Models of the Probability Distribution of Sea-Surface Wind Speeds

Adam Monahan

monahana@uvic.ca

School of Earth and Ocean Sciences
University of Victoria

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Outline

- **Motivation**

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- **Characterisation of wind speed pdfs**

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- **Empirical models of wind speed pdfs**

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- Mechanistic Model

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- Empirical models of wind speed pdfs
- Mechanistic Model
- Conclusions

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Motivation: Air/sea fluxes

- Ocean and atmosphere interact through respective boundary layers, exchanging

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- Sea surface winds ("eddy averaged") are an important determinant of turbulence in both boundary layers

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- Ocean and atmosphere interact through respective boundary layers, exchanging
  - momentum
  - energy
  - freshwater
  - gases & aerosols
- Sea surface winds ("eddy averaged") are an important determinant of turbulence in both boundary layers
- Turbulence feeds back on surface winds, primarily through surface momentum flux

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Motivation: GCMs

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- Calculation of averaged fluxes requires full surface wind pdf

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$\Rightarrow$ Motivates development of parameterisation of wind speed pdf in terms of vector winds

2. Wind speed pdf arises from vector wind pdf through nonlinear coordinate transformation

$\Rightarrow$ higher order vector wind moments may affect lower order wind speed moments

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Skewness and Kurtosis

- **Skewness**: measure of asymmetry of pdf

$$\text{skew}(x) = \left\langle \left(\frac{x - \langle x \rangle}{\text{std}(x)} \right)^3 \right\rangle$$

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- **Kurtosis**: measure of flatness of pdf

$$\text{skew}(x) = \left\langle \left(\frac{x - \langle x \rangle}{\text{std}(x)} \right)^4 \right\rangle - 3$$

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Sea-Surface Winds: Notation

Notation:

$\mathbf{u}$	vector wind (u, v)
u	along mean wind component
v	cross mean wind component
w	wind speed $\left(u^2 + v^2\right)^{1/2}$

Sea-Surface Winds: Data

- Will consider 6-hourly 10m ocean winds from NCEP/NCAR Reanalysis (1948-2005)

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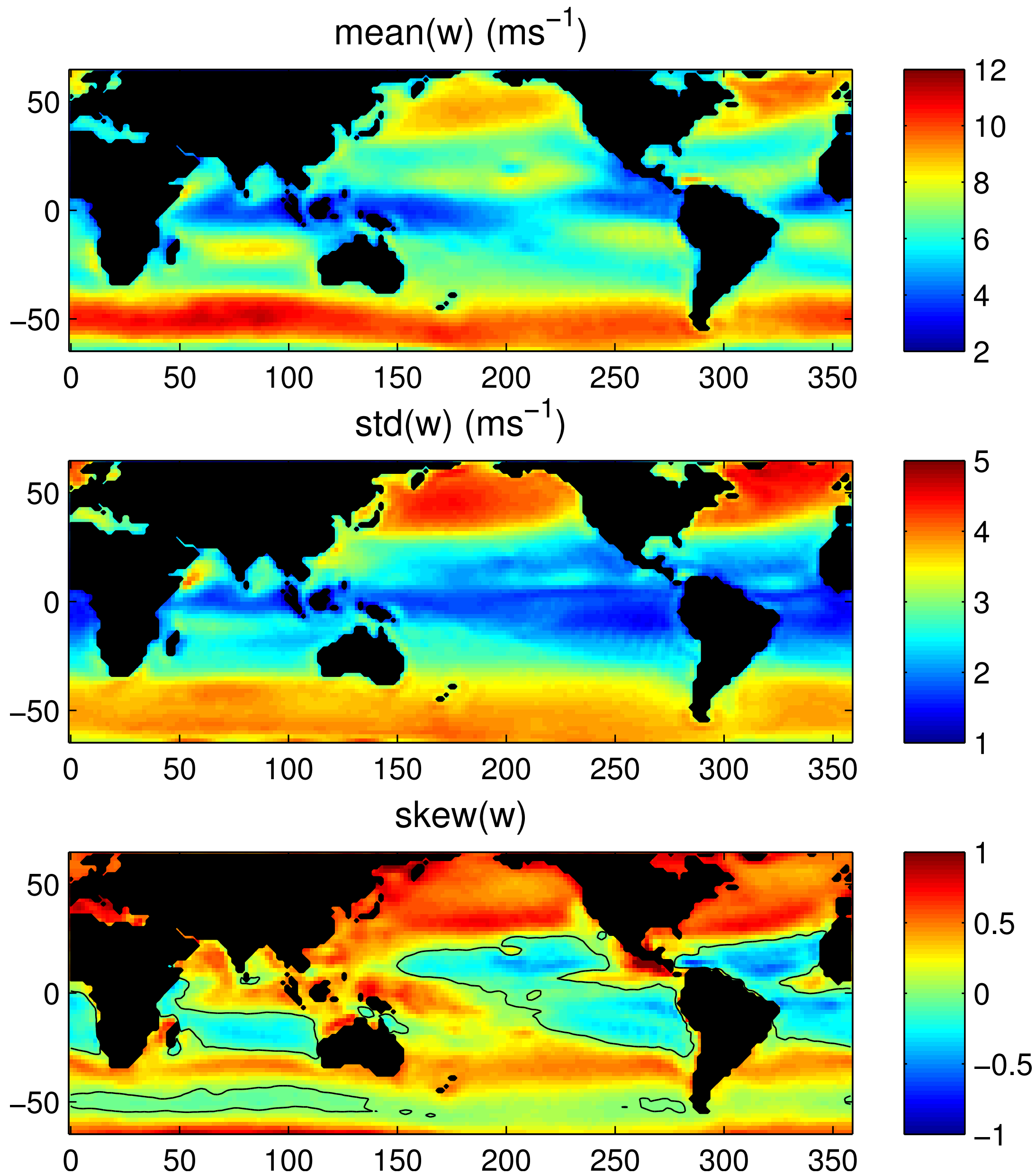
Sea-Surface Wind Speeds: Moments



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Wind Speed pdfs: Weibull

- The pdf of wind speed w has traditionally been represented by 2-parameter Weibull distribution:

$$p_w(w) = \begin{cases} \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left(-\left(\frac{w}{a}\right)^b\right) & w > 0 \\ 0 & w < 0 \end{cases}$$

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- a is the scale parameter (pdf centre)
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- $p_w(w)$ is unimodal

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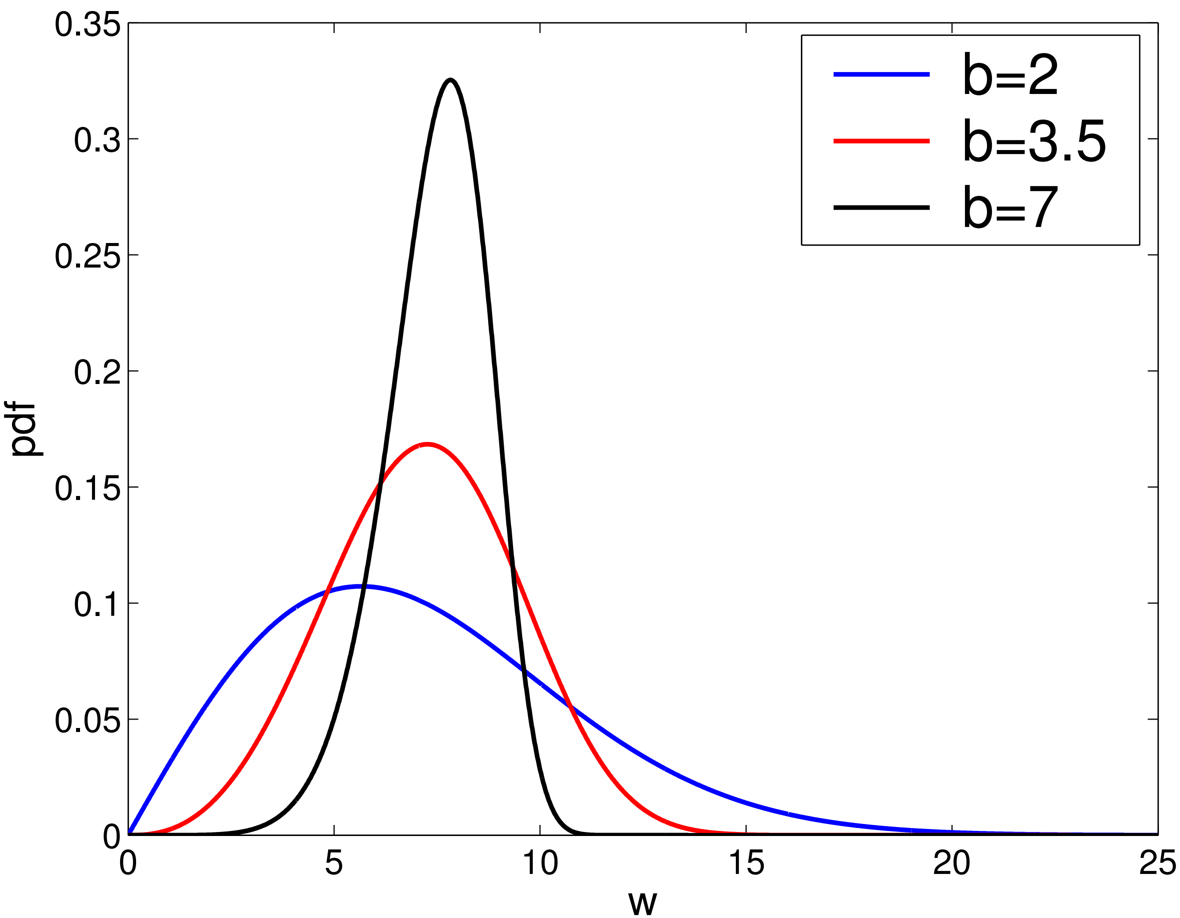
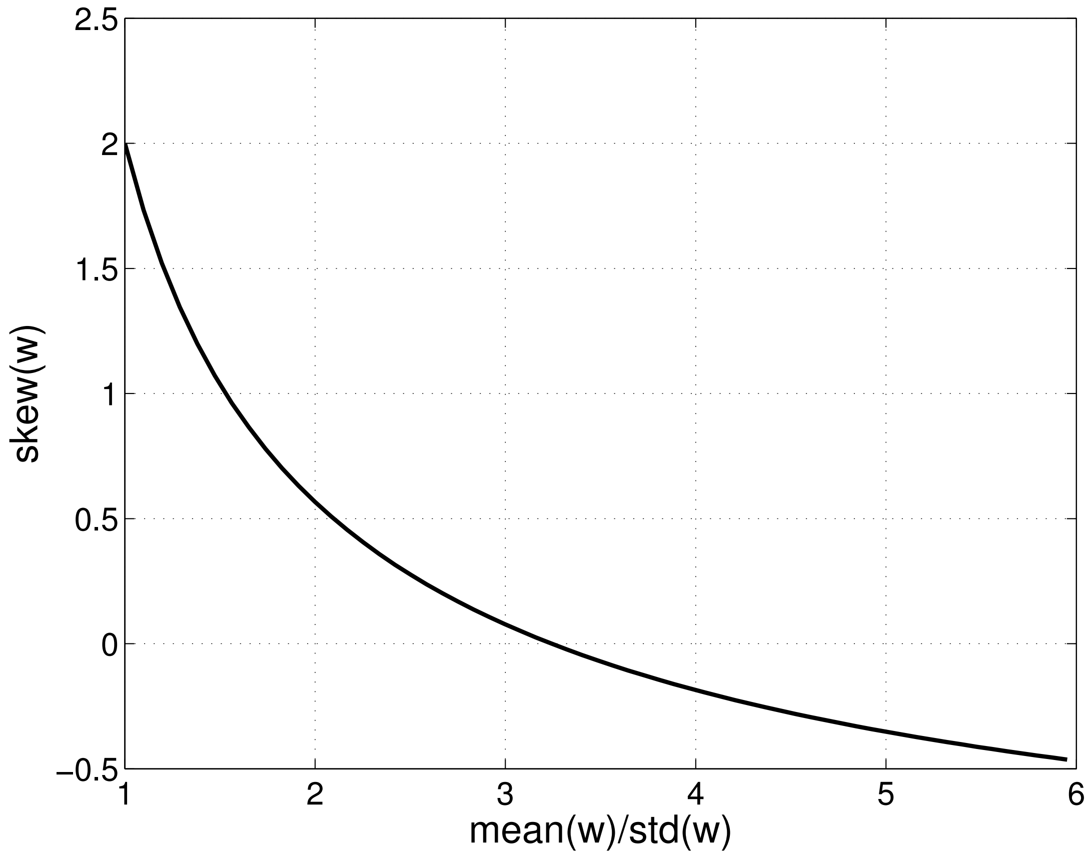
Wind Speed pdfs: Weibull

- Weibull pdfs for $a = 8$



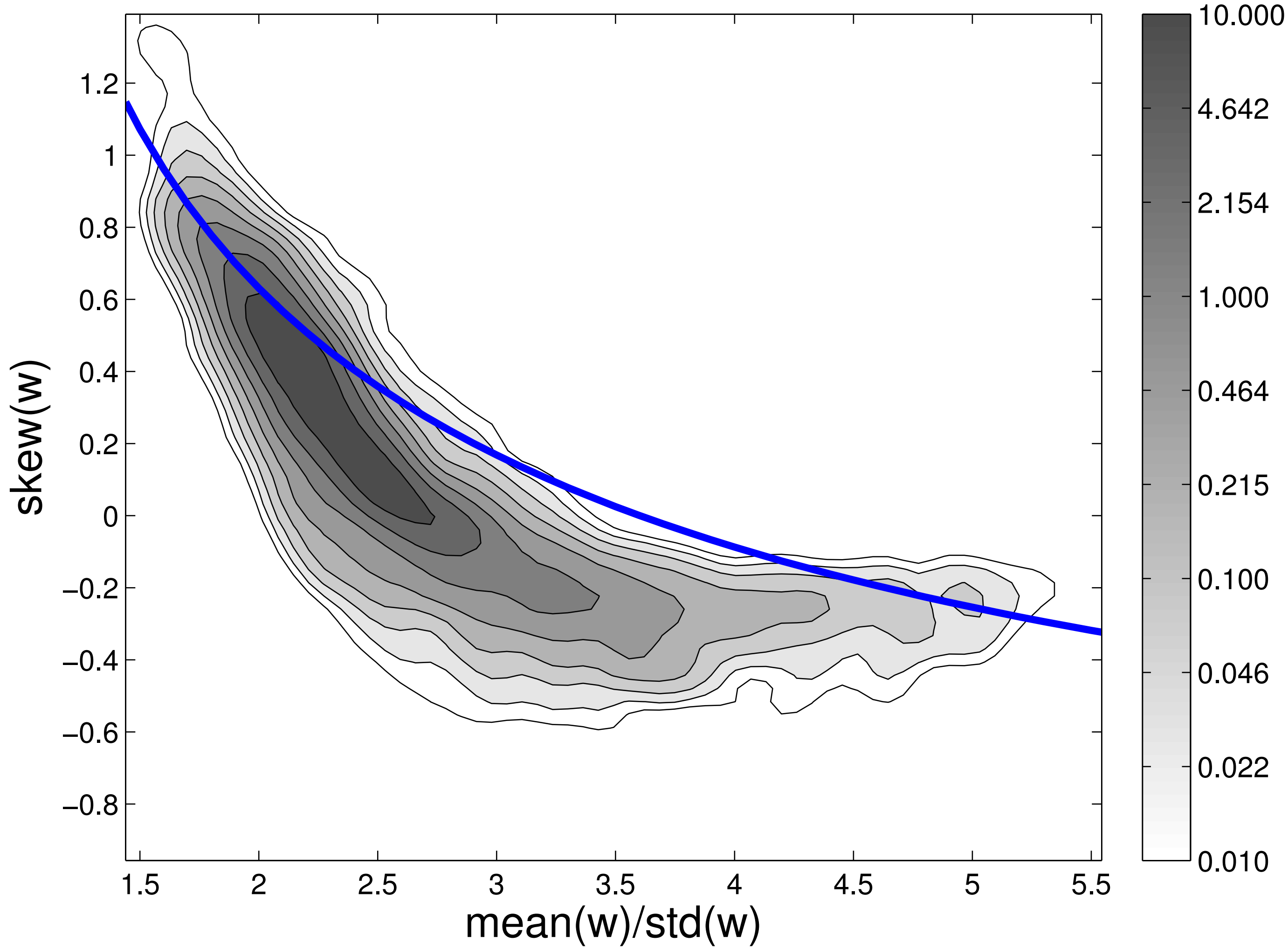
Wind Speed pdfs: Weibull

- For a Weibull distribution, skew(w) is a decreasing function of mean(w)/std(w)



Wind Speed pdfs: Observed

- Observed speed moments fall around Weibull curve



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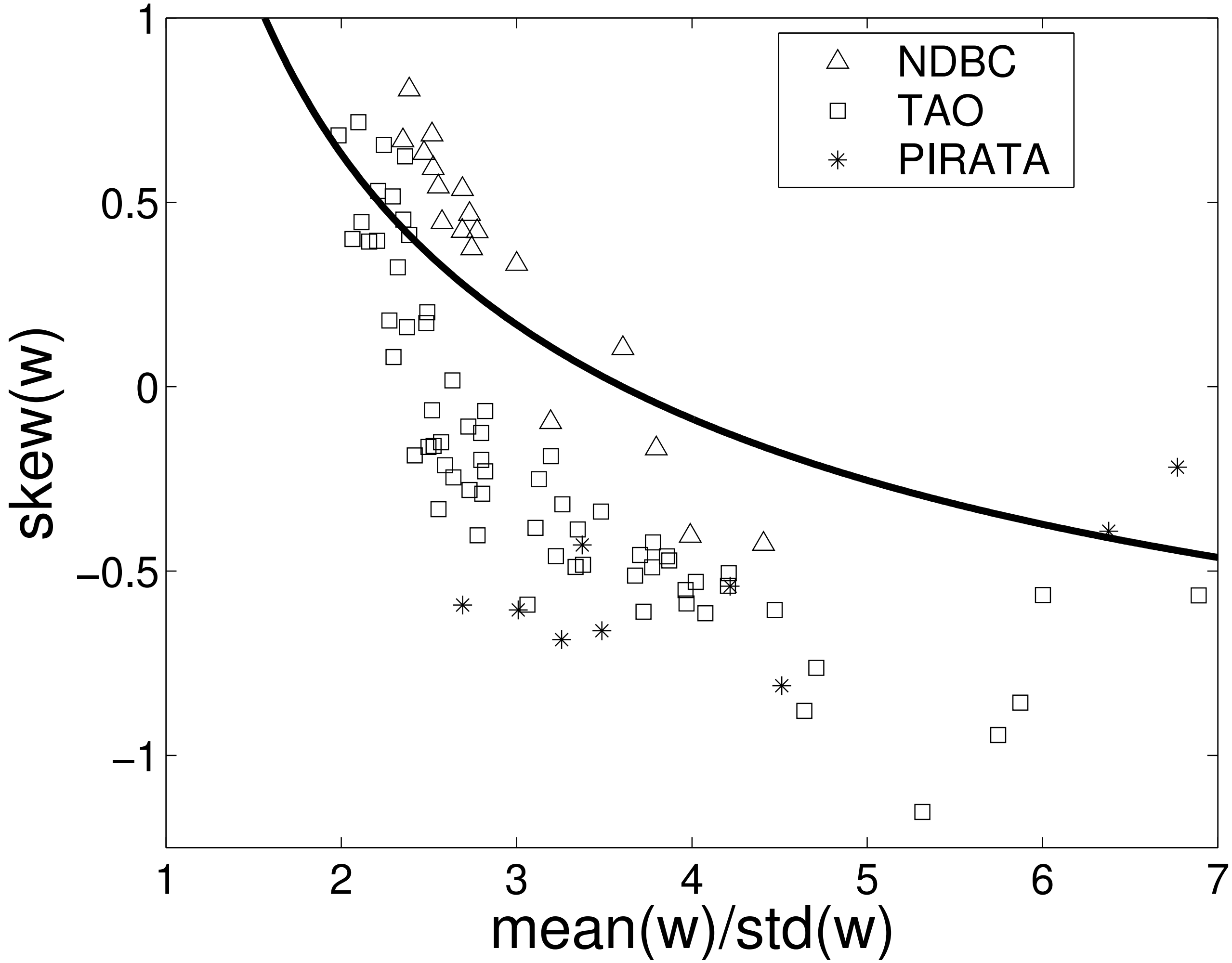

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- Degree of “non-Weibullness” can be measured by relative entropy:

$$\boxed{\rho = \int p_w(w) \ln \left(\frac{p_w(w)}{q_w(w)} \right) dw}$$

where:

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$q_w(w)$ = best-fit Weibull pdf

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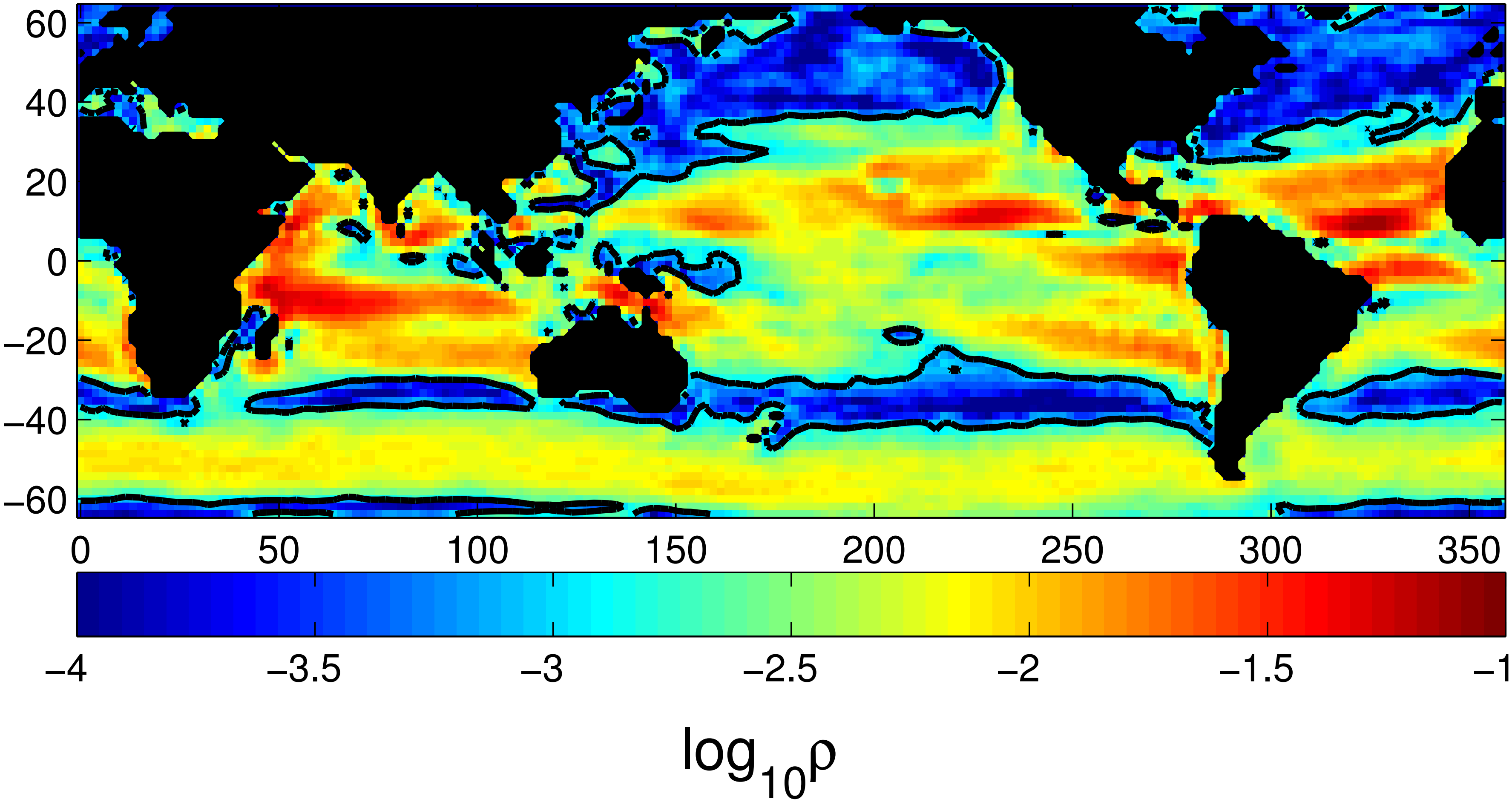
Wind Speed pdfs: Relative Entropy



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Wind Speed pdfs: Empirical Models

- Strategy: systematically construct wind speed pdfs from joint pdf of vector components, $p_{uv}(u, v)$

$$p_w(w) = w \int_{-\pi}^{\pi} p_{uv}(w \cos\theta, w \sin\theta)\, d\theta$$

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$$p_w(w) = w \int_{-\pi}^{\pi} p_{uv}(w \cos\theta, w \sin\theta) d\theta$$

- Approach simplified by assuming u, v independent, so

$$p_{uv}(u, v) = p_u(u) p_v(v)$$

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Wind Speed pdfs: Gaussian Vector Winds

- Simplest model assumes isotropic Gaussian fluctuations in vector winds:

$$
\begin{aligned}
p_u(u) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(u-\overline{u})^2}{2\sigma^2}\right) \\
p_v(v) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{v^2}{2\sigma^2}\right)
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$$

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$$p_v(v) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{v^2}{2\sigma^2}\right)$$

- Integrating over wind direction:

$$p_w(w) = \frac{w}{\sigma^2} \exp\left(-\frac{w^2+\overline{u}^2}{2\sigma^2}\right) I_0\left(\frac{w\overline{u}}{\sigma^2}\right)$$

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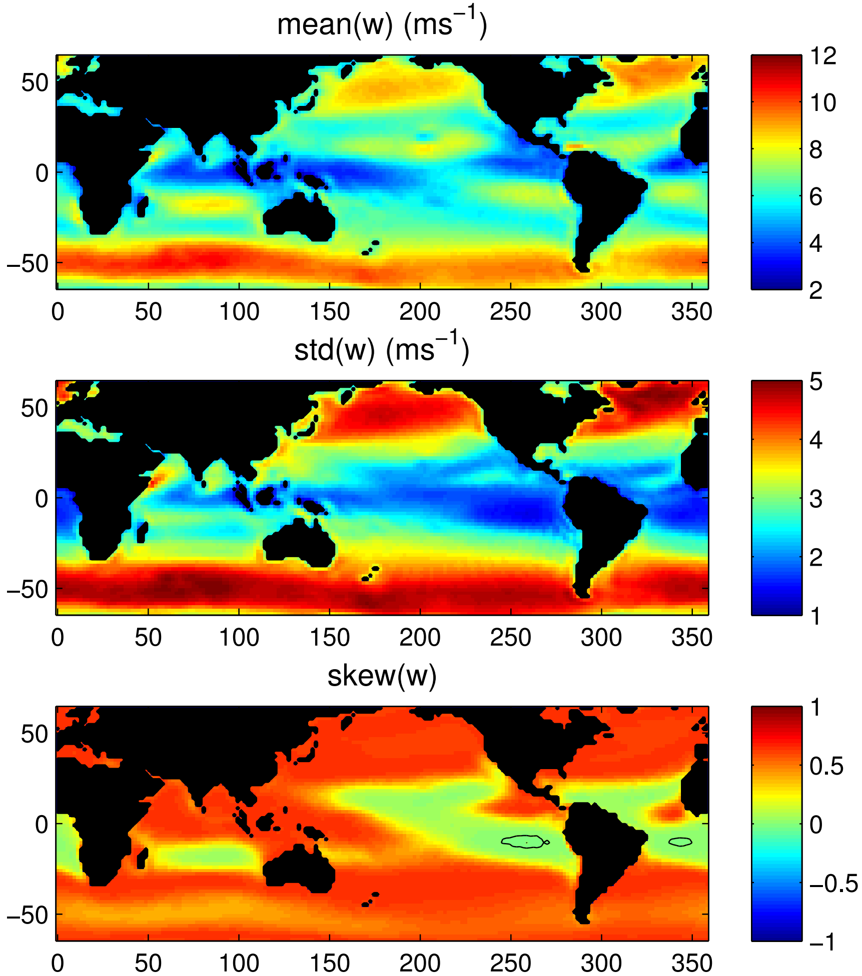
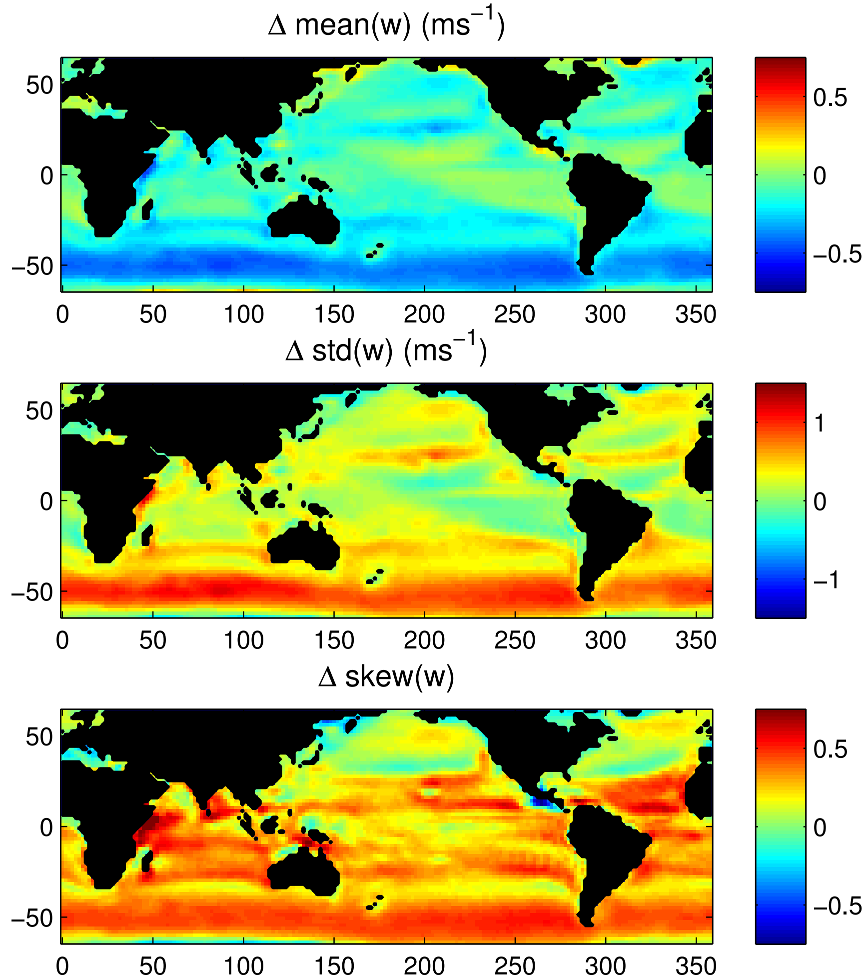
Wind Speed pdfs: Gaussian Vector Winds

Model



Model-Observations



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Wind Speed pdfs: Gaussian Vector Winds

- Isotropic Gaussian model $\Rightarrow$ large biases

Wind Speed pdfs: Gaussian Vector Winds

- Isotropic Gaussian model $\Rightarrow$ large biases
- Relax assumption of isotropy:
  $\sigma_u = \text{std}(u), \sigma_v = \text{std}(v) \Rightarrow$

$$\boxed{\begin{aligned} p_w(w) &= \frac{w}{\sigma_u\sigma_v}\exp\left(-\frac{w^2+\overline{u}^2}{2\sigma_u^2}\right)\times \\ &\left\{I_0\left(\frac{w\overline{u}}{\sigma_u^2}\right)+\sum_{k=1}^{\infty}\left[\frac{w}{\overline{u}}\left(1-\frac{\sigma_u^2}{\sigma_v^2}\right)\right]^k\frac{\Gamma(k+1/2)}{\sqrt{\pi}k!}I_k\left(\frac{\overline{u}w}{\sigma_u^2}\right)\right\}\end{aligned}}$$

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p_w(w) \;=\; & \frac{w}{\sigma_u \sigma_v} \exp\left(-\frac{w^2 + \overline{u}^2}{2\sigma_u^2}\right) \times \\
& \left\{ I_0\left(\frac{w\overline{u}}{\sigma_u^2}\right) + \sum_{k=1}^{\infty} \left[\frac{w}{\overline{u}}\left(1 - \frac{\sigma_u^2}{\sigma_v^2}\right)\right]^k \frac{\Gamma(k+1/2)}{\sqrt{\pi}k!} I_k\left(\frac{\overline{u}w}{\sigma_u^2}\right) \right\}
\end{aligned}
$$

- Actually makes approximation somewhat *worse*

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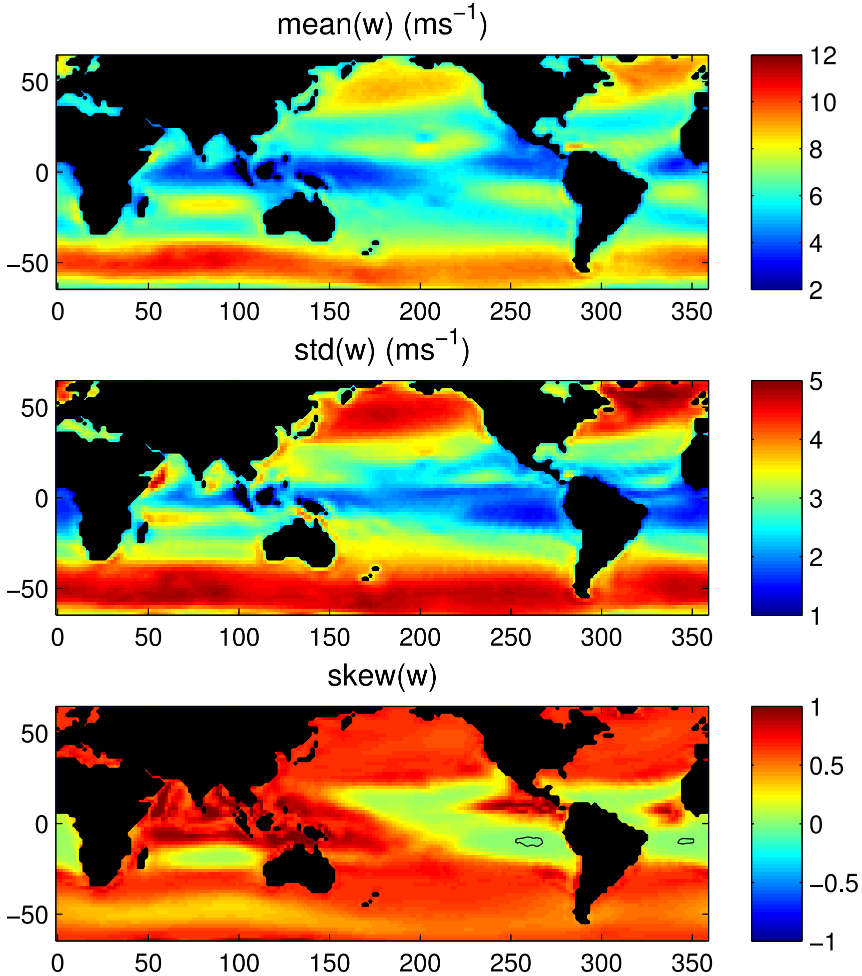
Wind Speed pdfs: Gaussian Vector Winds

Model



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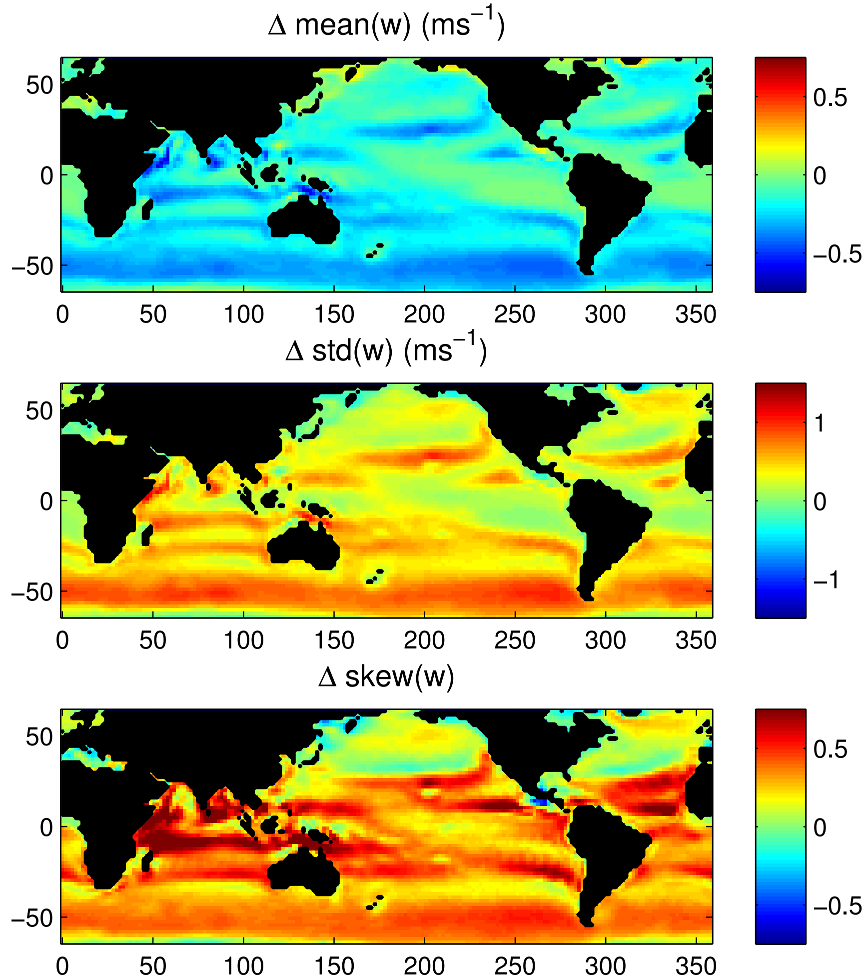

Wind Speed pdfs: What now?

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- Along-mean-wind component characterised by anticorrelated mean and skewness fields
- Coupling of moments follows from nonlinear dependence of surface drag on surface winds
- Symmetric fluctuations in forcing
$\Rightarrow$ asymmetric response, skewed toward rest

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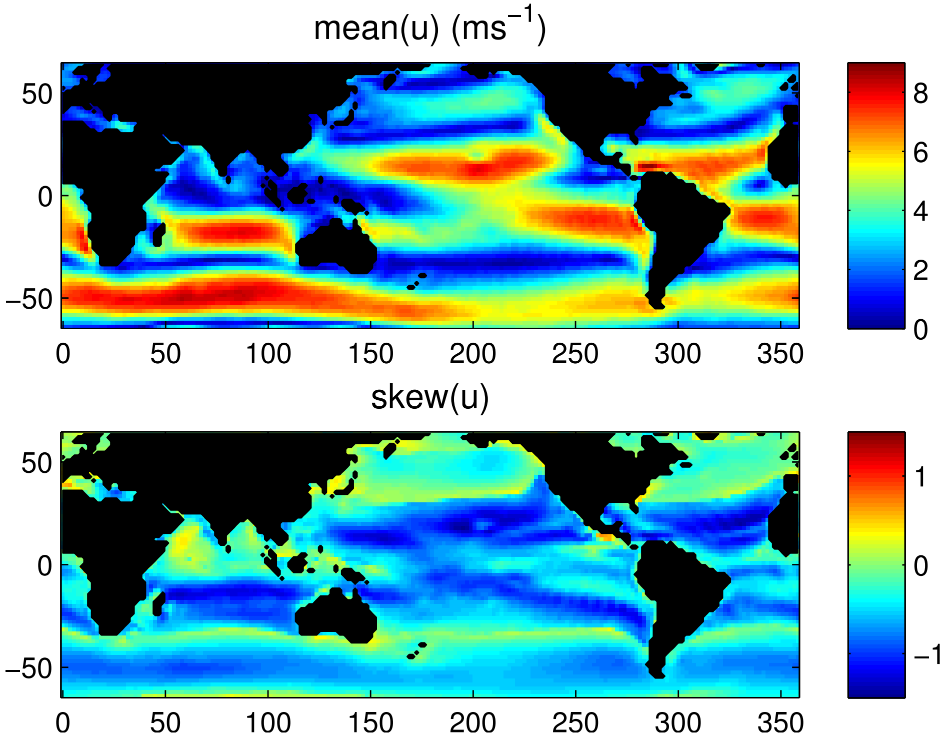
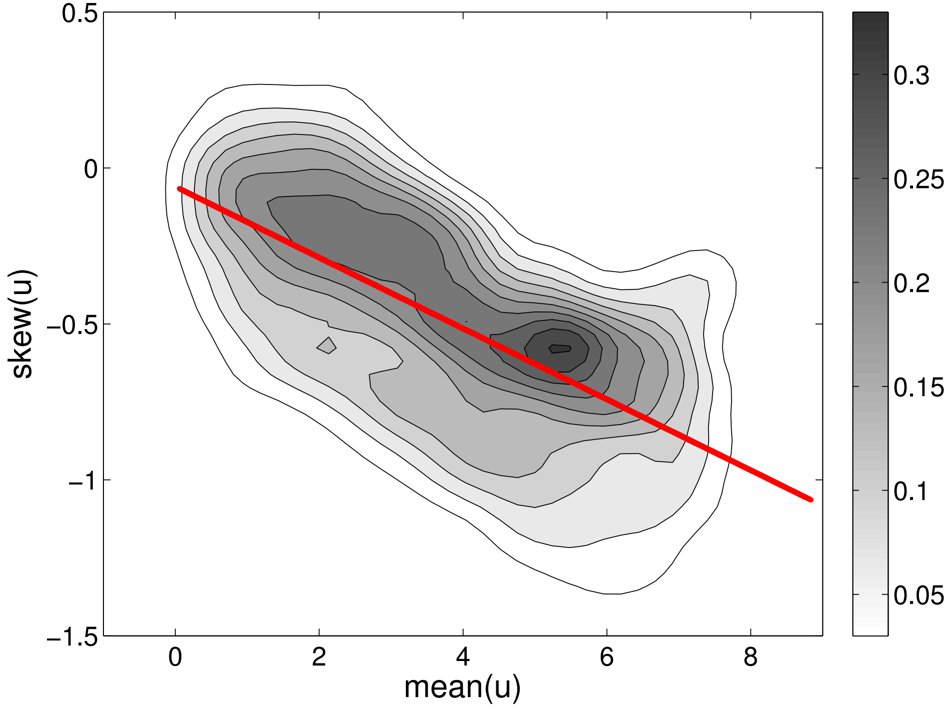
Vector Wind Moments





$$\text{skew}(u) \simeq (\text{-}0.11\ \text{sm}^{-1})\ \text{mean}(u)\ \text{-}\ 0.06$$

Vector Wind pdfs: Skewed pdfs

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- Question is: how important are details of skewed pdf of u for pdf of w?

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Bigaussian pdf

- Two half-Gaussians:

$$p_u(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \begin{cases} \exp\left(-\frac{(u-\mu)^2}{2\sigma^2(1-\epsilon)^2}\right) & u < \mu \\ \exp\left(-\frac{(u-\mu)^2}{2\sigma^2(1+\epsilon)^2}\right) & u > \mu \end{cases}$$

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- Moments:

$$\begin{aligned} \text{mean}(u) &= \mu + \sqrt{\frac{8}{\pi}}\sigma\epsilon \\ \text{std}(u) &= \sigma\left[1 + \left(3 - \frac{8}{\pi}\right)\epsilon^2\right]^{1/2} \\ \text{skew}(u) &\simeq \sqrt{\frac{8}{\pi}}\frac{\epsilon}{\text{std}^3(u)} \end{aligned}$$

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Centred Gamma pdf

- With $z = (u - \overline{u})/\sigma$:

$$p_u(u) = \begin{cases} \frac{|\beta|}{\sigma\Gamma(\beta^2)} \left[\beta(z+\beta)\right]^{\beta^2-1} \exp\left[-\beta(z+\beta)\right] & z+\beta > 0 \\ 0 & z+\beta < 0 \end{cases}$$

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- Moments:

$$\begin{aligned} \text{mean}(u) &= \overline{u} \\ \text{std}(u) &= \sigma \\ \text{skew}(u) &= 2/\beta \end{aligned}$$

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Gram-Charlier Expansion

- With $z = (u - \overline{u})/\sigma$:

$$p_u(u) = \frac{1}{\sqrt{2\pi\sigma^2}}\left[1 + \frac{\nu}{6}H_3(z) + \frac{\kappa}{24}H_4(z)\right]\exp\left(-\frac{z^2}{2}\right)$$

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Maximum Entropy pdf

- Subject to the constraints:

$$\begin{aligned} \text{mean}(u) &= \overline{u} \\ \text{std}(u) &= \sigma \\ \text{skew}(u) &= \nu \\ \text{kurt}(u) &= \kappa \end{aligned}$$

find the pdf $p_u(u)$ which maximises the entropy

$$H = -\int p_u(u) \ln p_u(u) du$$

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Maximum Entropy pdf

- Solution takes the form

$$p_u(u) = \frac{1}{Z}\exp\left(\sum_{i=1}^{4}\lambda_i u^i\right)$$

Maximum Entropy pdf

- Solution takes the form

$$\boxed{p_u(u) = \frac{1}{Z} \exp\left(\sum_{i=1}^{4} \lambda_i u^i\right)}$$

- Lagrange multipliers $\{\lambda_i\}$ found as solutions to unconstrained dual variational problem

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Maximum Entropy pdf

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$$p_u(u) = \frac{1}{Z} \exp\left(\sum_{i=1}^{4} \lambda_i u^i\right)$$

- Lagrange multipliers $\{\lambda_i\}$ found as solutions to unconstrained dual variational problem
- Maximum entropy pdf is "least biased" among all pdfs with given moments, in a rigorous information theoretic sense

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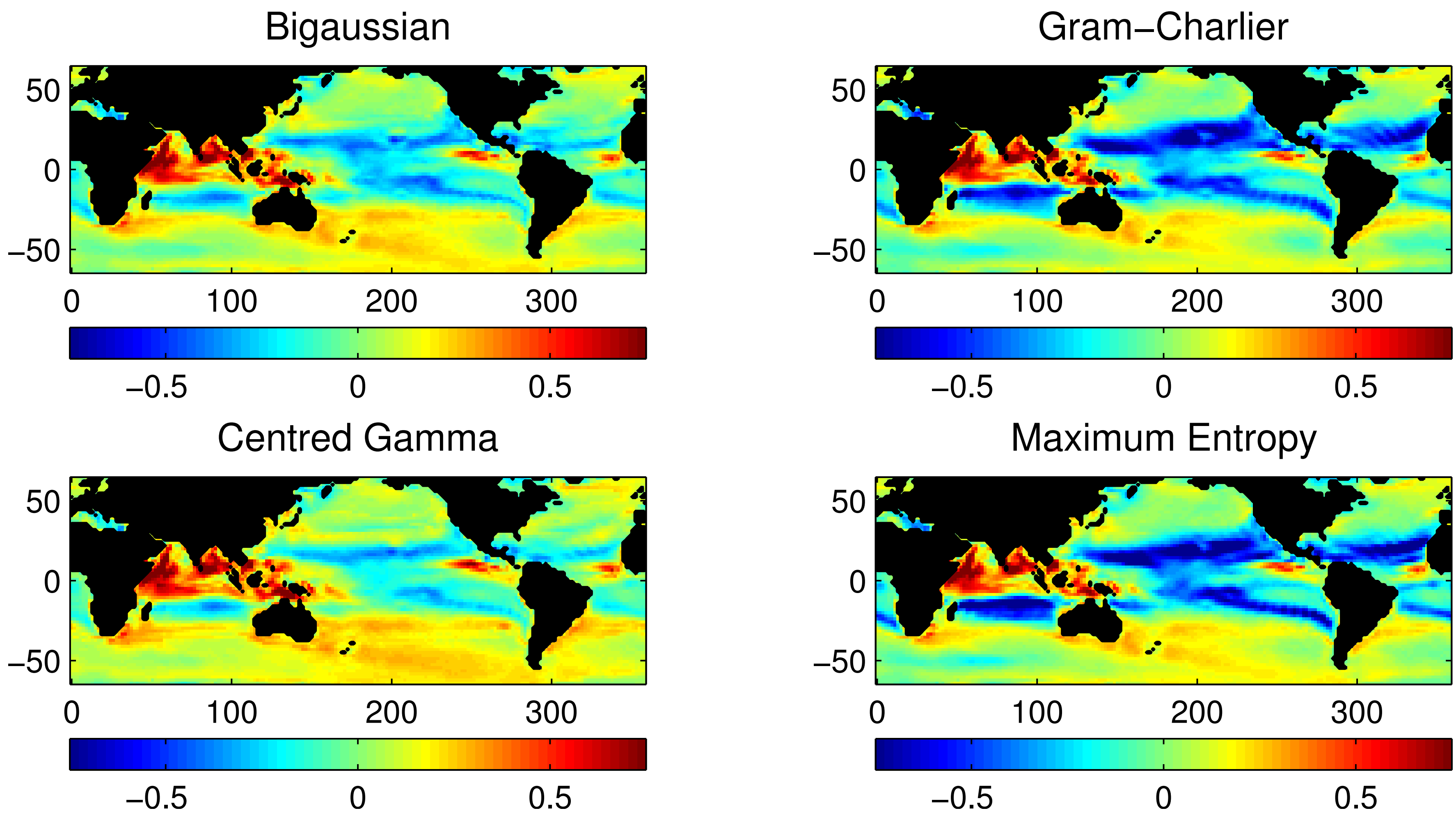
Empirical Wind Speed pdfs: Intercomparison

- Skew(u) from observations, no kurt(u) information



Modelled skew(w) - observed skew(w)

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- **BUT**: for Gram-Charlier & maximum entropy pdfs, have set kurt$(u) = 0$

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  - pdf parameterisation improves over Gaussian model
  - Bigaussian, centred gamma better than Gram-Charlier, maximum entropy
- **BUT**: for Gram-Charlier & maximum entropy pdfs, have set kurt$(u) = 0$
- In fact, kurtosis of along-mean-wind component non-trivial

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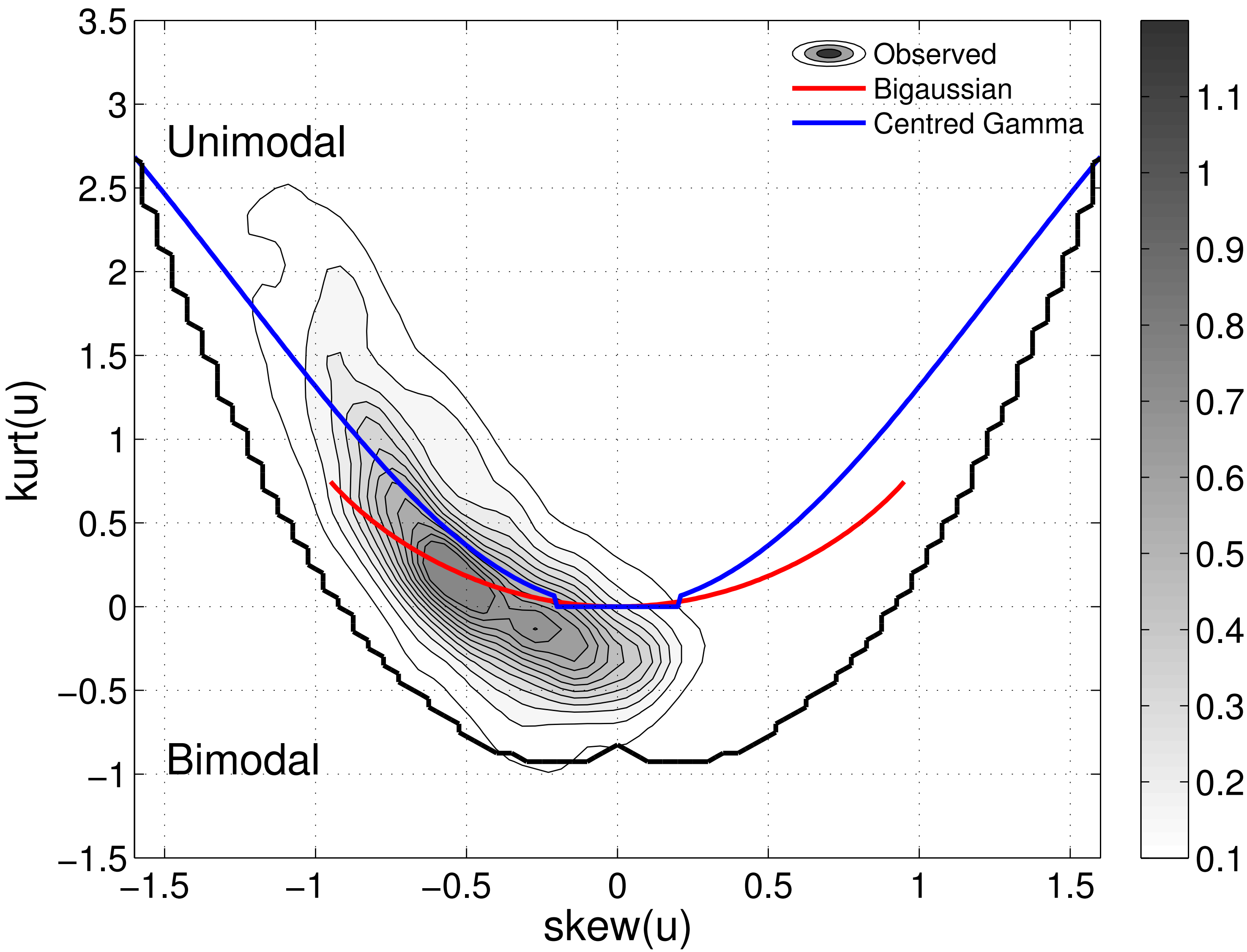
Skew(u) vs. Kurt(u)



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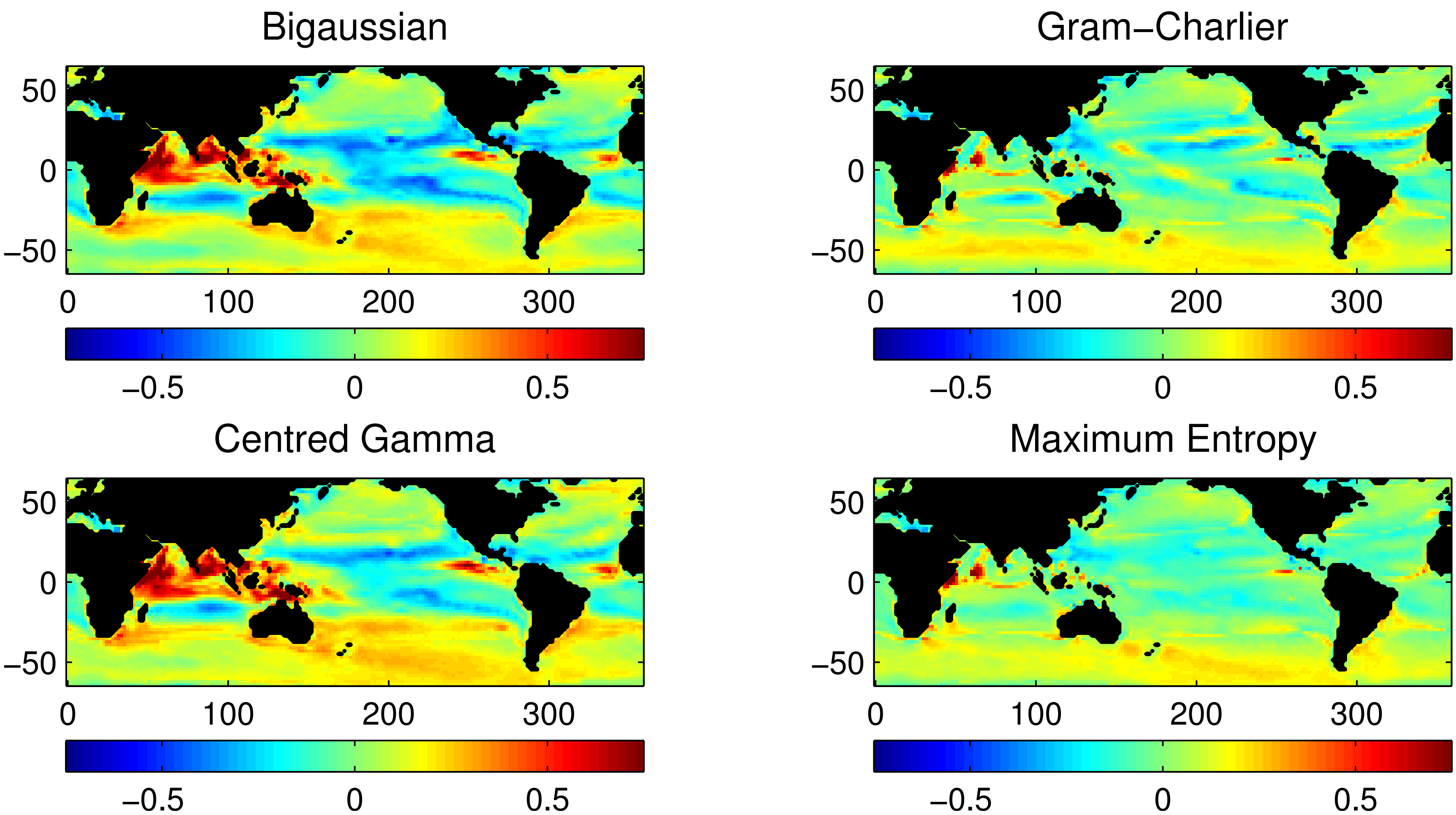
Empirical Wind Speed pdfs: Intercomparison

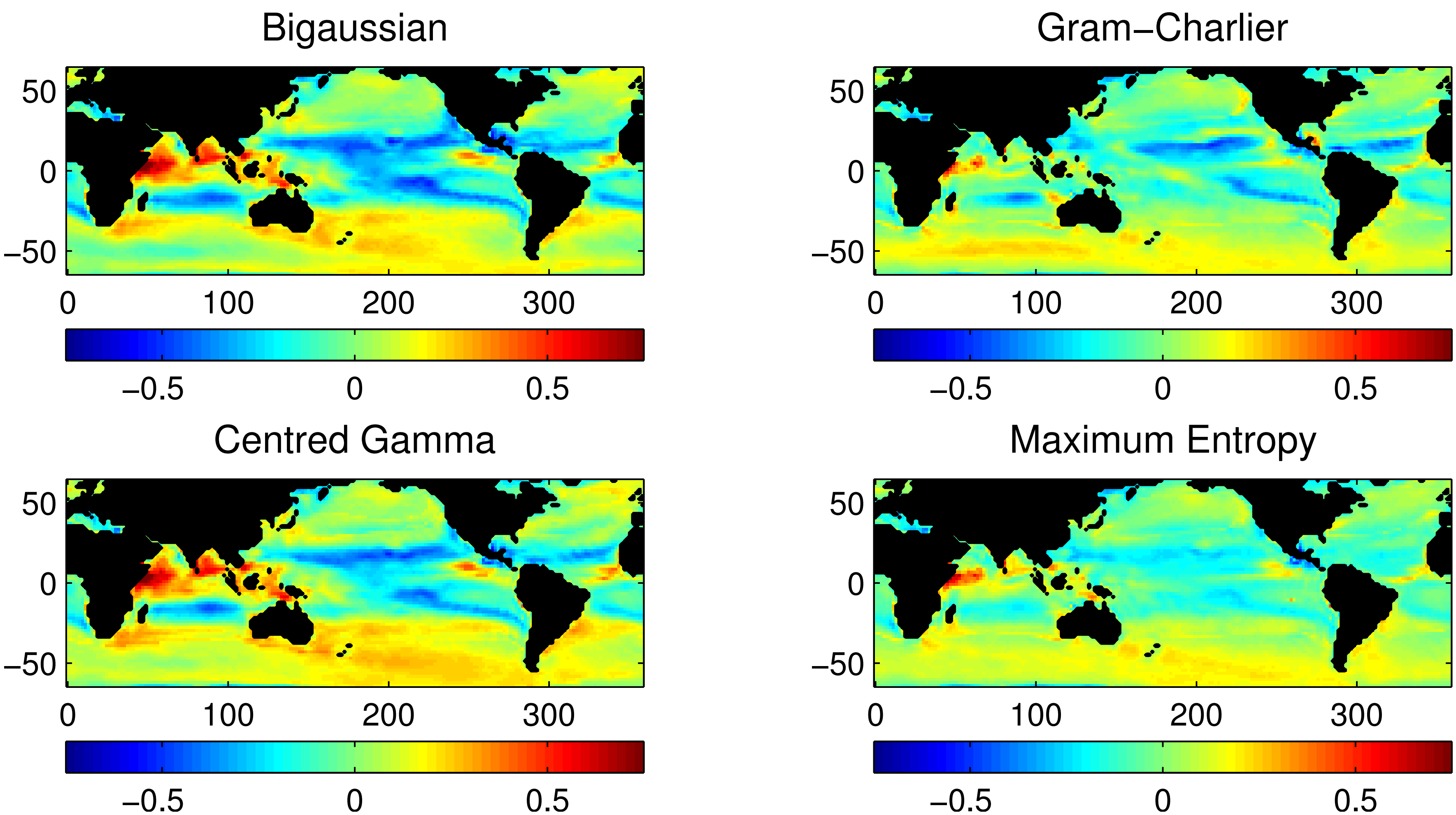
- Skew(u) and kurt(u) from observations



Modelled skew(w) - observed skew(w)

Empirical Wind Speed pdfs: Intercomparison

- Isotropic fluctuations



Modelled skew(w) - observed skew(w)

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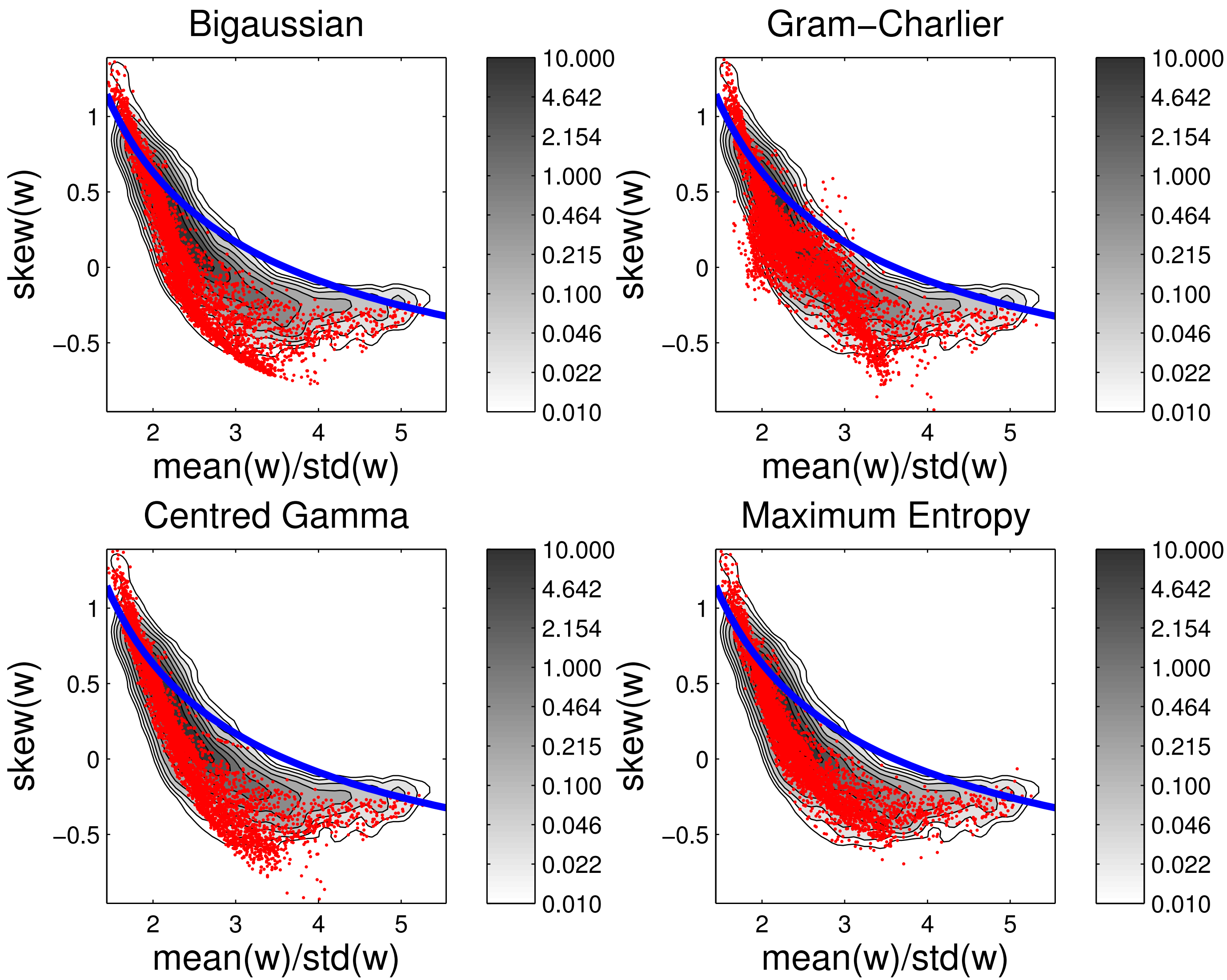
- Relationships between moments



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Wind Speed pdfs

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  can be parameterised in terms of

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- Relationship between moments can be found **empirically** or **mechanistically**

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Mechanistic Model

- Mechanistic model follows from boundary-layer dynamics

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- Horizontal momentum equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}' u_3'})}{\partial z}$$

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$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{c_d}{h} w\mathbf{u} + \frac{K}{h^2}(\mathbf{U} - \mathbf{u})$$

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Mechanistic Model: SDE

- Define:

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$$\begin{aligned} \Pi_u(t) &= \langle \Pi_u \rangle + \sigma \dot{W}_1(t) \\ \Pi_v(t) &= \sigma \dot{W}_2(t) \end{aligned}$$

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- Finally, we obtain stochastic differential equation

$$\begin{aligned} \frac{du}{dt} &= \langle \Pi_u \rangle - \frac{c_d}{h} wu - \frac{K}{h^2} u + \sigma \dot{W}_1 \\ \frac{dv}{dt} &= -\frac{c_d}{h} wv - \frac{K}{h^2} v + \sigma \dot{W}_2 \end{aligned}$$

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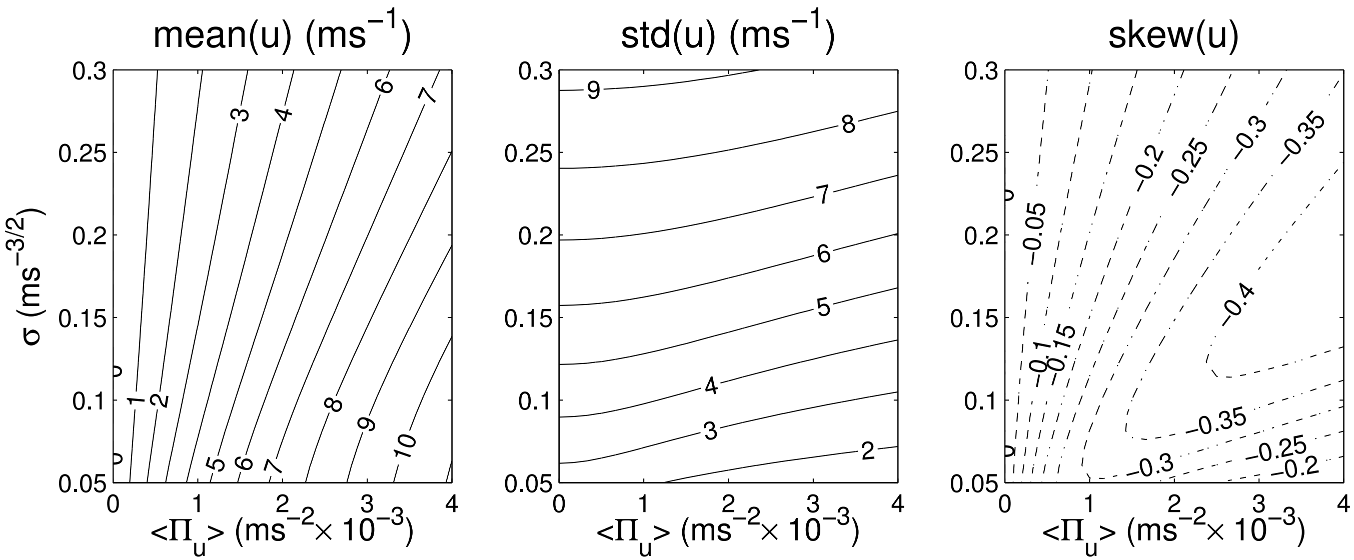
Mechanistic Model: pdf

- The stochastic differential equation has an associated Fokker-Planck equation for the stationary pdf, which yields the analytic solution:

$$p_{uv}(u,v) = \mathcal{N}_1 \exp\left(\frac{2}{\sigma^2}\left\{\langle \Pi_u \rangle u - \frac{K}{2h^2}(u^2+v^2) - \frac{1}{h}\int_0^{\sqrt{u^2+v^2}} c_d(w')w'^2\, dw'\right\}\right)$$

Mechanistic Model: Predictions



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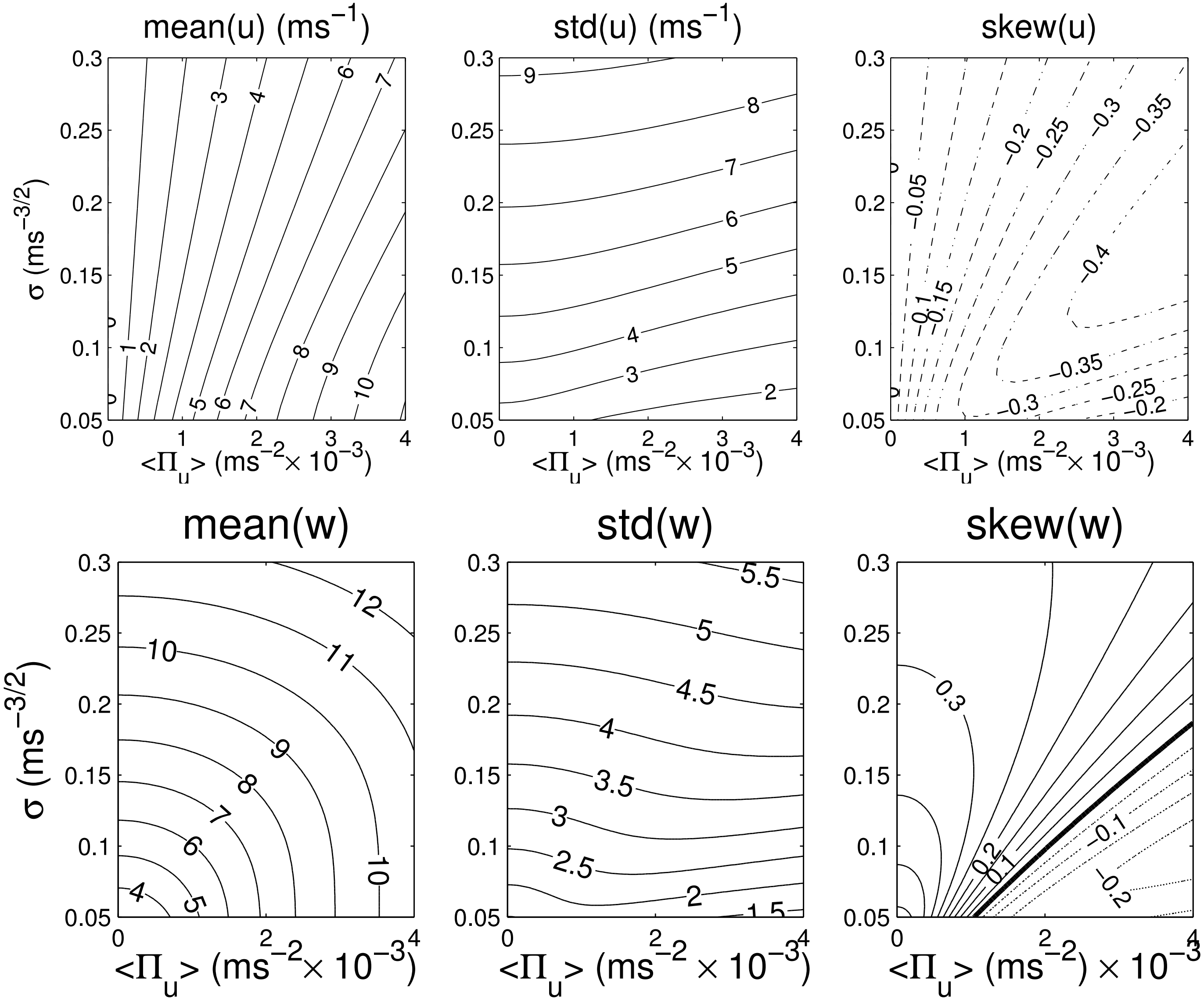
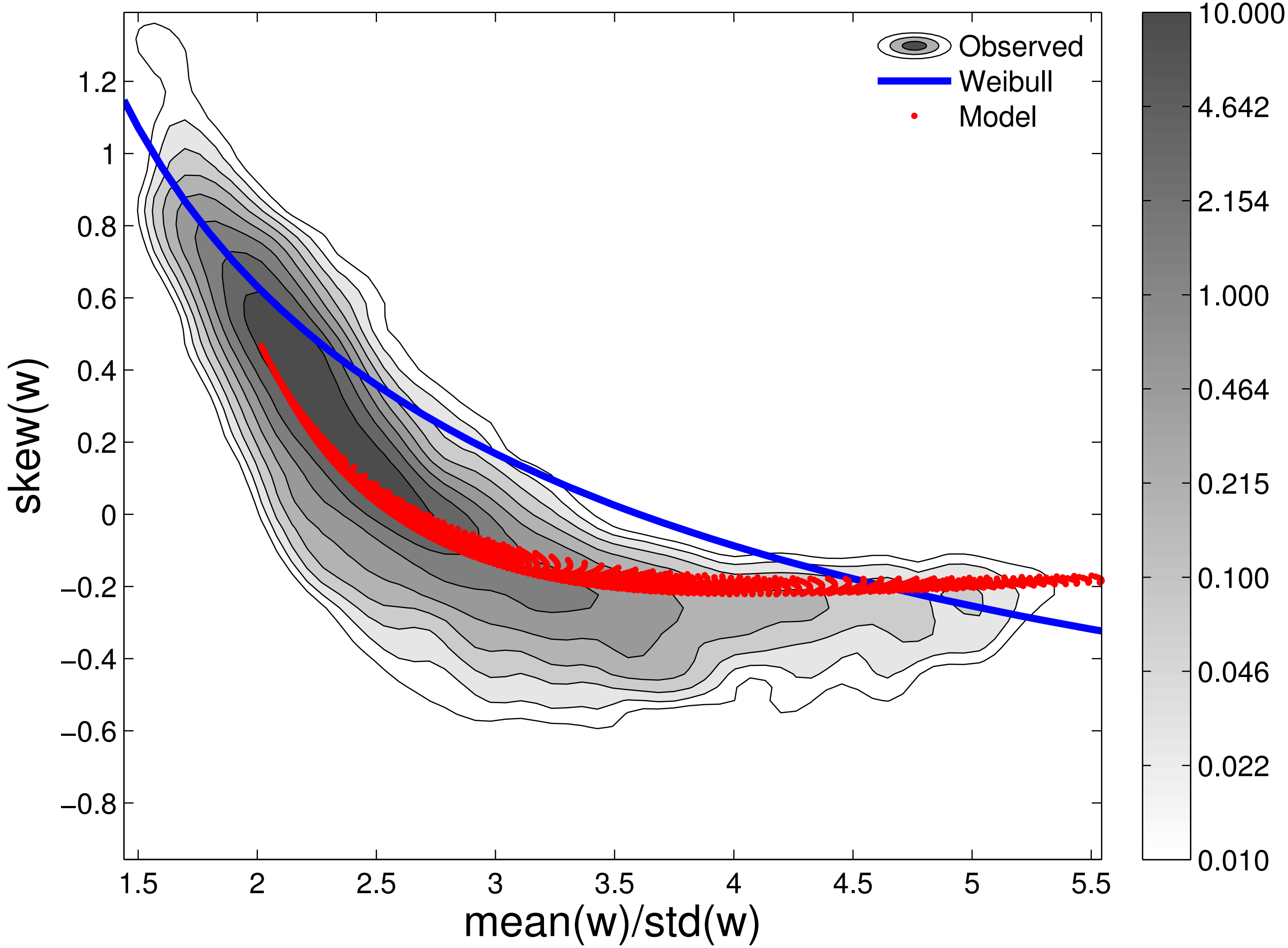


Mechanistic Model: Comparison with Observations



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- Isotropic, independent fluctuations
- Qualitative success of model suggests it has captured essential physics: still improvements to be made

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