

# Estimating Atmospheric Response Using Fluctuation-Response Based Methods

Grant Branstator, NCAR

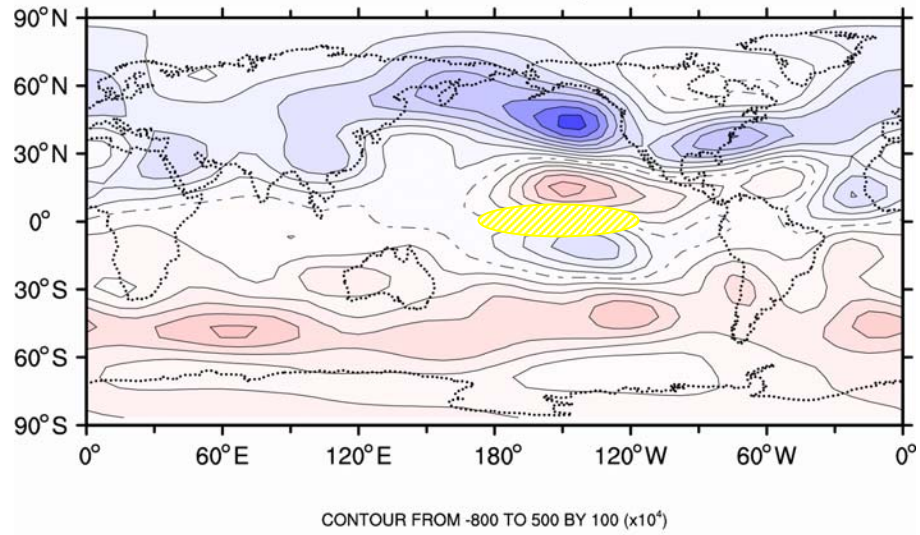
Andrey Gritsun, INM/RAS

- Linear response operator,  $L$

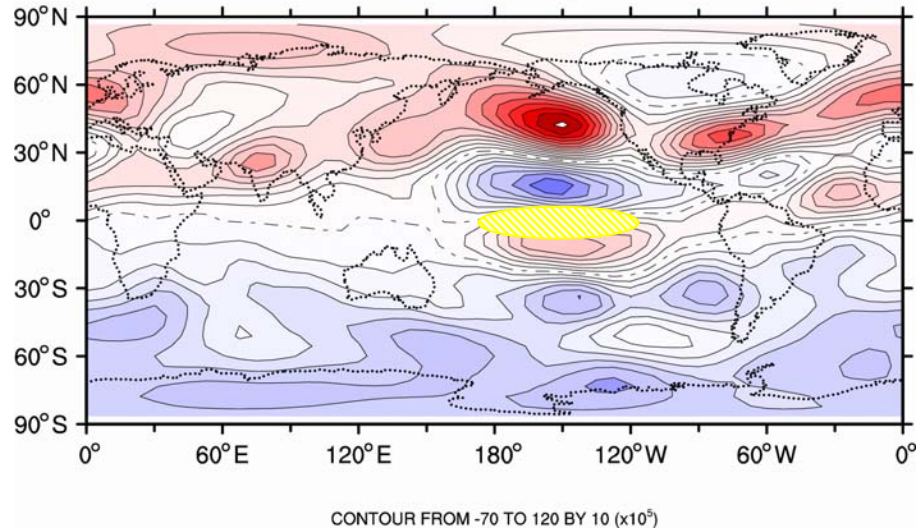
$$Lr' = s'$$

- Why not use an AGCM?
- Is a linear operator of any value?

observed warm Pacific psi 300 DJF



observed cold Pacific psi 300 DJF



$$\frac{\partial \zeta}{\partial t} = -\vec{v}_\psi \cdot \nabla(\zeta + f) - \dots$$

$$() = \frac{1}{T} \int_0^T () dt + ()'$$

$$\frac{\partial \zeta'}{\partial t} = -\bar{\vec{v}}_\psi \cdot \nabla(\bar{\zeta} + f) - \bar{\vec{v}}_\psi \cdot \nabla \zeta' - \vec{v}'_\psi \cdot \nabla \bar{\zeta} - \vec{v}'_\psi \cdot \nabla \zeta' \dots$$

$$\bar{\vec{v}}_\psi \cdot \nabla(\bar{\zeta} + f) = -\overline{\vec{v}'_\psi \cdot \nabla \zeta'} + \dots$$

$$\frac{\partial \zeta'}{\partial t} = -\bar{\vec{v}}_\psi \cdot \nabla \zeta' - \vec{v}'_\psi \cdot \nabla \bar{\zeta} - (\vec{v}'_\psi \cdot \nabla \zeta' - \overline{\vec{v}'_\psi \cdot \nabla \zeta'}) \dots$$

$$= -\bar{\vec{v}}_\psi \cdot \nabla \zeta' - \vec{v}'_\psi \cdot \nabla \bar{\zeta} + \text{damping} + \text{noise} \dots$$

$$\frac{\partial \zeta'}{\partial t} = (A + T)\zeta' + r'$$

$T\zeta'$



## Linear Inverse Model

Assume the observed system is linear, Markovian.

$$\frac{\partial s}{\partial t} + As = \epsilon$$

Then

$$C(t) = Cov(s; t) = \exp(-At)Cov(s; 0) = \exp(-At)C(0)$$

$$A = -\ell n[C(t)C(0)^{-1}]$$

Ensemble averaged forced response will be

$$\frac{\partial \langle s \rangle}{\partial t} + A \langle s \rangle = r$$

Steady response to steady forcing  $r_0$

$$\langle s \rangle = A^{-1}r_0$$

## Modified Linear Inverse Model (= FDT)

Assume the observed system is linear, Markovian

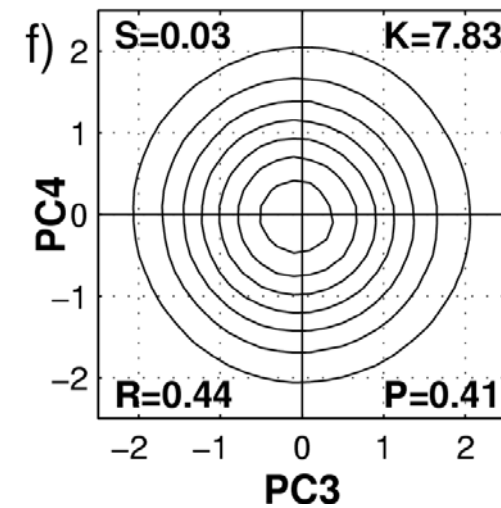
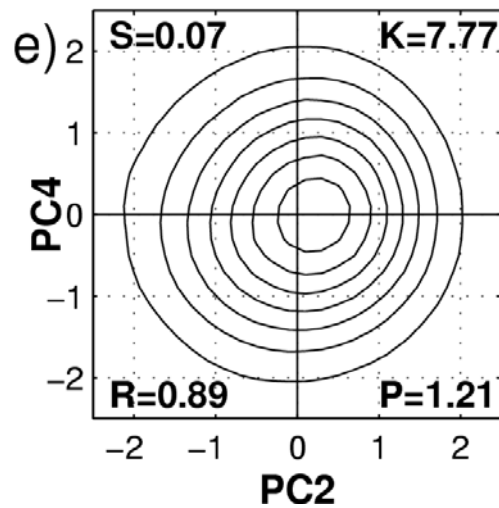
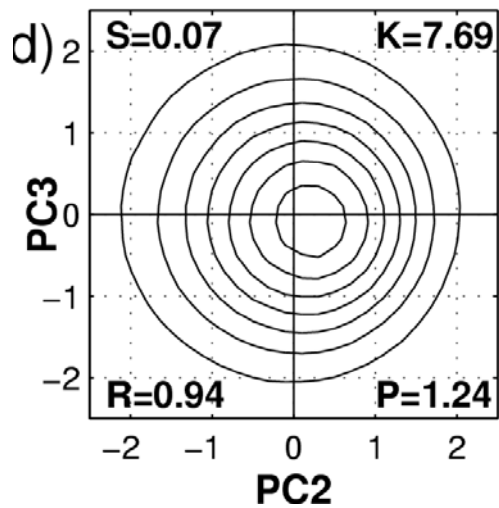
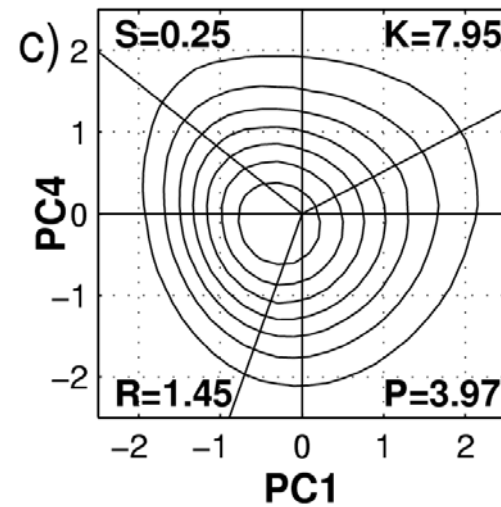
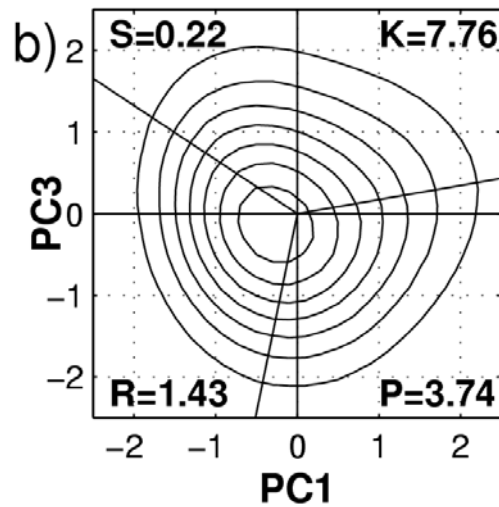
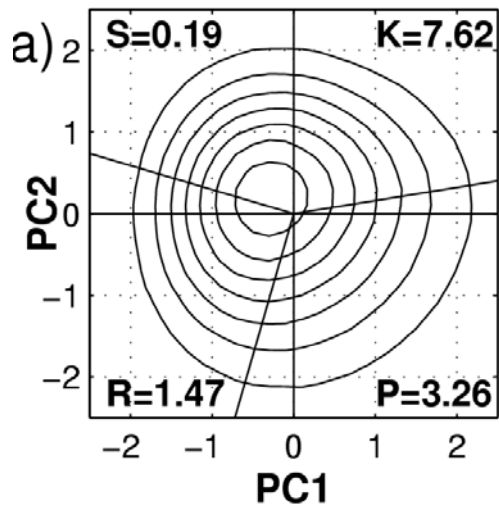
$$\frac{\partial s}{\partial t} + As = \epsilon$$

Then  $C(t) = \exp(-At)C(0)$

$$\text{So, } A^{-1} = \int_0^{\infty} C(t)C(0)^{-1}dt$$

And steady response to steady forcing  $r_o$

$$s = A^{-1}r_o$$



**Berner & Branstator (2006)**

$$L = \int_0^{\infty} C(t)C^{-1}(0)dt$$

## Application

Atmospheric general circulation model (NCAR's CCM0)

Primitive equations, circa 1980 physical parameterizations

Perpetual January, fixed boundary conditions

R15  
9 level } 18352 degrees of freedom

8 million 12hrly simulated states

# Reduce Dimensionality

## 1. Pick fields from

\* ps

 \* psi x 9

\* chi x 9

 \* T x 9

\* water vapor mixing ratio x 9

## 2. Truncate each field using EOFs

\* psi 100x9 (>90%)

\* T 496x9 (100%)

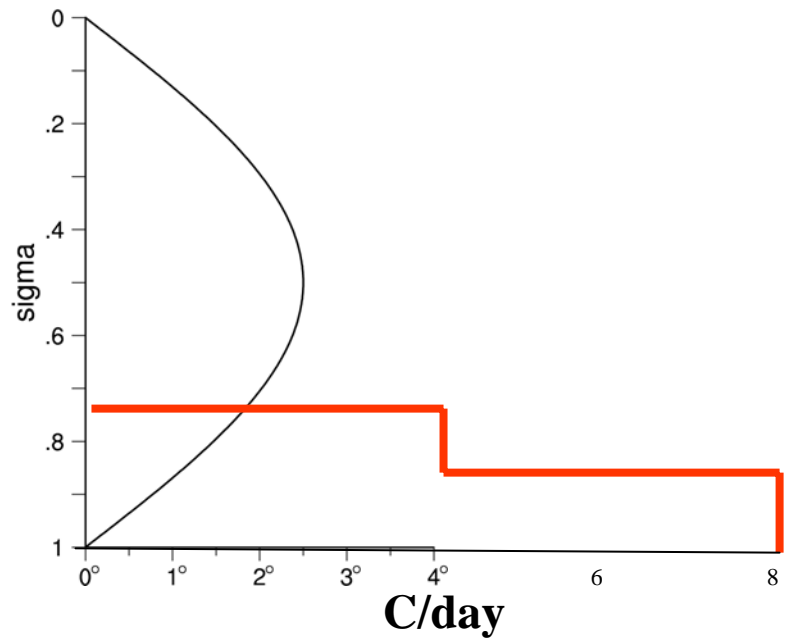
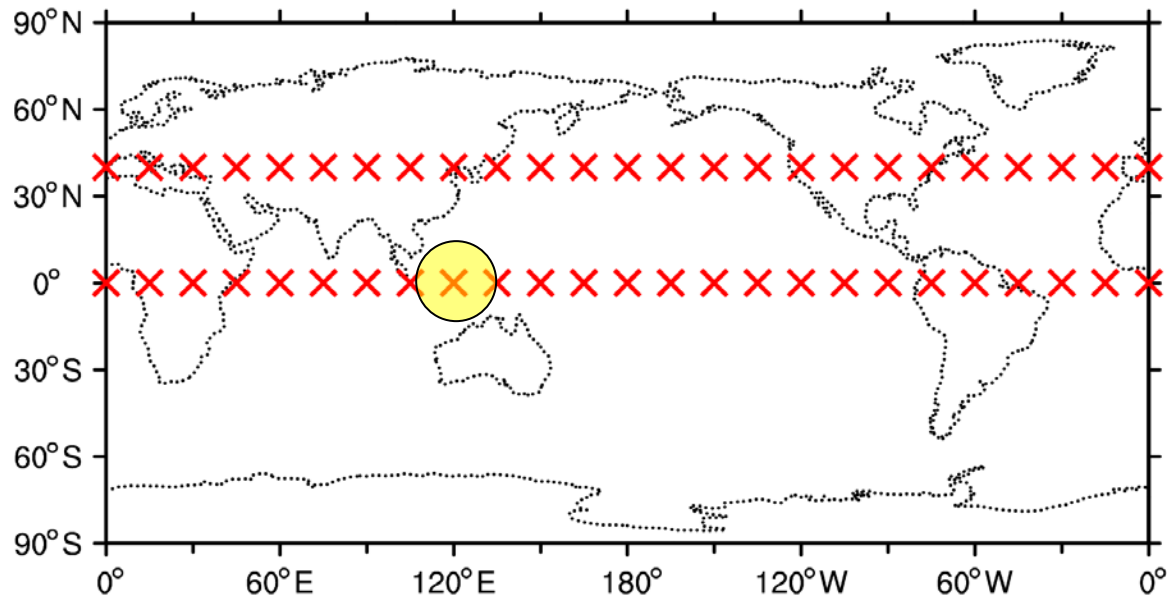
## 3. Form multivariate (truncated) fields,

normalize by std dev, giving temperature triple weighting

calculate EOFs

truncate (1800 EOFs, >95%)

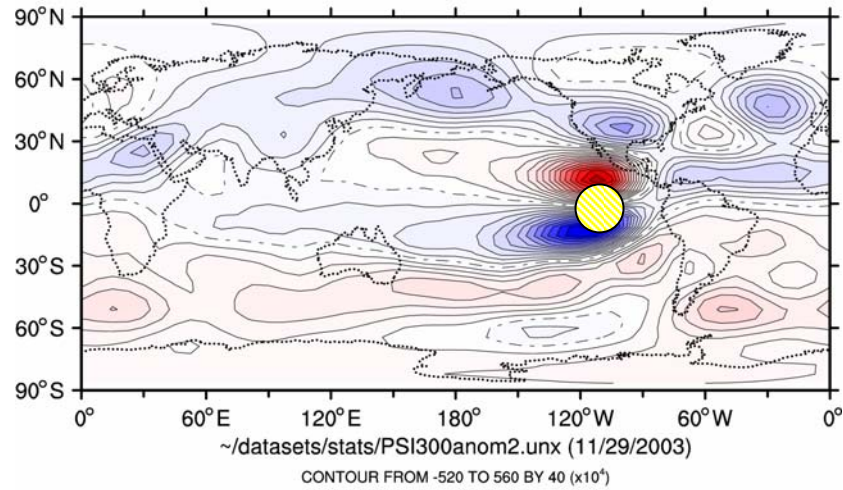




### 300mb PSI

Anomaly +2.5deg 105W

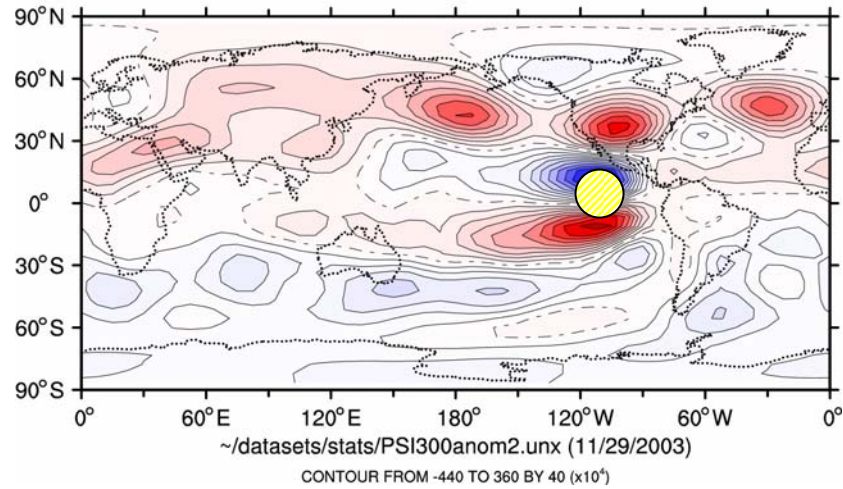
/MAI/DYNAMIC/FF20xx - /MAI/PJ300PAVG.100000



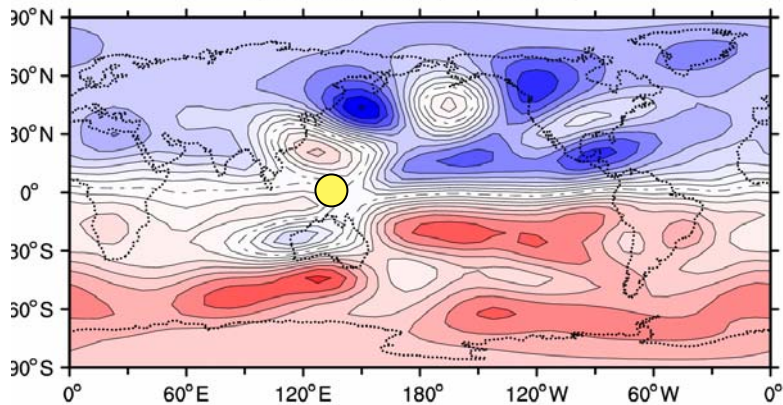
### 300mb PSI

Anomaly -2.5deg 105W

/MAI/DYNAMIC/J85xxx - /MAI/PJ300PAVG.100000

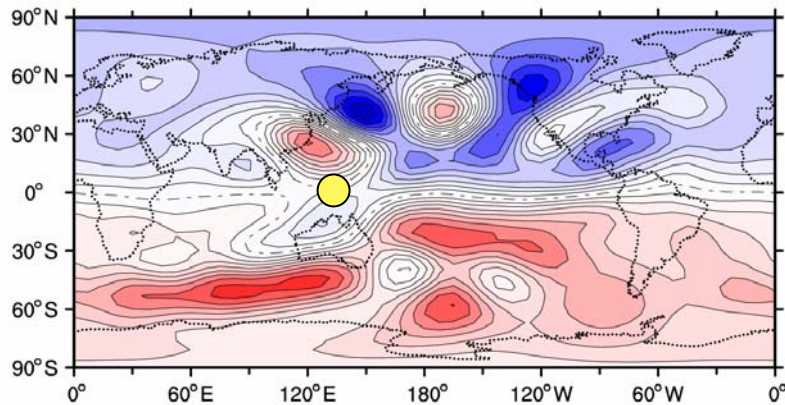


CCM0 strong forcing  
psi336  
( 135.00, 0.00) 2.5C/day



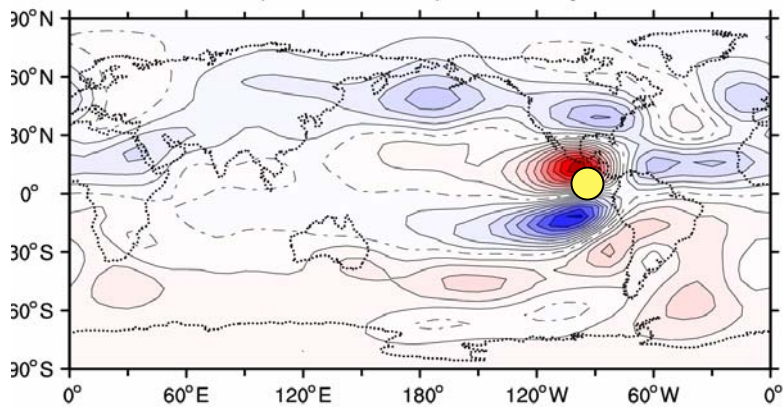
CONTOUR FROM -500 TO 400 BY 50 (x10<sup>4</sup>)  
FD vs CCM0 multifield cor= 0.81

FD  
psi336  
( 135.00, 0.00) 2.5C/day



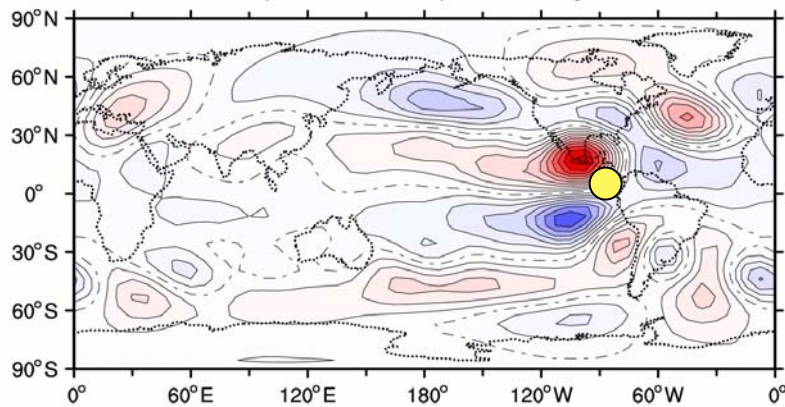
CONTOUR FROM -500 TO 400 BY 50 (x10<sup>4</sup>)  
FD vs CCM0 multifield cor= 0.81

CCM0 strong forcing  
psi336  
( -90.00, 0.00) 2.5C/day



CONTOUR FROM -450 TO 500 BY 50 (x10<sup>4</sup>)  
FD vs CCM0 multifield cor= 0.77

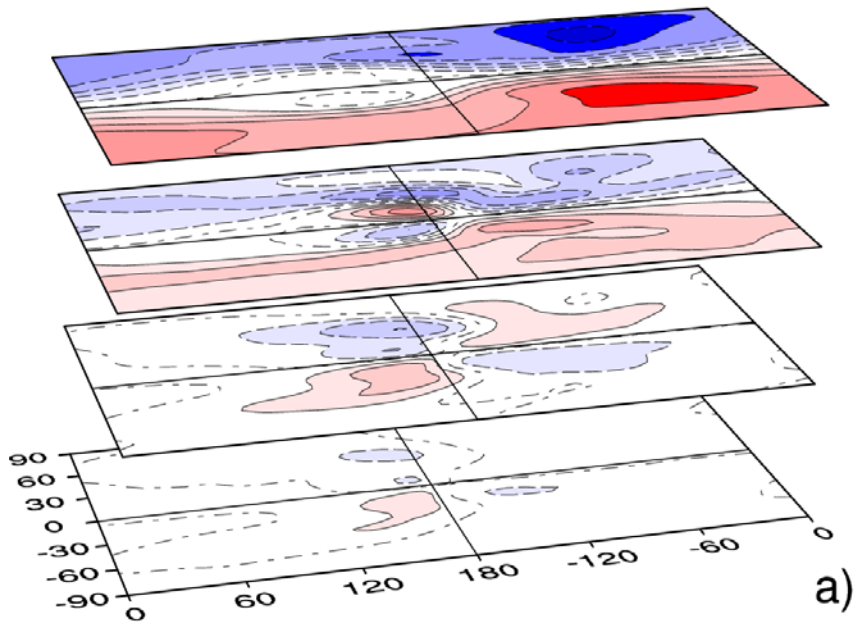
FD  
psi336  
( -90.00, 0.00) 2.5C/day



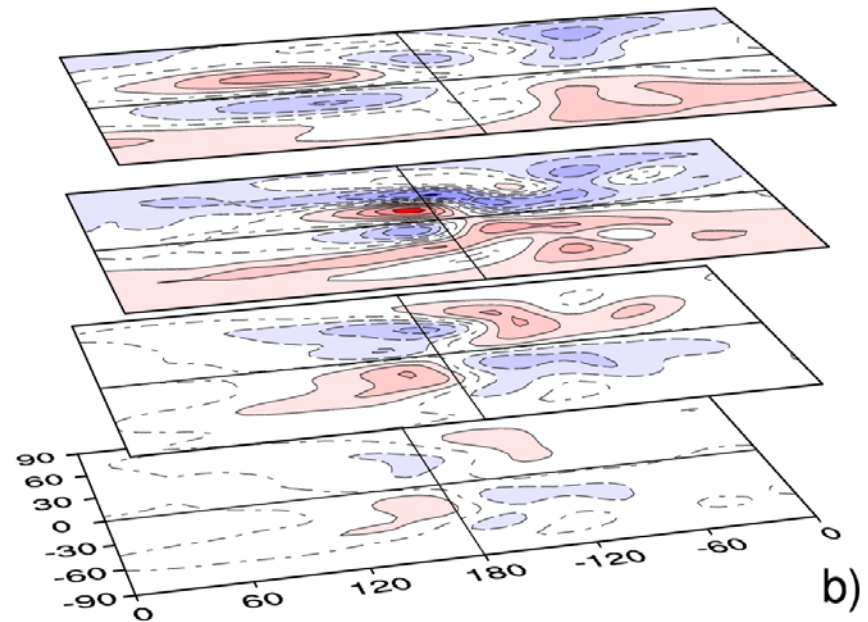
CONTOUR FROM -350 TO 500 BY 50 (x10<sup>4</sup>)  
FD vs CCM0 multifield cor= 0.77

24 case average response to  
sinusoidal equatorial heating  
(streamfunction)

CCM0  $\Psi$

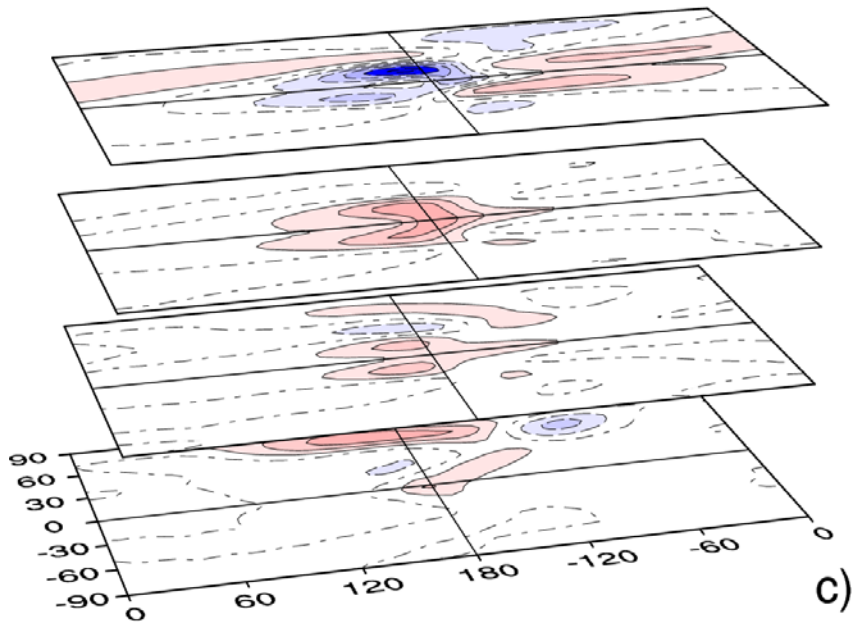


FD  $\Psi$

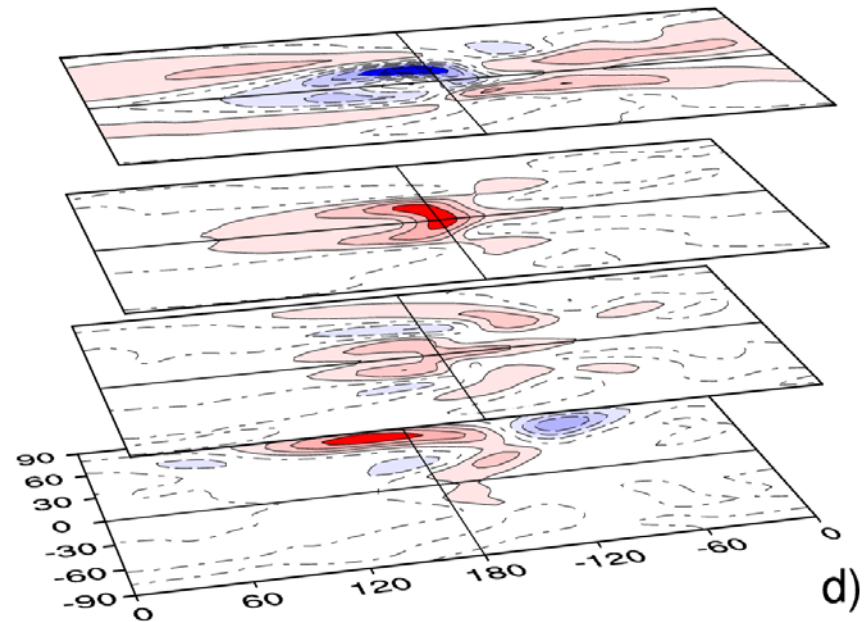


24 case average response to  
sinusoidal equatorial heating  
(temperature)

CCM0 T

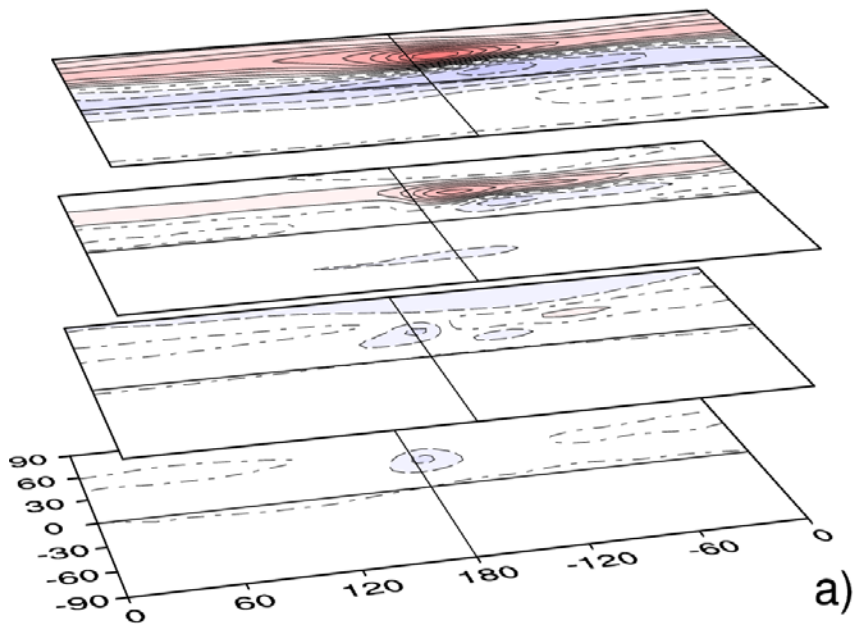


FD T

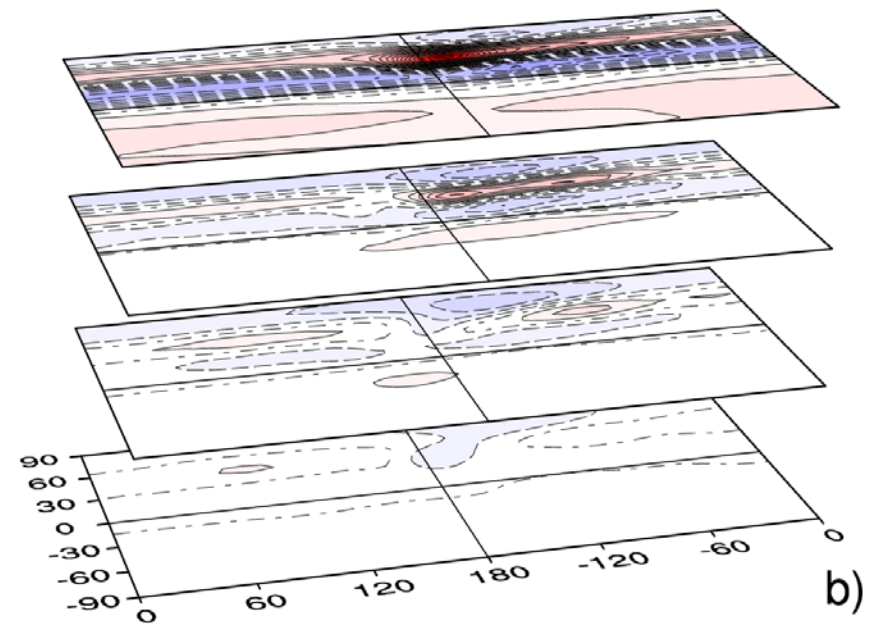


24 case average response to  
sinusoidal 40N heating  
(streamfunction)

CCM0  $\Psi$

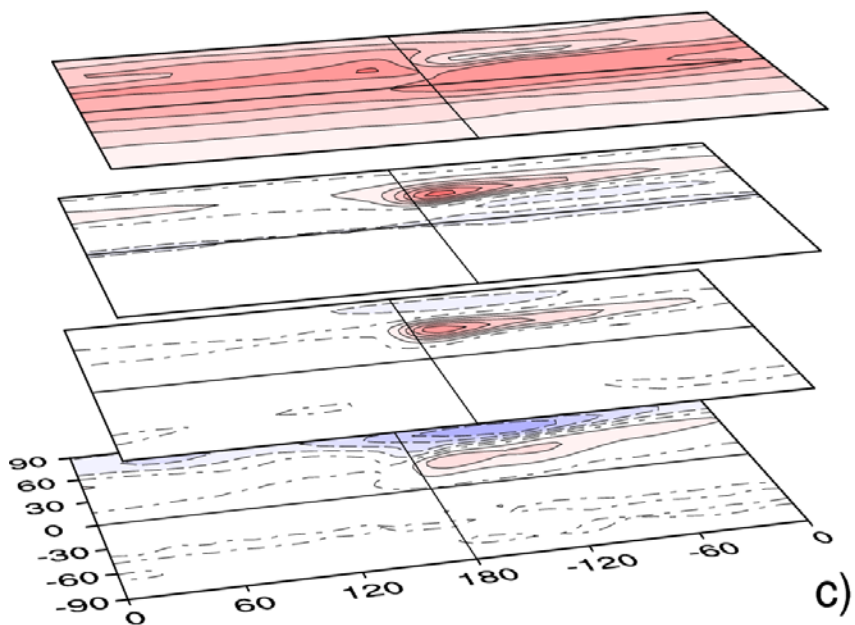


FD  $\Psi$

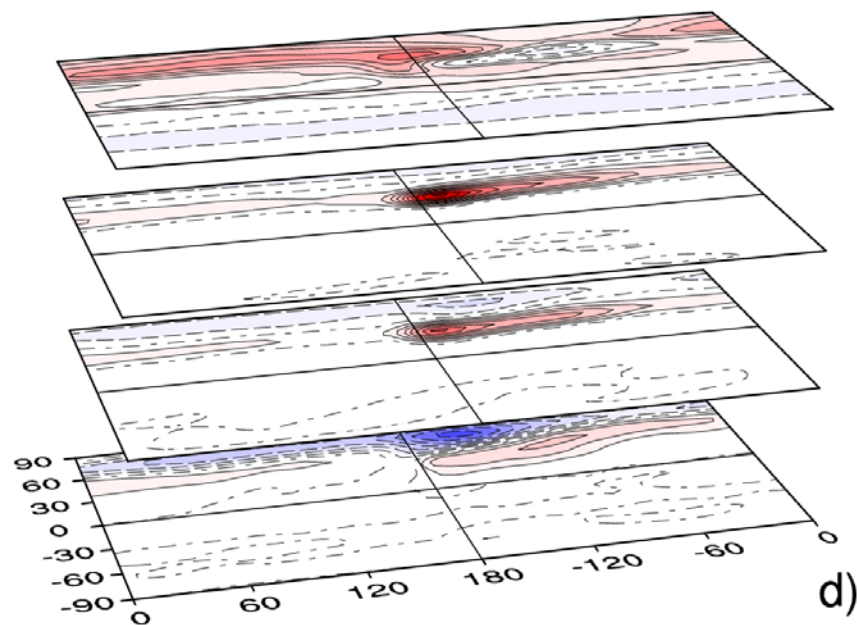


24 case average response to  
sinusoidal 40N heating  
(temperature)

CCM0 T

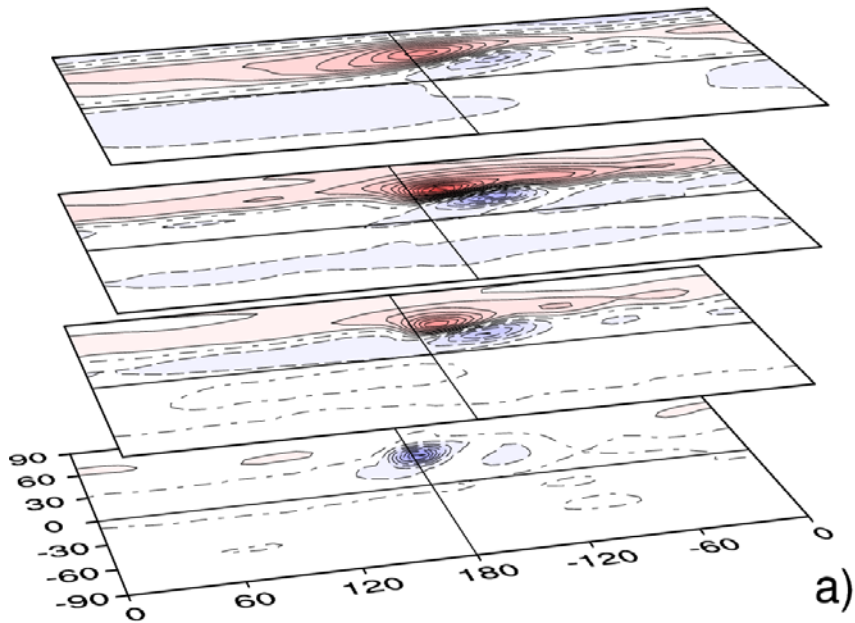


FD T

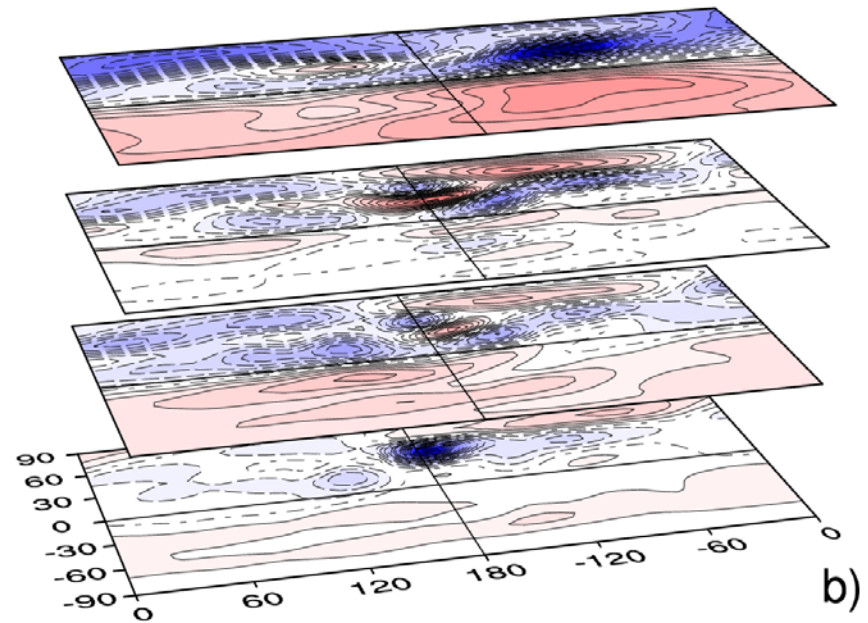


24 case average response to sinusoid  
low level 40N heating  
(streamfunction)

CCM0  $\Psi$

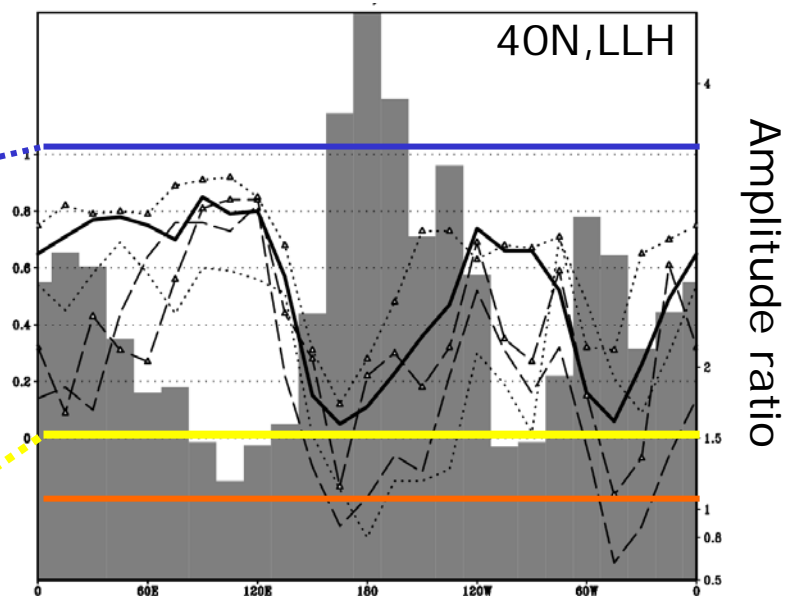
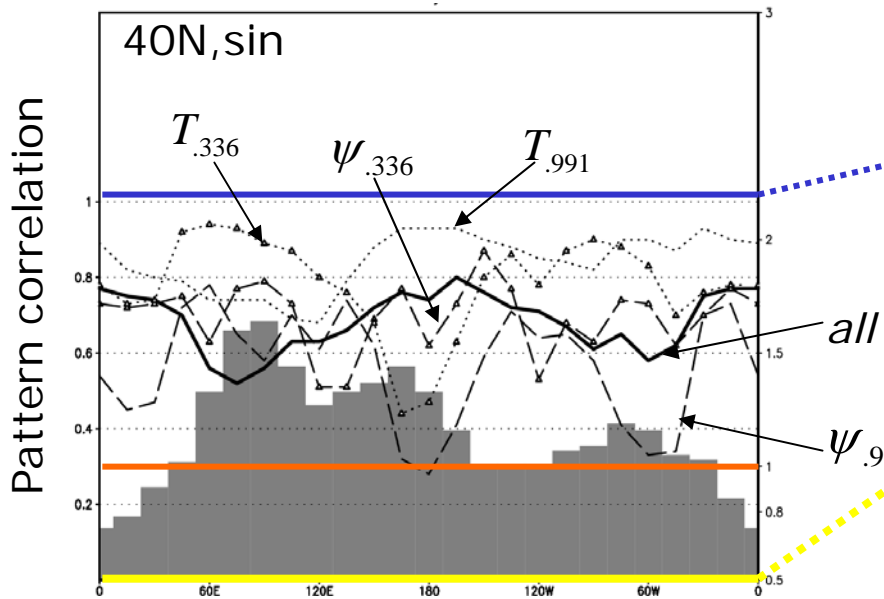
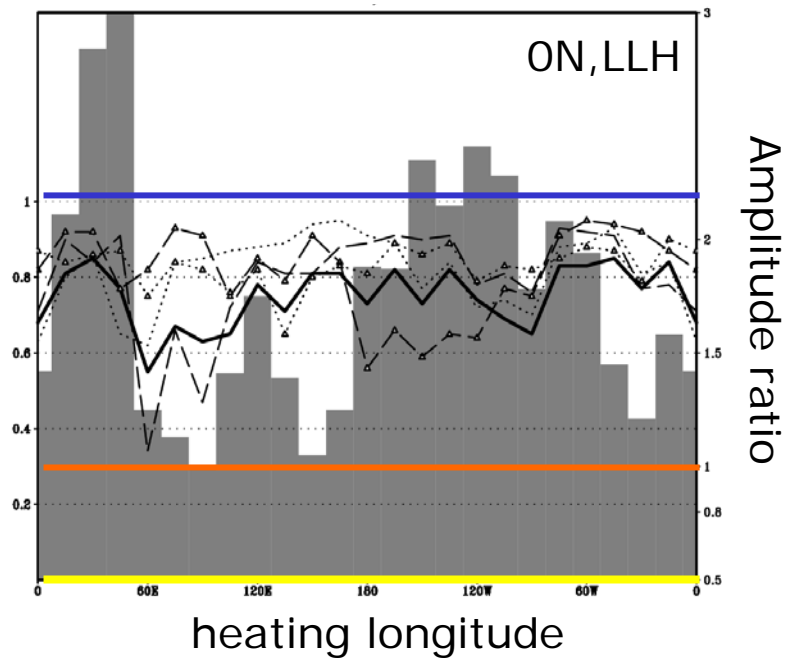
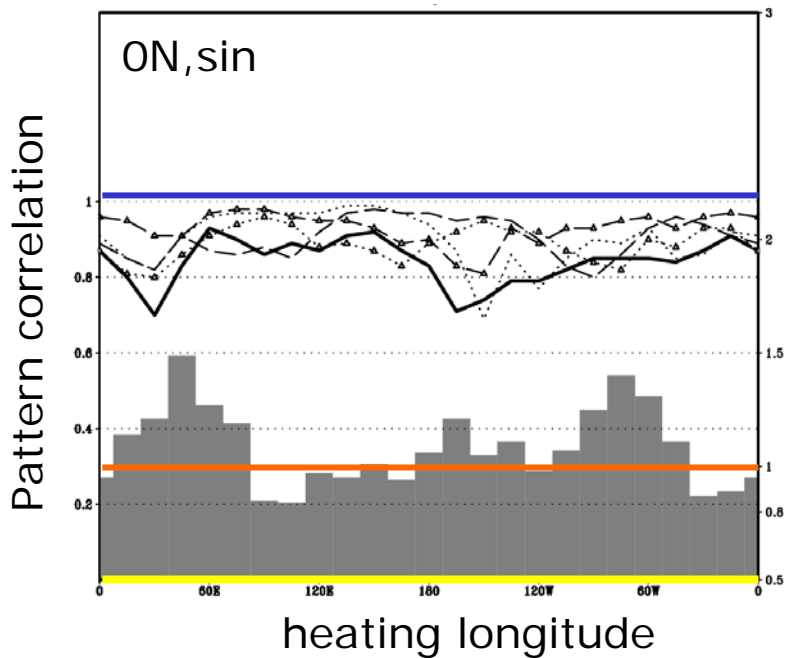


FD  $\Psi$

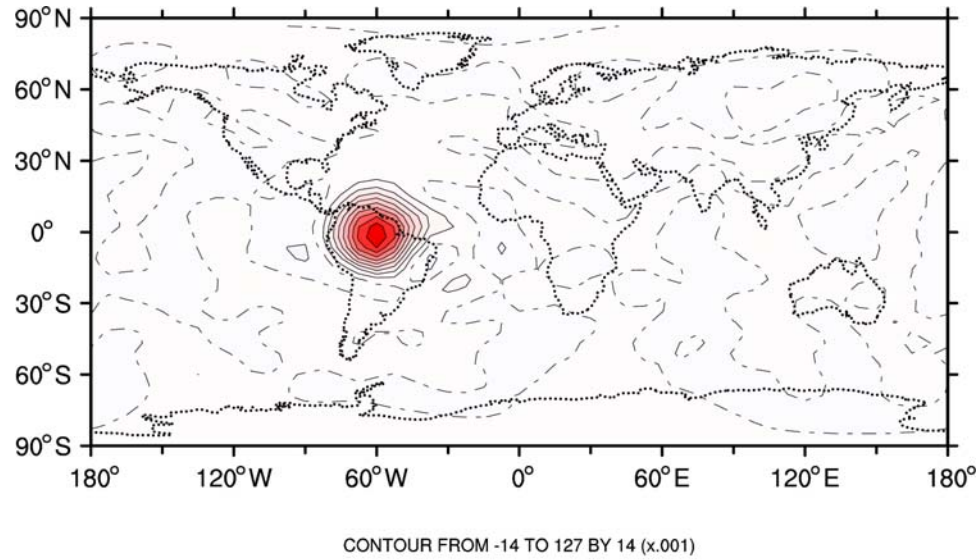




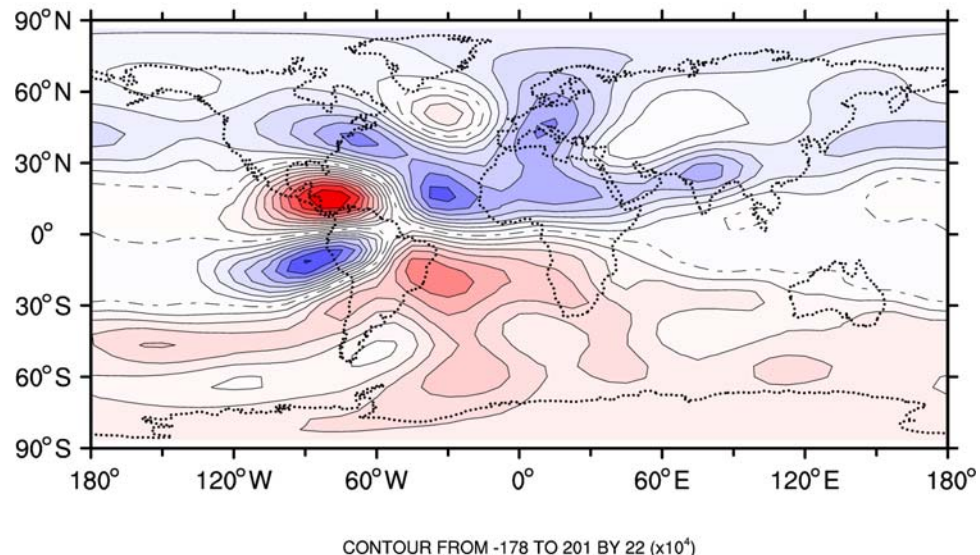
# FD skill for individual cases



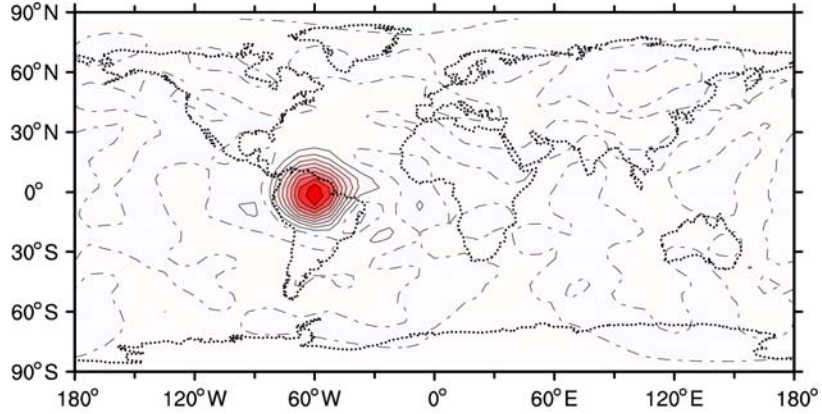
## 500mb heating inserted into CCM0



## 336mb streamfunction CCM0 response

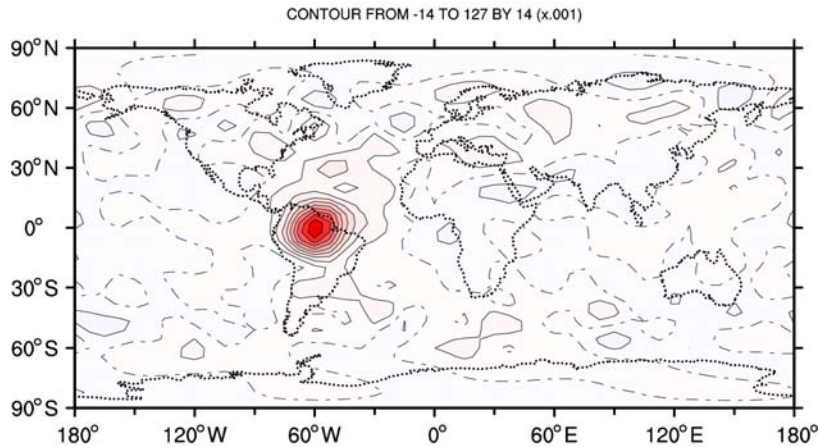


**500mb heating applied  
to CCM0**



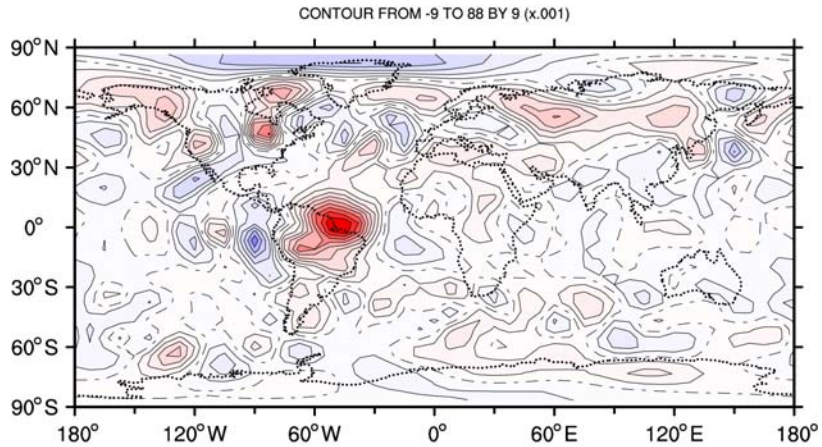
.014

**500mb heating derived  
from inverse of FDO**



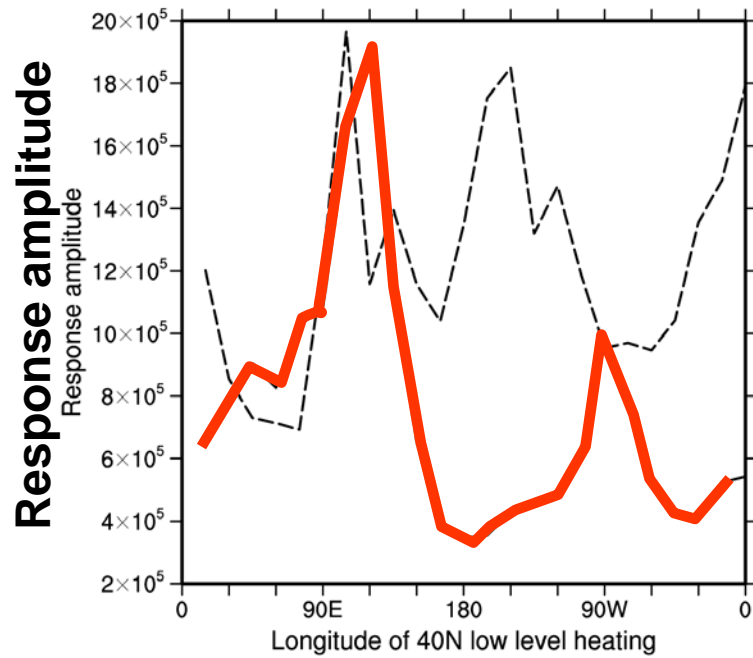
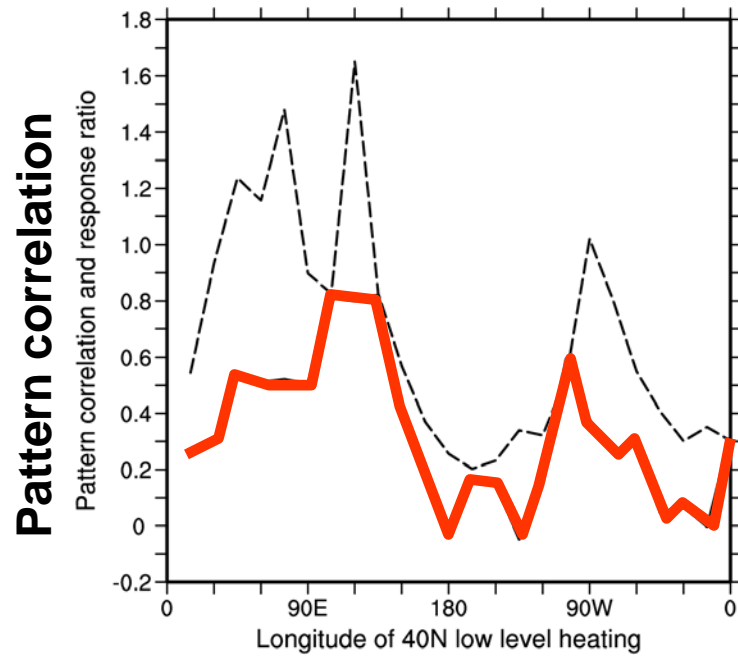
.009

**811mb heating derived  
from inverse of FDO**



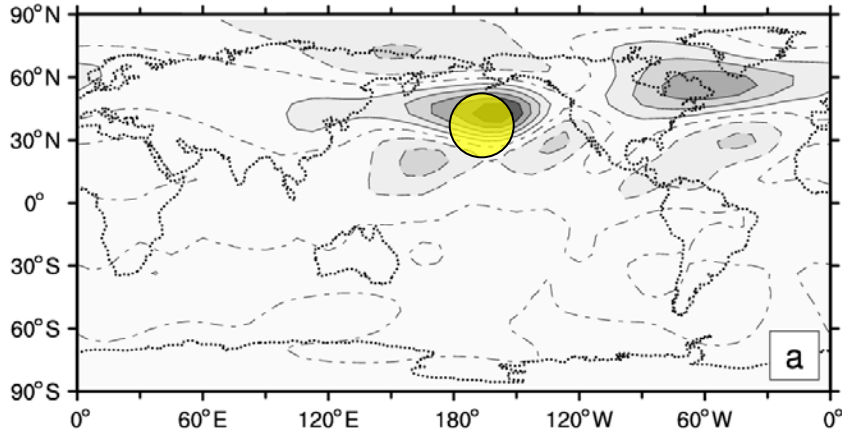
.005

CONTOUR FROM -295 TO 442 BY 49 (x.0001)

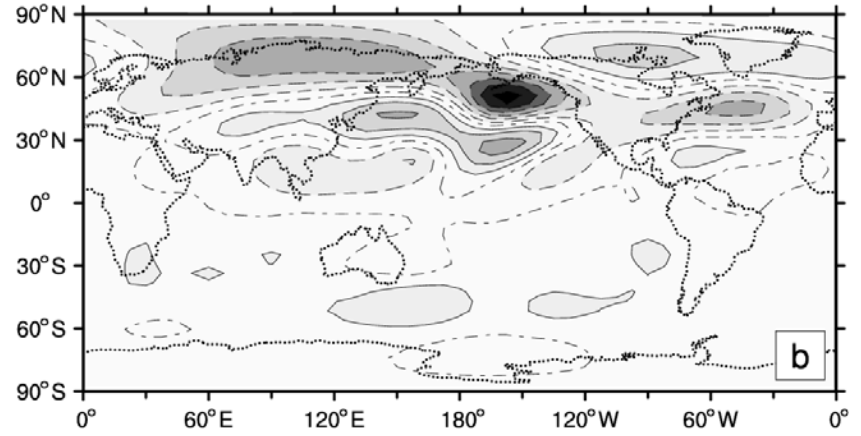


# (40N,165W) low level heating

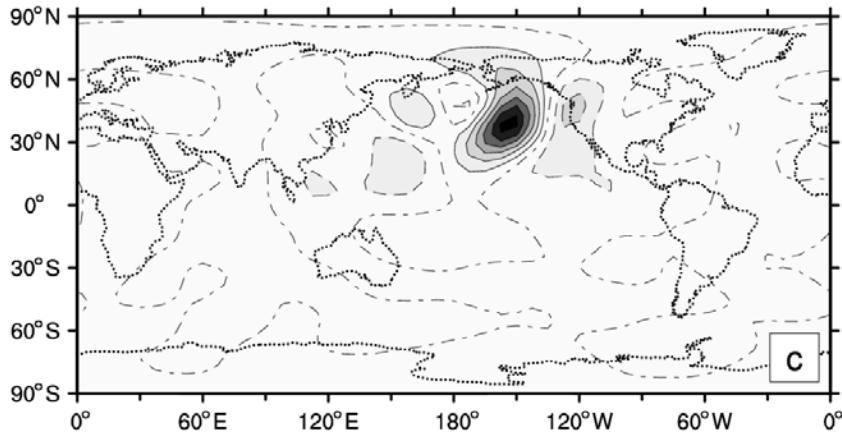
## $\Psi_{.336}$ CCM0 response



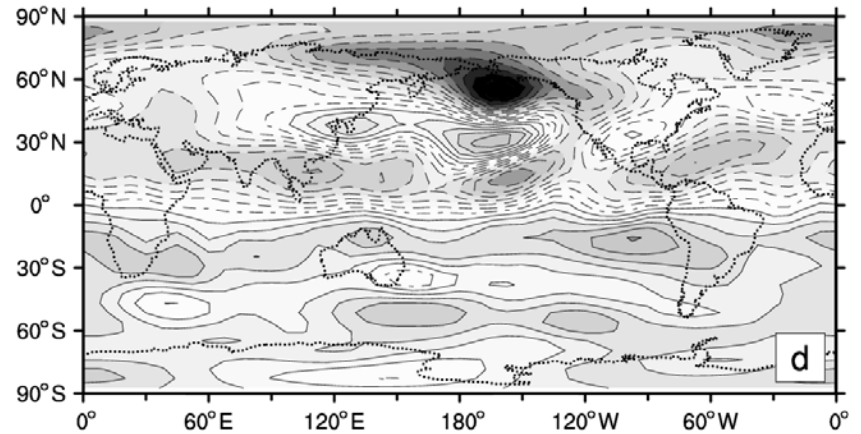
## $\Psi_{.336}$ FDT response



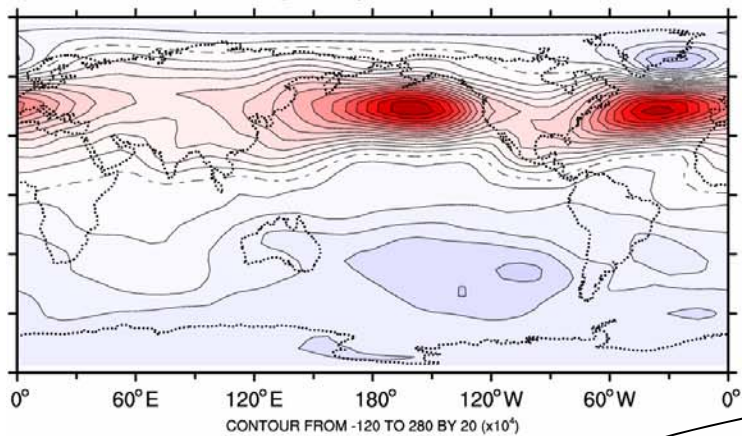
## spurious $\Psi_{.336}$ forcing



## CCM0 $\Psi_{.336}$ response to spurious forcing

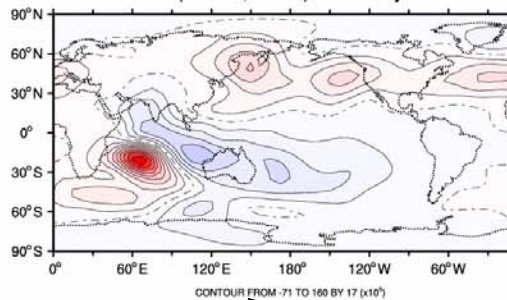


global psi991 EOF1  
regress psi level 1



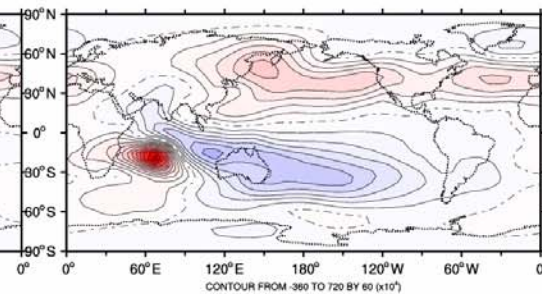
**FD**

psi991  
( 75.00,-10.00) 5.0C/day

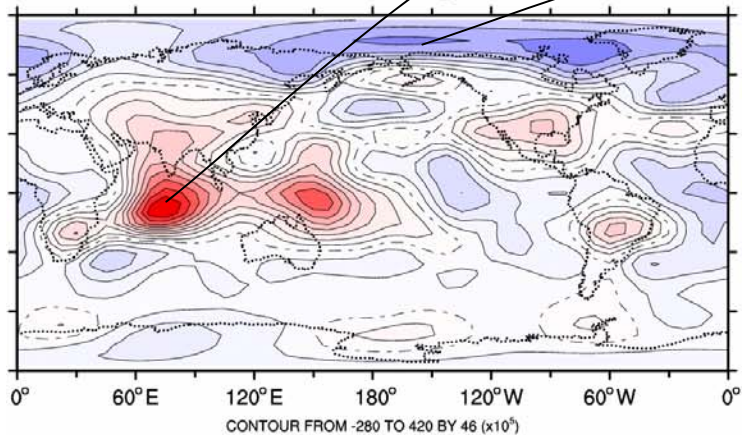


**CCM0**

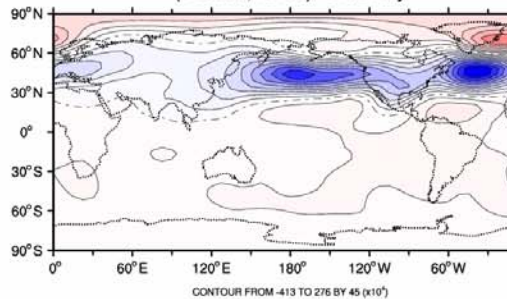
psi 0.991  
GBD03-GBD04



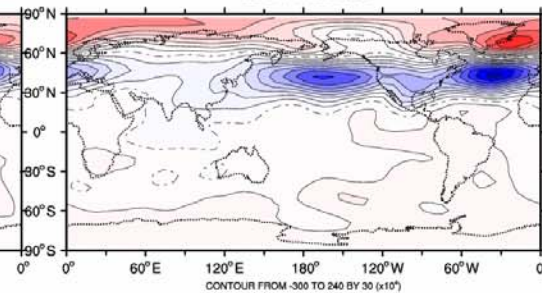
pattern response  
T500 forcing



psi991  
(-150.00, 75.00) 5.0C/day

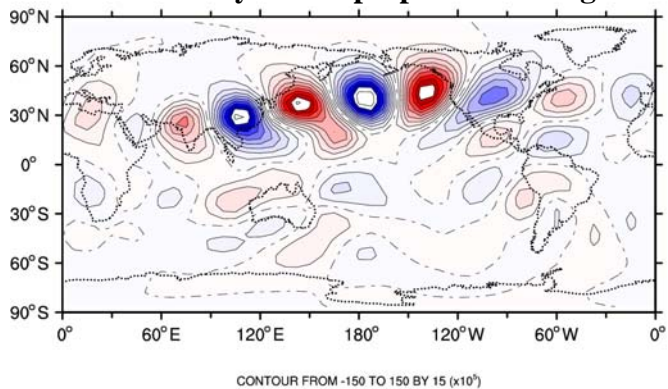


psi 0.991  
GBD14-GBD50

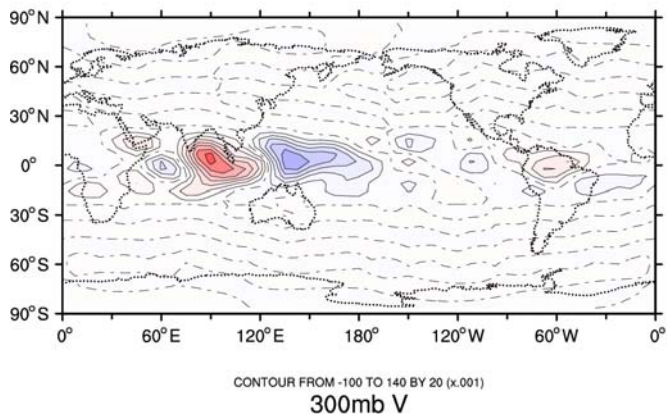


# v336 EOF1 for FDO randomly

## forced by midtropospheric heating



## Forcing associated with vEOF1

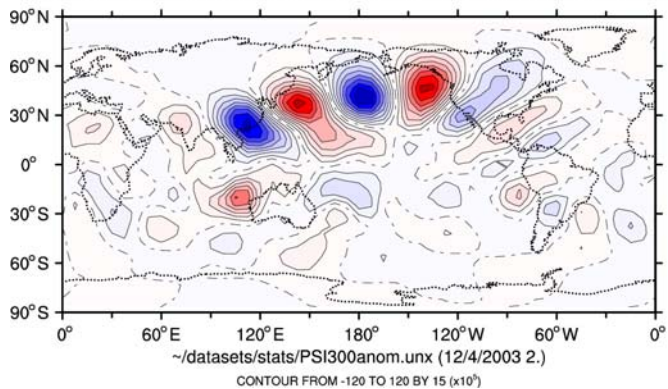


300mb V

Gr75 anomaly (10000 days) (Gr69 MFAC=0.1)

/BRANST/Gritsoun.vEOF1forcing.1-3.4000EOFbasis - /MAI/PJ300PAVG.100000

**CCM0  
v300  
response**



## Potential improvements & elaborations:

1. Data requirements / choice of basis and truncation
2. Nongaussianity
3. Estimating statistics beyond the mean
4. Time dependent response