Moist processes in the atmosphere:
From simple concepts to sophisticated parameterizations

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Why is it so hard to simulate the Earth climate system?
Why is it so hard to simulate the Earth climate system?

Because some of the key processes are even not on this diagram....
Kiehl and Trenberth, BAMS 1997

Fig. 7. The earth’s annual global mean energy budget based on the present study. Units are W m\(^{-2}\).
Indirect aerosol effects

1st Indirect effect
2nd Indirect Effect

cloud base

cloud updraft

maritime ("clean")
continental ("polluted")
p-T phase equilibrium diagram for water substance
ELEMENTARY CLOUD PHYSICS:

clouds form due to cooling of air (e.g., adiabatic expansion of a parcel of air rising in the atmosphere)

- condensation: water vapor $\rightarrow$ cloud droplets

heterogeneous nucleation on atmospheric aerosols called Cloud Condensation Nuclei (CCN); typically highly soluble salts (sea salt, sulfates, ammonium salts, nitrates)

only a very small percentage of CCN used by clouds (i.e., water clouds form just above saturation)
ELEMENTARY CLOUD PHYSICS, cont.:

• formation of ice particles

heterogeneous nucleation on atmospheric aerosols called Ice-forming Nuclei (IN); dominates for temperatures higher than about -40 deg C (233 K); dominated by the so-called contact mode where a supercooled droplet freezes upon contact with IN;

IN are typically silicate particles (clays) or other compounds with crystallographic lattice similar to ice, highly insoluble

IN are scarce, their number depends exponentially on temperature (typically, 1 per liter at -20 deg C, 10 per liter at -25 deg C).

homogeneous freezing is possible once droplet temperature is smaller than about -40 deg C.
From cloud droplets and ice crystals to precipitation:

**WARM RAIN:**

→ gravitational collision and coalescence between cloud droplets

**ICE PROCESSES:**

→ Findeisen-Bergeron process: water vapor pressure at saturation is lower over ice than over water; it follows that once ice crystal is formed from supercooled droplet, it grows rapidly through diffusion of water vapor at the expense of cloud droplets

→ riming: falling ice crystal collects supercooled droplets that freeze upon contact (graupel, hail, etc).
Modeling moist processes in the atmosphere:

- Gas dynamics for the air with moisture (i.e., containing water vapor, suspended small cloud particles, falling larger precipitation particles);

- Thermodynamics for the air containing water vapor (i.e., phase changes, latent heating, etc).
Gas dynamics for moist air:

- Water vapor is a minor constituent:
  
  mass loading is typically smaller than 1%; thermodynamic properties (e.g., specific heats etc) only slightly modified;

- Suspended small particles (cloud droplets, cloud ice):
  
  mass loading is typically smaller than a few tenths of 1%, particles are much smaller than the smallest scale of the flow; multiphase approach is not required

- Precipitation (raindrops, snowflakes, graupel, hail):
  
  mass loading can reach a few %, particles are larger than the smallest scale of the flow; multiphase approach needed only for very-small-scale modeling (e.g., DNS).
In the spirit of the Boussinesq approximation, moisture and condensate affect gas dynamics equations only through the buoyancy term.

\[
\frac{du}{dt} = -\frac{1}{\rho} \nabla p - gk + \ldots \text{(Coriolis, turbulence, etc)}
\]

\[
\rho = \rho_0(z) + \rho'
\]

\[
p = p_0(z) + p'
\]

\[
(\rho_0 + \rho') \frac{dw}{dt} = -\frac{\partial p_0}{\partial z} - \rho_0 g - \frac{\partial p'}{\partial z} - \rho' g + \ldots
\]

For small-Mach number flows (|u| \ll c_s; c_s - speed of sound):

\[
\frac{\rho'}{\rho_0} \approx -\frac{T'}{T_0}
\]

\[
\frac{du}{dt} = -\frac{1}{\rho_0} \nabla p' - gk \frac{T'}{T_0} + \ldots
\]
Density temperature $T_d$: the temperature dry air has to have to yield the same density as moist cloudy air

$$T_d = T \frac{1 + q/\epsilon}{1 + q + Q}$$

$T$ - air temperature

$q$ - water vapor mixing ratio ($\sim 10^{-3}$)

$Q$ - condensate mixing ratio (cloud water, rain, ice, snow, etc.; $\sim 10^{-3}$)

$$\epsilon = \frac{R_d}{R_v} \approx 0.622$$

$$T_d \approx T \left[ 1 + \left( \frac{1}{\epsilon} - 1 \right) q - Q \right]$$

$$T_d \approx T (1 + 0.61q - Q)$$
Thermodynamics:

Moist air is treated as a perfect gas

Phase changes lead to the release of latent heat and formation of condensed (liquid or solid) phase of the water substance (cloud droplets, raindrops, ice crystals, snow, etc)

Condensed phase is treated as continuous medium, i.e., described as density (of cloud droplets, raindrops, etc).

In practice, variables most often used to describe water vapor and condensate are not densities, but mixing ratios, i.e., densities normalized by the air density.
\[ \frac{\partial \rho_a}{\partial t} + \nabla (\rho_a \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_a}{dt} + \rho_a \nabla \mathbf{u} = 0 \]

\[ \frac{\partial \rho_v}{\partial t} + \nabla (\rho_v \mathbf{u}) = 0 \quad \text{or} \quad \frac{d\rho_v}{dt} + \rho_v \nabla \mathbf{u} = 0 \]

mixing ratio: \[ Q = \frac{\rho_v}{\rho_a} \]

\[ \frac{dQ}{dt} = \frac{1}{\rho_a} \frac{d\rho_v}{dt} - \frac{\rho_v}{\rho_a^2} \frac{d\rho_a}{dt} \equiv 0 \]
First Law of Thermodynamics:

\[ dq = du + p \, dv \]  

\( dq \) - heat (per unit mass) added to the system
\( du \) - increase of internal energy (per unit mass)
\( p \, dv \) - work (per unit mass) performed by the system

\[ du = c_v \, dT, \quad p v = R T, \quad v = 1/\rho, \quad c_v + R = c_p \]

\[ dq = c_p \, dT - \frac{RT}{p} \, dp \]

Introducing potential temperature as:

\[ \theta = T \left( \frac{p_{oo}}{p} \right)^{R/c_p} \]

where \( p_{oo} = \text{const (typically 1000 mb)} \), (1) can be written as:

\[ d\theta = \frac{\theta}{c_p T} \, dq \]
\[ \frac{d\theta}{dt} = \frac{\theta}{c_p T} S \]

where \( S = \frac{dq}{dt} \) in the heat source per unit mass [in J kg\(^{-1}\) s\(^{-1}\)]

\( S = 0 \) - adiabatic motions

\( S \neq 0 \) - motions with diabatic processes (heating due to radiative transfer, phases changes, chemical reactions, etc)

For phase changes of water substance:

\[ S = L \frac{dQ}{dt} \]

where \( L \) is the latent heat (of condensation, freezing, or sublimation), and \( \frac{dQ}{dt} \) is the change of corresponding water mixing ratio
BULK MODEL OF CONDENSATION:

\[
\frac{d\theta}{dt} = \frac{L_v}{c_p T} \frac{C_d}{\theta}
\]

\[
\frac{dq_v}{dt} = -C_d
\]

\[
\frac{dq_c}{dt} = C_d
\]

\(\theta\) - potential temperature

\(q_v\) - water vapor mixing ratio

\(q_c\) - cloud water mixing ratio

\(L_v\) - latent heat of condensation/evaporation

\(C_d\) - condensation rate

Note: \(\theta/T\) function of pressure only \((\approx \theta_o/T_o)\)

\(C_d\) is defined such that cloud is always at saturation, which is a very good approximation:

\[q_c = 0 \quad \text{if} \quad q_v < q_{vs}\]

\[q_c > 0 \quad \text{only if} \quad q_v = q_{vs}\]

where \(q_{vs}(p, T) \approx 0.622 \frac{e_s(T)}{p}\) is the water vapor mixing ratio at saturation
WARM RAIN BULK MODEL (Kessler 1969):

\[
\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} (C_d - EV AP)
\]

\[
\frac{dq_v}{dt} = -C_d + EV AP
\]

\[
\frac{dq_c}{dt} = C_d - AUT - ACC
\]

\[
\frac{dq_r}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho q_r v_t) + AUT + ACC - EV AP
\]

\(\theta\) - potential temperature
\(q_v\) - water vapor mixing ratio
\(q_c\) - cloud water mixing ratio
\(q_r\) - rain water mixing ratio
\(C_d\) - condensation rate
\(EV AP\) - rain evaporation rate
\(AUT\) - “autoconversion” rate: \(q_c \rightarrow q_r\)
\(ACC\) - accretion rate: \(q_c, q_r \rightarrow q_r\)

\(v_t(q_r)\) - rain terminal velocity (typically derived by assuming a drop size distribution; e.g., the Marshall-Palmer distribution \(N(D) = N_o exp(-\Lambda D), N_o = 10^7 \text{ m}^{-4}\)).
BIN-RESOLVING WARM RAIN MODEL:

\[
\frac{d\theta}{dt} = \frac{L_v}{c_p T} \sum_{i=1}^{N} C_d^{(i)}
\]

\[
\frac{dq_v}{dt} = -\sum_{i=1}^{N} C_d^{(i)}
\]

for \( i = 1, N \):

\[
\frac{dq_{c}^{(i)}}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho q_{c}^{(i)} v_t(r^{(i)}) \right] + \sum_{i=1}^{N} C_d^{(i)} + F_+^{(i)} - F_-^{(i)}
\]

\( \theta \) - potential temperature
\( q_v \) - water vapor mixing ratio
\( q_c^{(i)} \) - cloud water mixing ratio for drops in size bin \( i \) \( (i = 1,N, N \sim 100) \)

\( C_d^{(i)} \) - condensation/evaporation rate for drops in size bin \( i \); depends on super/undersaturation \( S = q_v/q_{vs} - 1 \) and drop size \( r^{(i)} \).

\( F_+^{(i)} \) - source due to collisions between \( j \) and \( k \) resulting in drops in \( i \)
\( F_-^{(i)} \) - sink due to collisions between \( i \) and all other bins
Modeling of indirect effects in Stratocumulus

Stevens et al., 2006, MWR
Kinematic (prescribed-flow) model of microphysical processes in Stratocumulus (2D: x-z)

Vertical velocity
- Run up to quasi-steady-state is obtained (typically couple hours)…

Horizontal velocity
Cloud water (after 3hrs)

Maritime (clean)                          Continental (polluted)
Supersaturation (after 3hrs)

Maritime (clean)  Continental (polluted)
Cloud droplet \((r < 20\) microns) number concentration

Maritime (clean)

Continental (polluted)
Cloud droplet ($r < 20$ microns) mean radius

Maritime (clean)  

Continental (polluted)
Drizzle ($r > 25$ microns) mixing ratio

Maritime (clean)                  Continental (polluted)
Drizzle ($r > 25$ microns) mean radius

Maritime (clean)                                           Continental (polluted)
Comments:

-Bulk warm rain microphysics is a relatively straightforward and computationally efficient approach (just 2 variables for the condensed water);

-Problems begin for shallow clouds when microphysical details decide whether precipitation develops or not (e.g., stratocumulus, shallow convection);

-When coupled to the radiative transfer, information about cloud droplet size is needed; bulk warm rain model is not able to provide this;

-Detailed (bin-resolved) microphysics solves the above two problems, but it is very expensive (~100 extra variables) and still leaves some issues (discussed later);

-A reasonable compromise is to predict both the mass and the number of particles (thus, 4 variables used for the condensed water); “two-moment bulk microphysics schemes”.
p-T phase equilibrium diagram for water substance
Ice processes:

Three main problems:
Ice processes:

Three main problems: ice initiation, ice initiation, and ice initiation;
(primary, secondary)
Ice processes:

Three main problems: ice initiation, ice initiation, and ice initiation;
(primary, secondary)

Equilibrium approach possible only in special cases (e.g., deep convection), questionable in shallow clouds (like arctic Scu);

Unlike warm-rain microphysics, where cloud droplets and rain/drizzle drops are well separated in the radius space, growth of ice phase is continuous in size/mass space. Both diffusional and acretional growth important;

Complexity of ice crystal shapes (“habits”).
Equilibrium approach  
(simple extension of a warm-rain scheme)

**Simple Bulk Ice Model (Grabowski 1998):**

\[
\frac{d\theta}{dt} = \frac{L_v \theta}{c_p T} (\text{COND} - \text{DIFF})
\]

\[
\frac{dq_v}{dt} = -\text{COND} + \text{DIFF}
\]

\[
\frac{dq_c}{dt} = \text{COND} - \text{AUTC} - \text{ACCR}
\]

\[
\frac{dq_p}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho q_p u_t) + \text{AUTC} + \text{ACCR} - \text{DIFF}
\]

\(\theta\) - potential temperature  
\(q_v\) - water vapor mixing ratio  
\(q_c\) - cloud condensate (water or ice) mixing ratio  
\(q_p\) - precipitation water (rain or snow) mixing ratio  
\(\text{COND}\) - condensation rate (saturation adjustment)  
\(\text{DIFF}\) - diffusional growth rate  
\(\text{AUTC}\) - “autoconversion” rate: \(q_c \rightarrow q_p\)  
\(\text{ACCR}\) - accretion rate: \(q_c, q_p \rightarrow q_p\)  

saturation: \(q_{vs} = \alpha q_{vw} + (1 - \alpha) q_{vi}\)  
cloud water: \(q_w = \alpha q_c\); cloud ice: \(q_i = (1 - \alpha) q_c\)  
rain: \(q_r = \alpha q_p\); snow: \(q_s = (1 - \alpha) q_p\)  
\(\text{DIFF} = \text{DIFF}_r + \text{DIFF}_s\)  
\(\text{AUTC} = \text{AUTC}_r + \text{AUTC}_s\)  
\(\text{ACCR} = \text{ACCR}_r + \text{ACCR}_s\)  
\(u_t = \alpha u_t(q_r) + (1 - \alpha) u_t(q_s)\)
BULK MODEL WITH ICE MICROPHYSICS:

- potential temperature $\theta$:

$$
\frac{d\theta}{dt} = \frac{L_v \theta_e}{c_p T_e} S_1 + \frac{L_s \theta_e}{c_p T_e} S_2 + \frac{L_f \theta_e}{c_p T_e} S_3
$$

- water vapor mixing ratio $q_v$:

$$
\frac{dq_v}{dt} = S_v
$$

- cloud condensate variables $q_c^i$, $i = 1, N_c$ (typically, $N_c = 2$: cloud water, cloud ice):

$$
\frac{dq_c^i}{dt} = S_c^i
$$

- precipitating water variables $q_p^i$, $i = 1, N_p$:
  (typically, $N_p = 3$: rain, snow, graupel/hail):

$$
\frac{dq_p^i}{dt} = \frac{1}{\rho} \frac{\partial}{\partial z} (\rho q_p^i v_i^z) + S_p^i
$$

$S$ – various sources/sinks due to phase changes.
FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.
Figs. 7. The earth's annual global mean energy budget based on the present study. Units are W m$^{-2}$. 
Radiative-convective quasi-equilibrium mimicking planetary energy budget using a cloud-resolving model

- Solar input: 342 Wm$^{-2}$
- Surface temperature = 15° C
- Surface relative humidity = 80%
- Surface albedo = 0.15
Numerical model:

- Dynamics: 2D super-parameterization model (Grabowski 2001) with simple bulk microphysics (warm-rain plus ice; Grabowski 1998)

- Radiation: NCAR’s Community Climate System Model (CCSM) (Kiehl et al 1994) in the Independent Column Approximation (ICA) mode

- 100 columns ($\Delta x=2$ km) and 61 levels (stretched; 12 levels below 2 km; top at 24 km)
How do dynamic and radiation transfer models communicate?

- Dynamics provides profiles of temperature, moisture, and cloud properties in each model column (condensate mixing ratios, optical properties of cloud particles - effective radii\(^1\) of cloud droplets and ice crystals in particular)

- Radiation calculates radiative fluxes based on solar input, surface characteristics, and properties within the column; divergence of solar and longwave fluxes are fed to the dynamics

\(^1\) – effective radius is the ratio between the third and the second moments of droplet/ice crystal size distribution
Effective radius for ice particles...

This formula is used in all simulations (i.e., no indirect impact on ice particles).
Effective radius for water droplets...

\[ r_{\text{eff}} \sim r_v \quad \text{(e.g., Martin et al. JAS 1994)} \]

\[ r_v \equiv \langle r^3 \rangle^{1/3} - \text{mean volume radius;} \]

\[ \rightarrow \text{cloud water } q_c \sim N r_v^3 \]

"bulk model provides } } q_c; \text{ how to estimate } r_v ?}
Shallow convective clouds are strongly diluted by entrainment

Siebesma et al. JAS 2003
Bulk mixing between cloudy and cloud-free air (adiabatic, isobaric)

What is wrong with this picture?

t – temperature
q – water vapor mixing ratio
l – cloud water mixing ratio
Homogeneous mixing:
all droplets are exposed to the same conditions during mixing

Extremely inhomogeneous mixing:
some droplets evaporate completely, the rest does not change size at all
Simulations of decaying moist turbulence (final stages of cloud entrainment)

Andrejczuk et al., JAS, 2004

T=293 K (both cloud and clear air)

RH=65%

$q_c = 3.2$ g/kg

(filaments neutrally buoyant)

Velocity scale for low TKE (x10 for high TKE)
Evolution of the cloud water

High TKE

Moderate TKE

Low TKE
Laboratory experiment

Model simulation

60 cm

60 cm
Microphysical transformations during cloud-clear air mixing (Andrejczuk et al. JAS 2004)

HOMOGENEOUS MIXING

EXTREMELY INHOMOGENEOUS MIXING

r-N diagram
prescribe number of droplets:

\[ r_v^3 \sim \frac{q_c}{N} \]

*HOMOGENEOUS MIXING*

prescribe size of droplets:

\[ r_v = r_{ad}(z) \]

*EXTREMELY INHOMOGENEOUS MIXING*
1\textsuperscript{st} and 2\textsuperscript{nd} indirect effects of water clouds in a cloud-resolving model:

<table>
<thead>
<tr>
<th></th>
<th>“pristine”</th>
<th>“polluted”</th>
</tr>
</thead>
<tbody>
<tr>
<td>cloud droplet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>concentration</td>
<td>100</td>
<td>1,000</td>
</tr>
<tr>
<td>(cm\textsuperscript{-3})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1\textsuperscript{st} effect: a simple parameterization of the effective radius

2\textsuperscript{nd} effect: Berry’s parameterization of the conversion from cloud water to rain
Model simulations:

- 100 per cc versus 1,000 per cc
- Mixing:
  - homogeneous (h)
  - extremely inhomogeneous (ei)
  - between “h” and “ei” (the same changes in N and $r^3$ (rN))

Each simulation run for 120 days, results averaged over the last 60 days
Homogeneous mixing (droplet concentration the same everywhere...)
<table>
<thead>
<tr>
<th></th>
<th>PRISTINE 100 per cc</th>
<th></th>
<th>POLLUTED 1,000 per cc</th>
<th></th>
<th>KT97</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h</td>
<td>rN</td>
<td>ei</td>
<td>h</td>
<td>rN</td>
</tr>
<tr>
<td>TOA albedo</td>
<td>0.35</td>
<td>0.31</td>
<td>0.29</td>
<td>0.41</td>
<td>0.38</td>
</tr>
<tr>
<td>OLR (W m(^{-2}))</td>
<td>242</td>
<td>242</td>
<td>243</td>
<td>240</td>
<td>241</td>
</tr>
<tr>
<td>radiative cooling</td>
<td>-101</td>
<td>-100</td>
<td>-100</td>
<td>-101</td>
<td>-100</td>
</tr>
<tr>
<td>of atmosphere (W m(^{-2}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solar energy</td>
<td>163</td>
<td>176</td>
<td>182</td>
<td>141</td>
<td>153</td>
</tr>
<tr>
<td>absorbed at surface (W m(^{-2}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface net</td>
<td>72</td>
<td>73</td>
<td>73</td>
<td>70</td>
<td>71</td>
</tr>
<tr>
<td>longwave flux (W m(^{-2}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface sensible</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>heat flux (W m(^{-2}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface latent</td>
<td>73</td>
<td>73</td>
<td>73</td>
<td>75</td>
<td>74</td>
</tr>
<tr>
<td>heat flux (W m(^{-2}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface energy</td>
<td>-2</td>
<td>10</td>
<td>17</td>
<td>-23</td>
<td>-12</td>
</tr>
<tr>
<td>budget (W m(^{-2}))</td>
<td>(7)</td>
<td>(5)</td>
<td>(5)</td>
<td>(9)</td>
<td>(9)</td>
</tr>
</tbody>
</table>

Table 2: Energy fluxes averaged over 60-day period (days 61-120) for various simulations assuming PRISTINE and POLLUTED cloud conditions. Columns marked “h”, “ei”, and “rN” show results from simulations where, respectively, homogeneous, extremely inhomogeneous, and intermediate mixing scenarios were assumed to prescribe the effective radius of cloud droplets. Values in brackets in the surface energy budget show standard deviations for the 60-day averaging period. Estimates of global mean energy budgets from Kiehl and Trenberth (1997) are shown in KT97 column.
Key results:

1. Water and energy fluxes surprisingly similar to Kiehl & Trenberth (1997)!

2. Insignificant effects on longwave fluxes, surface sensible and latent heat fluxes (no 2\textsuperscript{nd} indirect effect)!

3. Significant impact on solar radiation: changes in “planetary” albedo and surface energy budget:

<table>
<thead>
<tr>
<th></th>
<th>Clean 100 per cc</th>
<th>Polluted 1000 per cc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface energy</td>
<td>h rN ei</td>
<td>h rN ei</td>
</tr>
<tr>
<td>budget (W/m\textsuperscript{2})</td>
<td>-2 10 17</td>
<td>-23 -12 -2</td>
</tr>
</tbody>
</table>

4. The 1\textsuperscript{st} indirect effect (Twomey effect) can be offset by assumptions about mixing!
But…

- dynamic model is 2D,
- microphysics is extremely simple,
- clouds are under-resolved (especially shallow convection),
- radiative transfer is 2D…
Assumptions about spatial variability of effective radius in warm convective clouds seem to impact the surface energy budget at a level comparable to the 1st indirect effect itself.

This conclusion comes from low-spatial resolution 2D model and two-stream independent column radiation transfer model. LES-type simulations applying 3D radiative transfer are needed.

Combination of in-situ aircraft and ground-based remote sensing observations, together with laboratory studies and numerical modeling should provide sufficient data to develop useful parameterizations of microphysical properties of shallow convective clouds.
Why is it so hard to simulate the Earth climate system?

Because some of the key processes are even not on this diagram....
Conclusions:

Cloud microphysics is one of key elements of climate and climate change.

Cloud microphysics is tightly coupled to cloud dynamics; super-parameterization was proposed to provide a better framework for the cloud microphysics in the clouds-in-climate problem.

Lower tropospheric clouds are especially important because of their strong impact on solar radiation reaching the surface and small impact on longwave radiation. Their microphysical properties are critical.