs parametrized because need a factor 100 higher resolution.
How long will that take?

Oliver Bühler, Courant Institute, New York University, 2005

Computer power and resolution increase

Moore's law: computing power doubles every 18 months.

To reduce grid size by a factor 10 requires 10x10x10x10=10000 times more computing power in a three-dimensional time-dependent simulation (grid points x time steps with CFL condition)

Let N by number of necessary doubling events:

$$2^N = 10^4 \qquad N \log 2 = 4 \log 10 \qquad N = 13.3$$

$$\boxed{T = N * 18\,\text{months} = 19.95\,\text{years}}$$

To increase resolution by a linear factor of 10 takes 20 years

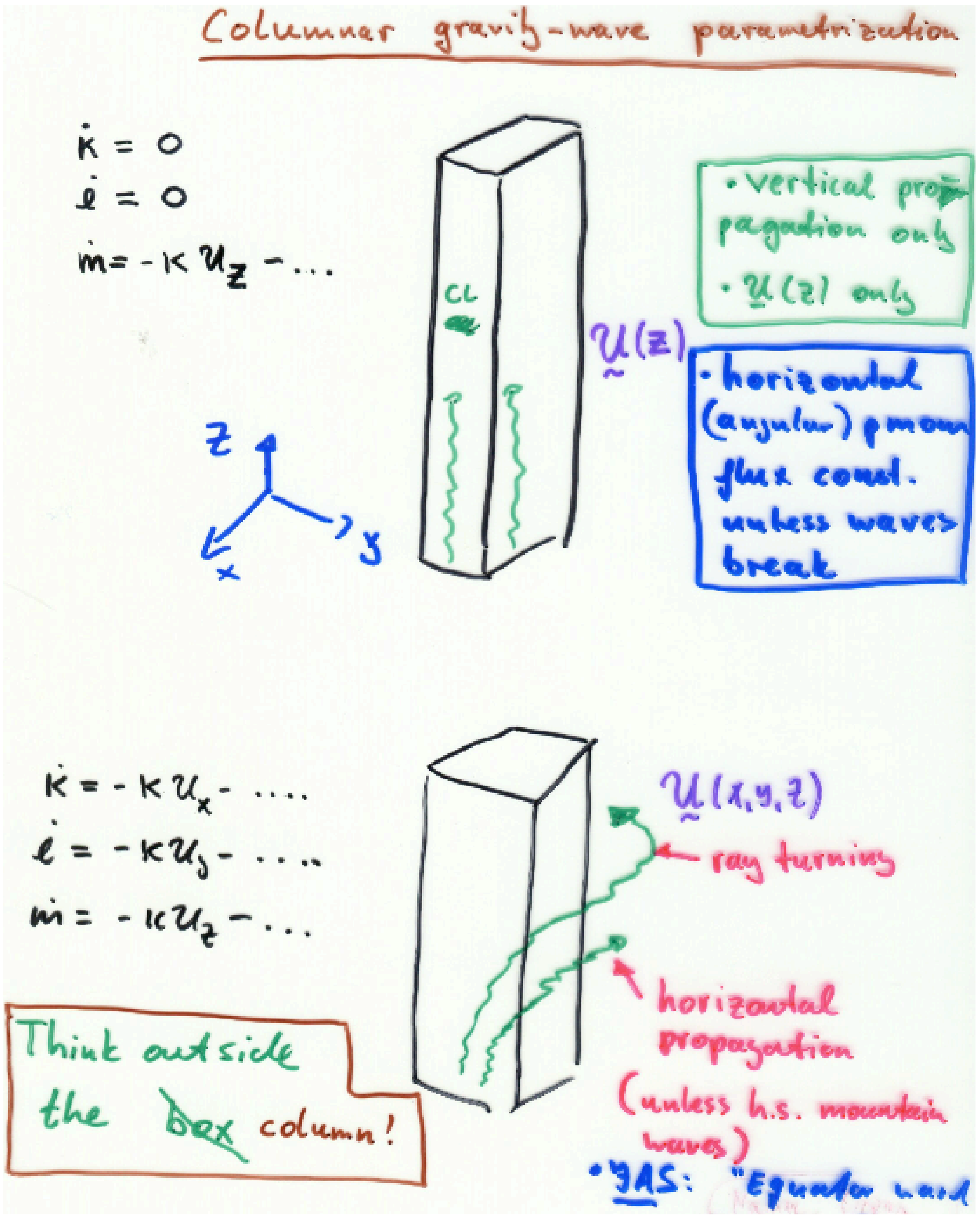
Moore is not enough.........

Multiscale-based career advice

For gravity waves need factor 100

This will take 40 years, ok for a career!

Columnar gravity wave parametrization



Parametrization is applied independently in each vertical model column

Time-dependence is ignored

Vertical wave propagation and vertical mean-flow derivatives are taken into account

Horizontal wave propagation and horizontal mean-flow derivatives are ignored: no refraction by mean flow

Many effects are neglected. Which are the important ones?

Some neglected effects are known to be important.
For instance, intermittency.

Known unknown: intermittency

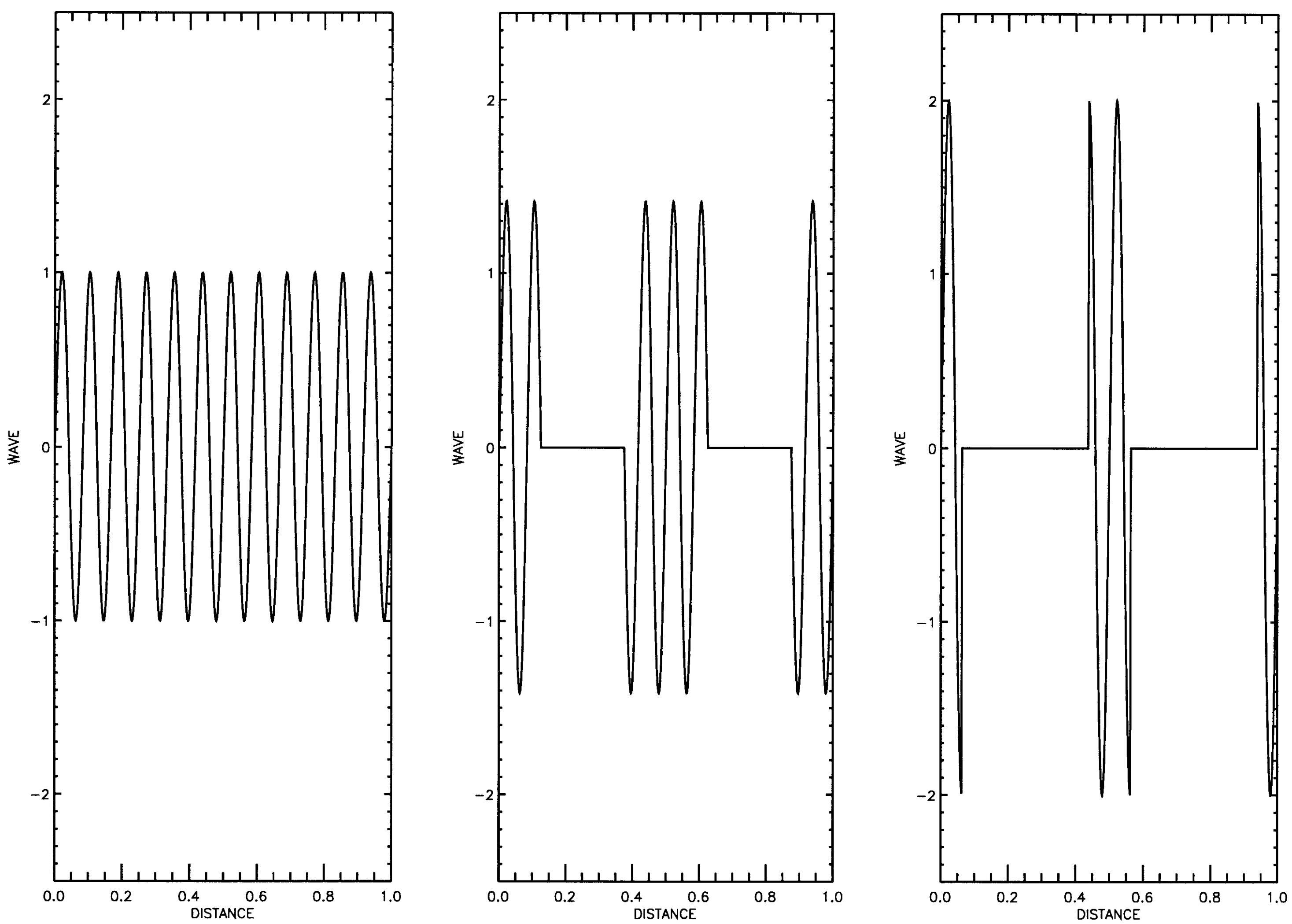


FIG. 4. Wave packets due to hypothetical intermittent wave source at $y_0 = 0$. (left) Steady, nonintermittent source. (middle) Intermittent source with $\gamma = 50\%$. (right) Intermittent source with $\gamma = 25\%$. The wave amplitude increases by a factor of $\sqrt{2}$ from wave to wave, keeping the expected wave energy flux the same in all three cases.

Bühler, 2003 JAS

Intermittency increases wave amplitudes at fixed mean wave activity flux

Wave breaking is highly amplitude-dependent

"Intermittency factor" adjusted to increase predicted amplitudes to obtain breaking

Better to model wave generation as stochastic process (future work)

Unknown unknowns in parametrization

Requirements for better parametrization

Theoretical constraints	almost none; plenty of ideas
Computational constraints	Must be in-column & inexpensive
Observational constraints	Must not require more information

Three-dimensional refraction is a candidate

Claim: singular geometric perturbation

Key to strong effects:
ignoring the horizontal refraction is a <u>singular perturbation</u>

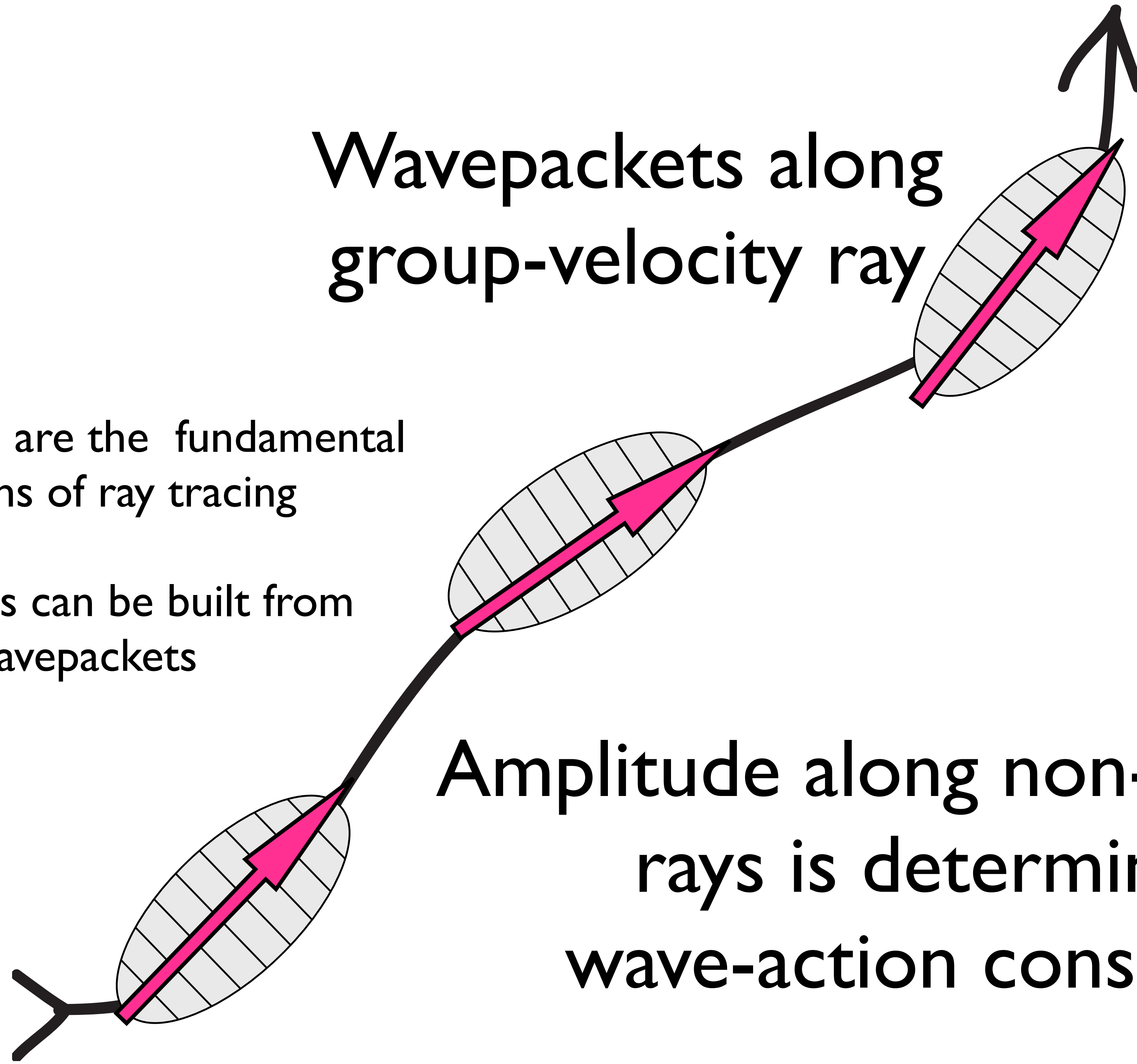
3d ray tracing admits exponentially fast wave breaking without critical layers (Jones 1969, Badulin & Shira 1993). Wave capture.

3d wave-mean interactions exhibit new features such as remote recoil and missing forces (Bühler & McIntyre 2003) Wave-vortex duality.

What are the new effects?

An adventure in ray tracing

Wavepackets along group-velocity ray



Wavepackets are the fundamental solutions of ray tracing

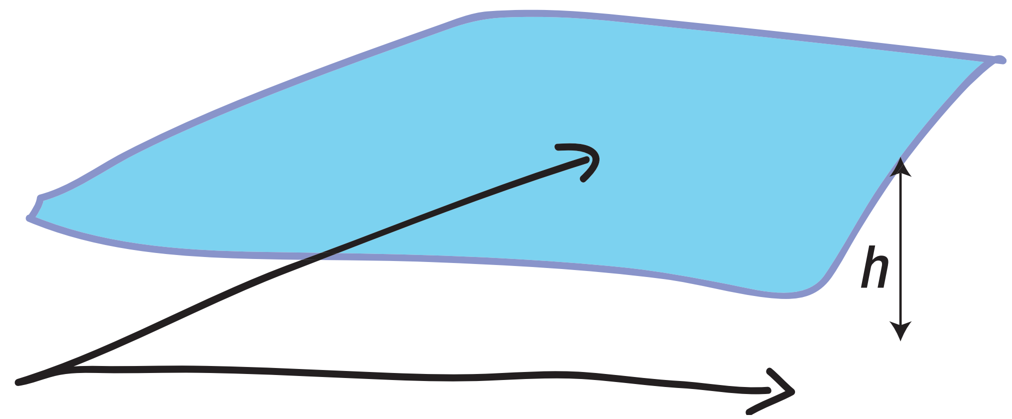
Wavetrains can be built from wavepackets

Amplitude along non-intersecting rays is determined by wave-action conservation

Oliver Bühler, Courant Institute, New York University, 2005

Warm-up example: shallow water system

Single layer of hydrostatic incompressible fluid



Variables

$$\boldsymbol{x} = (x, y) \qquad \boldsymbol{u} = (u, v)$$

$$\text{depth} \quad h \qquad \frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})$$

Mass continuity

$$\frac{\mathrm{D}h}{\mathrm{D}t} + h\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

Momentum conservation

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + g\boldsymbol{\nabla} h = 0$$

Linear small-scale wavepacket

(Example in shallow water)

$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}' + O(a^2) \qquad \mathrm{D}_t\boldsymbol{u}' + g\boldsymbol{\nabla}h' = 0 \qquad \mathrm{D}_t = \frac{\partial}{\partial t} + (\boldsymbol{U}\cdot\boldsymbol{\nabla})$$

$$h = H + h' + O(a^2) \qquad \mathrm{D}_t h' + H\boldsymbol{\nabla}\cdot\boldsymbol{u}' = 0$$

$$\boxed{a \ll 1}$$

Slowly varying wavetrain $h' = Ha(\mu\boldsymbol{x}, \mu t)\,\exp(i\theta),$ $\boxed{\mu \ll 1}$

$$\boldsymbol{k} = \boldsymbol{\nabla}\theta, \quad \omega = -\theta_t,$$

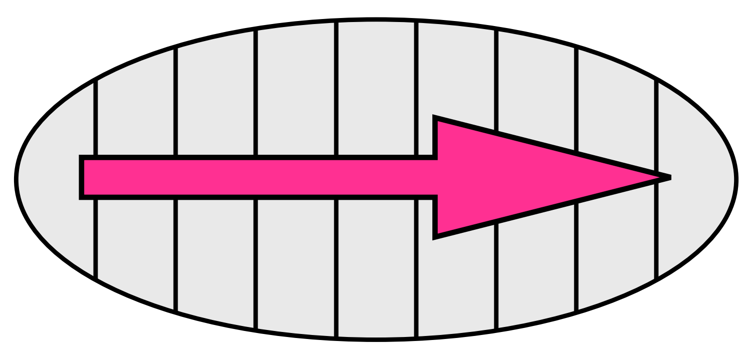
Dispersion relation

$$\boxed{\hat{\omega}^2 = gH|\boldsymbol{k}|^2}$$

background flow, if any

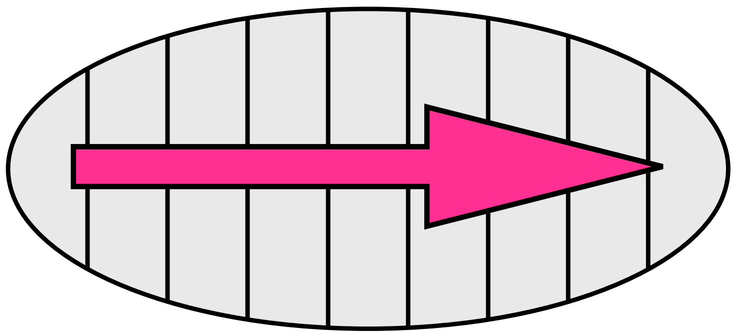
$$\omega = \boldsymbol{U}\cdot\boldsymbol{k} + \hat{\omega}$$

Wavepacket:



phase lines

Geometric ray tracing - phases



phase lines
of a wavepacket

$$\Omega(\boldsymbol{k}, \boldsymbol{x}, t) = \boldsymbol{U} \cdot \boldsymbol{k} + \hat{\omega}$$

$$\boxed{\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = +\frac{\partial \Omega}{\partial \boldsymbol{k}} \quad \text{and} \quad \frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\frac{\partial \Omega}{\partial \boldsymbol{x}}}$$

Group velocity

$$\boldsymbol{u}_g = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{U} + \hat{\boldsymbol{u}}_g$$

Ray time derivative

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (\boldsymbol{u}_g \cdot \boldsymbol{\nabla})$$

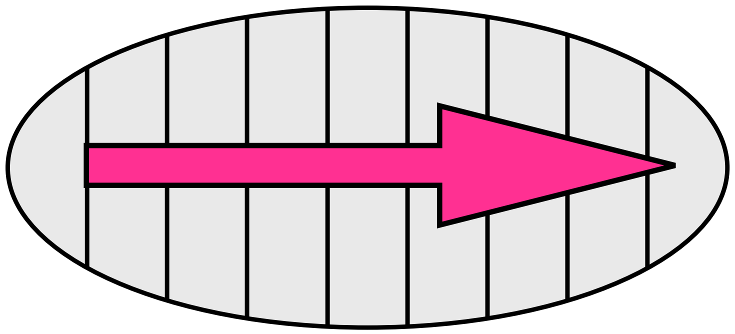
Simple case $\hat{\omega}(\boldsymbol{k})$:

$$\boxed{\frac{\mathrm{d}k_{\underline{i}}}{\mathrm{d}t} = -\frac{\partial U_j}{\partial x_{\underline{i}}} k_j}$$

Wavenumber changes
due to background
inhomogeneity
--> refraction

Oliver Bühler, Courant Institute, New York University, 2005

Physical ray tracing - amplitudes



"Mean" is the average over rapidly varying wave phase

$$h = \overline{h} + h'$$
$$\overline{h'} = 0$$

Wave energy $$E = \frac{1}{2} H \left(\overline{\boldsymbol{u}'^2} + g\overline{h'^2}/H \right)$$

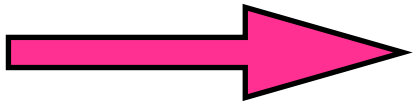
Wave action $$A = \frac{E}{\hat{\omega}}$$

$$\boxed{\frac{\partial A}{\partial t} + \boldsymbol{\nabla} \cdot (A\boldsymbol{u}_g) = 0}$$

Amplitude prediction from wave action conservation

Another important wave property:

Pseudomomentum $\mathbf{p} = \boldsymbol{k}\, A$



Pseudomomentum changes with wavenumber due to refraction

Understanding wavenumber refraction

Wavenumber

$$\frac{\mathrm{d}k_i}{\mathrm{d}t} = -\frac{\partial U_j}{\partial x_i} k_j$$ is equivalent to $$\boxed{\frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\boldsymbol{\nabla}\boldsymbol{U}\cdot\boldsymbol{k}}$$

Passive tracer

$\phi(\boldsymbol{x}, t)$ such that

$$\mathrm{D}_t\phi = 0$$

$$\boxed{\mathrm{D}_t\left(\boldsymbol{\nabla}\phi\right) = -\boldsymbol{\nabla}\boldsymbol{U}\cdot\left(\boldsymbol{\nabla}\phi\right)}$$

$\boldsymbol{k}$ and $\boldsymbol{\nabla}\phi$ evolve similarly

(i.e. wave phase and passive tracer evolve similarly)

Intrinsic difference

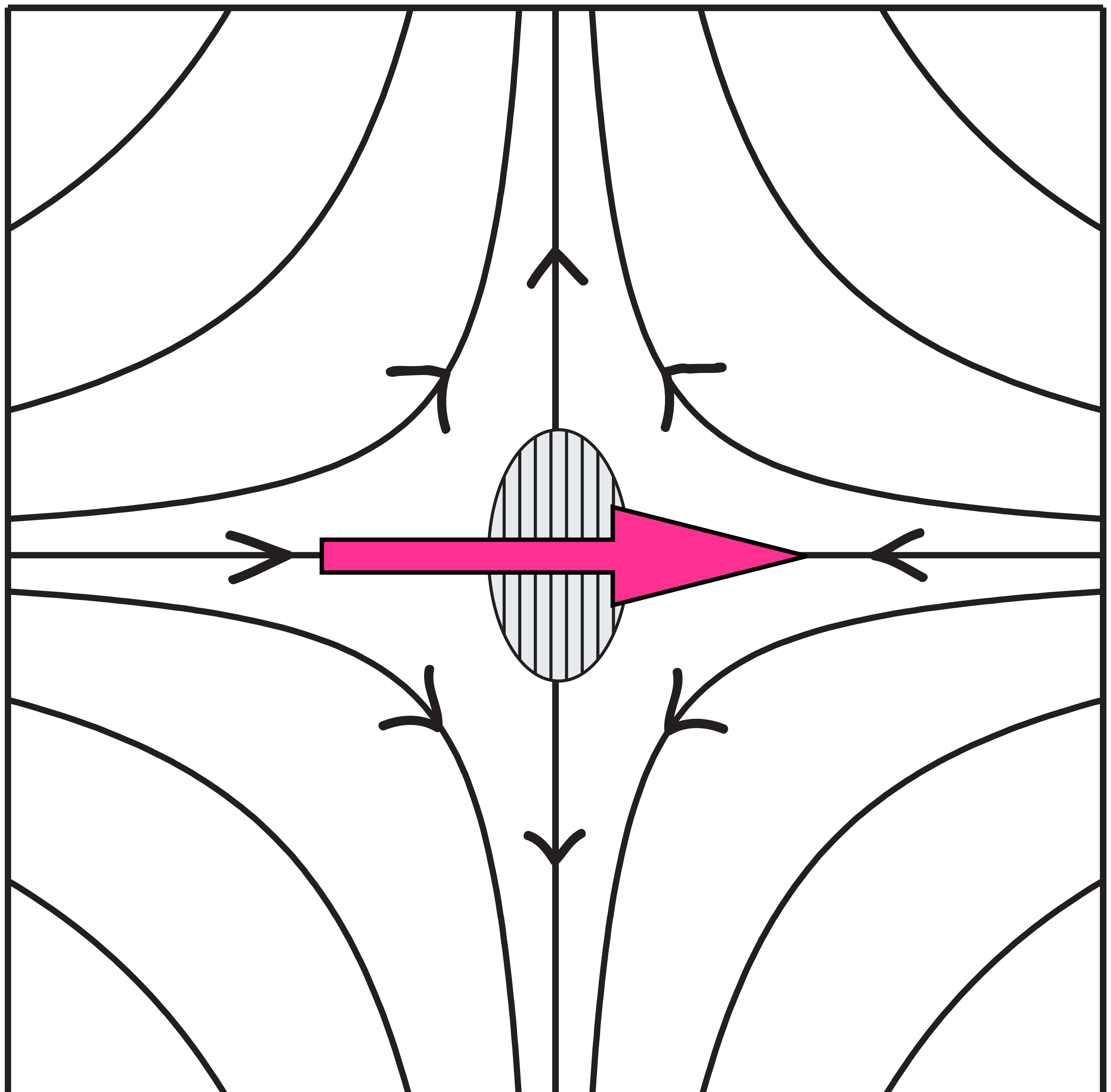
$$\boldsymbol{u}_g = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{U} + \hat{\boldsymbol{u}}_g$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t} - \mathrm{D}_t = \left(\hat{\boldsymbol{u}}_g\cdot\boldsymbol{\nabla}\right)}$$

measures the misfit

Wavepacket exposed to pure strain in analogy with passive advection

$\mathbf{p} = \boldsymbol{k}\, A$



Wavepacket is squeezed in x and stretched in y. Action is constant

Wavenumber vector **k** is increases in size

Pseudomomentum **p** increases as well

Is there a "Batchelor" regime for wave phase?

Oliver Bühler, Courant Institute, New York University, 2005

Not in shallow water...

Answer is <u>no</u> in shallow water with sub-critical steady background flow

$$\omega = \boldsymbol{U} \cdot \boldsymbol{k} + \hat{\omega} = \text{const.}$$

$$\hat{\omega} = \sqrt{gH}|\boldsymbol{k}|$$

$$\boxed{|\boldsymbol{k}| \leq \frac{\omega}{\sqrt{gH} - |\boldsymbol{U}|}}$$

wavenumbers are bounded unless

$U^2 > gH$ *(geophysically less relevant regime)*

Same answer for rotating shallow water, <u>but not for 3d flow!</u>

Boussinesq system

(no Coriolis force in talk, but in paper)

Variables $\boldsymbol{x} = (x, y, z)$ $\boldsymbol{u} = (u, v, w)$

Incompressible $\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$

Momentum conservation $$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + \boldsymbol{\nabla} P = \underbrace{b\widehat{\boldsymbol{z}}}$$ buoyancy acceleration

Stratification $$\frac{\mathrm{D}}{\mathrm{D}t}(b + N^2 z) = \frac{\mathrm{D}b}{\mathrm{D}t} + N^2 w = 0$$

constant value defines 3d stratification surfaces

Plane Boussinesq gravity waves

$$\nabla \cdot \boldsymbol{u} = 0 \quad \text{implies} \quad \boldsymbol{k} \cdot \boldsymbol{u}' = 0$$

Dispersion relation

$$\boxed{\hat{\omega}^2 = N^2 \frac{k^2 + l^2}{k^2 + l^2 + m^2}}$$

Wavenumber vector

$$\boldsymbol{k} = (k, l, m)$$

Frequency is <u>independent</u> of

$$\kappa = \sqrt{k^2 + l^2 + m^2}$$

Group velocity magnitude

$$|\hat{\boldsymbol{u}}_g|^2 = \frac{N^2 - \hat{\omega}^2}{\underline{\kappa^2}}$$

1) Unbounded wavenumber growth is possible at fixed frequency

2) Group velocity inversely proportional to wavenumber at fixed frequency

Oliver Bühler, Courant Institute, New York University, 2005

Three-dimensional ray tracing

Horizontal background flow

$$\boldsymbol{U} = (U, V, 0) \quad \text{and} \quad U_x + V_y = 0$$

Gradient of background flow treated as steady for simplicity

$$\boldsymbol{\nabla U} \equiv \begin{pmatrix} U_x & V_x & W_x \\ U_y & V_y & W_y \\ U_z & V_z & W_z \end{pmatrix} = \begin{pmatrix} U_x & V_x & 0 \\ U_y & -U_x & 0 \\ U_z & V_z & 0 \end{pmatrix}$$

$$D = U_x^2 + \left(\frac{V_x + U_y}{2}\right)^2 - \left(\frac{V_x - U_y}{2}\right)^2$$

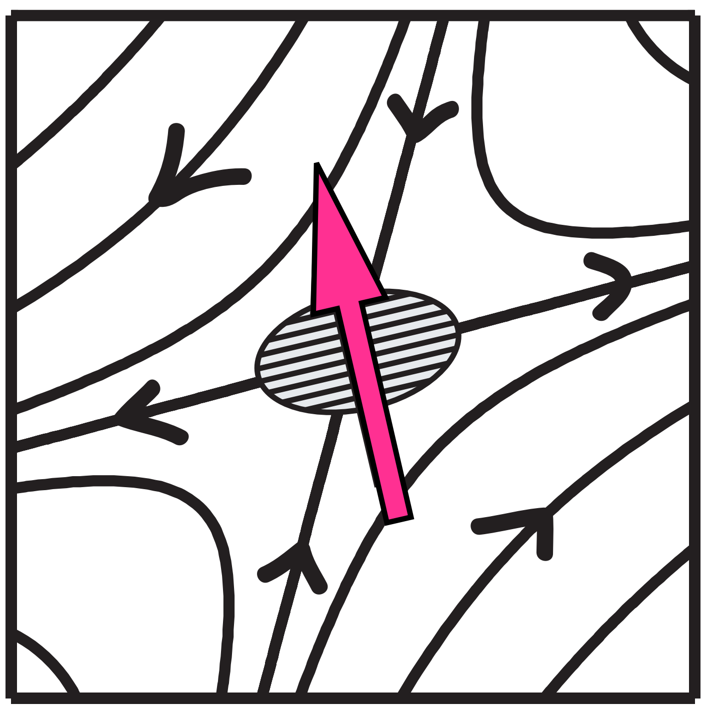
positive for generic case with open 2d stream lines in group-velocity frame

Wavenumber evolution decouples into horizontal and vertical components

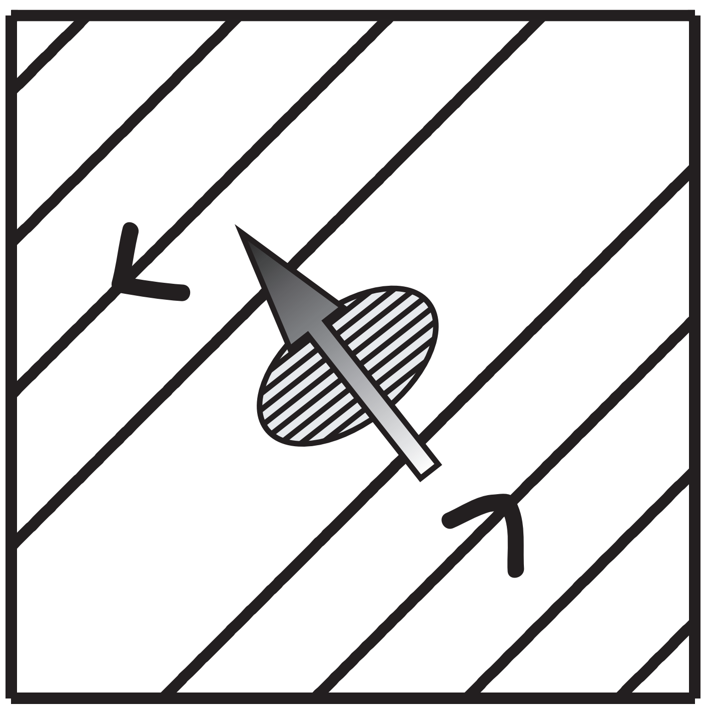
$$\frac{\mathrm{d_g}}{\mathrm{d}t}\begin{pmatrix} k \\ l \end{pmatrix} = -\begin{pmatrix} U_x & V_x \\ U_y & -U_x \end{pmatrix}\begin{pmatrix} k \\ l \end{pmatrix} \quad \text{and} \quad \frac{\mathrm{d_g}}{\mathrm{d}t} m = -U_z k - V_z l$$

easy 2d sub-problem of advection by area-preserving flow

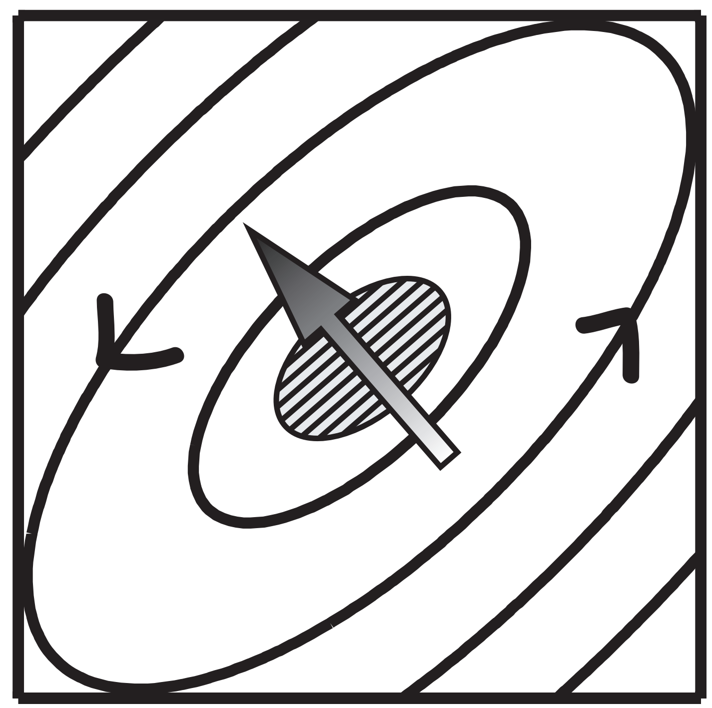
Streamlines in group-velocity frame



Hyperbolic D>0



Parabolic D=0



Elliptic D<0

The horizontal wavenumber vector aligns itself with the growing eigenvector, which is perpendicular to the extension axis

Final orientation is independent of initial orientation: wavepacket loses memory

Oliver Bühler, Courant Institute, New York University, 2005

Growing mode in three dimensions

$$\boldsymbol{k}_H(t) \propto (-V_x,\, U_x + \sqrt{D})\exp(\sqrt{D}\,t)$$

$$m(t) = -\frac{U_z k(t) + V_z l(t)}{\sqrt{D}} + O(1)$$

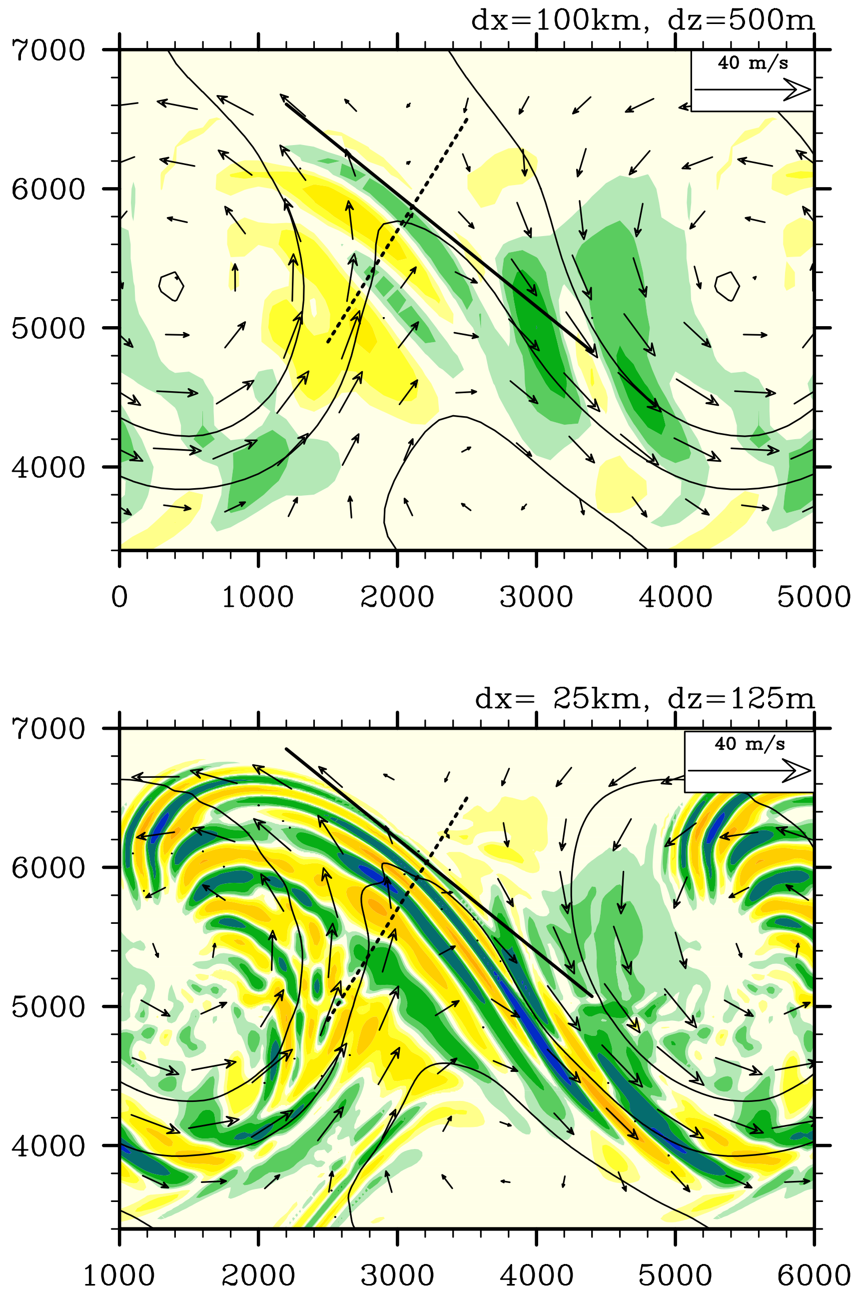
Intrinsic group velocity decreases as wavenumber grows; the wavepacket becomes frozen into the flow (Jones 1969, Badulin & Shrira 1993)

The reinforces the analogy between wave phase and passive tracer behaviour; reasonably to expect exponential straining to persist once it gets started

Wave capture:

Exponentially fast wave breaking scenario without critical layers!

Numerical example

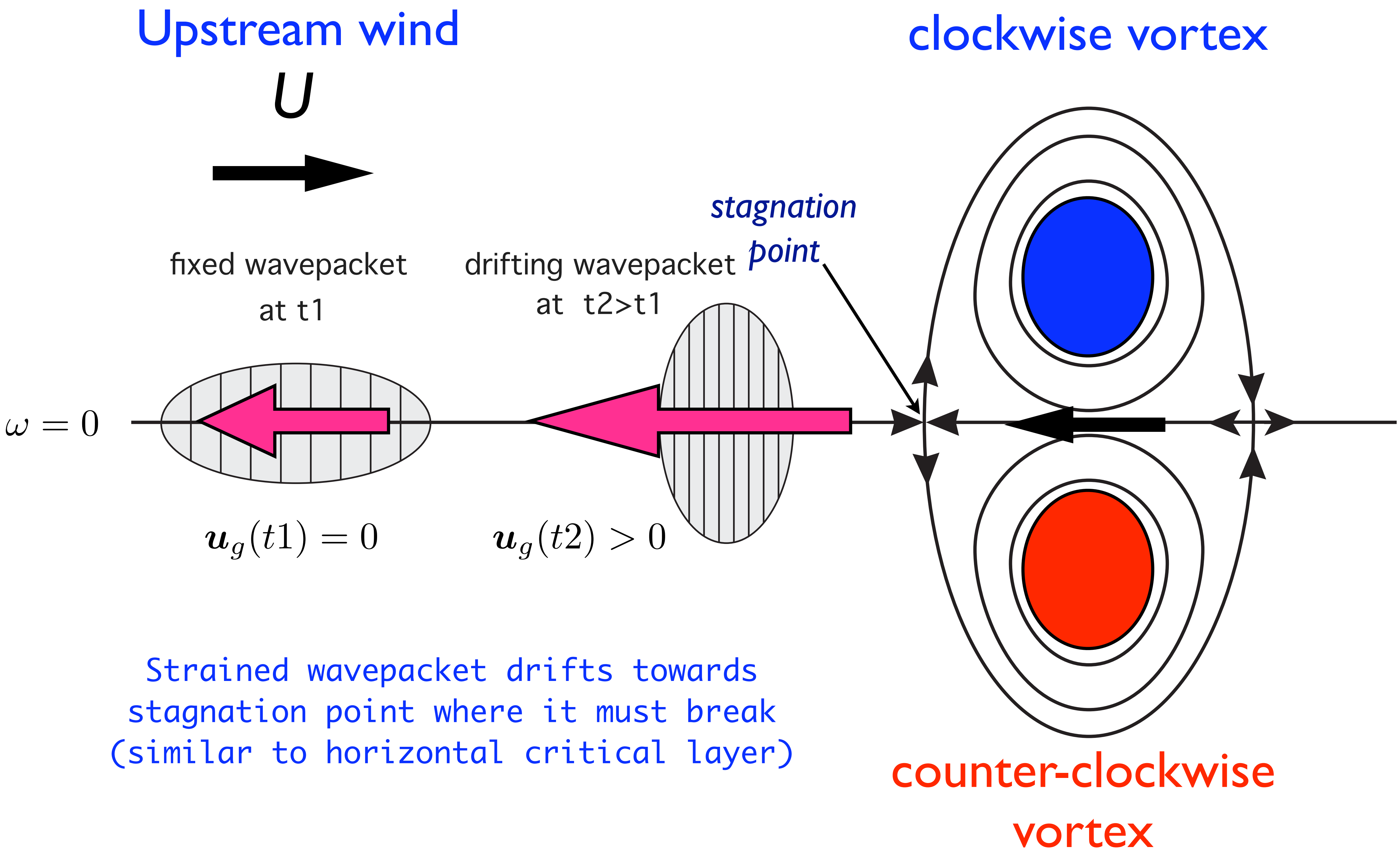


Plougonven & Snyder
GRL, 2005

Snapshots taken from numerical simulation of meandering jet stream

Interpreted based on wave straining

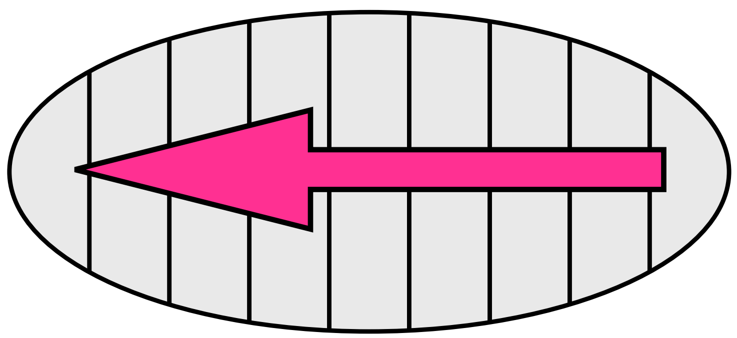
Theory example: wave capture by blocking dipole



Strained wavepacket drifts towards stagnation point where it must break (similar to horizontal critical layer)

Oliver Bühler, Courant Institute, New York University, 2005

Pseudomomentum surge and mean flow



$$\mathbf{p} = \boldsymbol{k}\, A$$

grows exponentially
during wave capture
(action is conserved,
but wavenumber grows)

Standard dissipative wave-mean interaction paradigm:
mean-flow momentum + wave pseudomomentum = constant.

Does the exponential pseudomomentum surge lead to a dramatic local mean-flow response ?

No, because the standard paradigm does not hold for horizontal wavepacket refraction...

Need to investigate $O(a^2)$ wave–mean interaction theory with refraction

Wave-mean interaction theory

Slowly varying Lagrangian mean flow with strong stratification is layerwise 2d, layerwise non-divergent, and at $O(a^2)$ is governed by (Bretherton 1969)

$$\boldsymbol{\nabla} \cdot \overline{\boldsymbol{u}}^L = 0 \quad \text{and} \quad \overline{w}^L = 0.$$

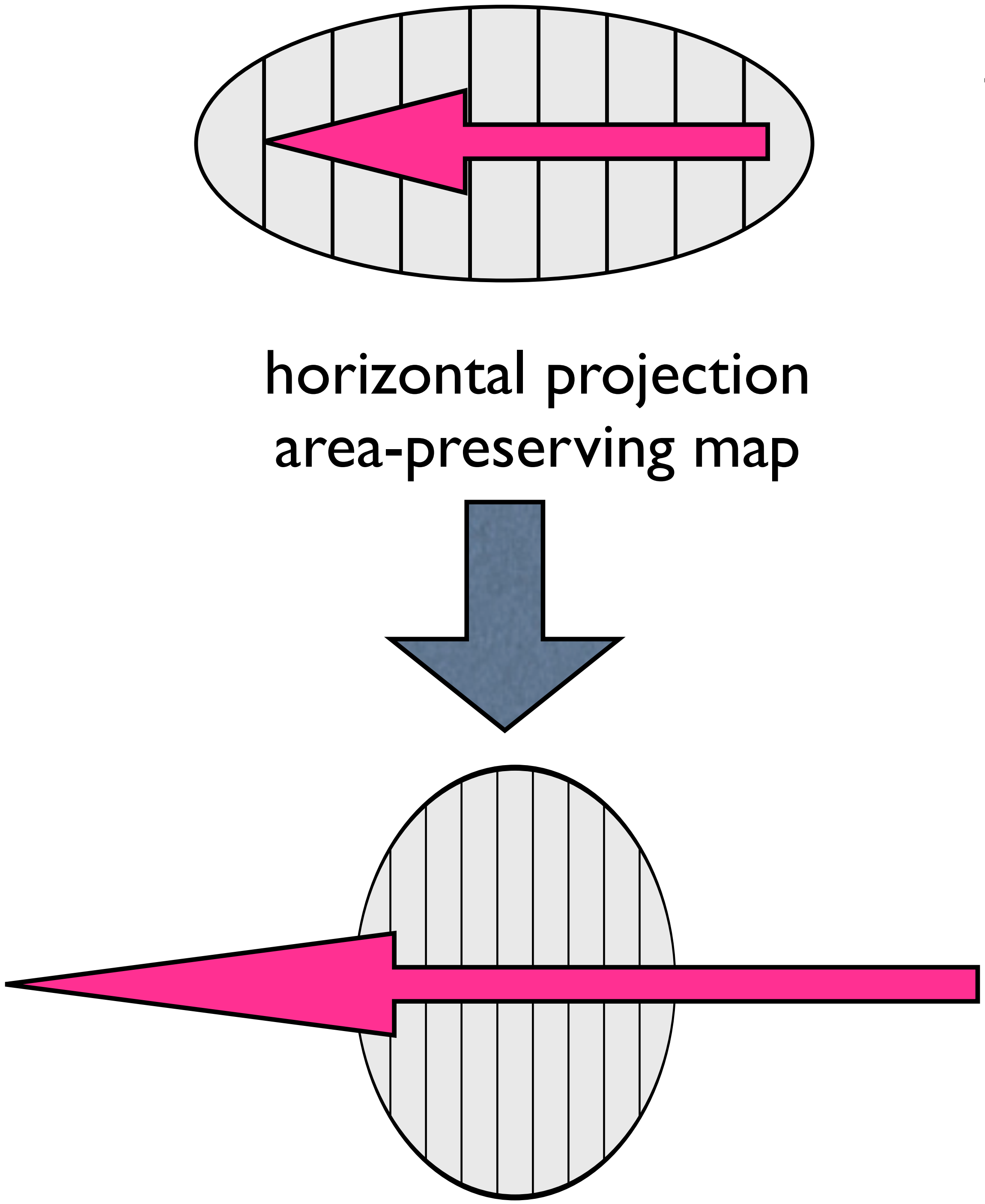
Can show that (AM 1978, BM 98, B00, BM05)

$$\boxed{\left(\frac{\partial}{\partial t} + \overline{\boldsymbol{u}}^L \cdot \boldsymbol{\nabla}\right) \left\{ \widehat{\boldsymbol{z}} \cdot \boldsymbol{\nabla} \times \left[\overline{\boldsymbol{u}}^L - \mathbf{p}\right] \right\} = 0}$$

(Lagrangian and Eulerian mean flows are equal to leading order for Boussinesq wavepackets, but this version holds more generally)

Hence only the vertical curl of pseudomomentum affects the mean flow

Vertical pseudomomentum curl



Neglect intrinsic group velocity

$$\mathbf{p} = \boldsymbol{k}\,A = \boldsymbol{\nabla}\theta\,A \quad \text{and therefore}$$

$$\widehat{\boldsymbol{z}} \cdot \boldsymbol{\nabla} \times \mathbf{p} = l A_x - k A_y \propto$$

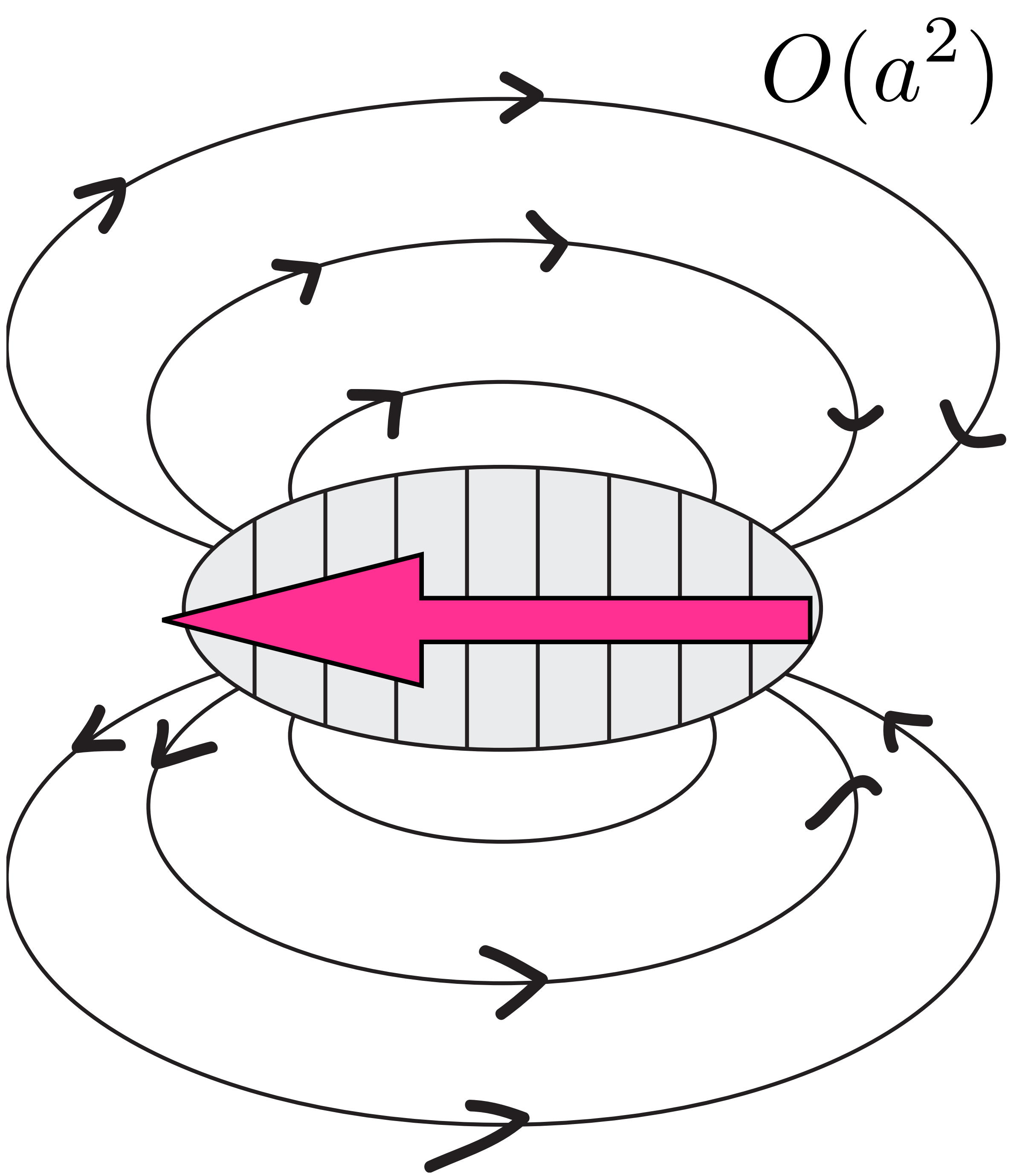
$$= \mathrm{d}A\,\mathrm{d}\theta = \ \mathrm{const.}$$

as both A and the wave phase are advected by area-preserving flow

Exponential surge in pseudomomentum but not in its vertical curl

Oliver Bühler, Courant Institute, New York University, 2005

Bretherton's flow (1969)



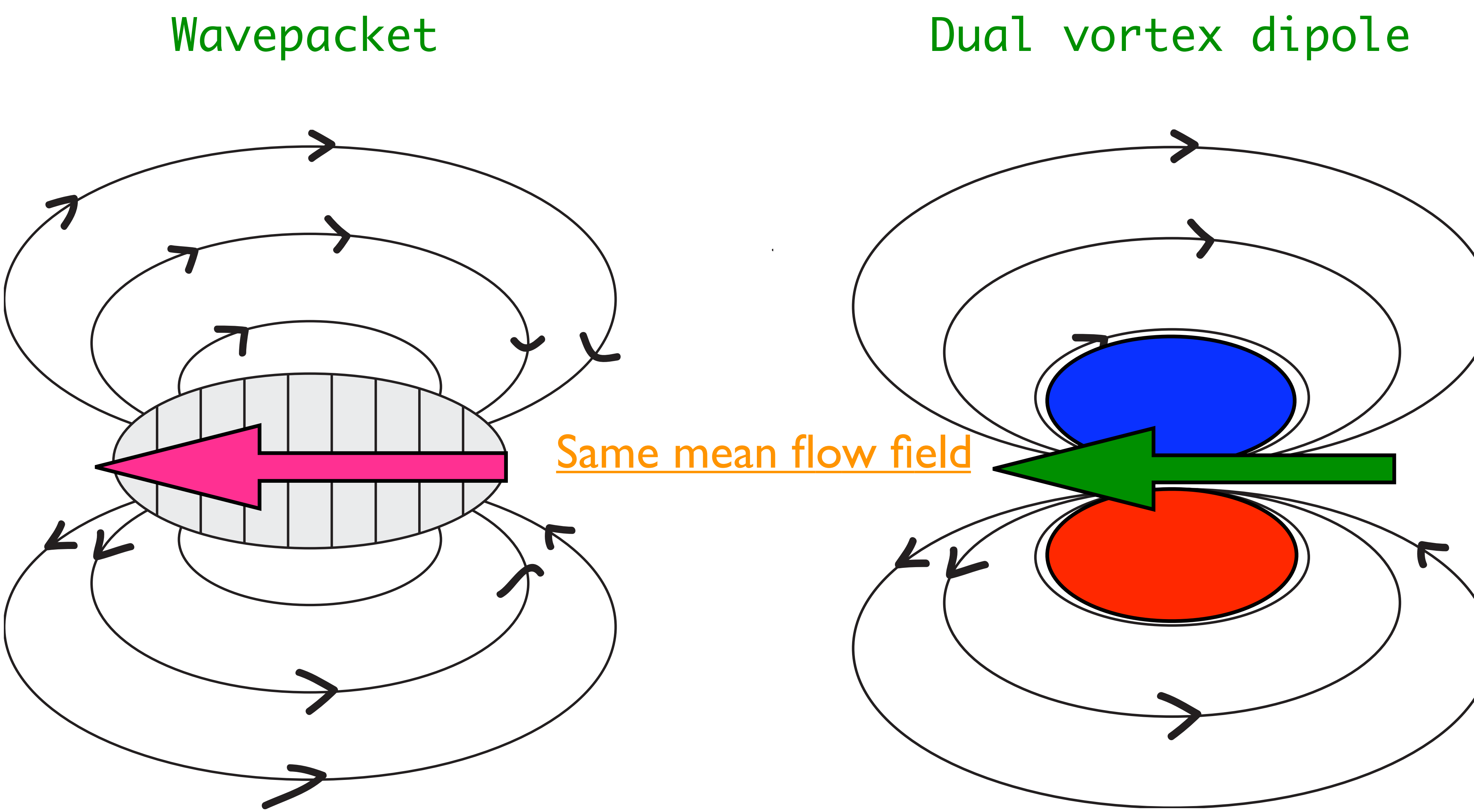
Large-scale dipolar return flow at second order in wave amplitude

Far-field mean velocity is non-divergent and decays with square of distance to wavepacket

The impulse (ie the skew linear moment of vorticity) of this layerwise 2d flow is well defined, but not its momentum

Feynman:
"children on a slide"

Wave-vortex duality



wave pseudomomentum = dual vortex impulse

.....suggests new thinking of interactions....

Oliver Bühler, Courant Institute, New York University, 2005

Straining of wavepacket and vortices

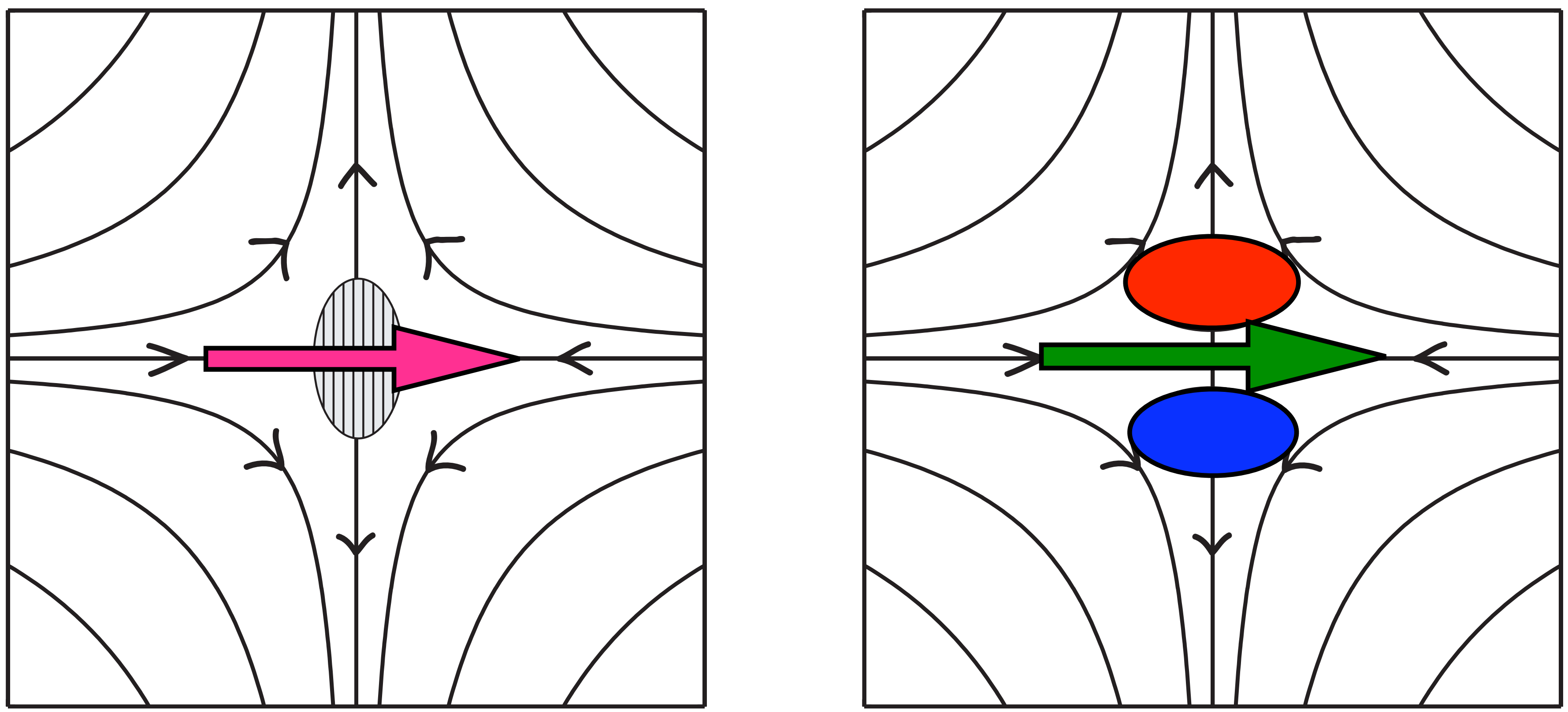


FIGURE 1. Left: wavepacket exposed to pure horizontal strain contracting along the x-axis and extending along the y-axis. The wavecrests align with the extension axis and their spacing is decreased, so that the wavenumber vector $\boldsymbol{k}$ points at right angles to the extension axis and grows in magnitude, as suggested by the large arrow. Right: a pair of oppositely signed vortices exposed to same strain. The arrow now indicates the vortex pair's Kelvin impulse.

Wavepacket and vortex dipole are strained in the same way

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Pseudomomentum + impulse = conserved

Potential vorticity $\qquad \overline{q}^L = \overline{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}^L = \widehat{\boldsymbol{z}} \cdot \boldsymbol{\nabla} \times (\overline{\boldsymbol{u}}^L - \mathbf{p}_H)$

GLM theory used here

Impulse $\qquad \mathbf{I}(t) = \iiint \mathbf{i}\,(\boldsymbol{x}, t)\, \mathrm{d}x\mathrm{d}y\mathrm{d}z \qquad$ where $\qquad \mathbf{i} = (y, -x, 0)\,\overline{q}^L$

skew linear moment of PV

Pseudomomentum $\qquad \mathbf{P}_H \equiv \iiint \mathbf{p}_H \, \mathrm{d}x\mathrm{d}y\mathrm{d}z$

$$\frac{\mathrm{d}\mathbf{P}_H}{\mathrm{d}t} = -\iiint (\boldsymbol{\nabla}_H \overline{\boldsymbol{u}}^L) \cdot \mathbf{p}_H \, \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

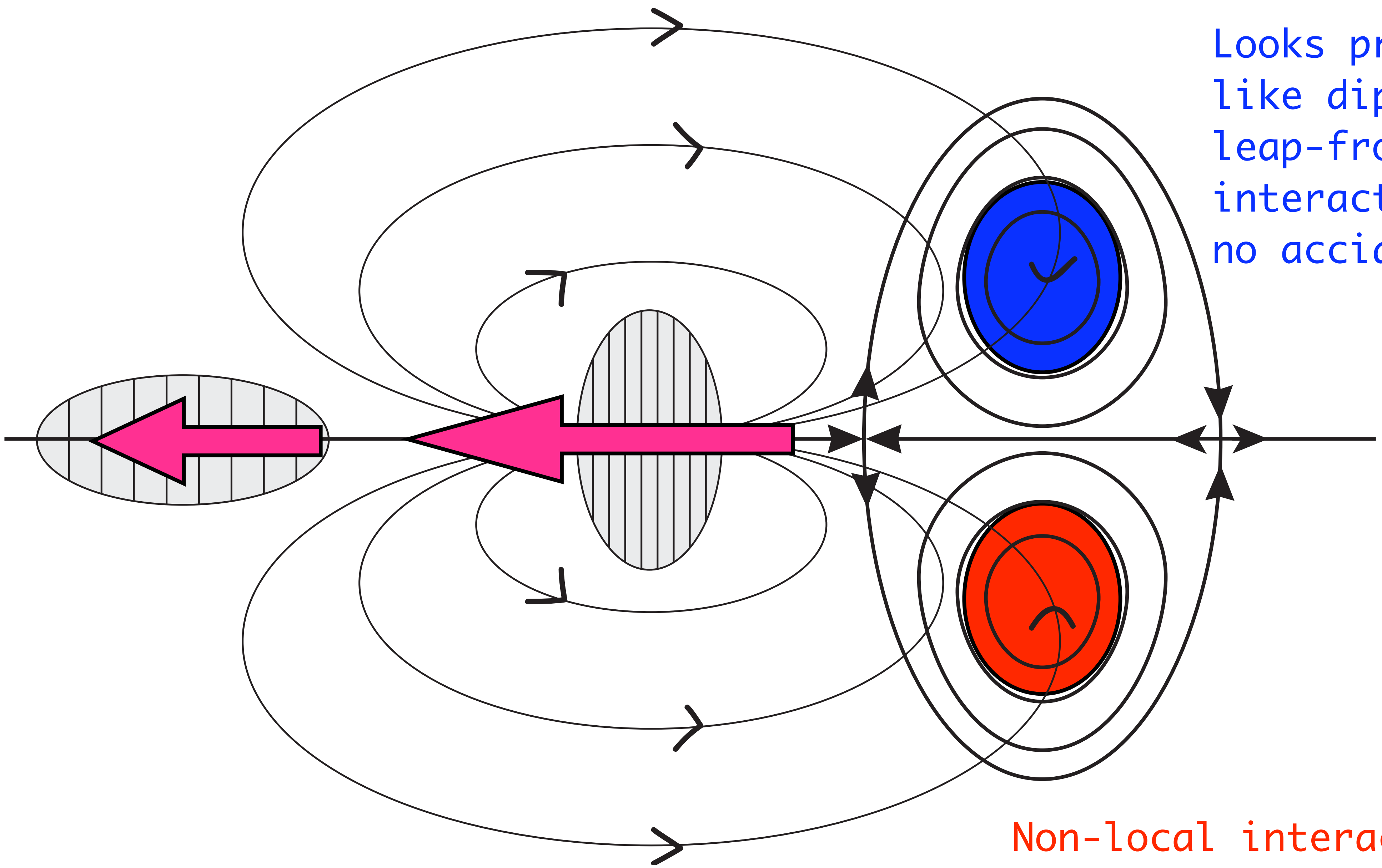
Refraction terms

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}t} = \iiint (\boldsymbol{\nabla}_H \overline{\boldsymbol{u}}^L) \cdot \mathbf{p}_H \, \mathrm{d}x\mathrm{d}y\mathrm{d}z \; .$$

$$\boxed{\mathbf{P}_H + \mathbf{I} = \text{constant}}$$

The resolution

Dipole straining increases wavepacket pseudomomentum.
Bretherton flow advects vortex dipole and reduces impulse.
Both compensate and the sum of P + I is conserved!



Looks precisely like dipole-dipole leap-frog interaction... no accident!

Non-local interaction at a distance

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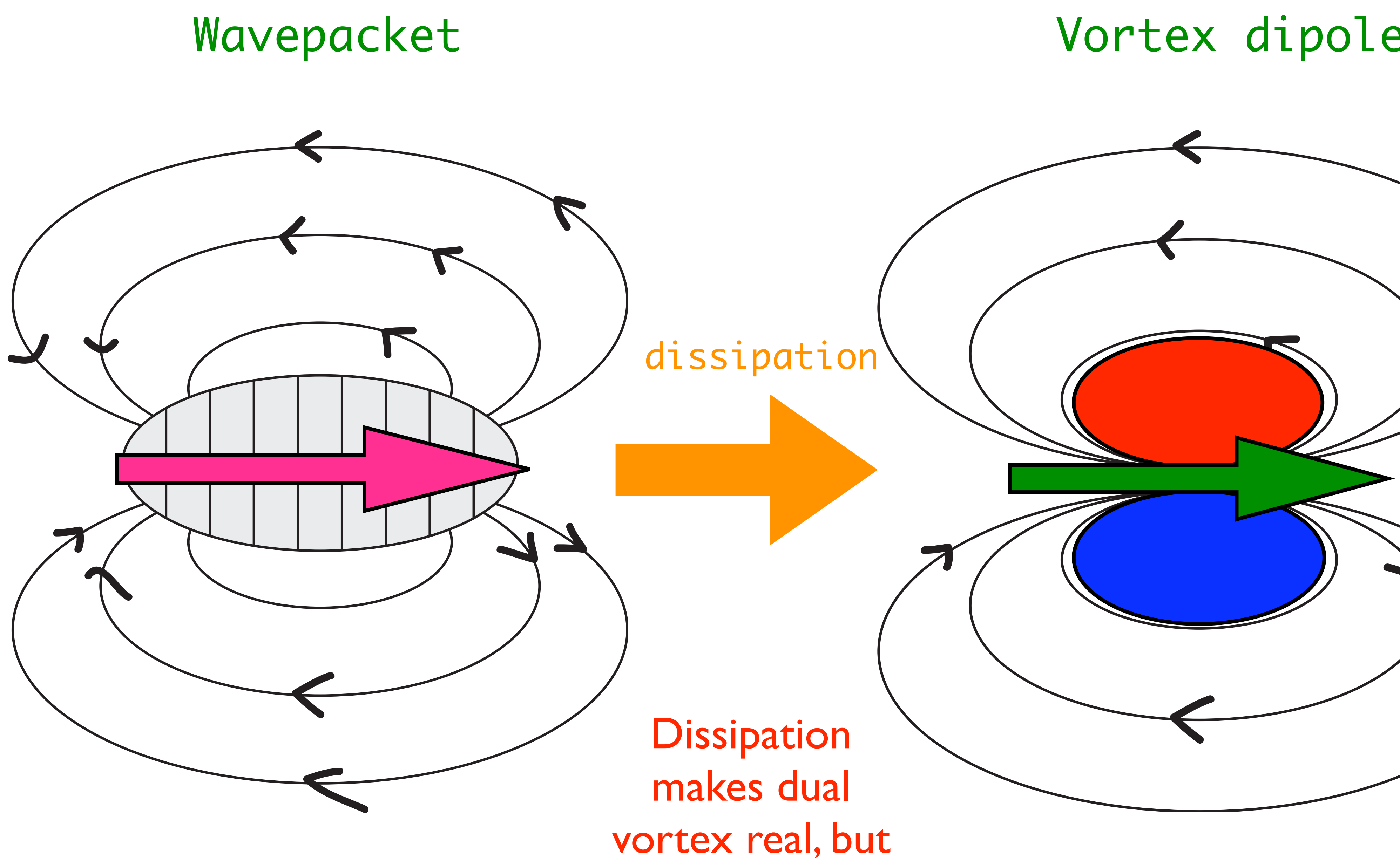
Duality and dissipation



Dissipation itself does not accelerate the mean flow!

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Towards a new parametrization scheme

Joint work with John Scinocca (CCCma, Univ. Victoria, CA)

1. based on existing columnar parametrization scheme

2. wave dissipation and breaking part unchanged, only non-dissipative propagation part is changed

3. requires no new assumptions on gravity wave launch spectrum (have only weak observational constraints)

4. requires horizontal derivatives of model fields, which is a bit non-trivial in operational GCMs because of parallel architecture

The key difference

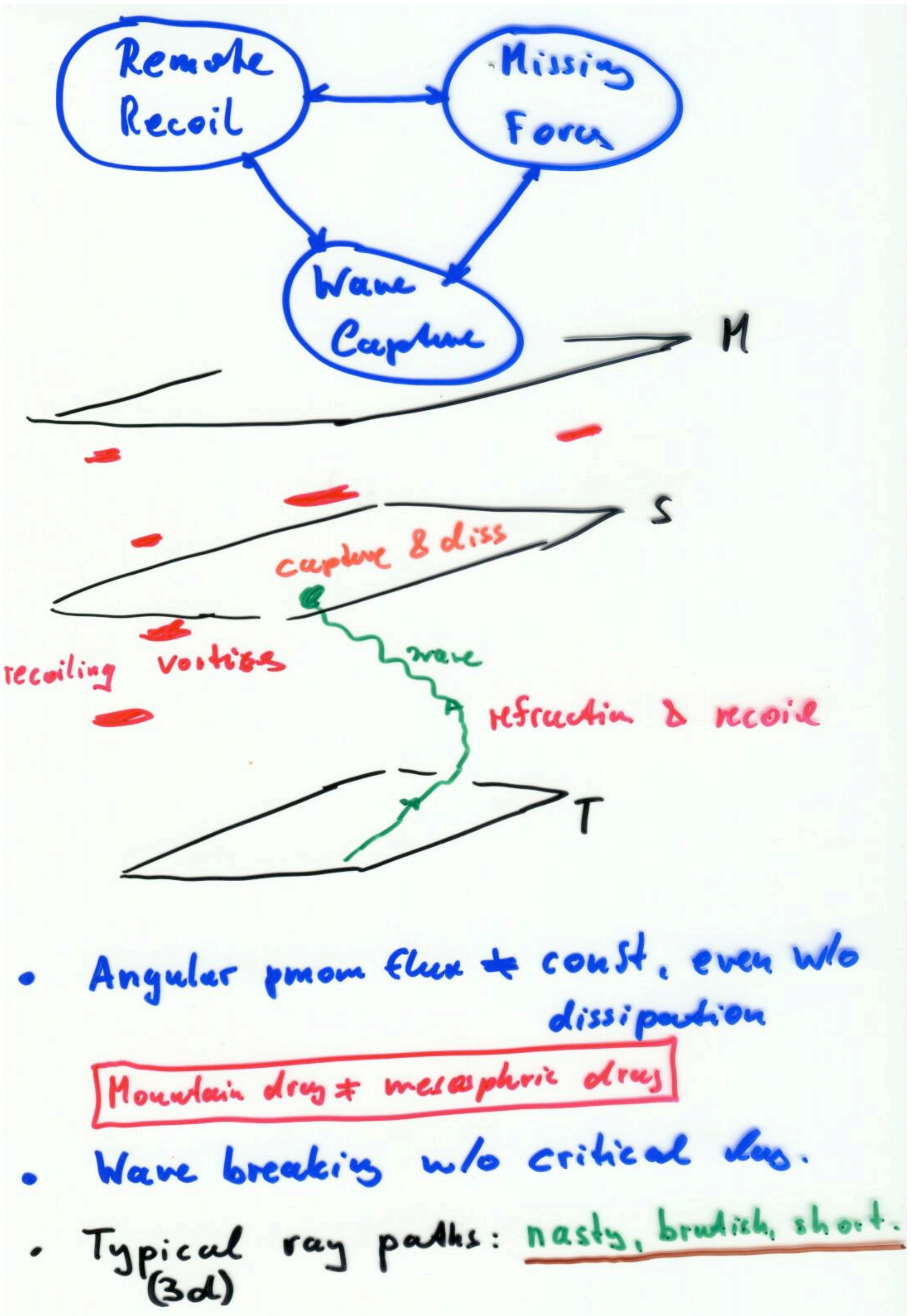
Old scheme is based on constant pseudomomentum flux

New scheme is based on constant wave action flux

pseudomomentum flux(z) = k(z) wave action flux

changes in k(z) due to horizontal refraction change the pseudomomentum flux(z) and hence produce new mean-flow forces

Concluding image



Wave action flux replaces pseudomomentum flux

Refraction leads to wave-mean momentum exchanges without dissipation

Mountain drag does <u>not</u> simply equal wave drag anymore (it never did)

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