The Impact of Stochastic Heat Fluxes on SST Variability and Atmosphere Ocean Coupling

Philip Sura

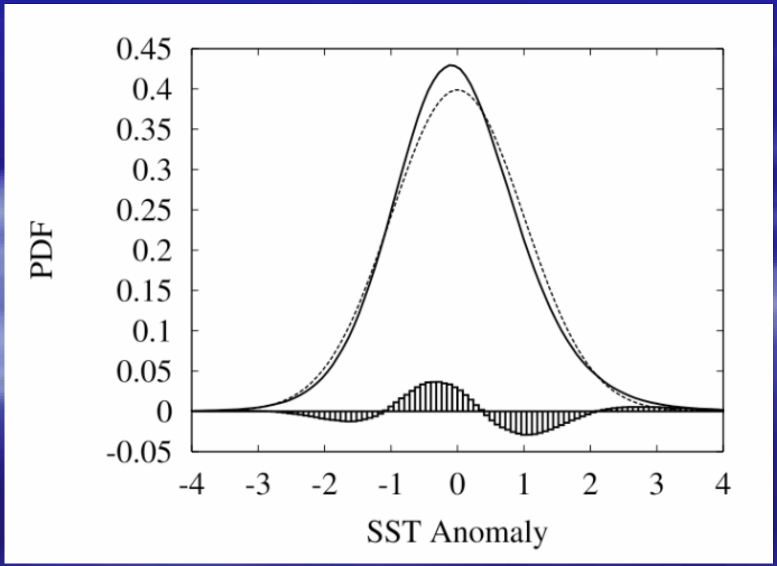
Matt Newman, Mike Alexander

NOAA-CIRES Climate Diagnostics Center

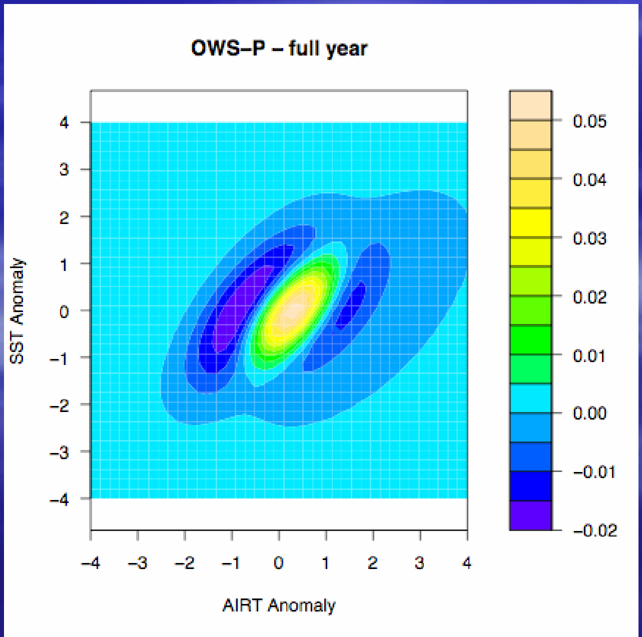
Introduction

- Daily SST/AIRT anomalies are non-Gaussian!
- Where does the non-Gaussianity comes from?
- How does the non-Gaussianity effect SST/AIRT variability?

1. The uncoupled problem



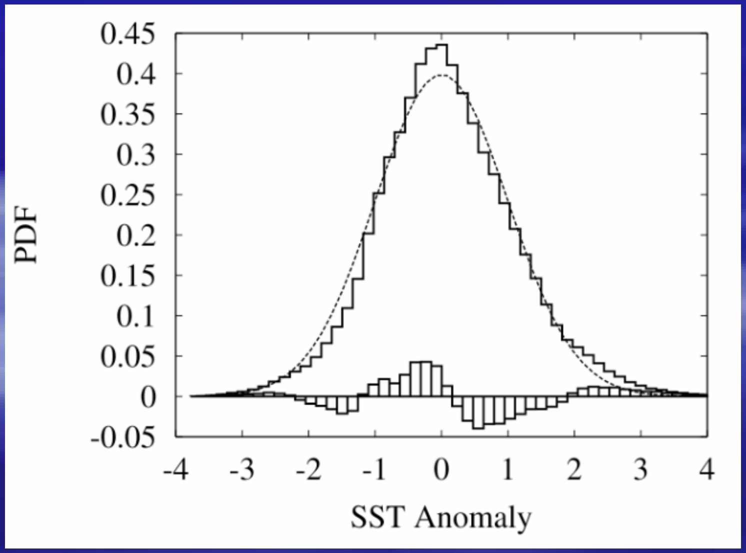
2. The coupled problem



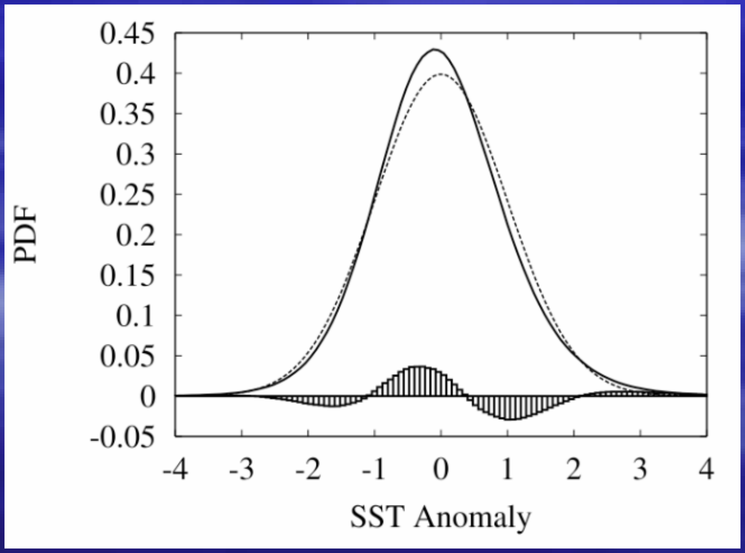
Multiplicative Noise and Non-Gaussian SST Variability

- Daily SST anomalies are non-Gaussian: Skew t-distribution.
- Where does the non-Gaussianity comes from?
- How does the non-Gaussianity effect SST variability?

PDFs of daily **SST anomalies** at OWS P (Gulf of Alaska)



Histogram



Parametric fit: Skew t

Modeling SST Anomalies

Neglecting many effects (advection, salinity, radiation) the heat budget equation for the SST is:

$$\frac{dT_o}{dt} = \frac{f(T_o, T_a, |\mathbf{U}|)}{h}$$

with the heat flux

$$f = \beta(T_a - T_o)|\mathbf{U}|$$

A Taylor expansion yields:

$$\frac{dT_o'}{dt} = \frac{f'}{h} + \frac{1}{h}\left\langle \frac{\partial f}{\partial T_o} \right\rangle T_o' + \frac{1}{h}\left(\frac{\partial f}{\partial T_o} \right)' T_o'$$

$$f' = \beta\left(\langle T_a \rangle - \langle T_o \rangle\right)|\mathbf{U}|'$$

$$\frac{\partial f}{\partial T_o} \equiv \left\langle \frac{\partial f}{\partial T_o} \right\rangle + \left(\frac{\partial f}{\partial T_o} \right)' = -\beta\langle |\mathbf{U}| \rangle - \beta|\mathbf{U}|'$$

Anomalous heat flux forcing:

$f'/h \approx 0.2$ K day^{-1}

Mean heat flux derivative:

$h^{-1}\langle \partial f / \partial T_o \rangle T_o' \approx 0.1$ K day^{-1}

Anomalous heat flux derivative:

$h^{-1}(\partial f / \partial T_o)' T_o' \approx 0.05$ K day^{-1}

Modeling SST Anomalies with Additive Noise

- Neglecting the anomalous heat flux derivative,
- Modeling the anomalous heat flux as white-noise yields:

$$\frac{dT_o'}{dt} = \frac{1}{h}\left\langle \frac{\partial f}{\partial T_o} \right\rangle T_o' + \frac{f'}{h}$$

$$\frac{dT_o'}{dt} = -\lambda T_o' + \eta_A$$

Frankignoul and Hasselmann (1977)

Pros:

- The red-noise spectrum is consistent with observations.

Cons:

- The PDF is strictly Gaussian and not consistent with observations.

Modeling SST Anomalies with Linear Multiplicative Noise

- Including the anomalous heat flux derivative,
- Modeling the anomalous heat flux and the anomalous heat flux derivative as white noise yields:

$$\frac{dT_o'}{dt} = \frac{1}{h}\left\langle \frac{\partial f}{\partial T_o} \right\rangle T_o' + \frac{1}{h}\left(\frac{\partial f}{\partial T_o} \right)' T_o' + \frac{f'}{h}$$

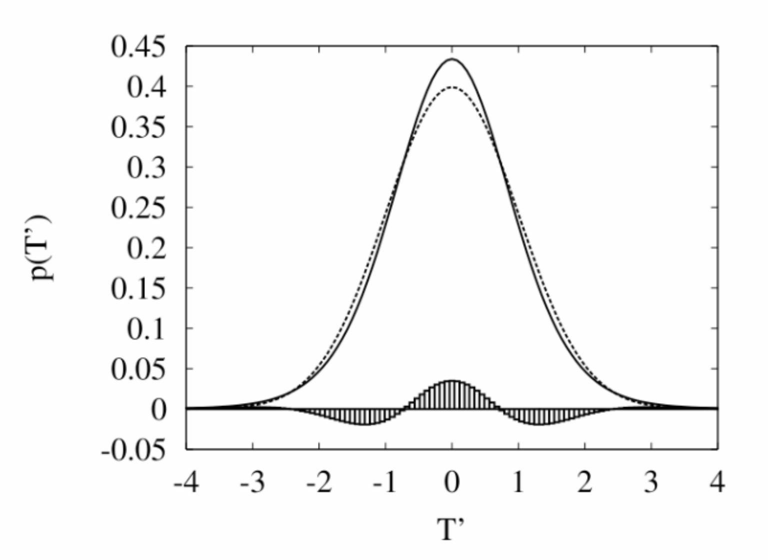
$$\frac{dT_o'}{dt} = -\lambda_{eff} T_o' + \eta_A + \eta_M T_o'$$

Pros:

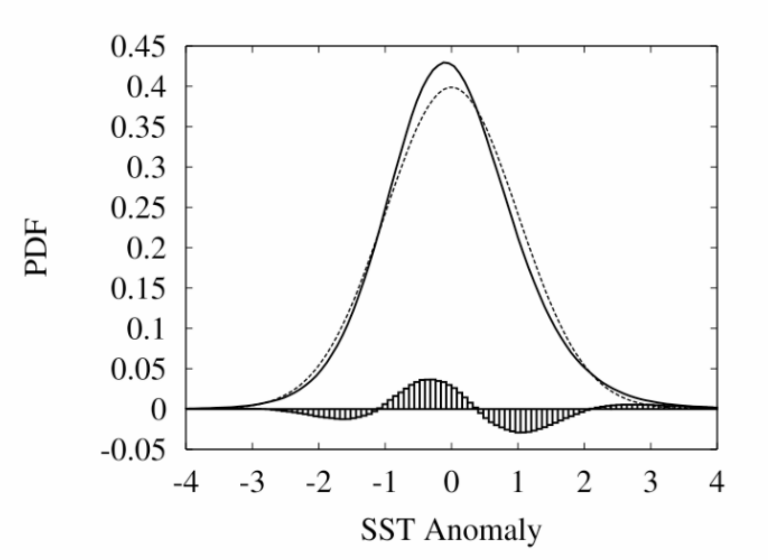
- The red-noise spectrum is consistent with observations.
- The PDF is almost consistent with observation.

Modeling SST Anomalies with Linear Multiplicative Noise

Model



Observations

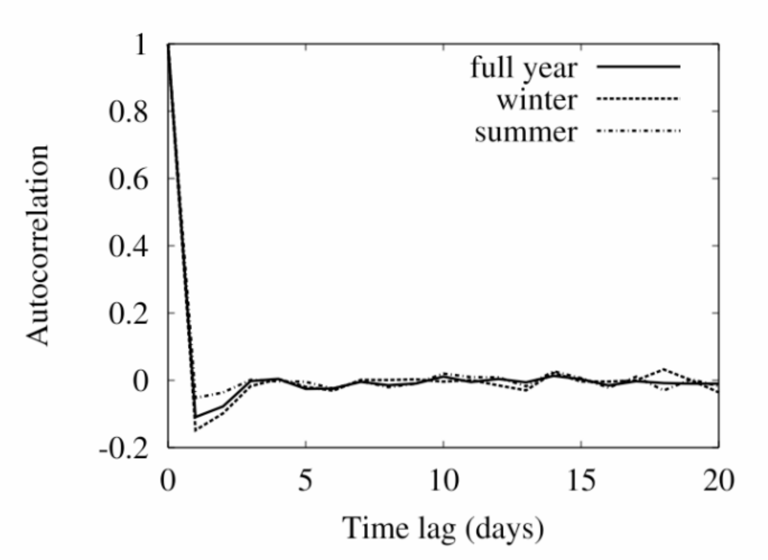


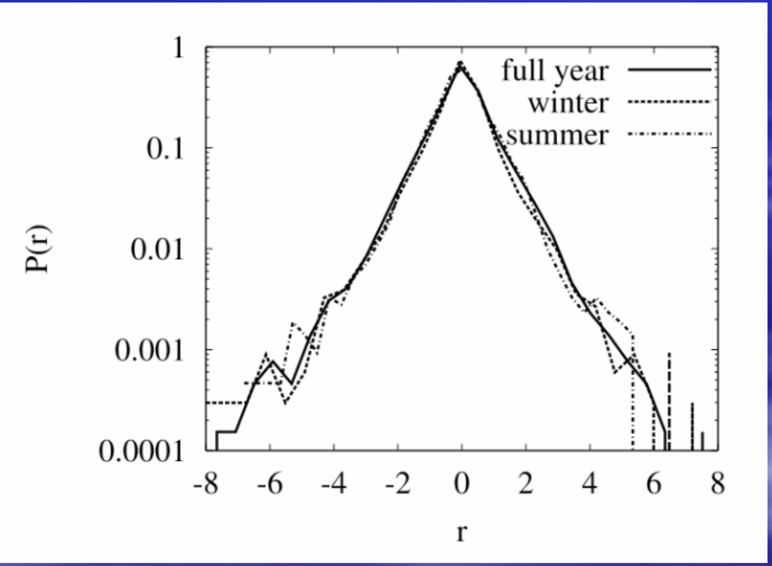
The anomalous heat flux derivative can explain the observed deviations (kurtosis) from Gaussianity.

Testing the White-Noise Approximation

$$T_o'(t+\Delta t) = -\lambda_{eff} T'_{o\,Obs}(t)\Delta t + T'_{o\,Obs}(t)$$

$$r \equiv T'_{o\,Obs}(t+\Delta t) - T_o'(t+\Delta t)$$





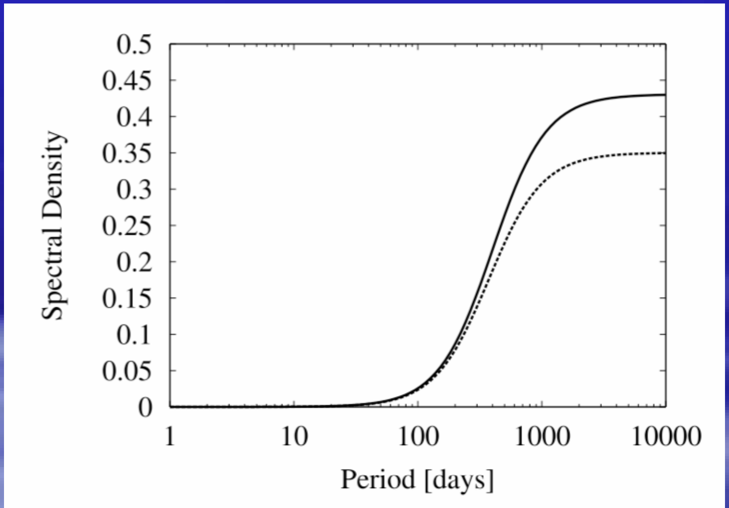
The residual is nearly uncorrelated and highly non-Gaussian on the resolved timescale. That is, the multiplicative white-noise approximation is justified.

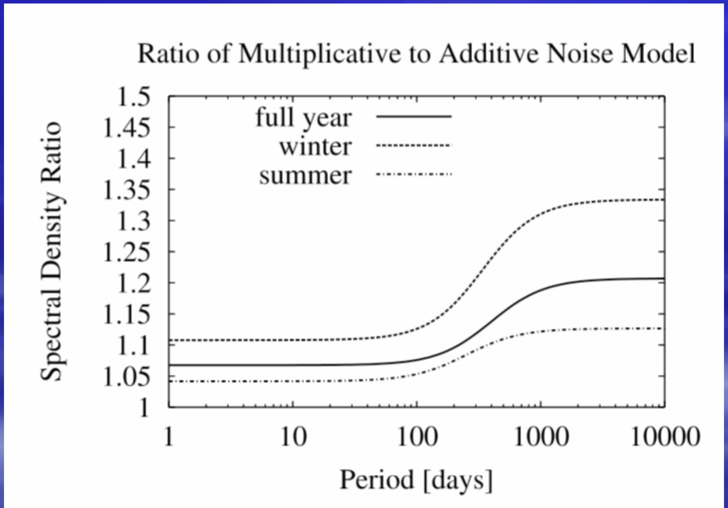
Impact of Linear Multiplicative Noise

The strength of the multiplicative noise is moderate:

$$\frac{\sigma_M}{\sigma_A} \approx 0.3 \qquad \frac{\lambda_{eff}}{\lambda} \approx 0.9$$

Does it matter?





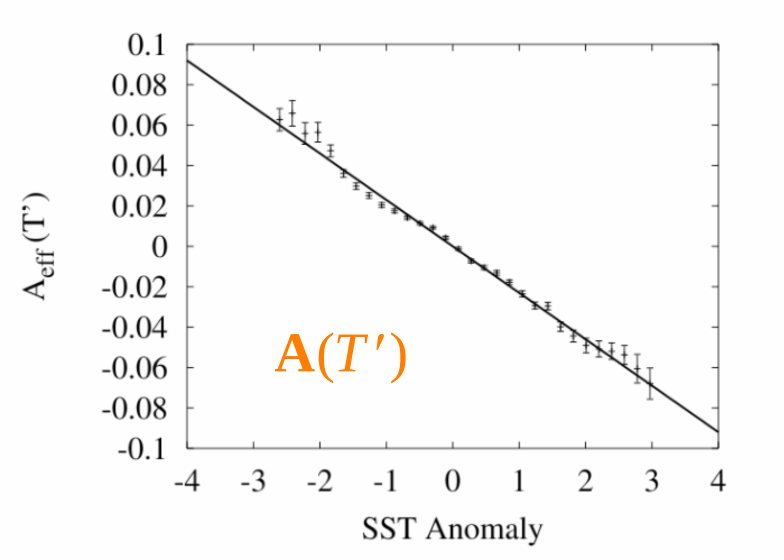
Multiplicative noise enhances low-frequency Anomalous SST variability by about 10%-30%.
Multiplicative noise has an impact.

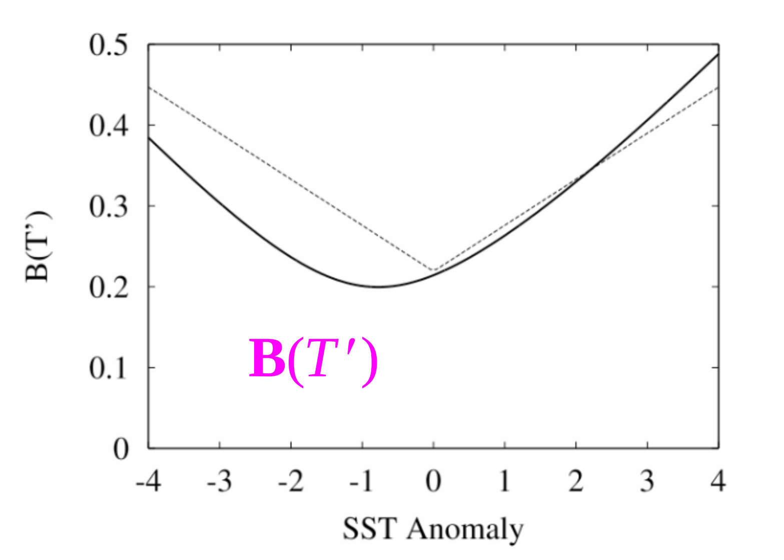
Nonlinear Inverse Modeling of SST Anomalies

Next we study the nonlinear inverse problem:

- Does nonlinear inverse modeling confirm our linear results?
- Does nonlinear inverse modeling improve our results?

$$\frac{dT_o'}{dt} = \mathbf{A}(T') + \mathbf{B}(T')\eta$$



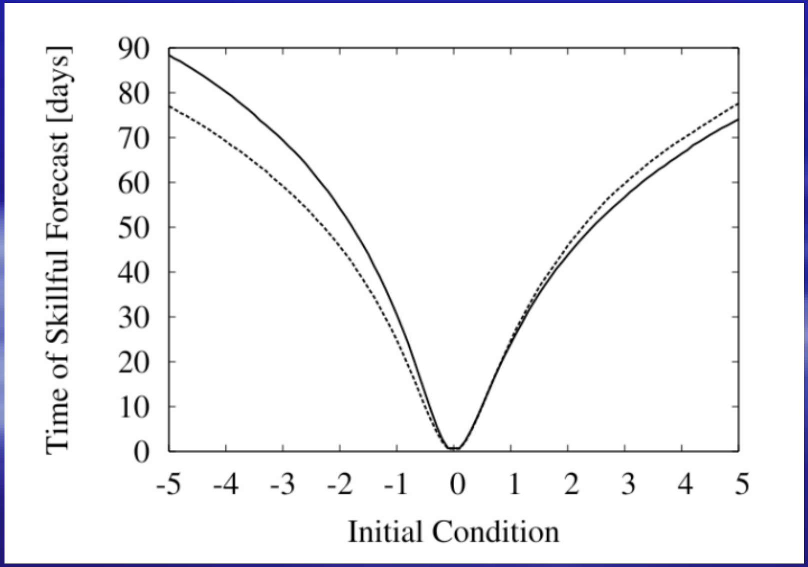


Impact of Nonlinear Multiplicative Noise

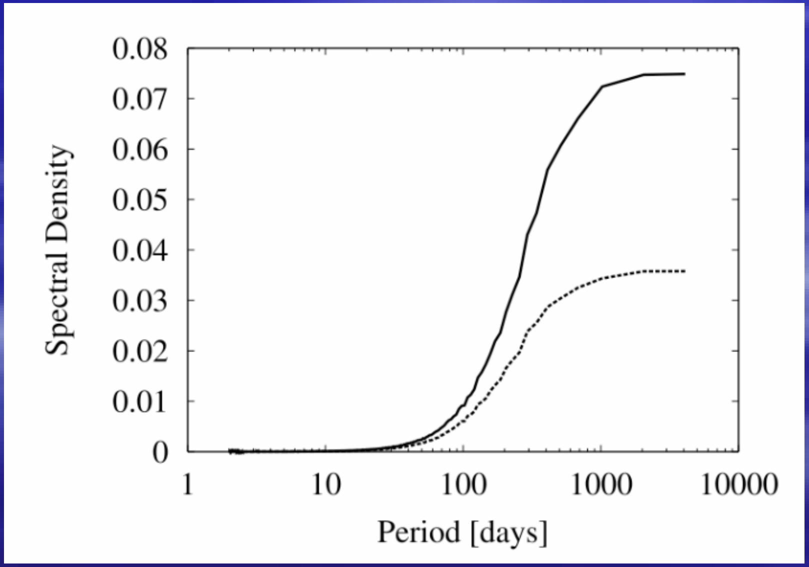
Nonlinear multiplicative noise :

- Changes SST predictability (left).
- Enhances low-frequency anomalous SST variability (right).

Predictability

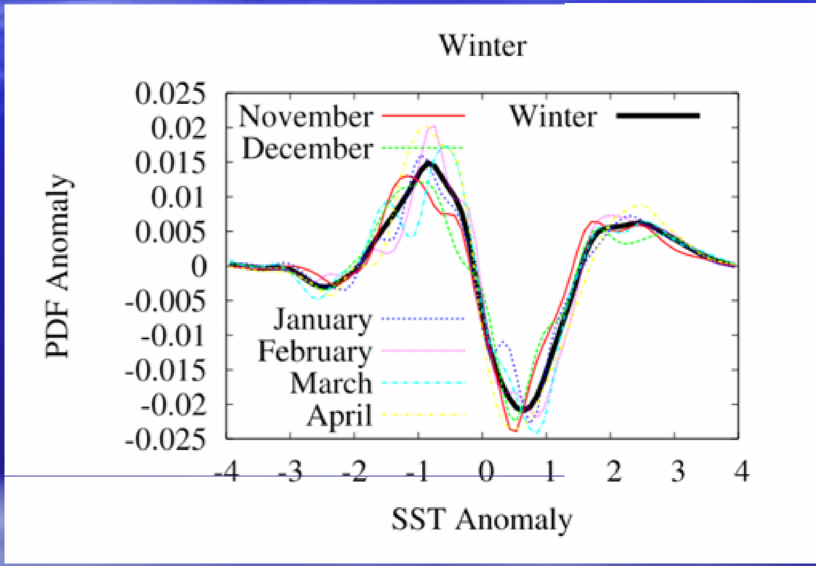


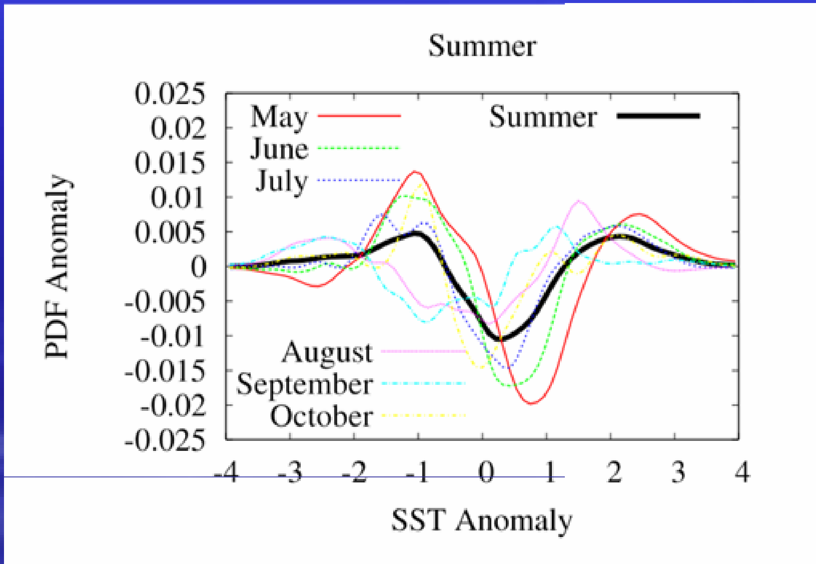
Spectra



PDFs in a Mixed Layer Model (MLM)

PDF anomalies of normalized daily SST anomalies for extended winter (left) and extended summer (right).



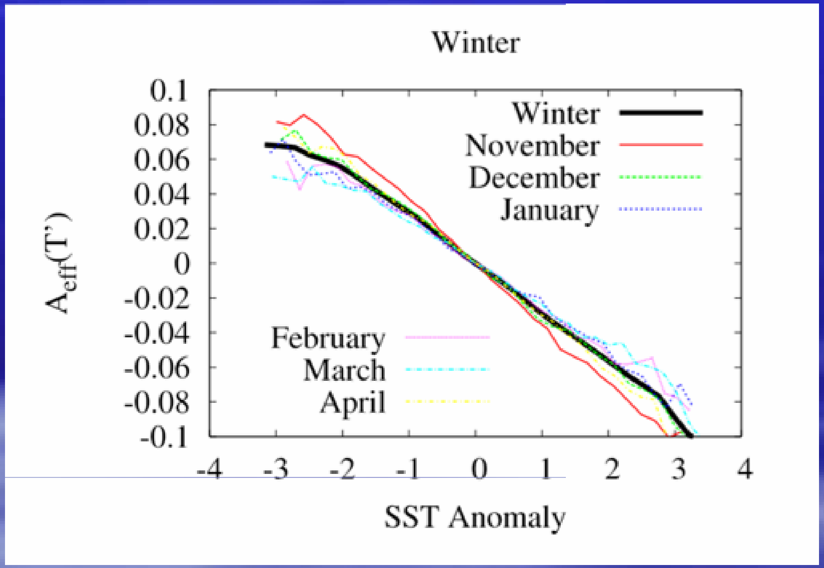


- MLM simulates the PDFs of SST anomalies well throughout the extended winter.
- MLM does a poorer job during the extended summer.

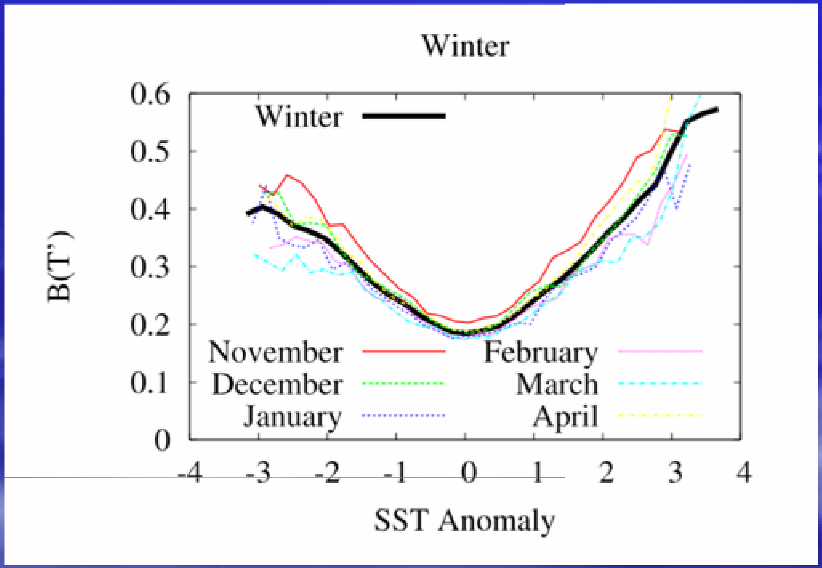
Drift and Noise in the MLM

$$\frac{dT_o'}{dt} = \mathbf{A}(T') + \mathbf{B}(T')\eta$$

A(T')



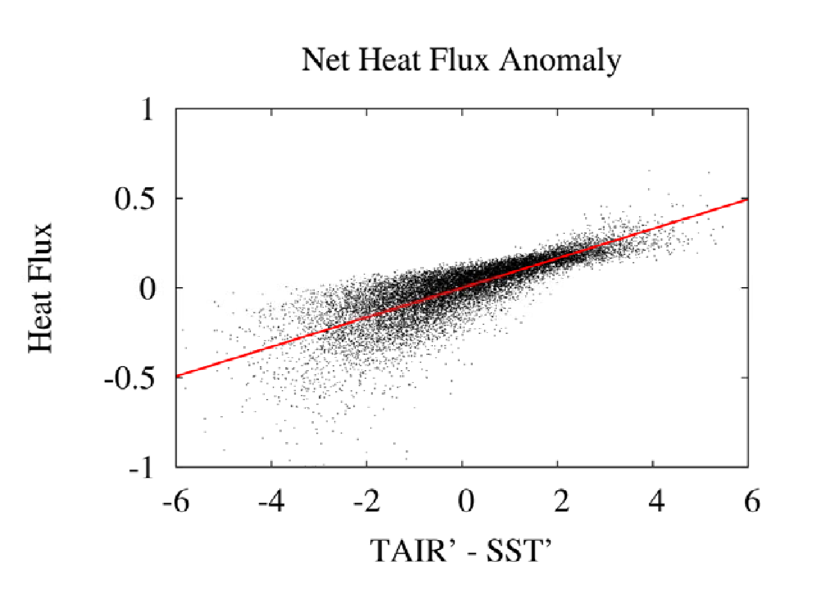
B(T')



The effective drift and the structure of the noise is generally similar to the stochastic model constructed from observations.

Multiplicative Noise in a Complex Model

Nonlinear inverse model is consistent with ocean model.



The skew is induced by:

- The mean air-sea temperature difference.
- The stability dependence of the bulk transfer coefficient.
- The nonlinearity of the Clausius-Clapeyron equation.

The high-frequency variability of boundary layer winds and related heat fluxes are crucial to model SST variability.

Modeling Atmosphere-Ocean Coupling

We now couple the ocean to the atmosphere through the heat flux, and add a simple radiative damping:

$$\gamma_a \frac{dT_a}{dt} = f(T_a, T_o, |\mathbf{U}|) - \lambda_a T_a$$
$$\gamma_o \frac{dT_o}{dt} = -f(T_a, T_o, |\mathbf{U}|) - \lambda_o T_o$$

with the heat flux $f = \beta(T_a - T_o)|\mathbf{U}|$

A Taylor expansion yields:

$$\frac{d\mathbf{T}'}{dt} = \mathbf{A}\mathbf{T}' + \mathbf{B}_M(\mathbf{T}')\eta_M + \mathbf{B}_A\eta_A$$

with

$$\mathbf{A} = \begin{pmatrix} -a & b \\ c & -d \end{pmatrix}$$

$$\mathbf{B}_M(\mathbf{T}') = \begin{pmatrix} B_{11}(T_o' - T_a + \Pi) & 0 \\ B_{21}(T_a' - T_o + \Pi) & 0 \end{pmatrix}$$

$$\Pi = \langle T_o \rangle - \langle T_a \rangle$$

(Barsugli and Battisti with multiplicative noise)

Drift and Noise in Coupled Model

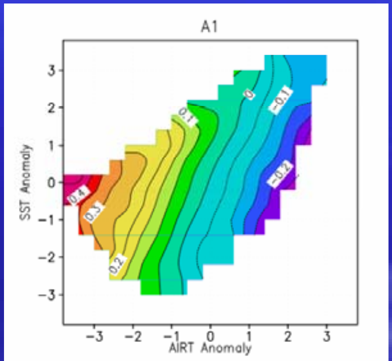
Parameters of coupled model can be estimated from data:

Effective Drift

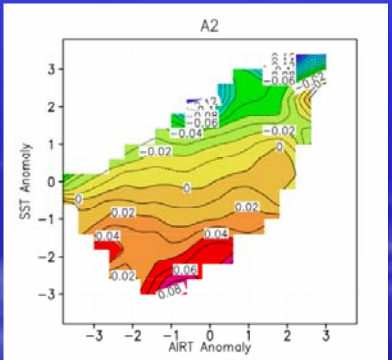
is indeed almost linear

$$\mathbf{A} = \begin{pmatrix} -0.39 & 0.11 \\ 0.02 & -0.06 \end{pmatrix}$$

AIRT-Component



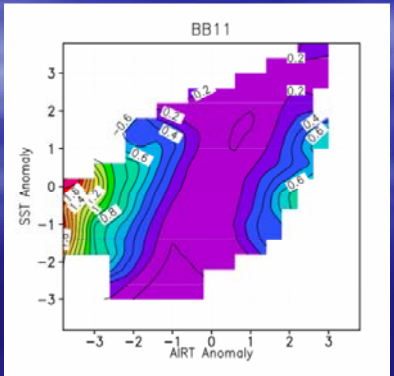
SST-Component



Multiplicative Noise

$$\mathbf{B}_M(\mathbf{T'}) = \begin{pmatrix} 0.3(T_o' - T_a + 0.74) & 0 \\ 0.03(T_a' - T_o - 0.74) & 0 \end{pmatrix}$$

$(BB^T)_{11}$-Component

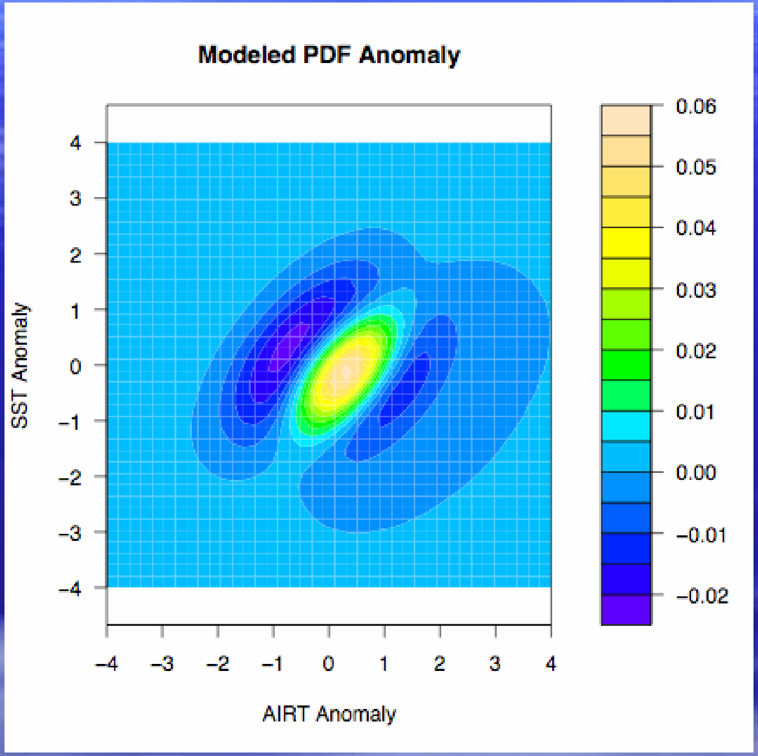


Additive Noise

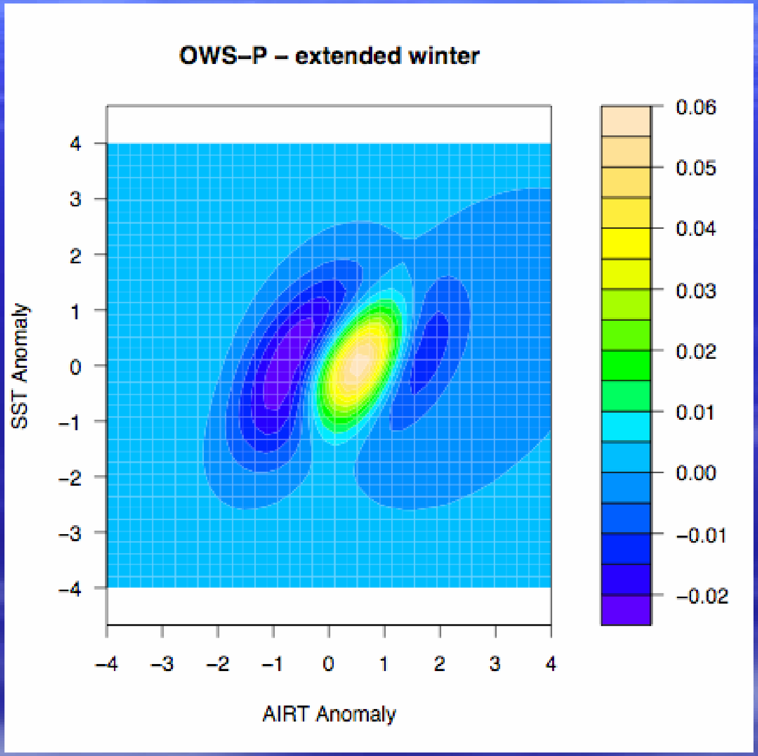
$$\mathbf{A}_A \mathbf{A}_A^T = -\mathbf{A}\mathbf{C}_o - \mathbf{C}_o\mathbf{A} - \left\langle \mathbf{B}_M \mathbf{B}_M^T \right\rangle$$

Joint PDFs (anomalies)

Model



Observations

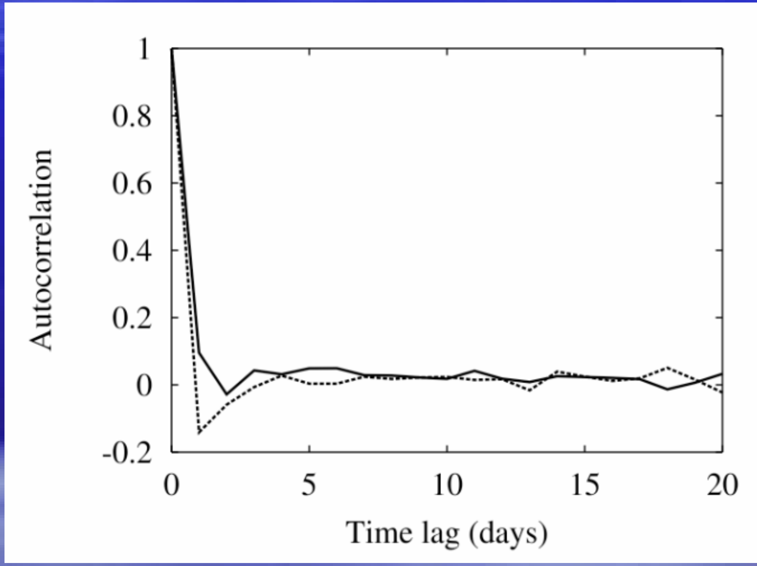


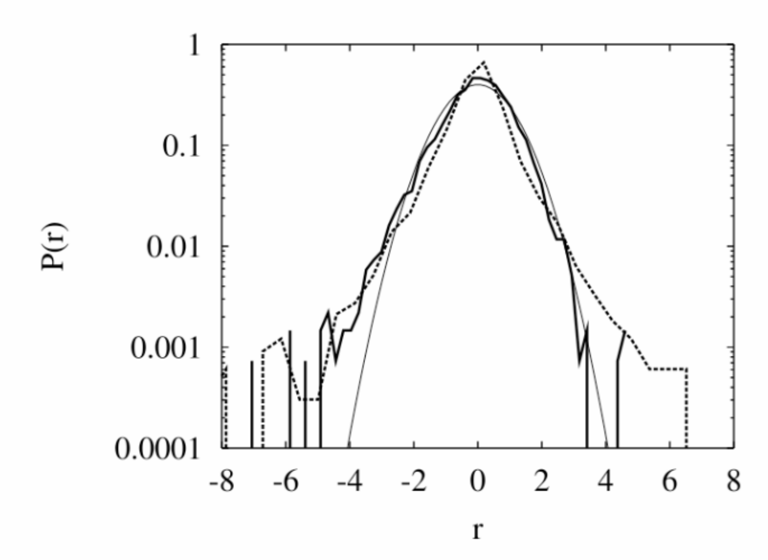
The state-dependent (multiplicative) anomalous (stochastic) heat flux can explain the observed deviations from Gaussianity.

Testing the White-Noise Approximation

$$\mathbf{T}'(t+\Delta t) = -\mathbf{A}\mathbf{T}'_{Obs}(t)\Delta t + \mathbf{T}'_{Obs}(t)$$

$$\mathbf{r} \equiv \mathbf{T}'_{Obs}(t+\Delta t) - \mathbf{T}'(t+\Delta t)$$

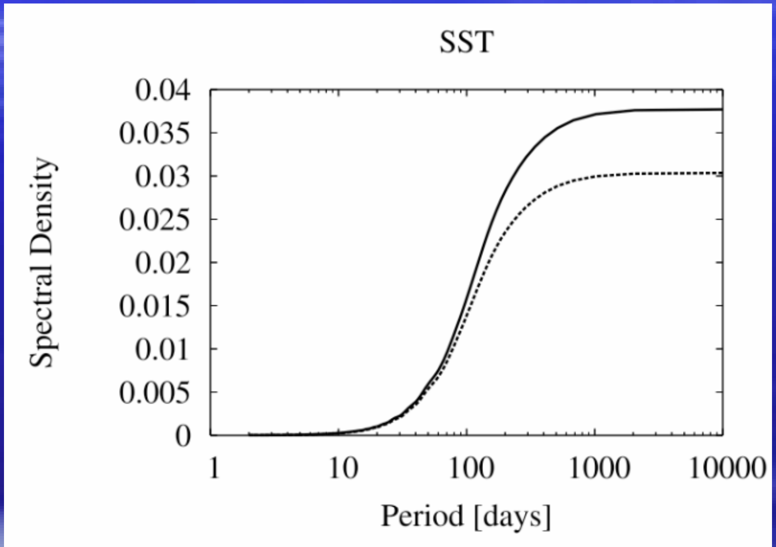


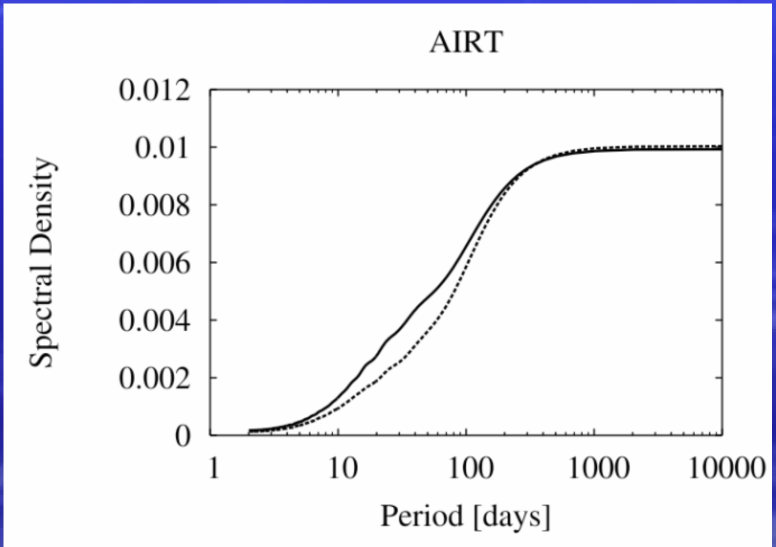


The residual is nearly uncorrelated and highly non-Gaussian on the resolved timescale. That is, the multiplicative white-noise approximation is justified.

Impact of Multiplicative Noise

No multiplicative noise: Reduced overall variance

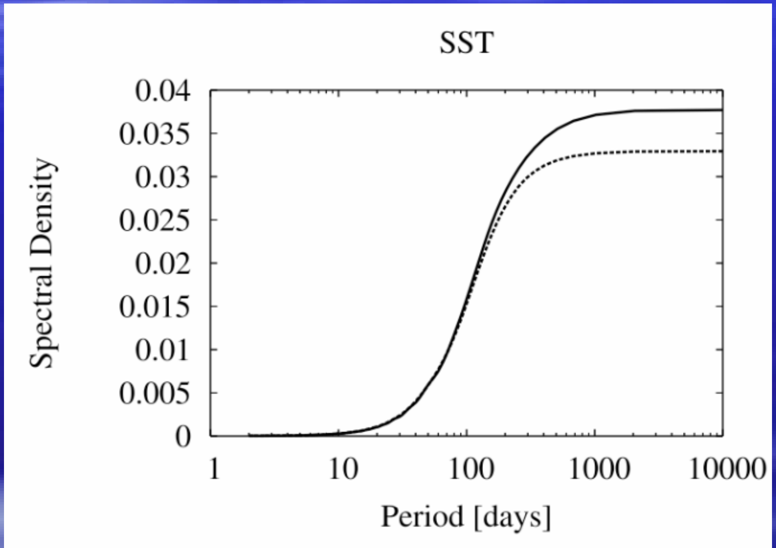


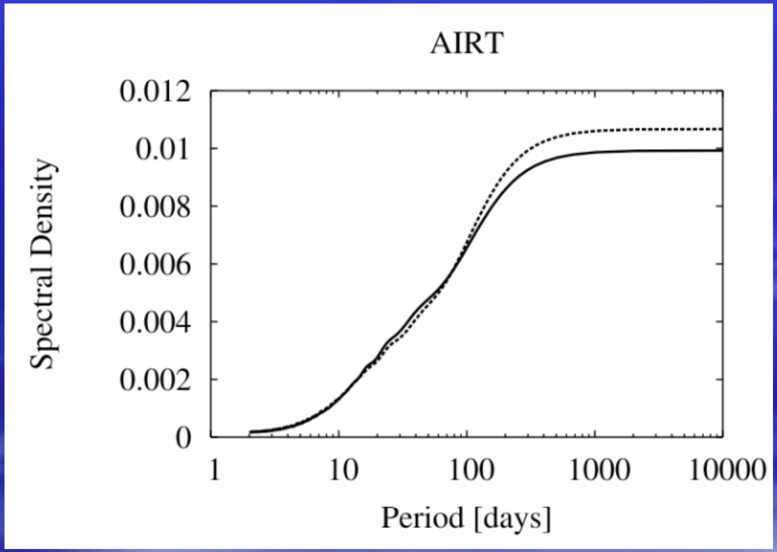


Multiplicative noise (noise induced drift) changes spectral response of anomalous SST/AIRT variability. Multiplicative noise has an impact.

Impact of Multiplicative Noise

No multiplicative noise: Retain overall variance





Multiplicative noise (noise induced drift) changes spectral response of anomalous SST/AIRT variability. Multiplicative noise has an impact.

Summary and Conclusions

- PDFs of SST/AIRT anomalies are non-Gaussian.
- Simple extension of Frankignoul and Hasselmann (1977) reproduces observed PDFs: Uncoupled and coupled.
- The high-frequency variability of boundary layer winds and related heat fluxes are crucial to model SST variability and local atmosphere-ocean coupling.
- A coupled model with incorrect atmospheric variability might incorrectly estimate SST variability and atmosphere-ocean coupling.