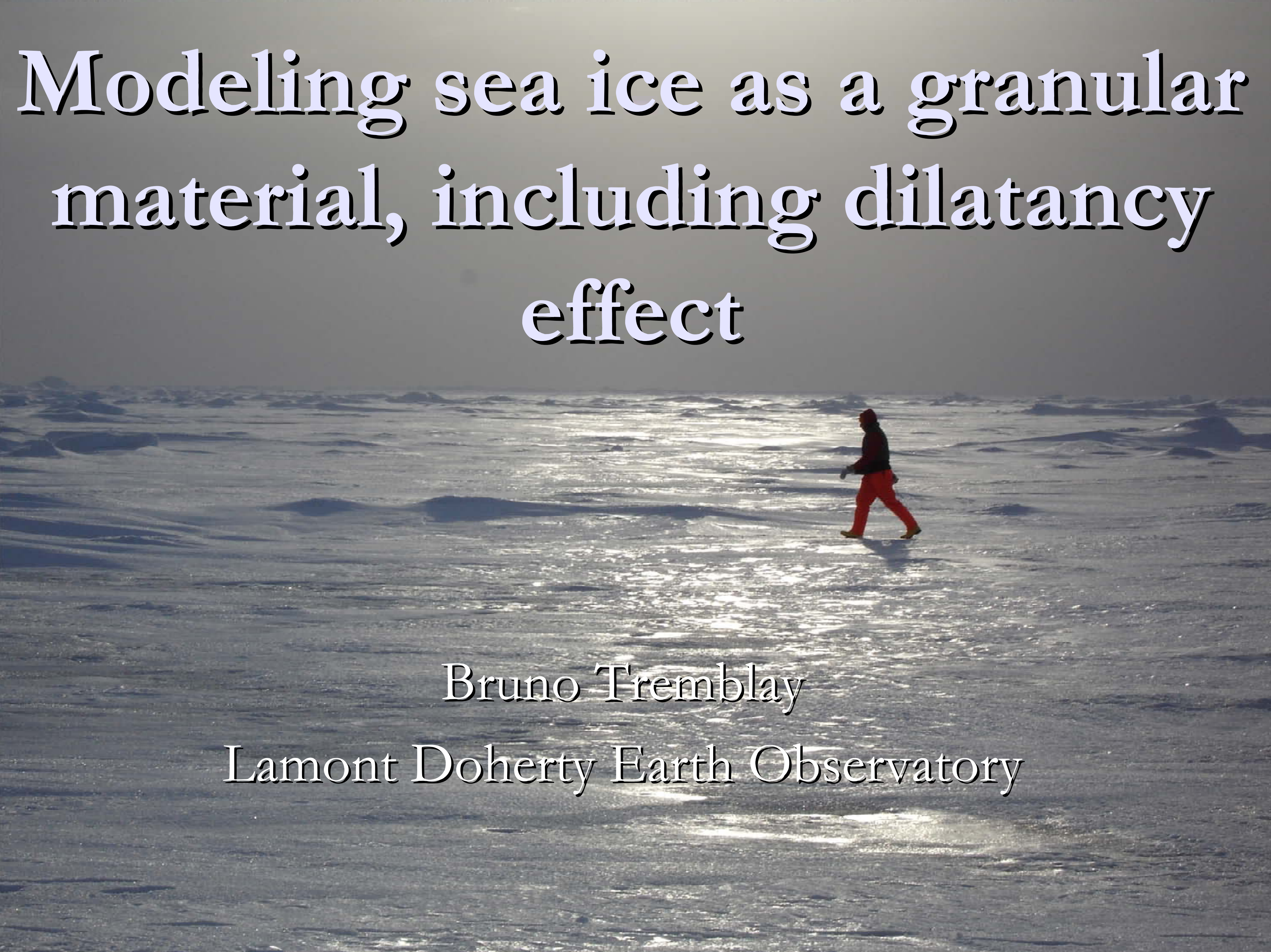
Modeling sea ice as a granular material, including dilatancy effect



Bruno Tremblay

Lamont Doherty Earth Observatory

Outline

- Introduction - Different Approach to Sea Ice Modeling
- Observations
- Granular Model
- Model Against Observations?

- Model Against Observations… continued

Momentum Equation

$$\rho h \frac{du}{dt} = \tau_a + \tau_w - \rho h f k \times u - \rho g h \nabla H + \nabla \cdot \sigma$$

Momentum Equation

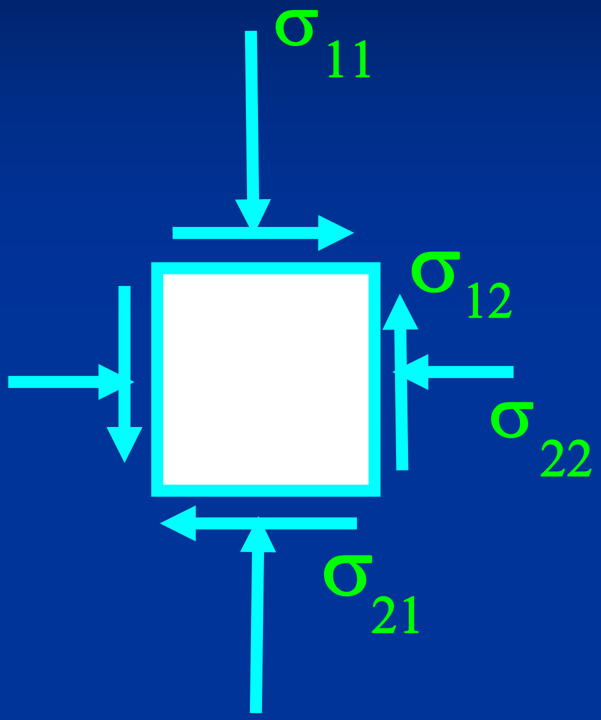
$$\rho h \frac{du}{dt} = \tau_a + \tau_w - \rho h f k \times u - \rho g h \nabla H + \nabla \cdot \sigma$$

Internal ice stress term

Modeling sea ice

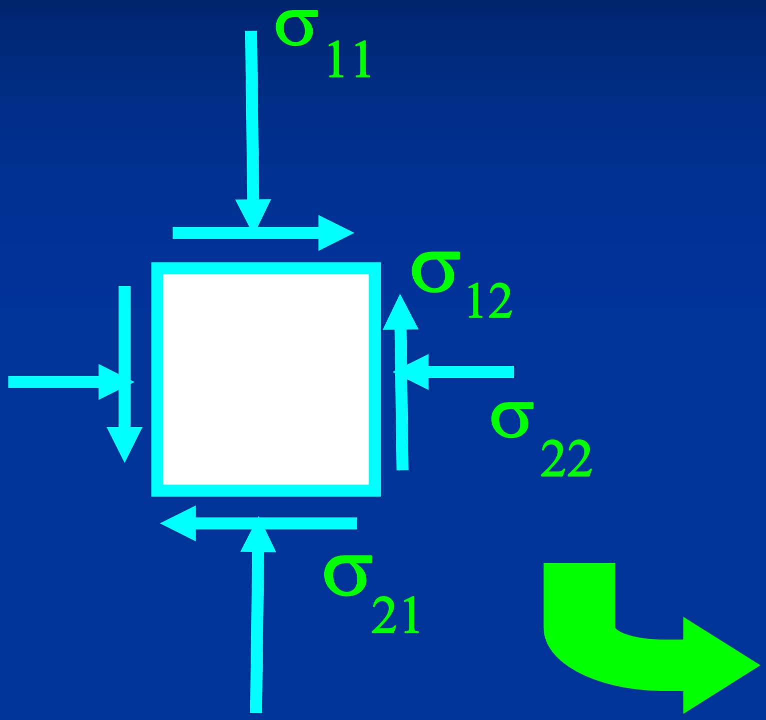
- Deformation
  - Elastic: small, reversible
  - Plastic: large, irreversible
- Elastic-Plastic (EP) [1974]
- Viscous-Plastic (VP) [1979]
- Elastic-Viscous-Plastic (EVP) [1998]
- Granular material model VP [1997]
  - yield curve + flow rule

Stress State



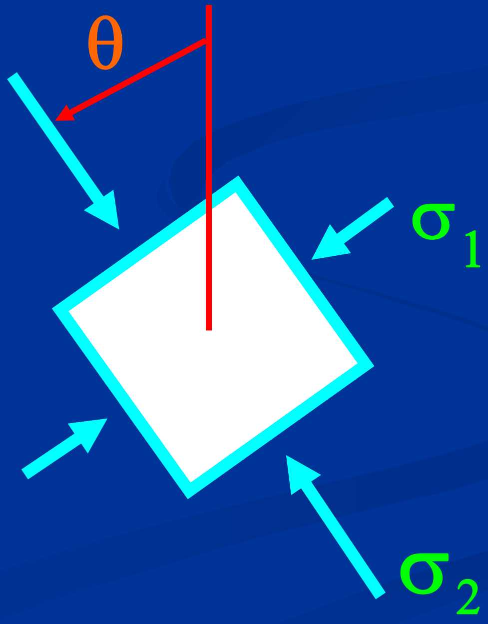
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

Stress State



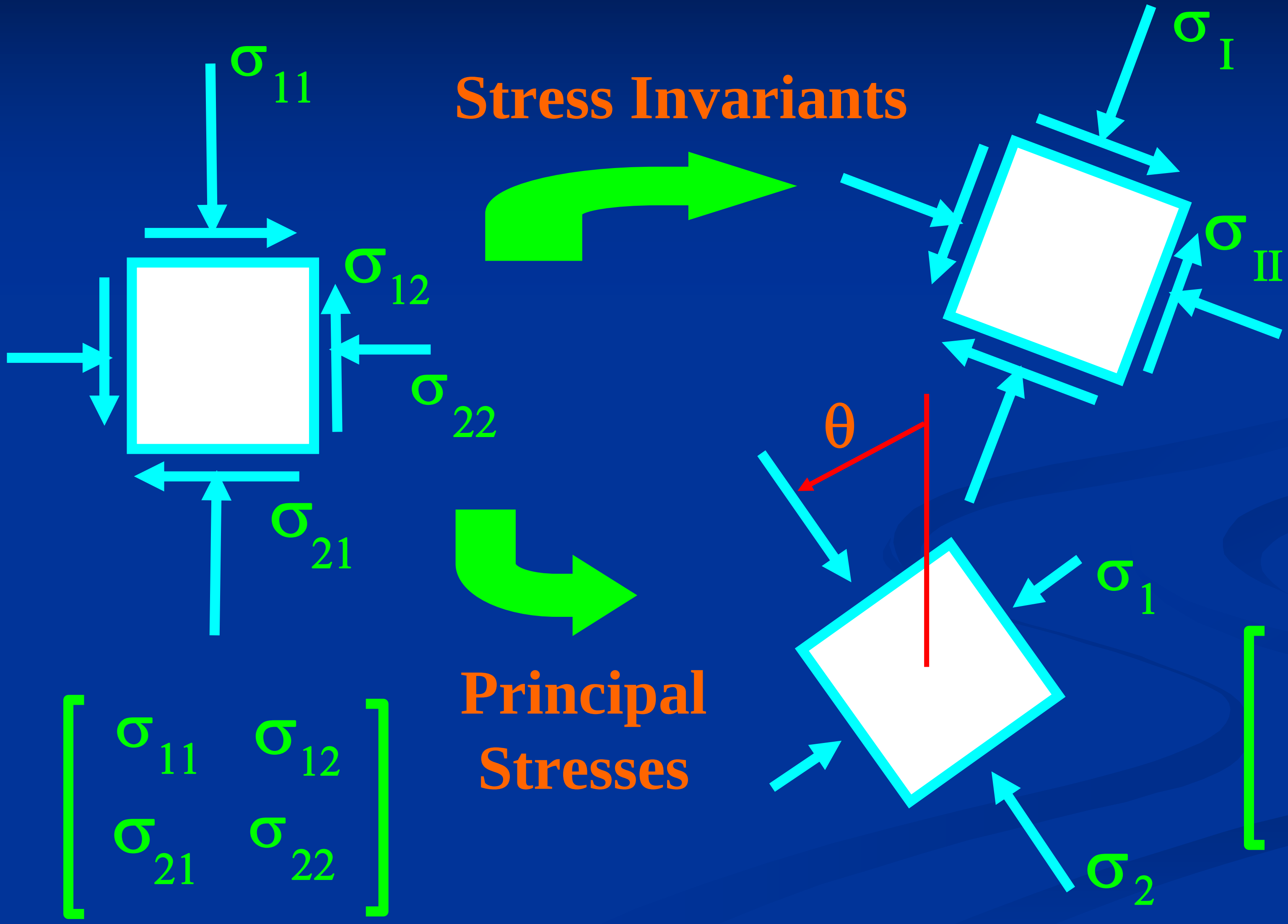
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

Principal Stresses



$$\begin{bmatrix} \sigma_{1} & 0 \\ 0 & \sigma_{2} \end{bmatrix}$$

Stress State



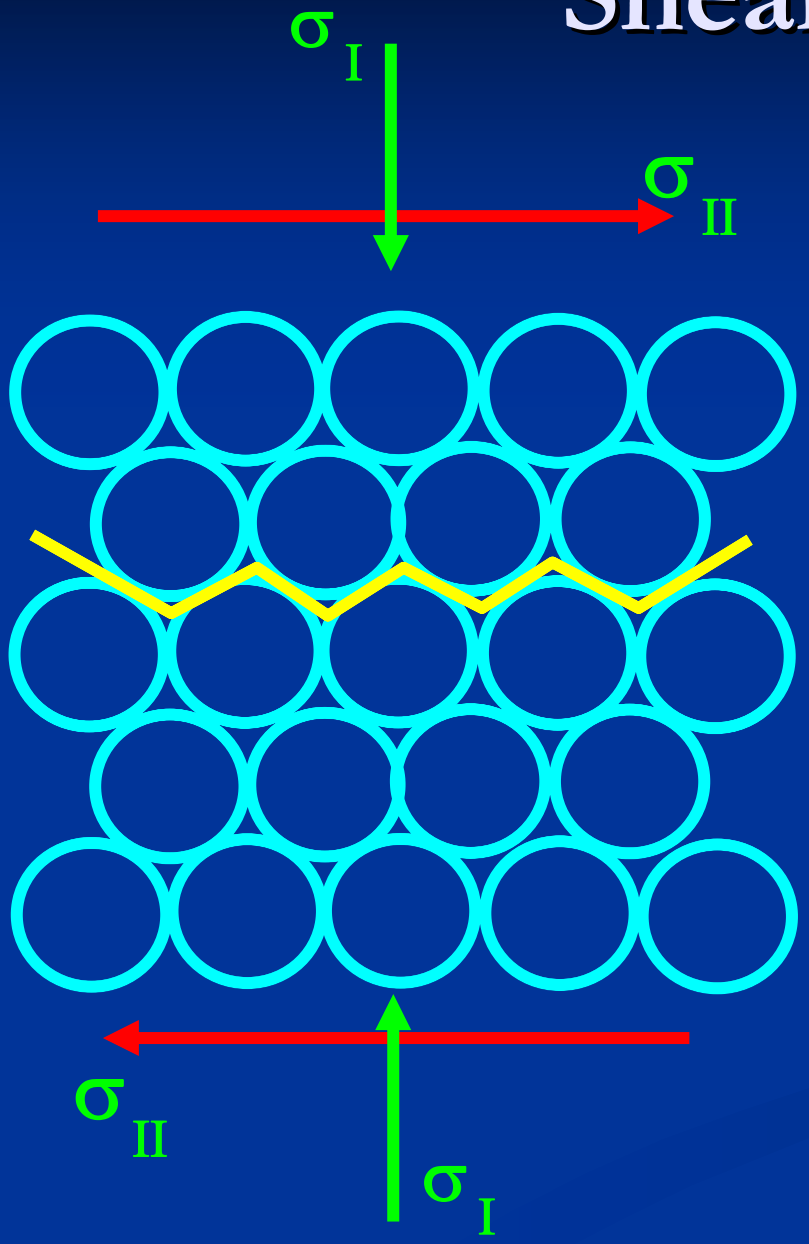
Stress Invariants

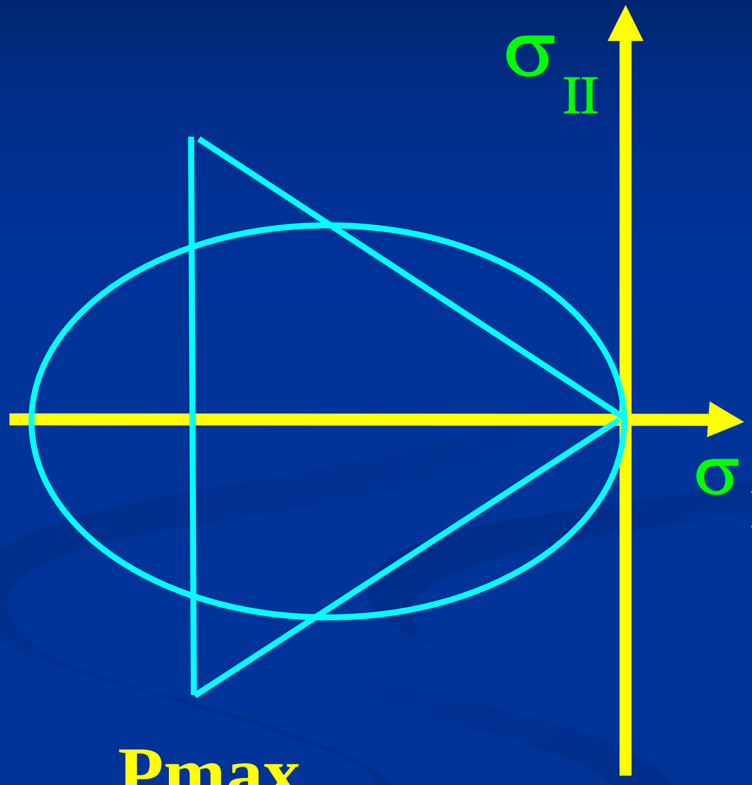
Principal Stresses

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{1} & 0 \\ 0 & \sigma_{2} \end{bmatrix}$$

Shear Test

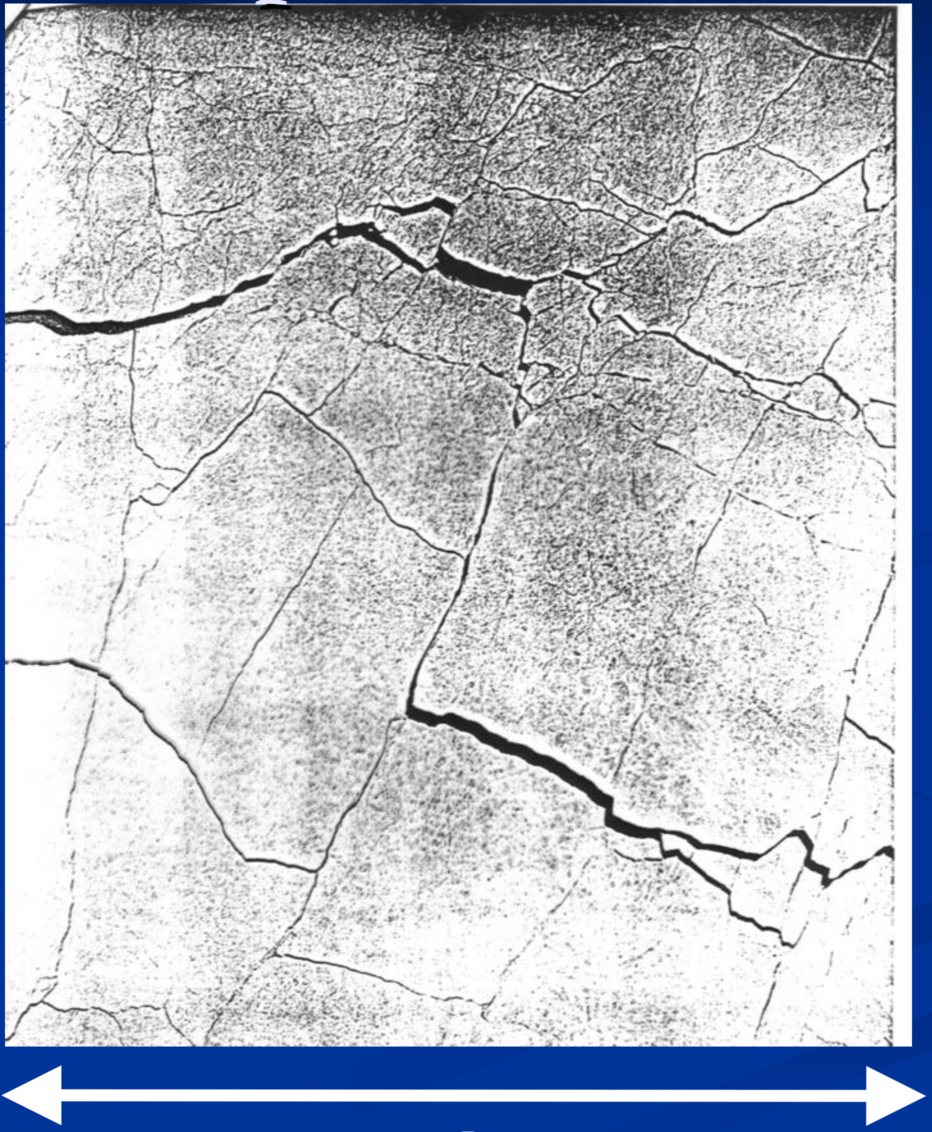




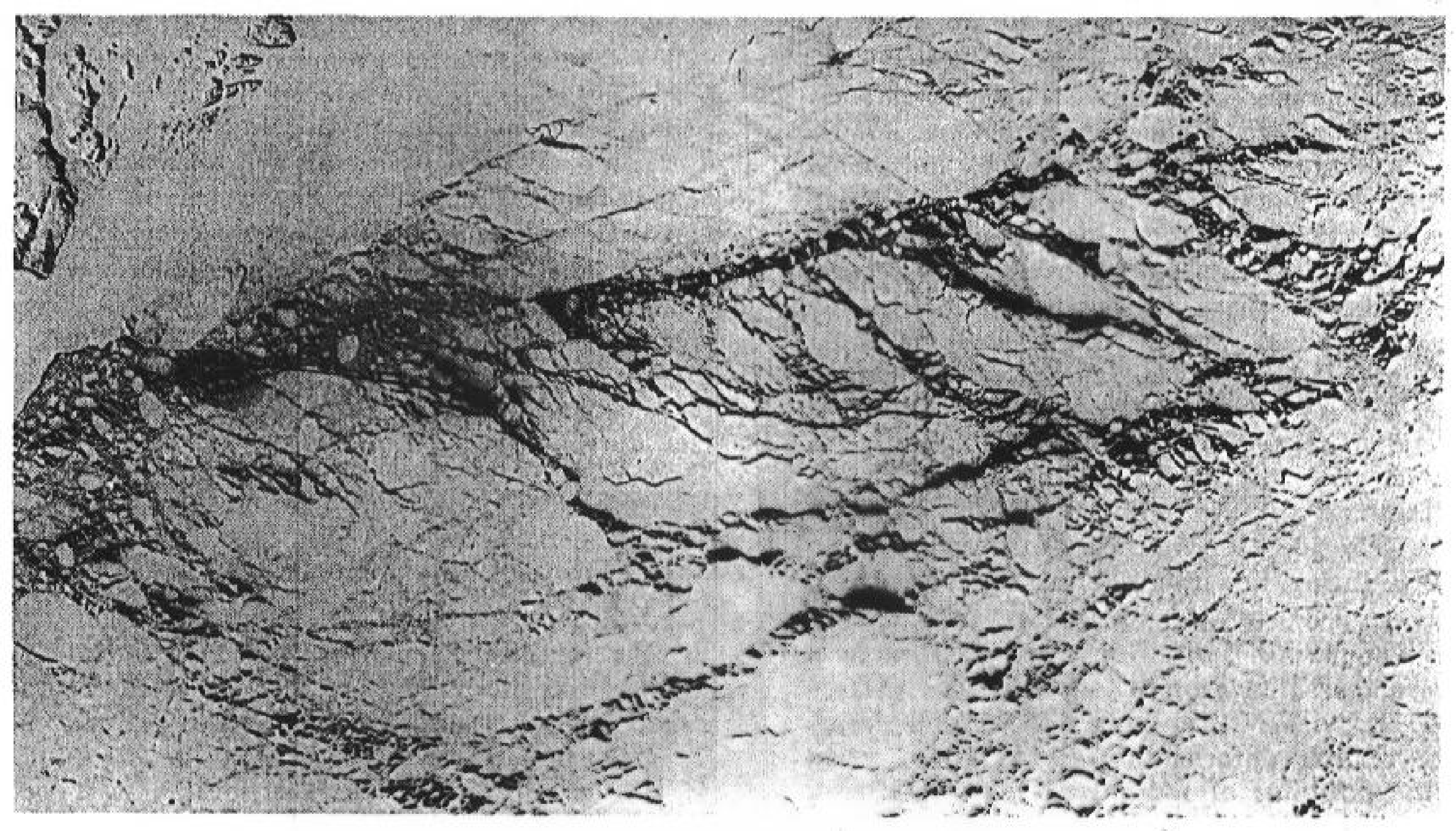
Pmax

Pmax = P* h f(A)

Fracture pattern in sea ice



Fracture pattern in sea ice

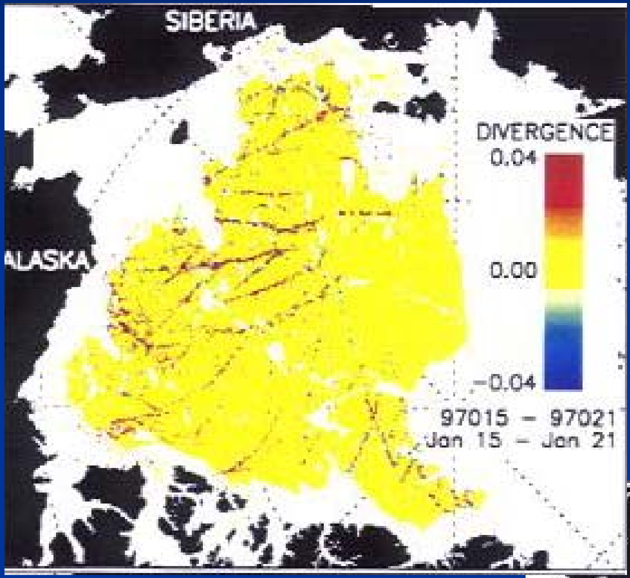


Scale: 200km

Lead pattern north of Greenland

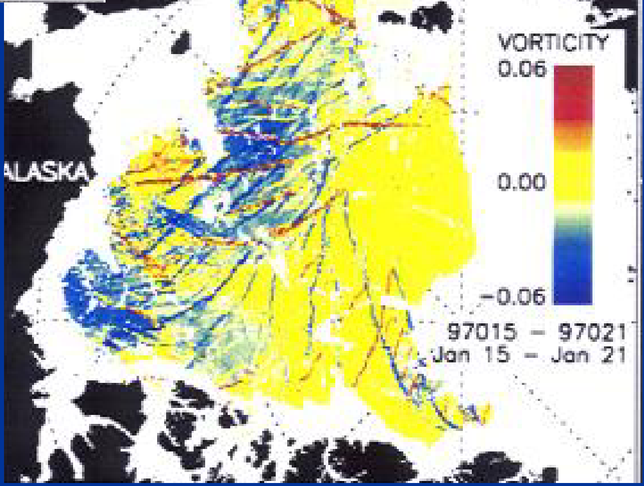
Scale: 500km

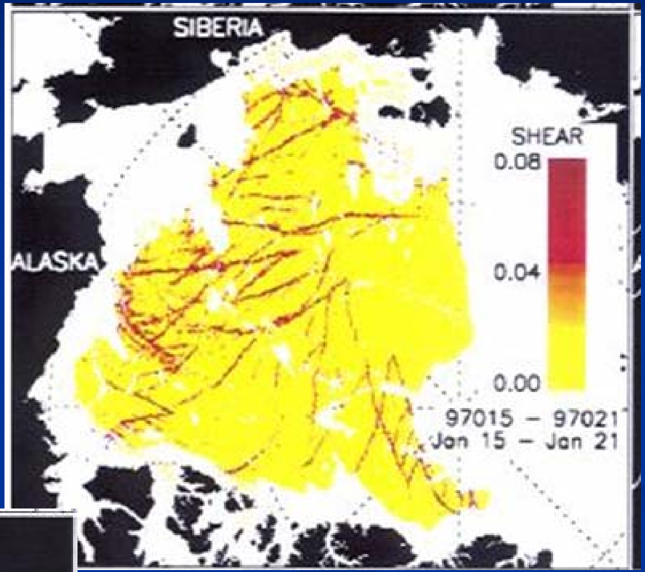
Sea ice deformation



divergence

vorticity

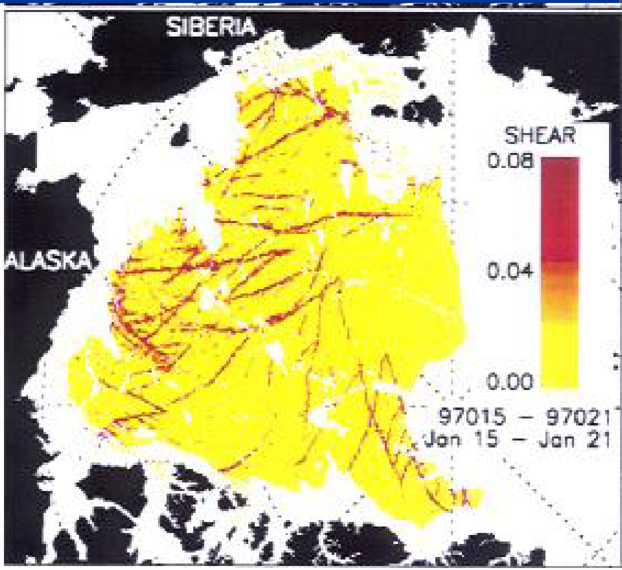


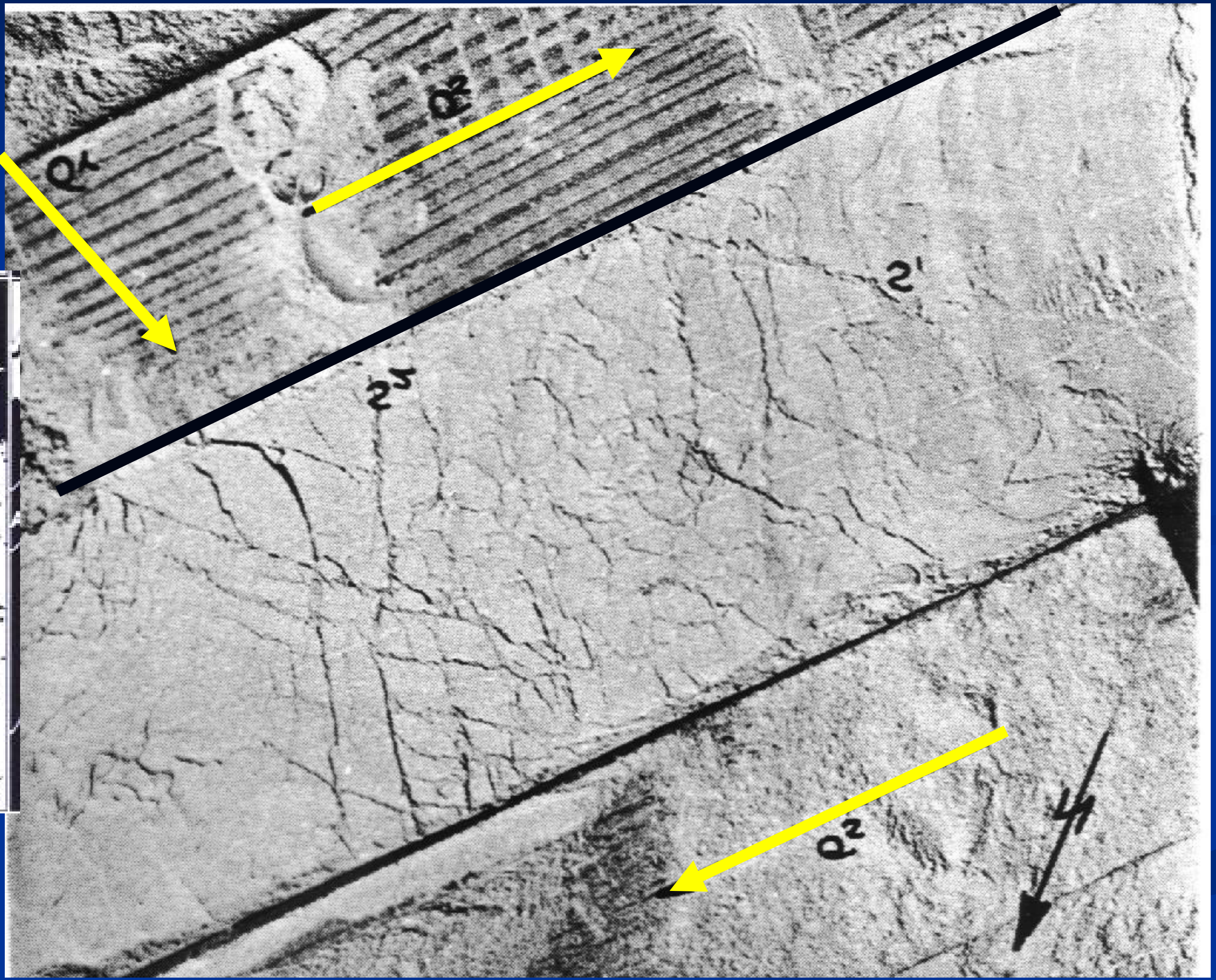


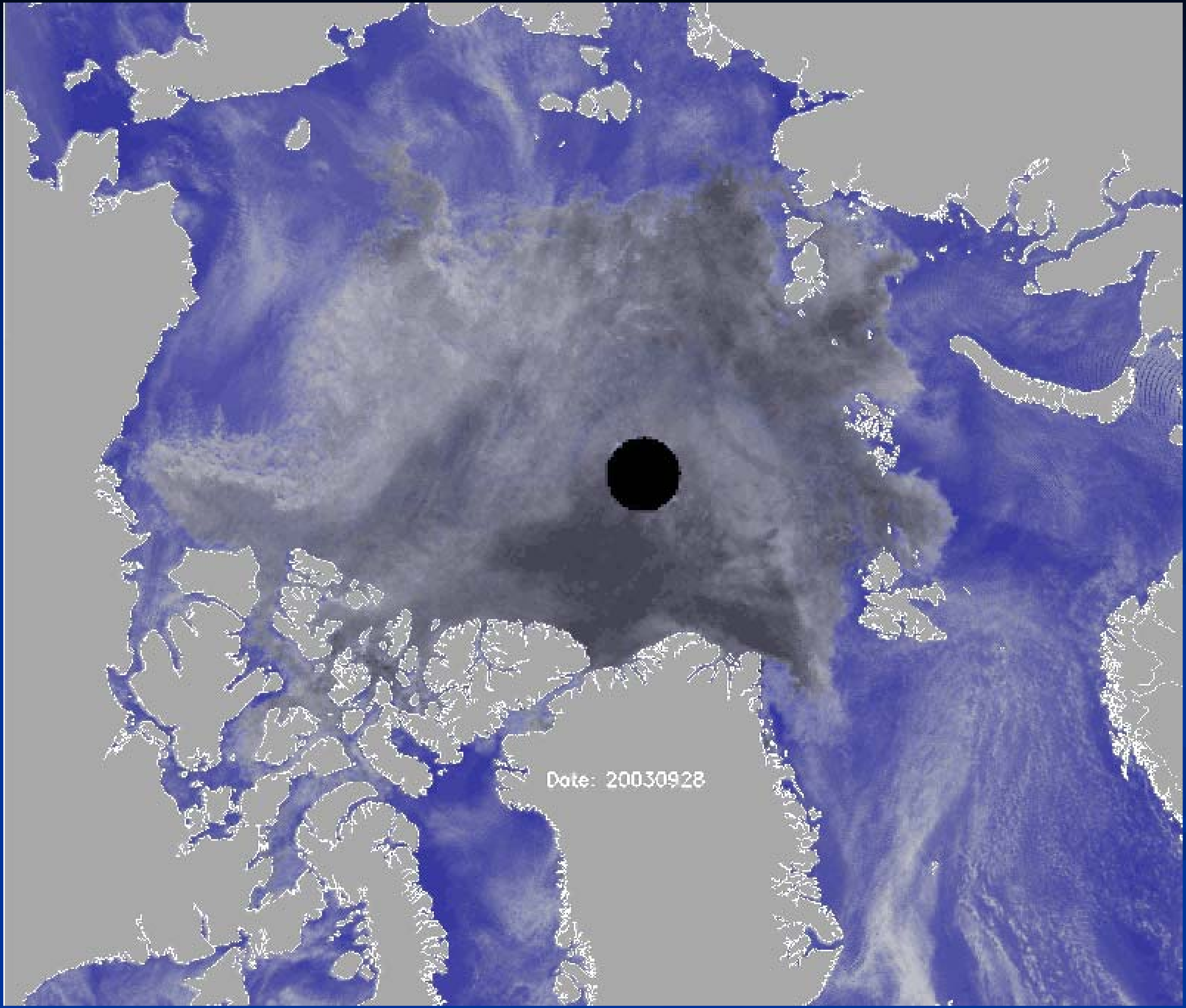
shear

RGPS data, Kwok

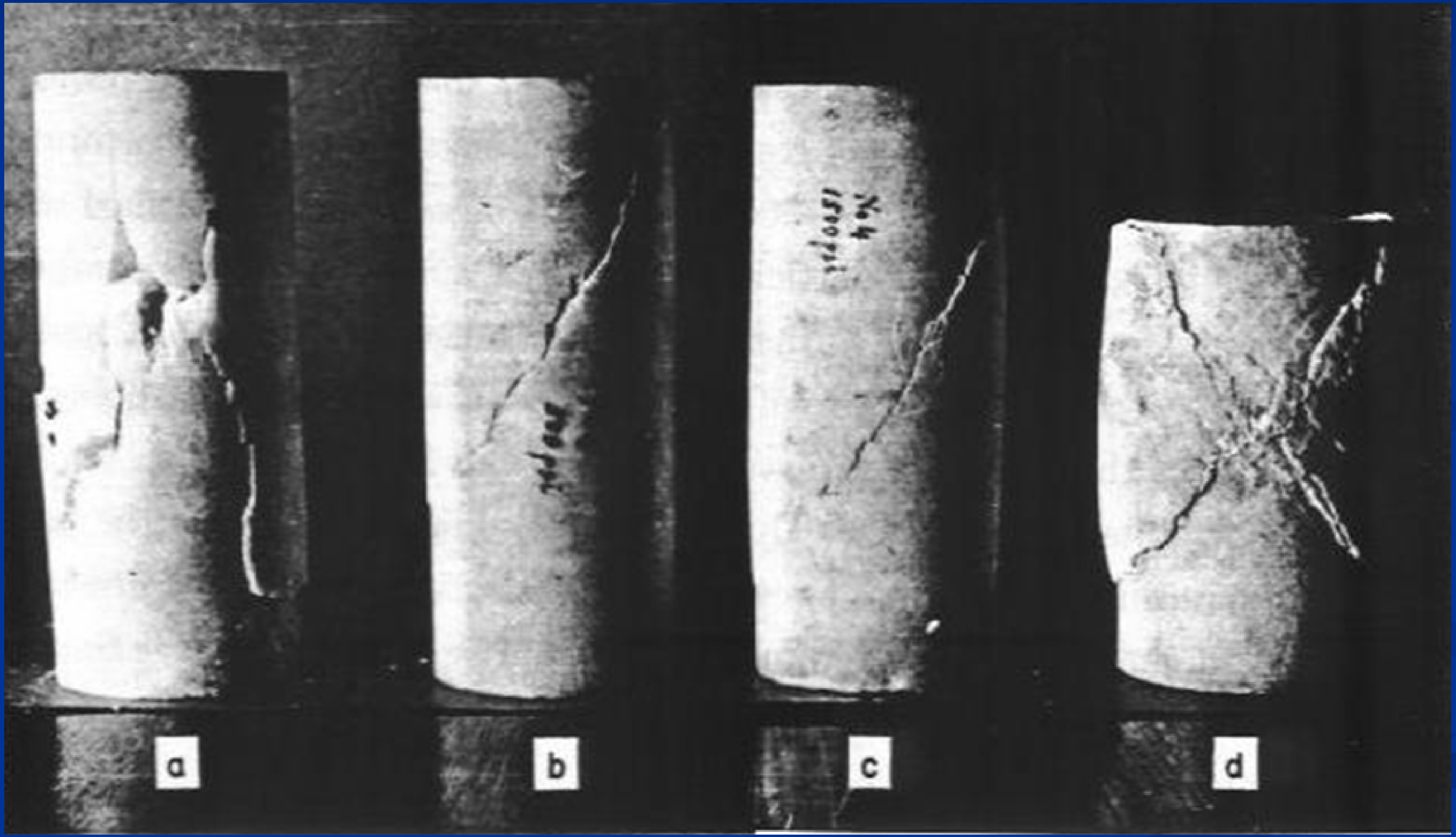
Clay under shear deformation



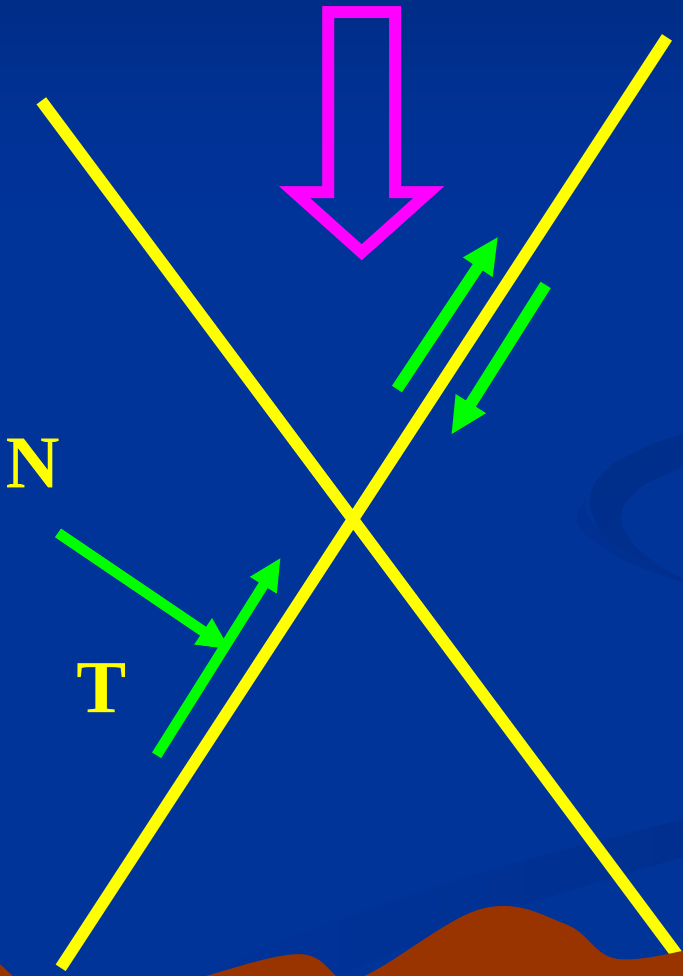



Date: 20030928

Compression test on concrete

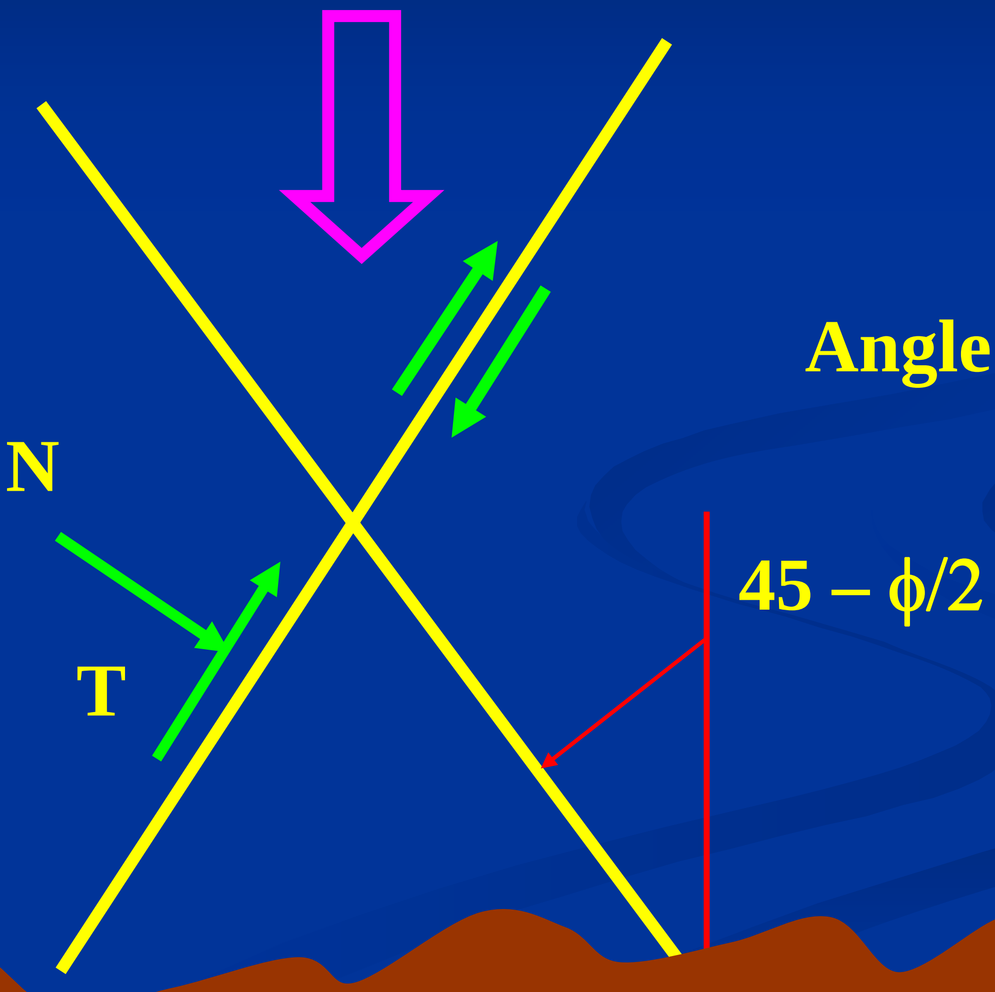


The Model



T = μ N

The Model

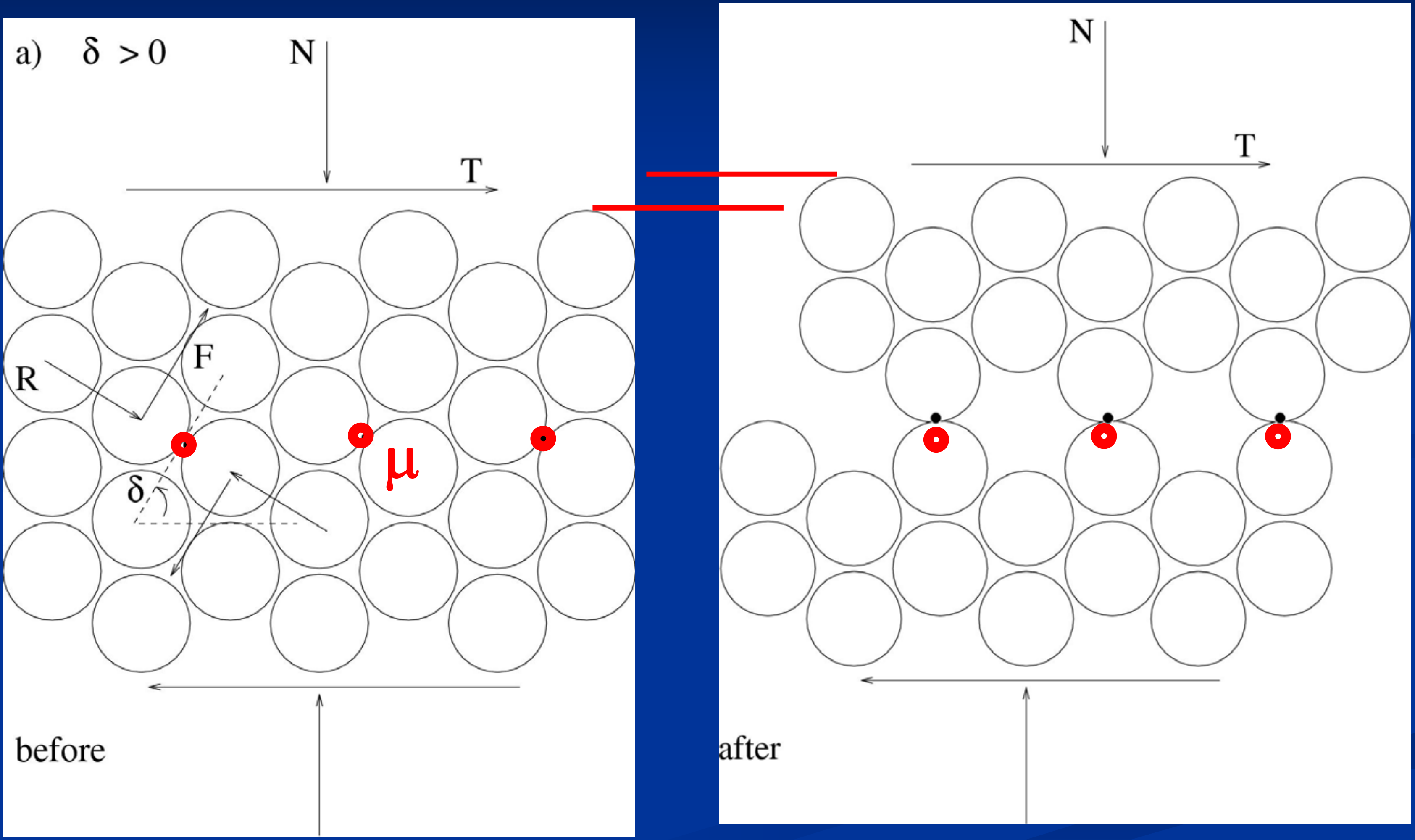


Angle of friction ϕ

Summary

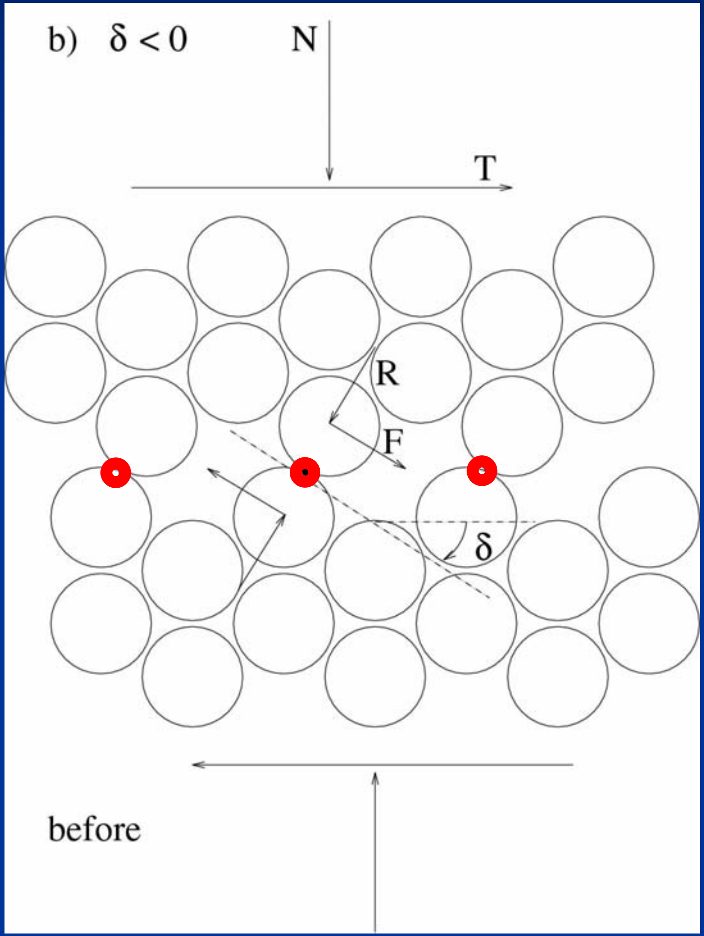
- You identify fracture patterns from satellite pictures
- You calculate the angle of friction from the fracture patterns
- You stick that into the model
- … and then the model tends to produce to fast ice drifts…

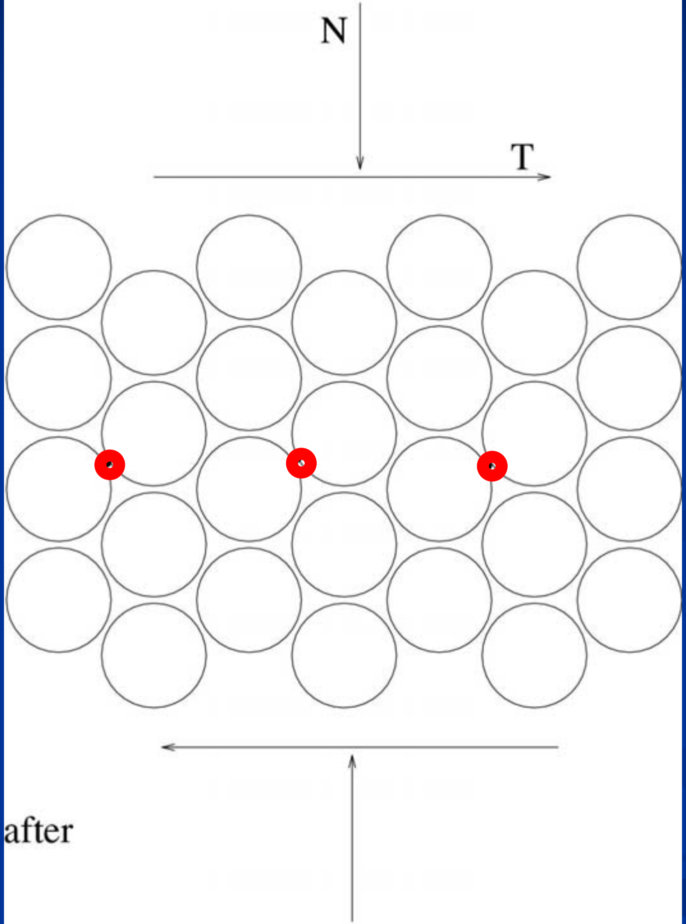
Dilatation δ



$$\phi = \delta + \mu$$

Dilatation





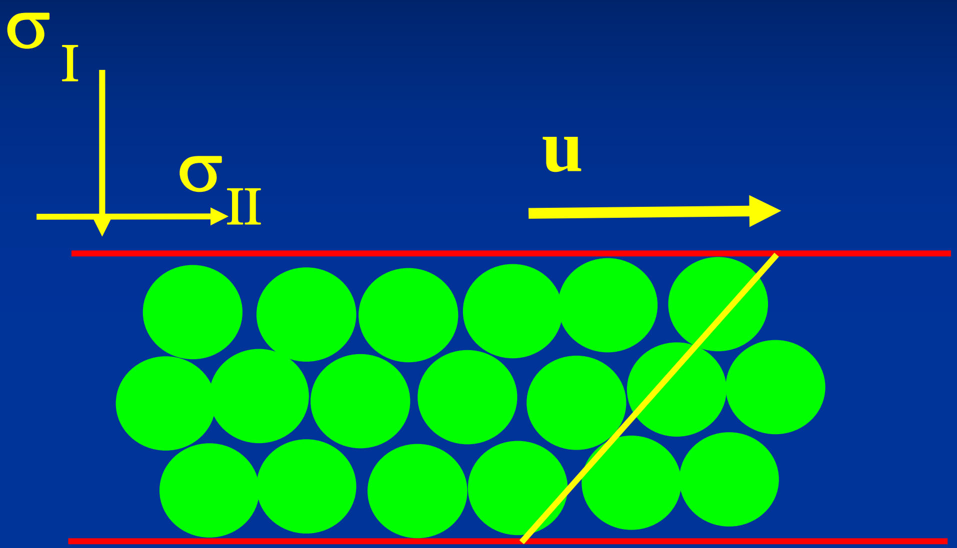
Rheology

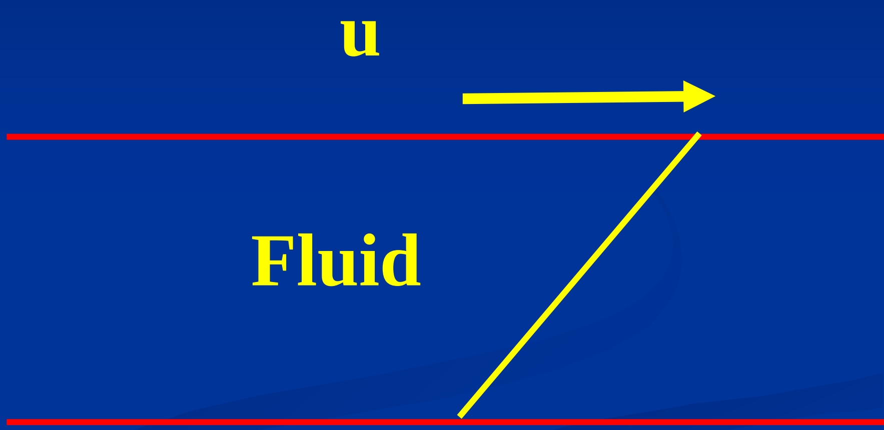
$$\sigma_{ij} = -p\delta_{ij} - \eta\dot{\epsilon}_{kk}\delta_{ij} + 2\eta\dot{\epsilon}_{i,j},$$

$$\eta = \min\left(\frac{p\,\sin\phi}{\sqrt{(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2}}, \eta_{\max}\right).$$

0 < p < P* h exp[-C (1-A)]

Or… in 1-D





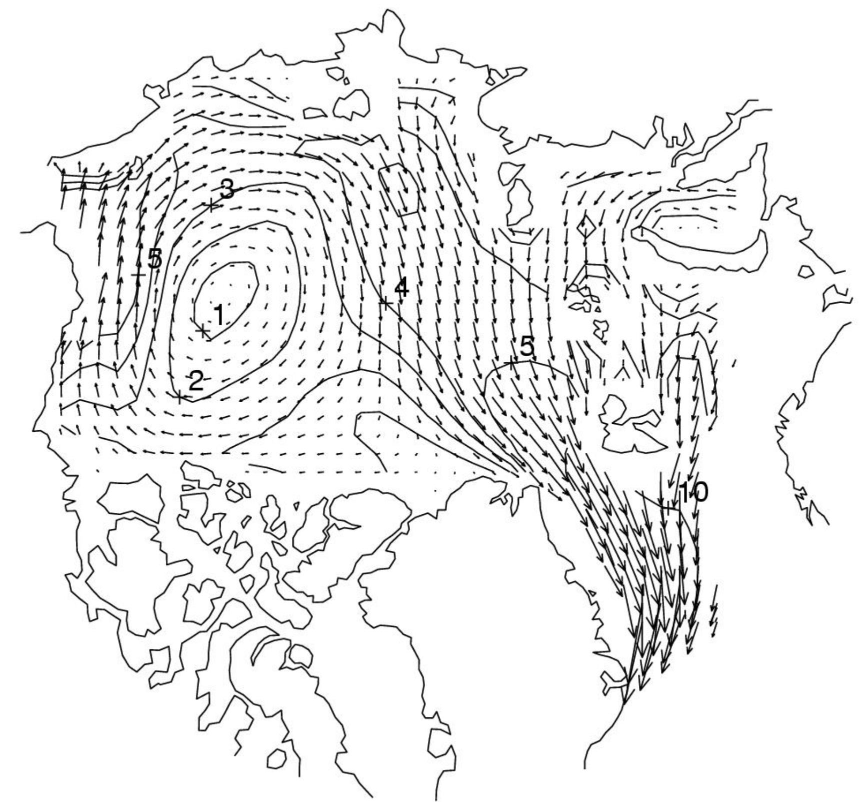
$$\sigma_{II} = \frac{\mu \quad \sigma_I}{du/dy} \quad du/dy$$

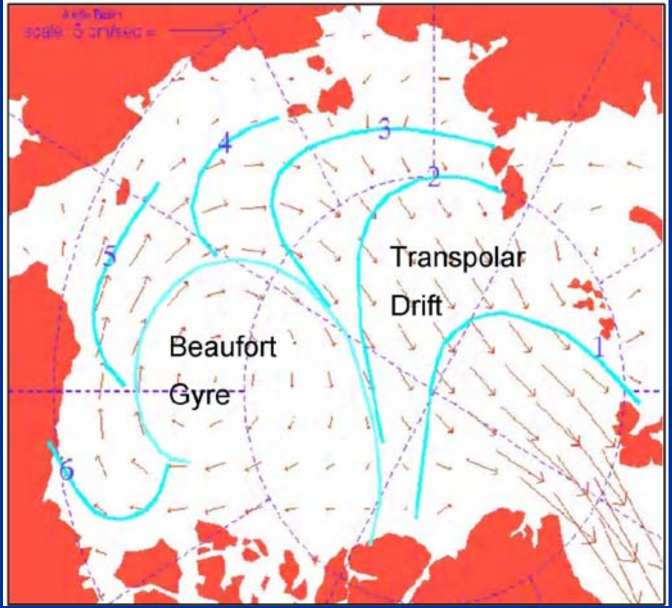
$$\tau = \mu \quad du/dy$$

Comparison with data?

- Sea ice drift
- Sea ice thickness/extent
- Shear lines

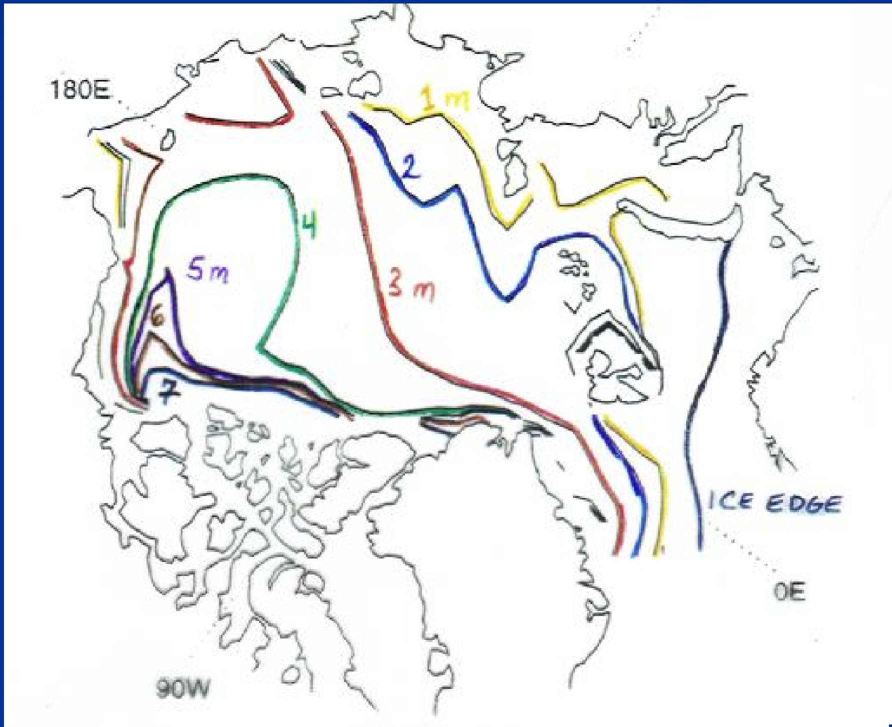
Mean sea ice drift





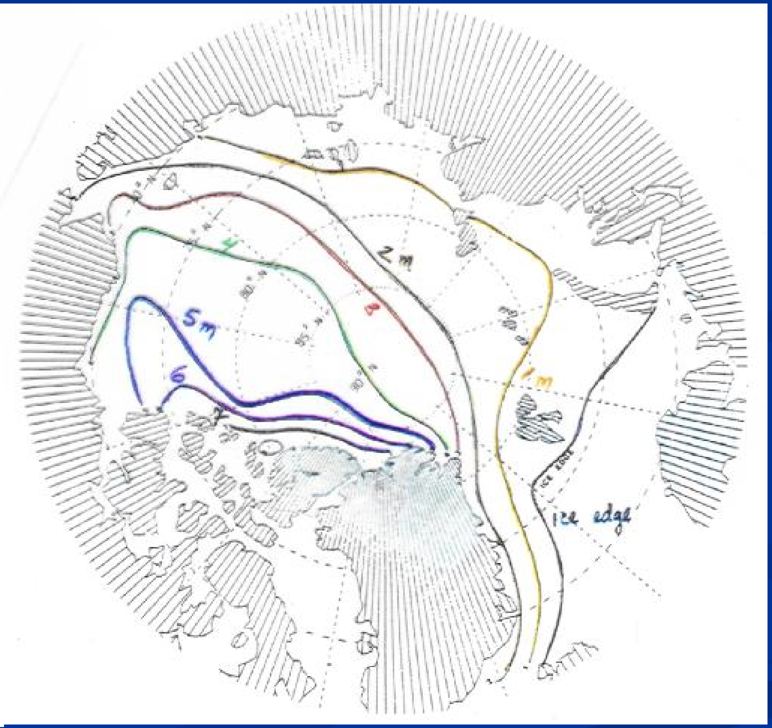
Sea ice thickness

simulated



Tremblay Mysak 1997

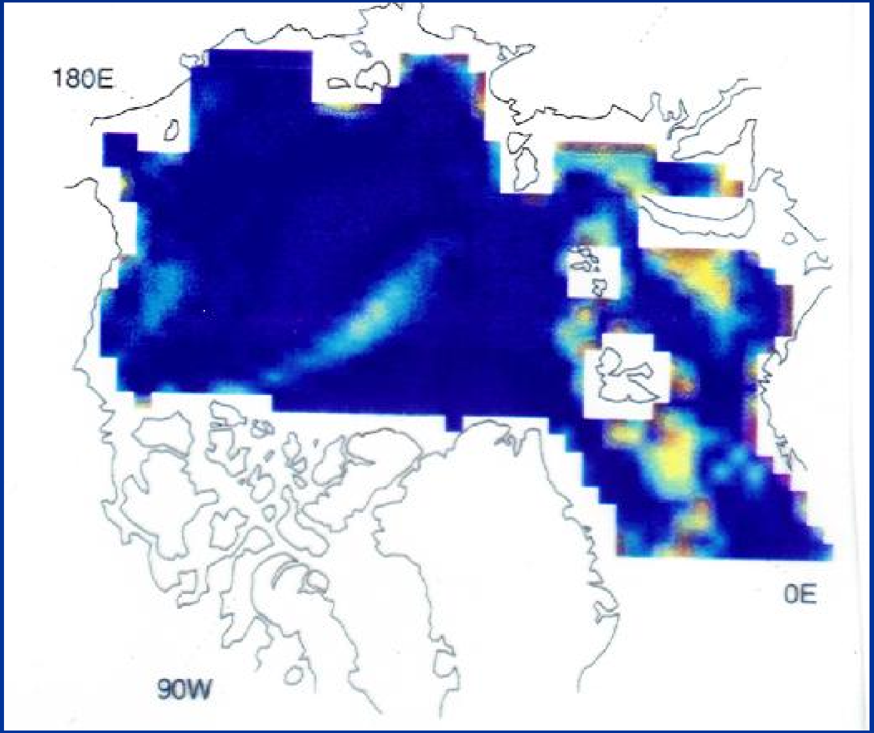
observed



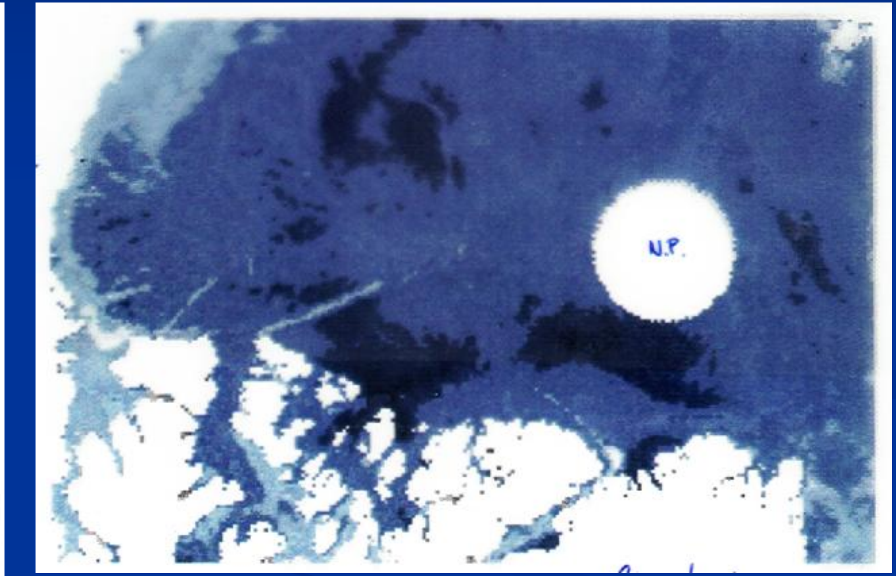
Bourke Garrett, 1986

Shear lines

simulated



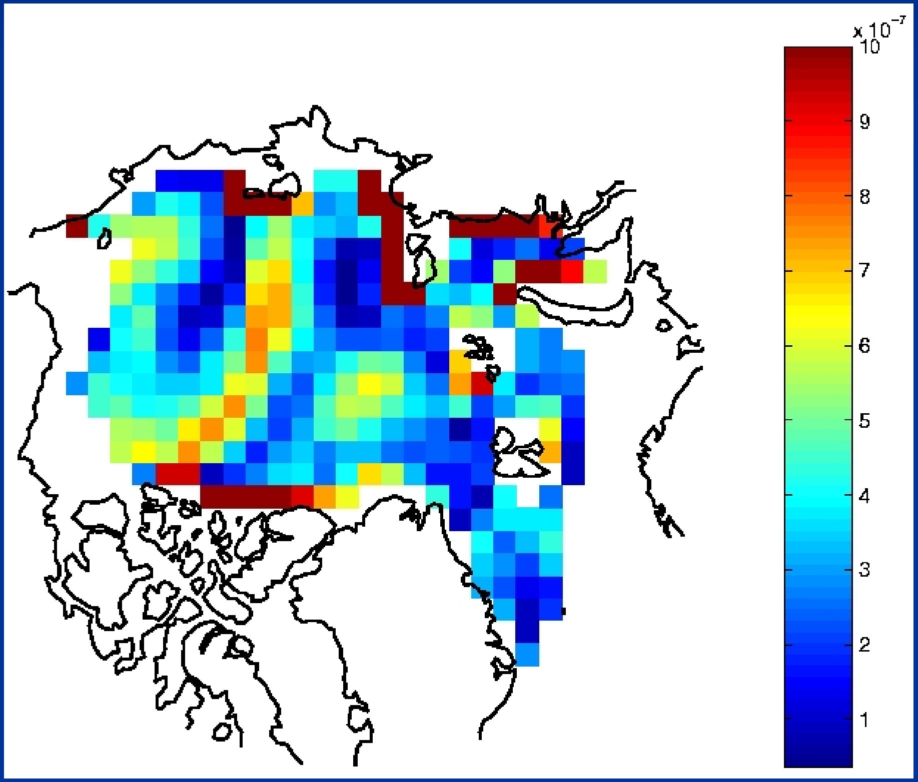
observed



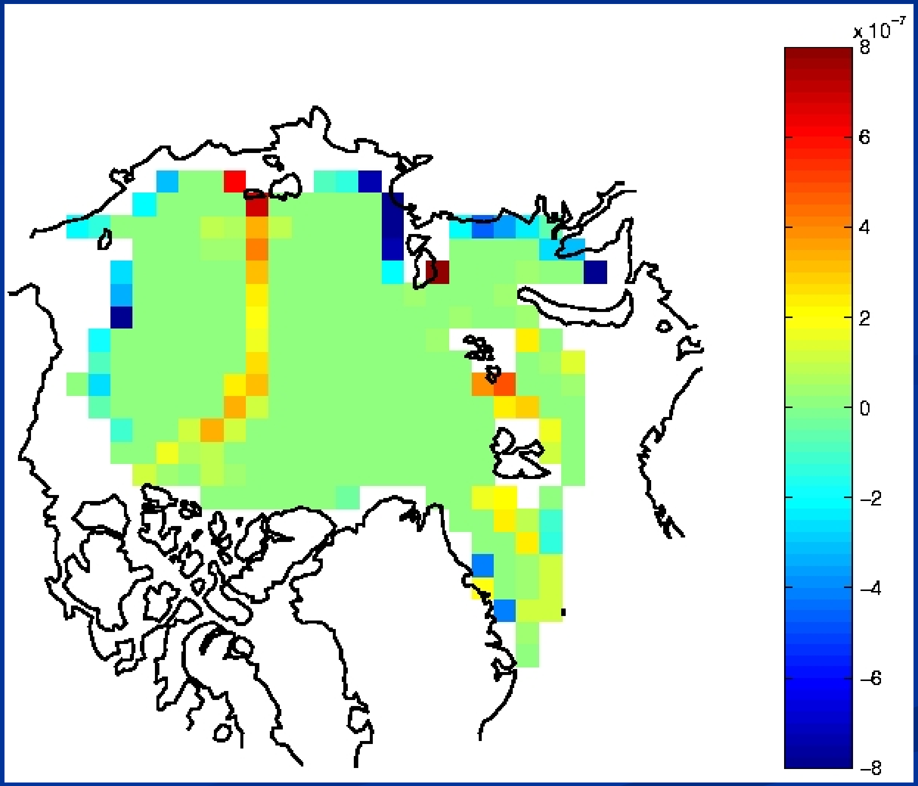
Tremblay Mysak 1997

SSMI passive microwave

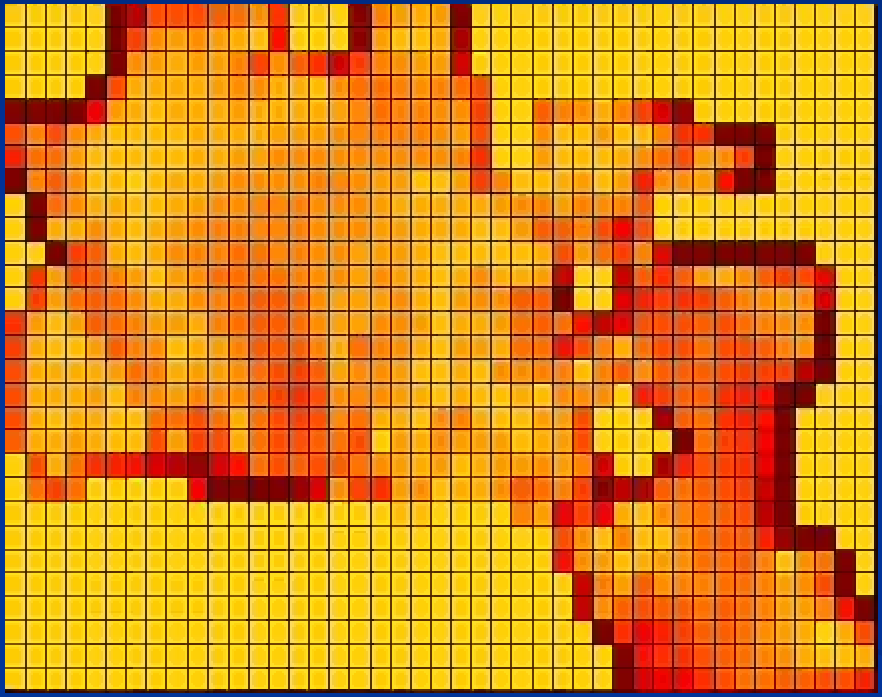
Shear lines



Shear



Divergence

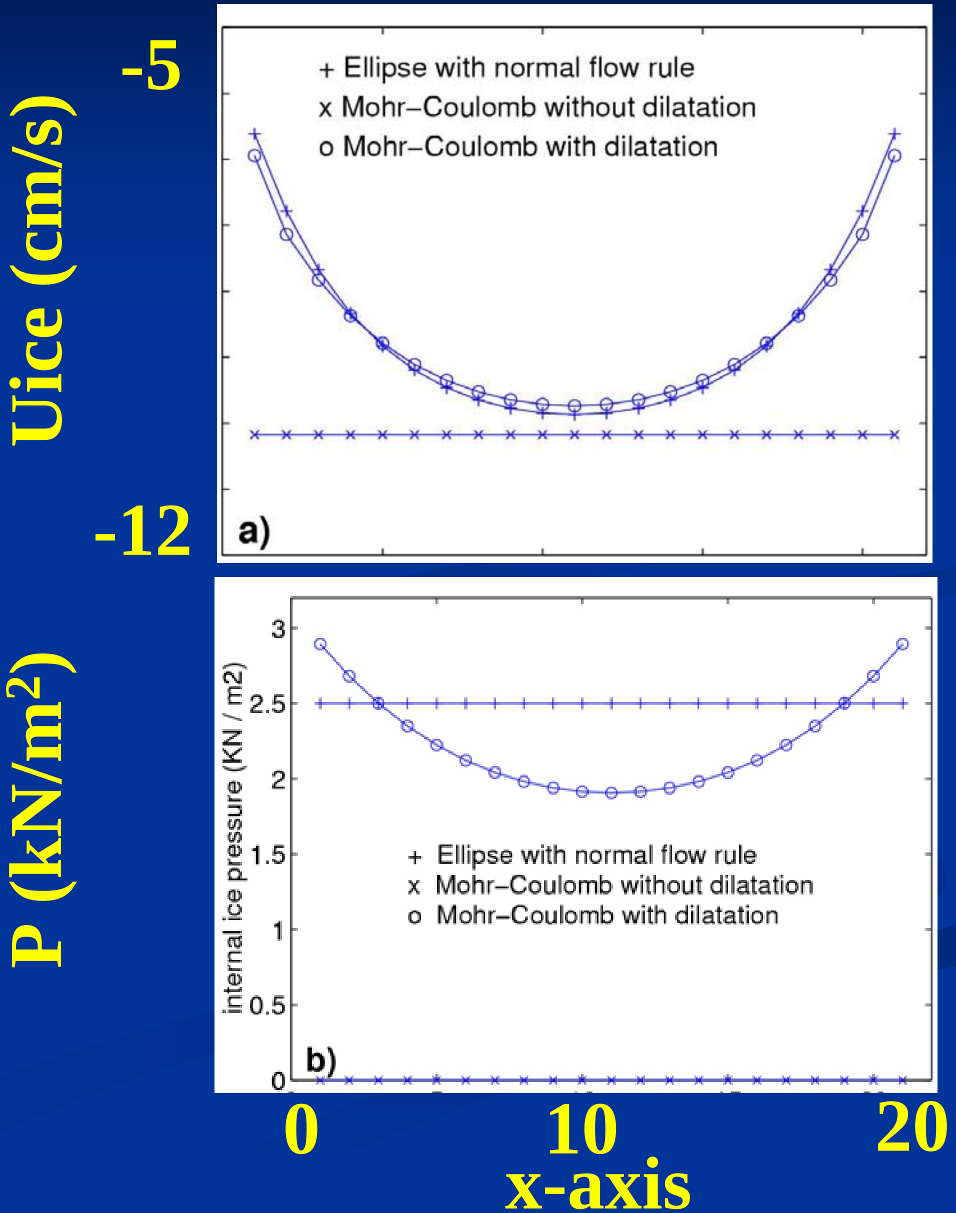


Dilatation effects?

- Flow down a channel
- Arctic simulation

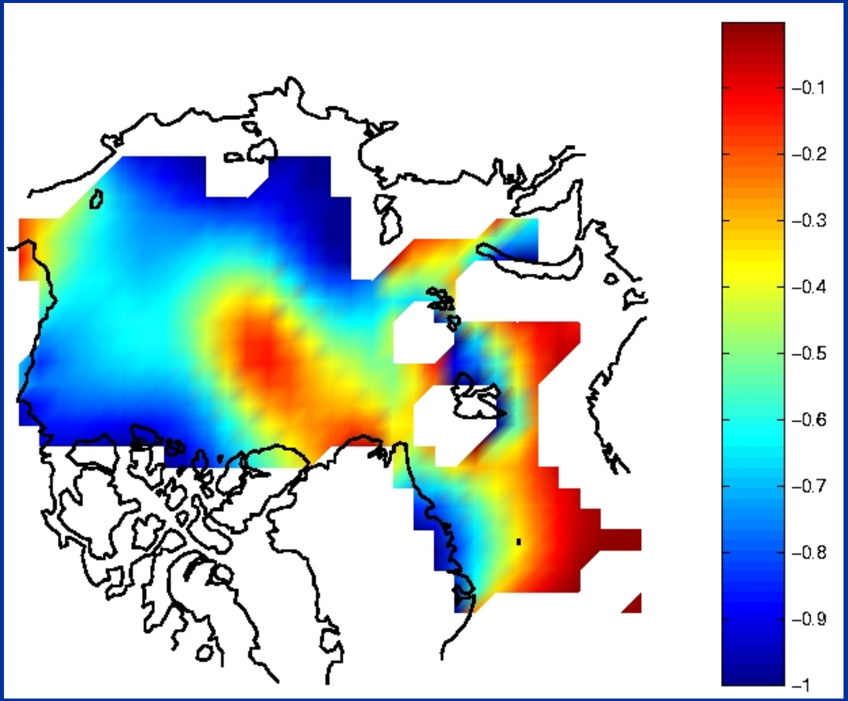
Results

y
x

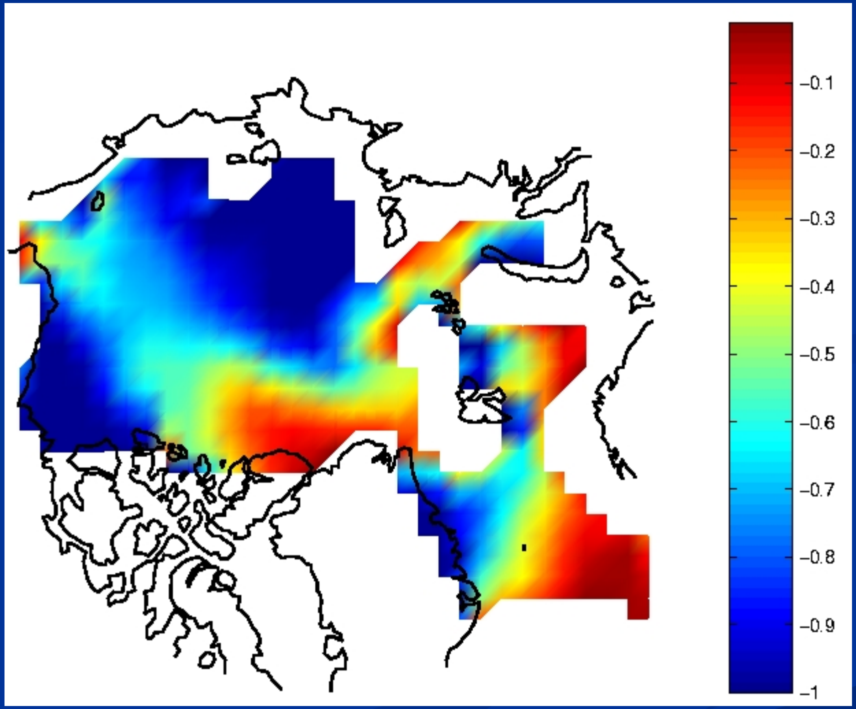


Uice (cm/s)
-5
-12
+ Ellipse with normal flow rule
x Mohr–Coulomb without dilatation
o Mohr–Coulomb with dilatation
a)
P (kN/m^2)
internal ice pressure (KN / m2)
3
2.5
2
1.5
1
0.5
0
+ Ellipse with normal flow rule
x Mohr–Coulomb without dilatation
o Mohr–Coulomb with dilatation
b)
0
10
20
x-axis

Internal Ice Pressure (normalized)



no dilatation

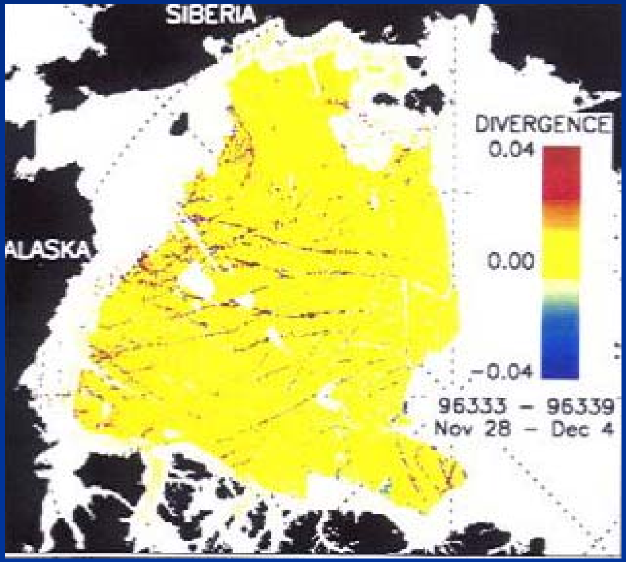


dilatation

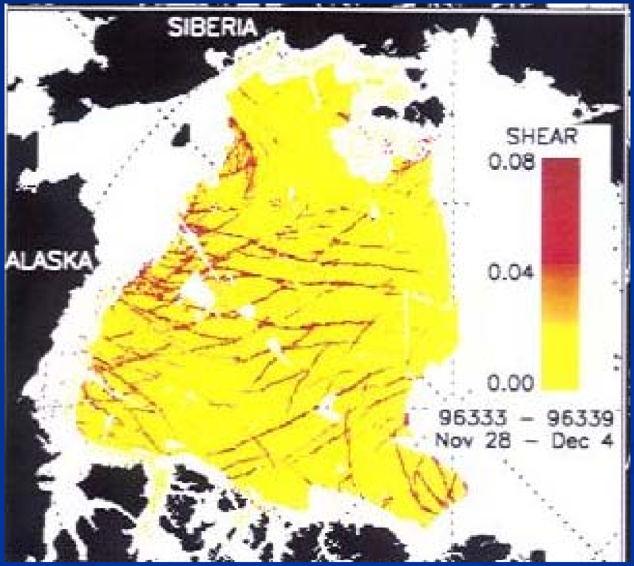
Plastic OR Viscous?

- Plastic OR Viscous?
- Are model iterated until convergence?

Sea ice deformation



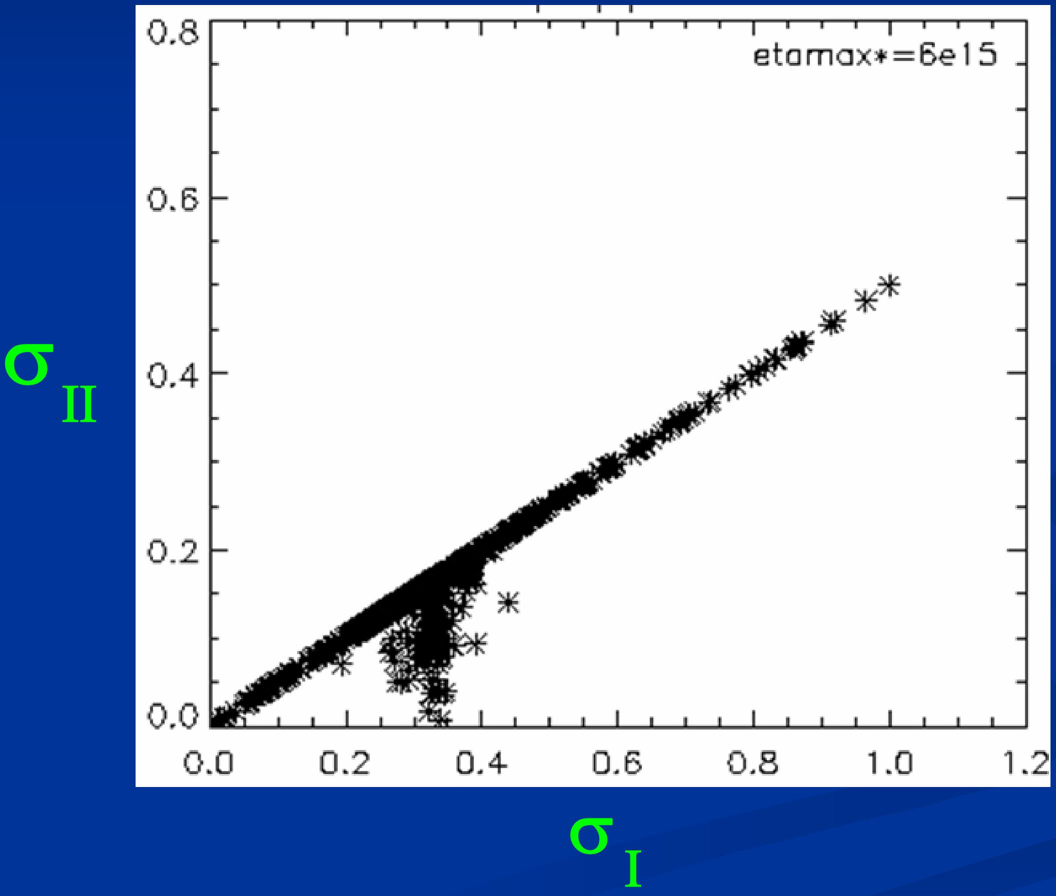
divergence



shear

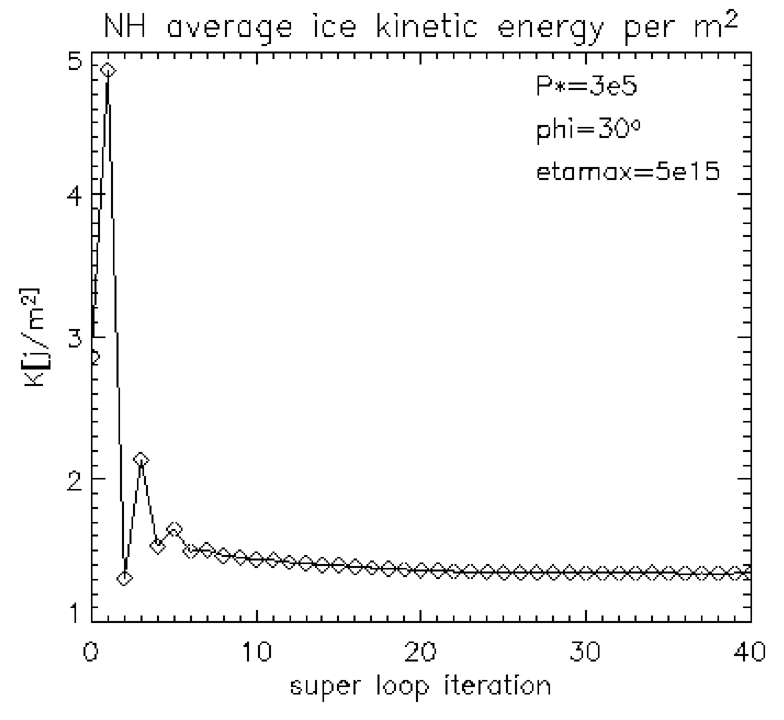
RGPS data, Kwok

Stress state on the yield curve

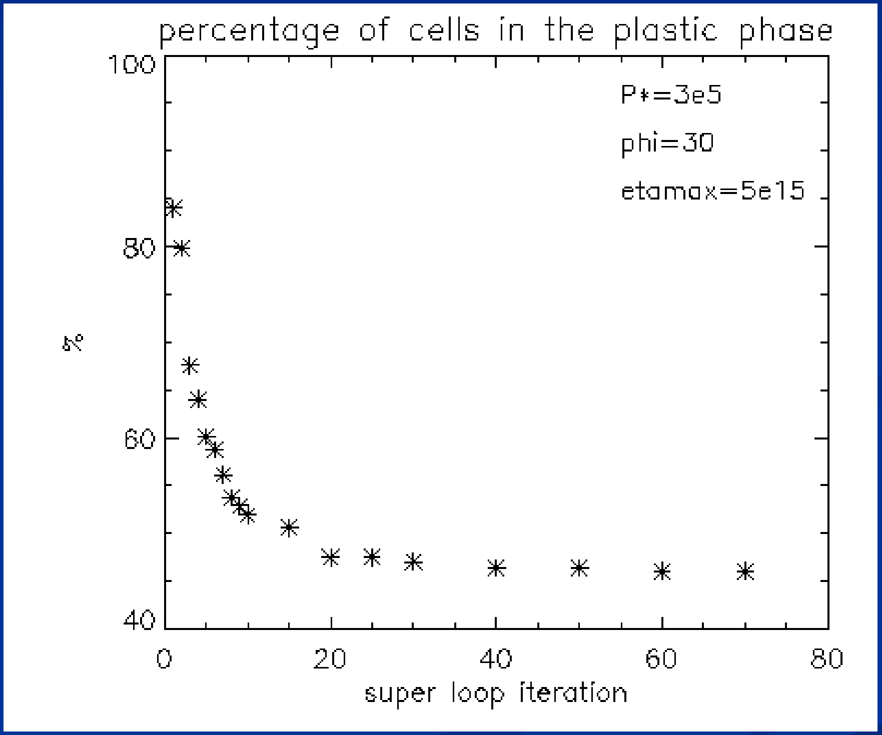


Model integration

Kinetic energy



% plastic phase

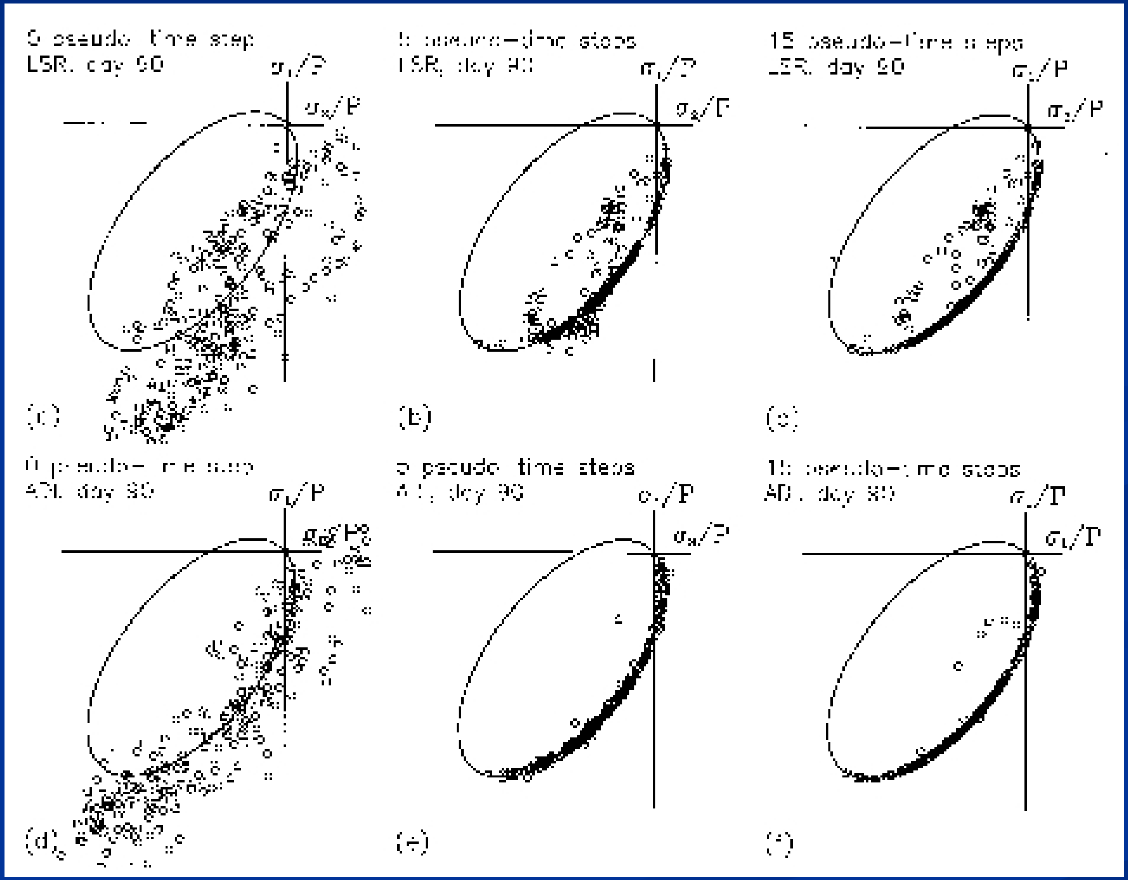


Super loop iteration

Stress states

Line Successive Relaxation

Alternating Direction Implicit



0 **5** **15**

Pseudo time step

Zhang Rothrock, 2000

Questions

- Should modeler use a larger P* in their sea ice simulation ? This would lead to more points in the viscous regime as observed…

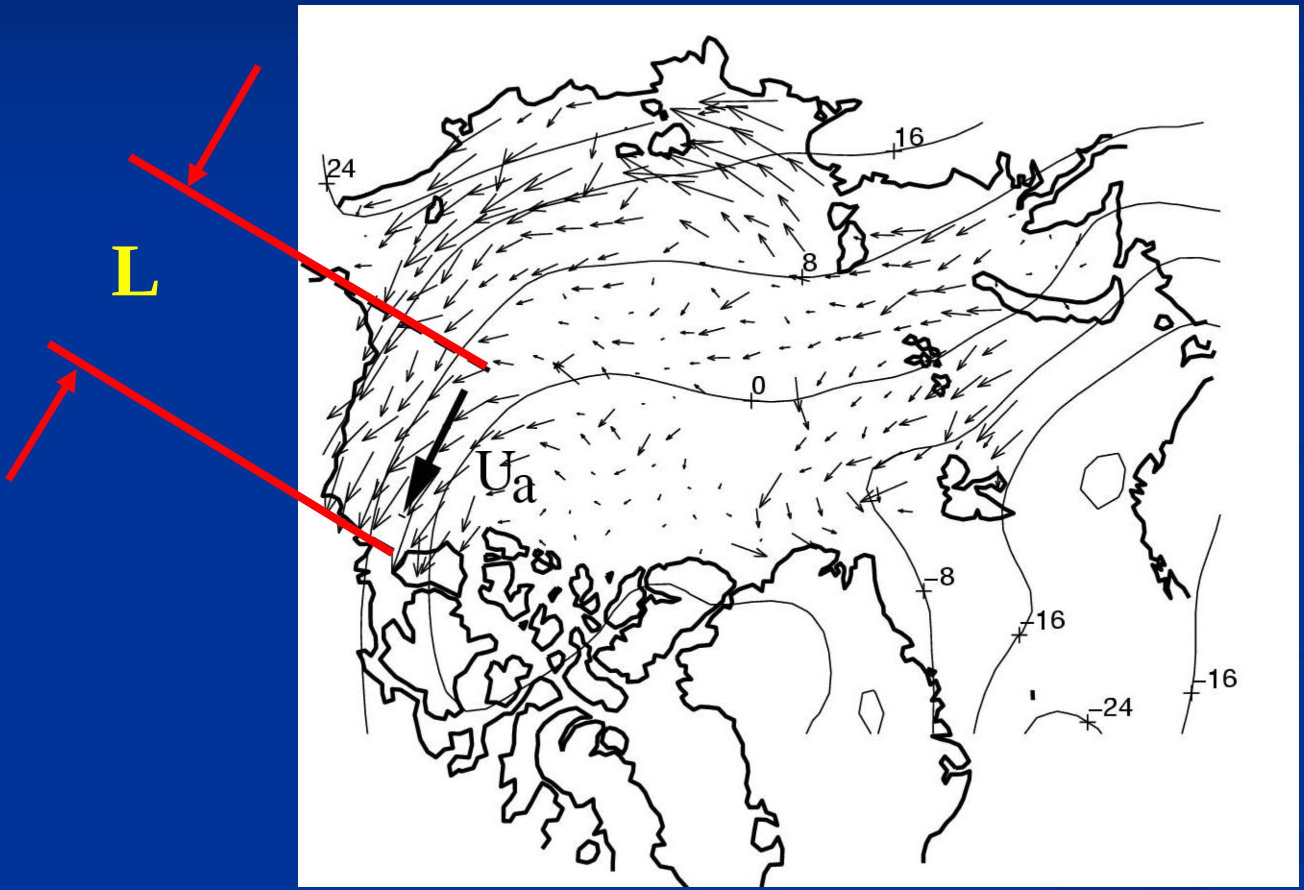
- What about the mean kinetic energy of the pack then?

- Since most of stress state are observed to be “non-plastic”, should we take more care about how we model the low deformation state? (viscous? elastic?)

Evidence for a higher P*

- From Model:
  - Kreyscher et al, 1998 → P* = 15,000 kN/m^2
  - Hibler Walsh 1982 → P* = 27,000 kN/m^2
- From Observations:

Pstar
from satellite/NCEP data



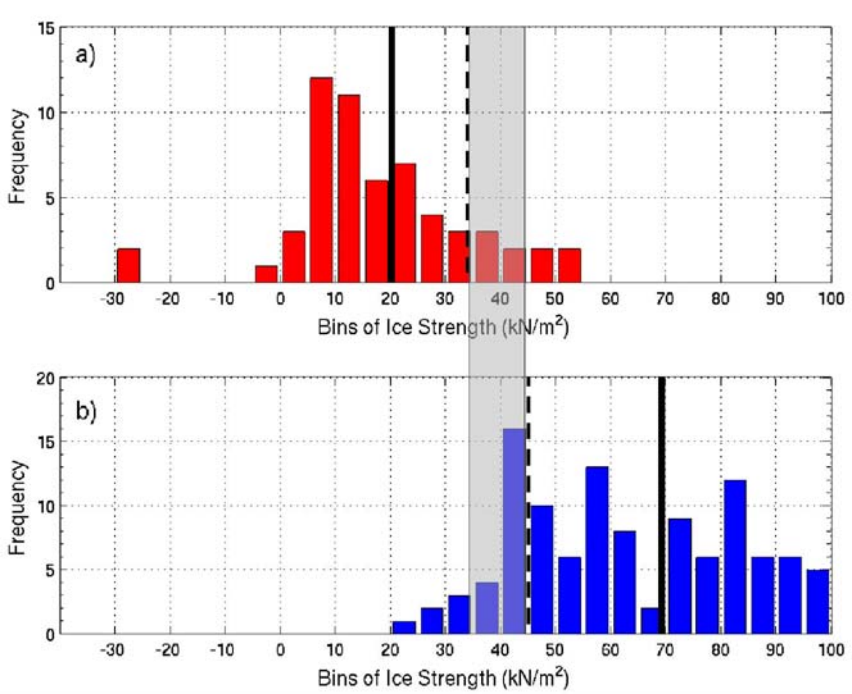
$$F = \tau a\, L$$

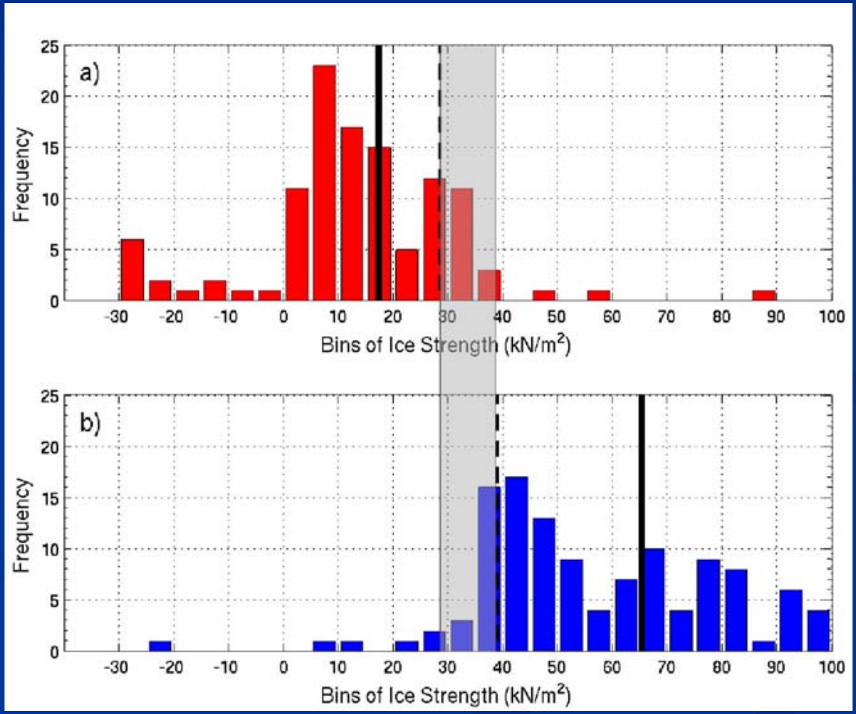
$$P^{*}h > \text{or} < F$$

Ice strength estimates
P*

1992-93 **1996-97**

frequency



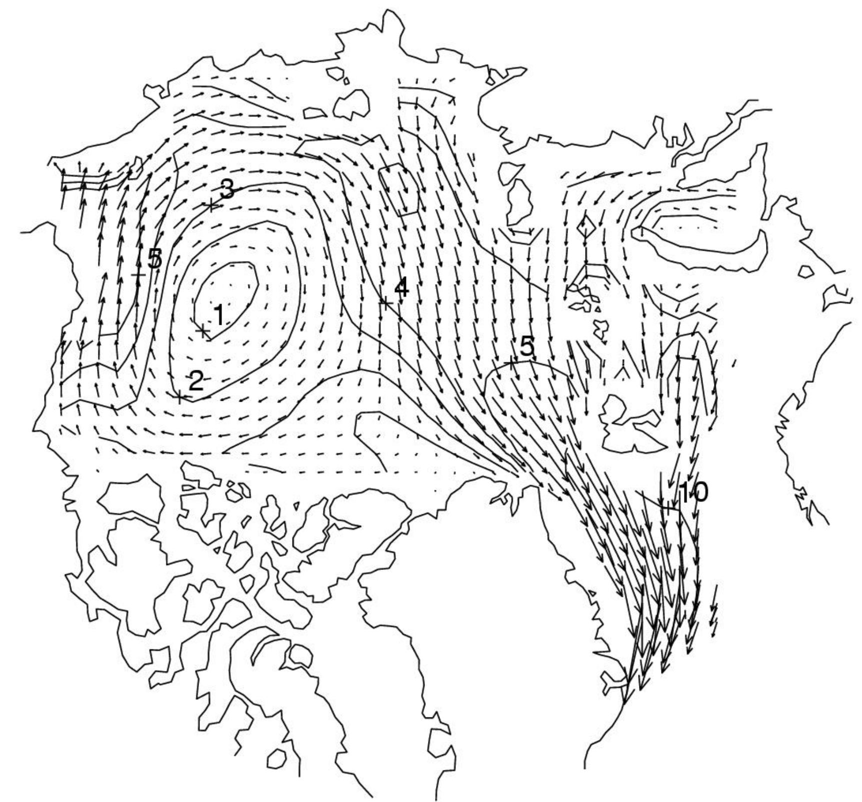


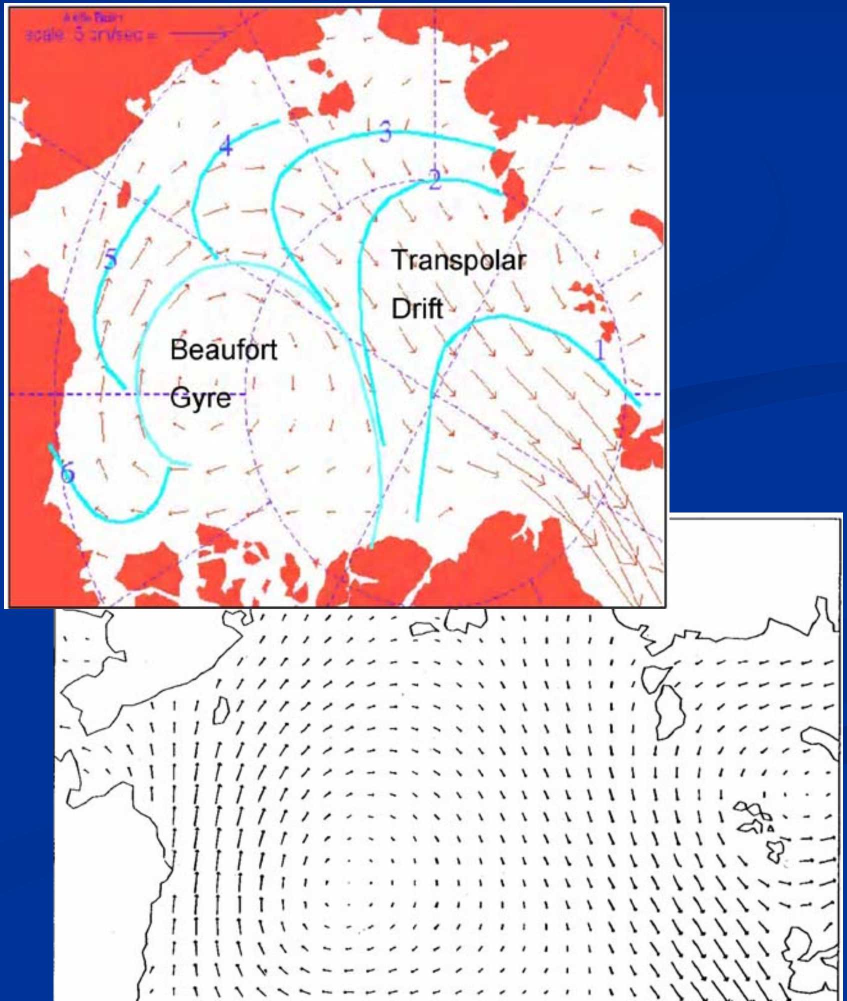
-30 0 100 -30 0 100

P* (kN/m^2)

Tremblay Hakakian, 2006

Mean sea ice drift





The Model