Orographic Precipitation II: Effects of Phase Change on Orographic Flow

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Condensation \equiv -\frac{D\rho_{vs}}{Dt} \approx -w(x, z) \frac{\partial\rho_{vs}}{\partial z}

\[ w = w(H, L, U, \text{Stability, Coriolis, 3D Effects}) \]

\( \rho_{vs} = \) saturation vapor density
Effects of Water in the Air on Buoyancy

\[ B = g \frac{\rho_{\text{env}} - \rho_{\text{par}}}{\rho_{\text{par}}} \]

\( \rho \) = density
“env” = environment
“par” = parcel
moist air is a mixture

\[ \rho = \rho_d + \rho_v + \rho_l \]

gas law

\[ p = (\rho_d R_d + \rho_v R_v)T \]

/ definitions

\[ \varepsilon \equiv \frac{R_d}{R_v} \quad ; \quad q_v \equiv \frac{\rho_v}{\rho_d} \quad ; \quad q_w \equiv \frac{\rho_v + \rho_l}{\rho_d} \]

gas law

\[ p = \rho R_d \left( \frac{1 + q_v \varepsilon^{-1}}{1 + q_w} \right) T \equiv \rho R_d \tilde{T} \]

\[ \rho \quad = \text{density} \]

subscript \( d \) = dry air

subscript \( v \) = water vapor

subscript \( l \) = liquid water
substitute $\tilde{T}$ for $\rho$

$$B = g \frac{T_{\text{par}} - T_{\text{env}}}{T_{\text{env}}}$$

$$\tilde{T} = \left( \frac{1 + q_v \varepsilon^{-1}}{1 + q_w} \right) T \approx (1 + 0.61 q_v - q_l) T$$

water vapor less dense than air

liquid water more dense than air
Main Effect on Buoyancy through Phase Change

\[ \delta \]

1st Law of Thermodynamics

\[ c_p \frac{DT}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = \frac{DQ}{Dt} = -L \frac{Dq_{vs}}{Dt} \]

condensation → latent heat release

\[ L \approx 600 \text{cal/g} \]
\[ c_p = .24 \text{cal/g/°C} \]
\[ \Delta q_{vs} = -1 \text{g/kg} = -.001 \Rightarrow \Delta T \approx 2.5 \text{°C} \]
Air Parcel Behavior with Phase Change

Latent Heat Release Reduces Stability
**Dynamics: Stable Flow**

**Weak Stability, No Blocking**

\[ w(x, z_0) \approx U \frac{\partial h}{\partial x} \rightarrow \int_{z_0}^{\infty} -w(x, z) \frac{\partial \rho_{vs}}{\partial z} \, dz \sim U \frac{\partial h}{\partial x} \]

**Strong Stability, Blocking**

\[ w(x, z_0) \neq U \frac{\partial h}{\partial x} \rightarrow \int_{z_0}^{\infty} -w(x, z) \frac{\partial \rho_{vs}}{\partial z} \, dz \sim ? \]
Weak Stability, No Blocking

Simplest Model $\Rightarrow$ Rainout $=$ Condensation

\[ R(x) = \int_{0}^{\infty} -w(x, z) \frac{\partial \bar{\rho}_{vs}}{\partial z} \, dz \]
Simple Model Overestimates $R$

\[
\frac{d\rho_{\text{liq}}}{dt} = \frac{d(\rho_c + \rho_r)}{dt} = -\frac{d\rho_{\text{vs}}}{dt} + \frac{\partial R}{\partial z}
\]

Introduce Time Lags

\[
\frac{d\rho_c}{dt} = -\frac{d\rho_{\text{vs}}}{dt} - \frac{\rho_c}{\tau_c}
\]

Conversion from Cloud droplets to Raindrops

\[
\frac{d\rho_r}{dt} = +\frac{\rho_c}{\tau_c} - \frac{\rho_r}{\tau_r}
\]

Precipitation

Smith and Barstaad (2004)
\[ \tau_{\text{microphysics}} \ll \tau_{\text{airflow}} \]

1000s \ll \frac{L}{U}

\[ U = 10 \text{m/s} \]

<table>
<thead>
<tr>
<th>L</th>
<th>( \frac{L}{U} )</th>
<th>( \tau_{\text{microphysics}} \sim \tau_c + \tau_r )</th>
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<td>100km</td>
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Transfer Function

\[ \hat{R}(k, l) = \frac{C_w i \sigma \hat{h}(k, l)}{[1 - im\lambda \rho ][1 + i\sigma \tau_c ][1 + i\sigma \tau_r ]} \]

Terrain transform

\[ \sigma = Uk + Vl \]

Transform of precipitation field

Airflow dynamics: Uplift penetration

Cloud physics: conversion

Cloud physics: fallout

Each bracket shifts and reduces precipitation
Triangle Ridge: Three models

- Raw upslope
- Airflow dynamics only
- Full linear model
• With Weak Static Stability No Blocking
  Linear Theory Applied to Oregon Climate

Smith, Bonneau and Barstaad (J. Atmos. Sci., 2004)
Air Parcel Behavior with Phase Change

Latent Heat Release can Produce Asymmetric Effects
Fundamental Nonlinearity

Upward Displacement: Parcel Remains Saturated

Downward Displacement: Parcel May Desaturate

\[ B = -N_m^2 \delta \quad \text{if } \delta > 0 \]
\[ B = -N_d^2 \delta \quad \text{if } \delta < 0 \]

Barcilon, Jusem and Drazin (1979)
Saturated Conditions Upstream, Unsaturated Conditions Downstream/ (Foehn)Wind

Miglietta and Rotunno (J. Atmos. Sci., 2005)
Piedmont Flood November 1994 (Obs Rain 06z 5 Nov –06z 6 Nov)

Rotunno and Ferretti (2001)
Nearly Moist Neutral
Air Parcel Behavior with Phase Change

Latent Heat Release can Produce Instability
Global Measure: Convective Available Potential Energy

\[ CAPE = \int_{z_0}^{z_1} B \, dz \]

\[ \int_{z_0}^{z_1} B dz = R_d \int_{p(z_0)}^{p(z_1)} \left( \overline{T}_{par} - \overline{T}_{env} \right) d \ln p \]

Emanuel (1994)
$T_{par} > T_{env}$
\[ w_{\text{max}} = \sqrt{2 \times CAPE} \approx 2 - 50 \text{ m/s} \]

\[ C(z_0) = \int_{z_0}^{\infty} -w(x, z) \frac{\partial \rho_{\text{vs}}}{\partial z} \, dz \]

\[ C(z_0) \approx w_{\text{max}} \rho_{\text{vs}}(z_0) = 2 \text{ m/s} \times \times 0.01 \text{ Kg/m}^3 = 72 \text{ mm/h} \]
Good News: Upslope Flow Can Provide Lift to Overcome Threshold (E.g. Stable Layers)
Bad News:
Upslope Wind Moves Cells Downwind ➔
Rain Accumulation Small
Typical Rain Cell Life Cycle

Byers and Braham (1948)

Cold Air Outflow
Good News: Cool Air Outflows May Initiate New Cells Upstream →

Chu and Lin (2000)
Bad News: Cool Air Outflows May Propagate Too Far Upstream

Chu and Lin (2000)
Good News: Rain accumulation large if wind varies with height such that cells are stationary wrt mountain.
Big Thompson Flood
Colorado, 1976

Caracena et al. (1979)
Caracena et al. (1979)
Summary

- Dynamics of orographic air flow strongly coupled to latent heating
- Stable Case: Latent heating renders flow less stable making possible flow over tall mountains condensing large amounts of water vapor. Microphysics present major uncertainties, however.
- Unstable Case: Convective cells may produce large amounts of condensed water, but motion of cells wrt to mountain makes detailed prediction difficult.