28 February 2008

Orientation of Eddy Fluxes in Geostrophic Turbulence

Balu Nadiga

Los Alamos National Laboratory
Los Alamos, NM 87545

(balu@lanl.gov)

http://public.lanl.gov/balu

Summary

- Two different points of view
  - Ensemble- or time-average
  - Scale-decomposition

- Net alignment of the eddy flux of PV with appropriate mean or large-scale gradient of PV required but found to be WEAK
  – Backscatter is almost as important as Damping

- With scale decomposition, strong correlation between the eddy-flux and a nonlinear combination of filtered gradients
  – Absent in Ensemble- or time-average based decomposition

$$\boxed{\frac{\partial q}{\partial t} + \nabla \cdot (\mathbf{u} q) = F + D, \quad \nabla \cdot \mathbf{u} = 0.}$$

Reynolds' Decomposition

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$$

$$\overline{\overline{\mathbf{u}}} = \overline{\mathbf{u}}, \quad \overline{\mathbf{u}'} = 0$$

Evolution of Mean-PV

$$\frac{\partial \overline{q}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}\, \overline{q}) = \overline{F} + \overline{D} - \nabla \cdot \Sigma$$

$$\boldsymbol{\Sigma} = \overline{\mathbf{u} q} - \overline{\mathbf{u}}\, \overline{q} = \overline{\mathbf{u}' q'}.$$

Scale Decomposition

$$\mathbf{u} = \mathbf{u}_{>l} + \mathbf{u}_{<l} = \mathbf{u}_l + \mathbf{u}_s.$$

$$\mathbf{u}_{ll} \neq \mathbf{u}_l, \quad \mathbf{u}_{sl} \neq 0$$

Evolution of Large-scale PV

$$\frac{\partial q_l}{\partial t} + \nabla \cdot (\mathbf{u}_l q_l) = F_l + D_l - \nabla \cdot \sigma$$

$$\sigma = (\mathbf{u} q)_l - \mathbf{u}_l q_l$$

Reynolds' Decomposition

Evolution of mean-enstrophy:

$$\overline{Z} = \overline{q}^2/2$$

$$\frac{\partial \overline{Z}}{\partial t} + \nabla \cdot \left(\overline{\mathbf{u}}\overline{Z} + \overline{q}\boldsymbol{\Sigma}\right) = \overline{F}\overline{q} + \overline{D}\overline{q} - \mathcal{T}_{\overline{Z}},$$

$$\mathcal{T}_{\overline{Z}} = -\boldsymbol{\Sigma} \cdot \nabla\overline{q} = -\overline{\mathbf{u}'q'} \cdot \nabla\overline{q}$$

Scale Decomposition

Large-scale enstrophy:

$$Z_l = q_l^2/2$$

$$\frac{\partial Z_l}{\partial t} + \nabla \cdot (\mathbf{u}_l Z_l + q_l \sigma) = F_l q_l + D_l q_l - \Pi_{Z_l},$$

$$\Pi_{Z_l} = -\sigma \cdot \nabla q_l$$

$$= -\left((\mathbf{u}q)_l - \mathbf{u}_l q_l\right) \cdot \nabla q_l$$

Reynolds' Decomposition

Mean-Eddy-Enstrophy

$$z = q'^2/2$$

$$\frac{\partial \overline{z}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}z}) = \overline{F'q'} + \overline{D'q'} + \mathcal{T}_{\overline{Z}}$$

Scale Decomposition

Small-Scale-Enstrophy

$$z_l = (q^2 - q_l^2)/2$$

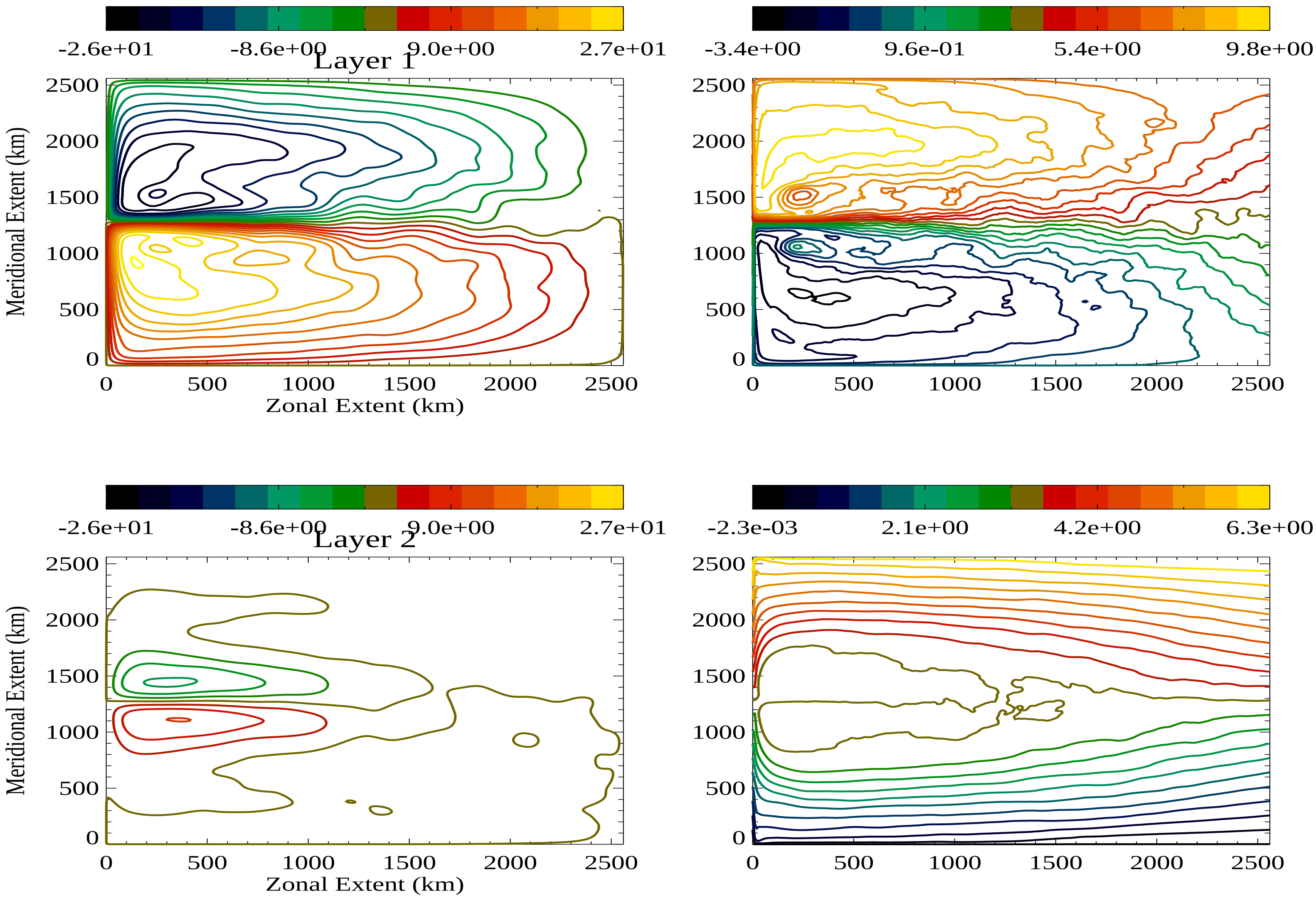
$$\frac{\partial z_l}{\partial t} + \nabla \cdot (uZ - \mathbf{u}_l Z_l - q_l \sigma) = Fq - F_l q_l + Dq - D_l q_l + \Pi_{Z_l}$$

Components of the subgrid pv flux

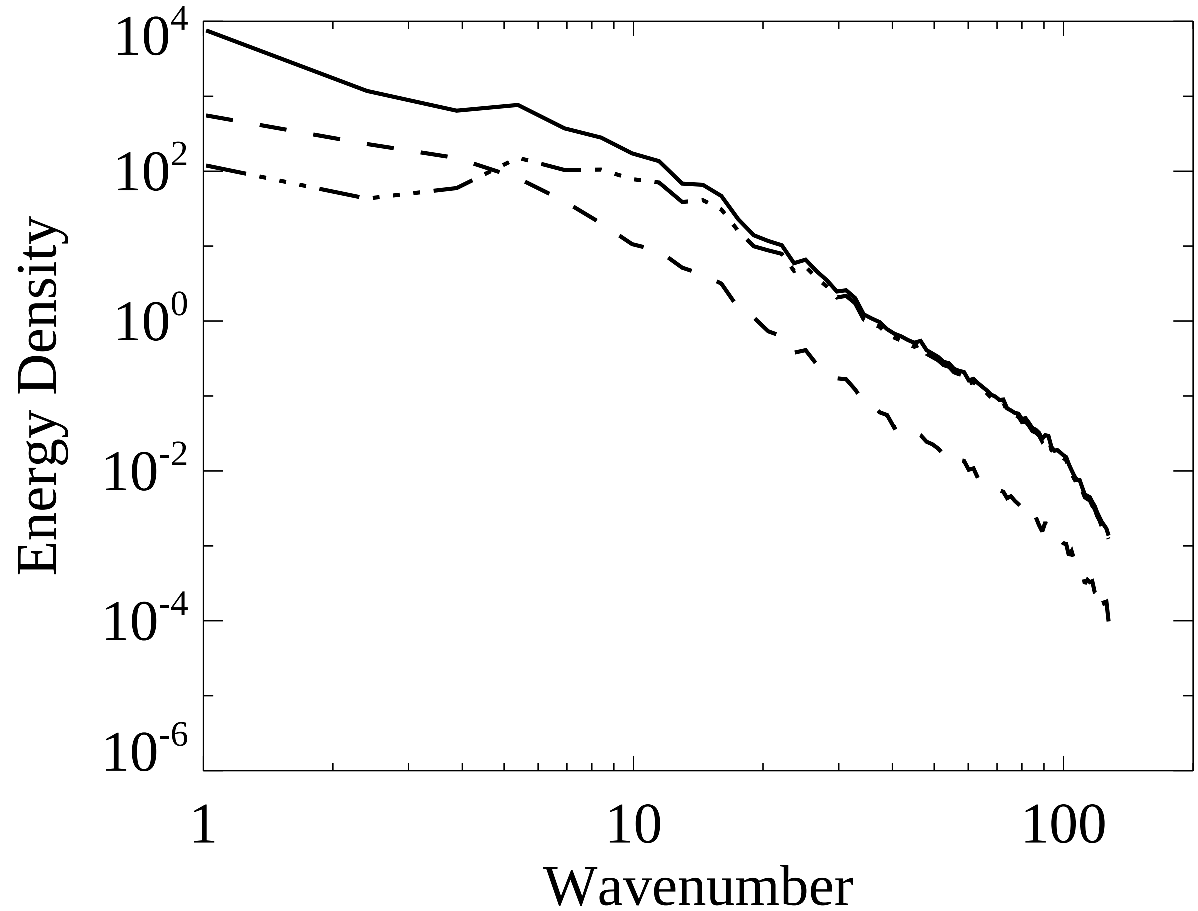
$$\sigma = \underbrace{(\mathbf{u}_l q_l)_l - \mathbf{u}_l q_l}_{\text{Leonard stress}} + \underbrace{(\mathbf{u}_l q_s)_l + (\mathbf{u}_s q_l)_l}_{\text{Cross-stress}} + \underbrace{(\mathbf{u}_s q_s)_l}_{\text{Reynolds stress}}$$

$$= \underbrace{(\mathbf{u}_l q_l)_l - \mathbf{u}_{ll} q_{ll}}_{\text{Leonard Stress}} + \underbrace{(\mathbf{u}_l q_s)_l + (\mathbf{u}_s q_l)_l - \mathbf{u}_{ll} q_{sl} - \mathbf{u}_{sl} q_{ll}}_{\text{Cross-stress}} + \underbrace{(\mathbf{u}_s q_s)_l - \mathbf{u}_{sl} q_{sl}}_{\text{Reynolds stress}}$$

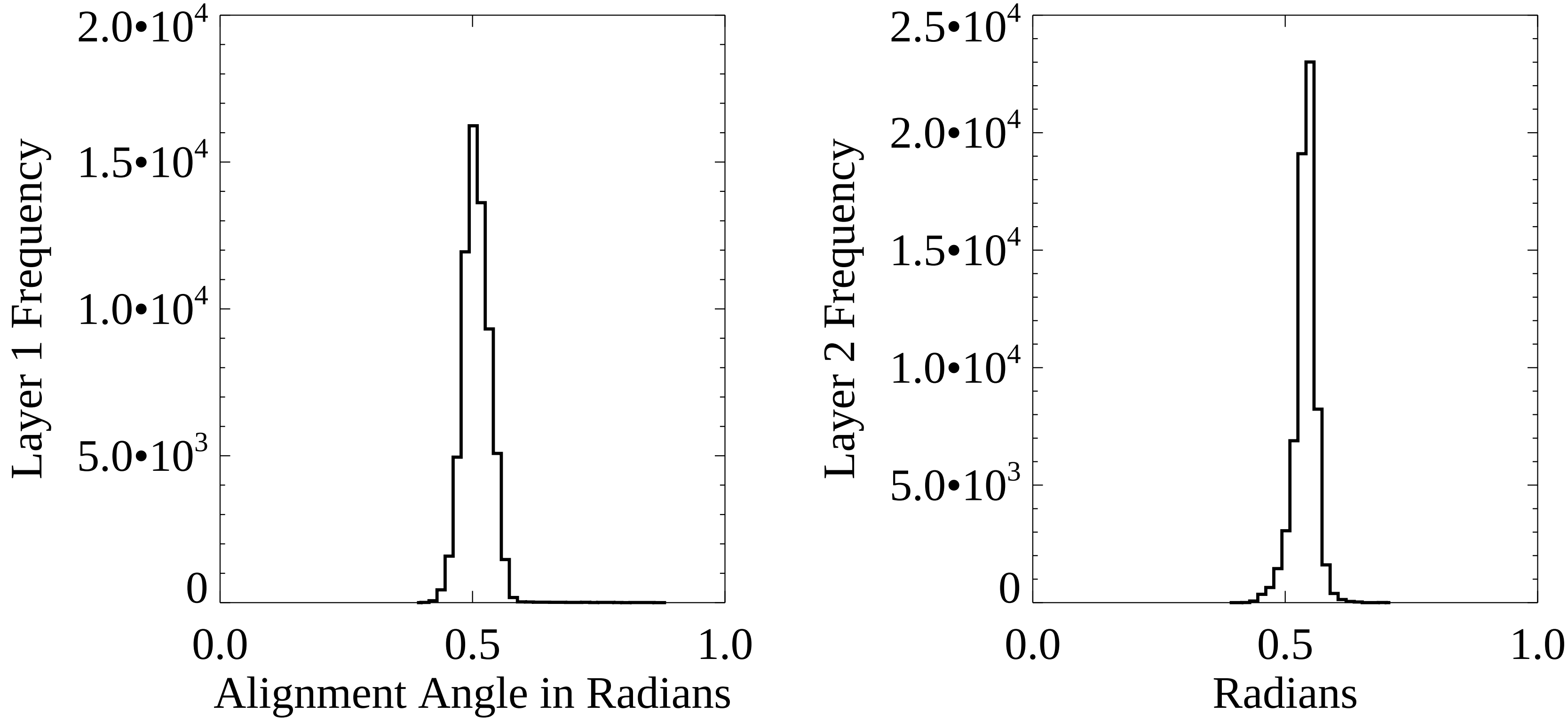
(Galilean Invariant Decomposition)



Time-mean circulation (left) and potential vorticity (right) in non-dimensional units. Top row is top layer. Bottom row is bottom layer



Spectra of total energy (solid line), barotropic energy (dashed line) and baroclinic kinetic energy (dot-dashed line) Note that 1) Only the total energy is inviscidly conserved. 2) All spectra fall-off steeply at the scale of filtering.



Distribution of angle between eddy-flux of pv and time-mean pv gradient using Reynolds decomposition in the top layer (left) and bottom layer (right). The required alignment of the eddy-flux down the gradient of mean pv is verified in the mean angle in the above plots being slightly greater than $\pi/2$.

$$\frac{\partial \overline{z}}{\partial t} + \nabla \cdot (\overline{\mathbf{u} z}) = \overline{F'q'} + \overline{D'q'} + \mathcal{T}_{\overline{Z}}$$

$$\mathcal{T}_{\overline{Z}} = -\boldsymbol{\Sigma} \cdot \nabla \overline{q} = -\overline{\mathbf{u}'q'} \cdot \nabla \overline{q}$$

- Steady forcing; statistical stationarity.

$$\text{Since } \int_D \overline{D'q'} < 0, \quad \int_D \mathcal{T}_{\overline{Z}} > 0$$

- Locally advection matters, particularly in basin config.
  So $\overline{\mathbf{u}'q'} \propto -\nabla \overline{q}$ is not good locally.

- But only divergent component affects $\overline{q}$ evolution. However, no UNIQUE decomposition of $\overline{\mathbf{u}'q'}$

$$\boxed{\frac{\partial q}{\partial t} + \nabla \cdot (\mathbf{u}q) = F + D, \quad \nabla \cdot \mathbf{u} = 0.}$$

Reynolds' Decomposition

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$$

$$\overline{\overline{\mathbf{u}}} = \overline{\mathbf{u}}, \quad \overline{\mathbf{u}'} = 0$$

Evolution of Mean-PV

$$\frac{\partial \overline{q}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}\,\overline{q}) = \overline{F} + \overline{D} - \nabla \cdot \Sigma$$

$$\mathbf{\Sigma} = \overline{\mathbf{u}q} - \overline{\mathbf{u}}\,\overline{q} = \overline{\mathbf{u}'q'}.$$

Scale Decomposition

$$\mathbf{u} = \mathbf{u}_{>l} + \mathbf{u}_{<l} = \mathbf{u}_l + \mathbf{u}_s.$$

$$\mathbf{u}_{ll} \neq \mathbf{u}_l, \quad \mathbf{u}_{sl} \neq 0$$

Evolution of Large-scale PV

$$\frac{\partial q_l}{\partial t} + \nabla \cdot (\mathbf{u}_l q_l) = F_l + D_l - \nabla \cdot \sigma$$

$$\sigma = (\mathbf{u}q)_l - \mathbf{u}_l q_l$$

Marshall and Shutts, 1981

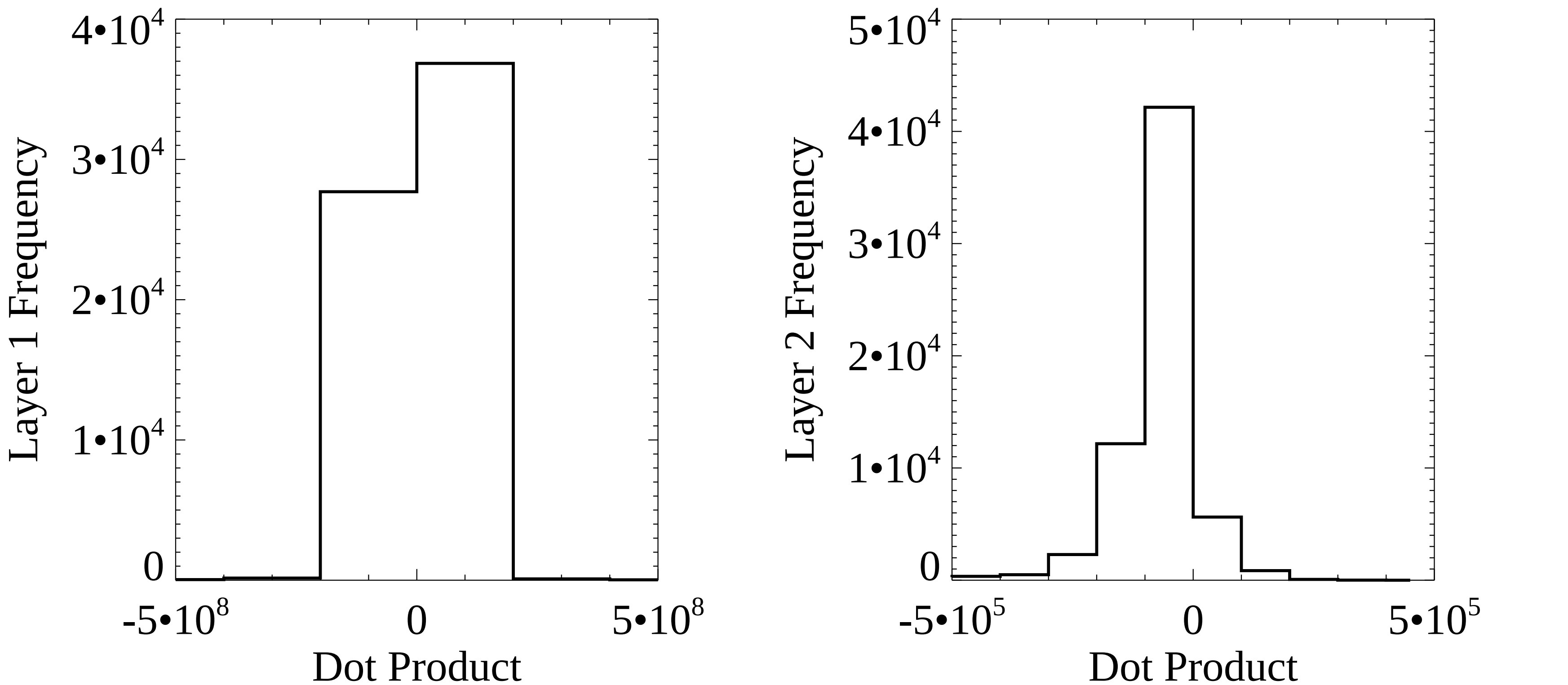
- If mean circulation contours do not deviate much from the mean pv contours, then a two way balance is possible

- Mean advection of perturbation enstrophy could be balanced by a rotational pv flux aligned along contours of perturbation potential enstrophy

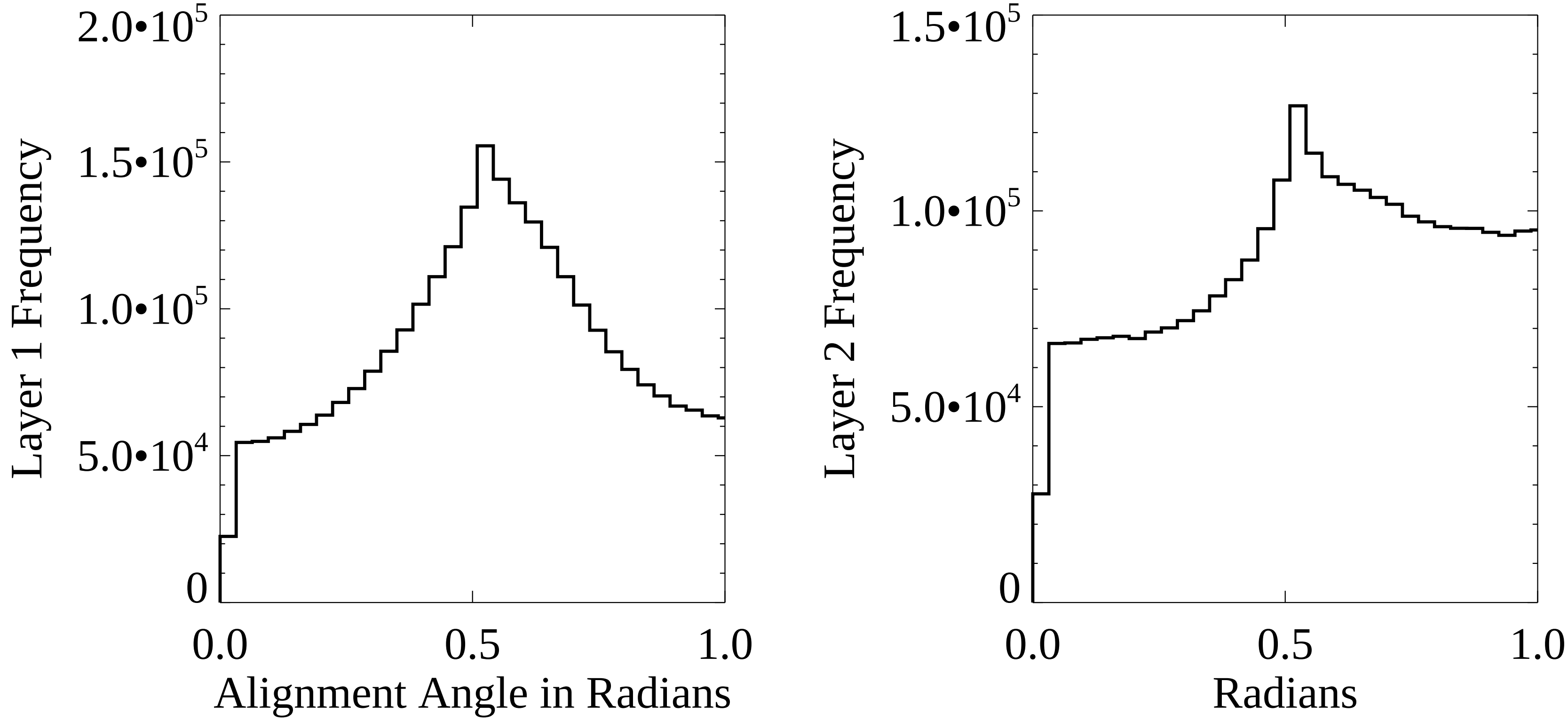
- Rest of eddy-pv flux could be downgradient (after neglecting triple correlations)

$$\nabla \cdot (\overline{\mathbf{u}}\,\overline{z}) = \left(\overline{\mathbf{u}'q'}\right)_{rot} \cdot \nabla \overline{q}$$

$$\overline{D'q'} = \left(\overline{\mathbf{u}'q'}\right)_{div} \cdot \nabla \overline{q},$$



Distribution of the dot product between a divergent component of eddy-pv flux and the mean-gradient. In effect a component of the eddy-flux that circulates around contours of perturbation potential enstrophy has been removed to obtain the 'divergent' component of eddy-pv flux.



Distribution of angle between sub-filter pv flux and large-scale pv gradient. The distributions still peak at $\pi/2$, i.e., the eddy flux is most often perpendicular to the large-scale gradient. However, the net downgradient alignment is more pronounced than in the case of the classical Reynolds decomposition.

Components of the subgrid pv flux

$$\sigma = \underbrace{(\mathbf{u}_l q_l)_l - \mathbf{u}_l q_l}_{\text{Leonard stress}} + \underbrace{(\mathbf{u}_l q_s)_l + (\mathbf{u}_s q_l)_l}_{\text{Cross-stress}} + \underbrace{(\mathbf{u}_s q_s)_l}_{\text{Reynolds stress}}$$

$$= \underbrace{(\mathbf{u}_l q_l)_l - \mathbf{u}_{ll} q_{ll}}_{\text{Leonard Stress}} + \underbrace{(\mathbf{u}_l q_s)_l + (\mathbf{u}_s q_l)_l - \mathbf{u}_{ll} q_{sl} - \mathbf{u}_{sl} q_{ll}}_{\text{Cross-stress}} + \underbrace{(\mathbf{u}_s q_s)_l - \mathbf{u}_{sl} q_{sl}}_{\text{Reynolds stress}}$$

(Galilean Invariant Decomposition)

Taylor series expansion of $\mathbf{u}_l(\mathbf{x}')$ about $\mathbf{u}(\mathbf{x})$ etc... in Leonard stress at first order:

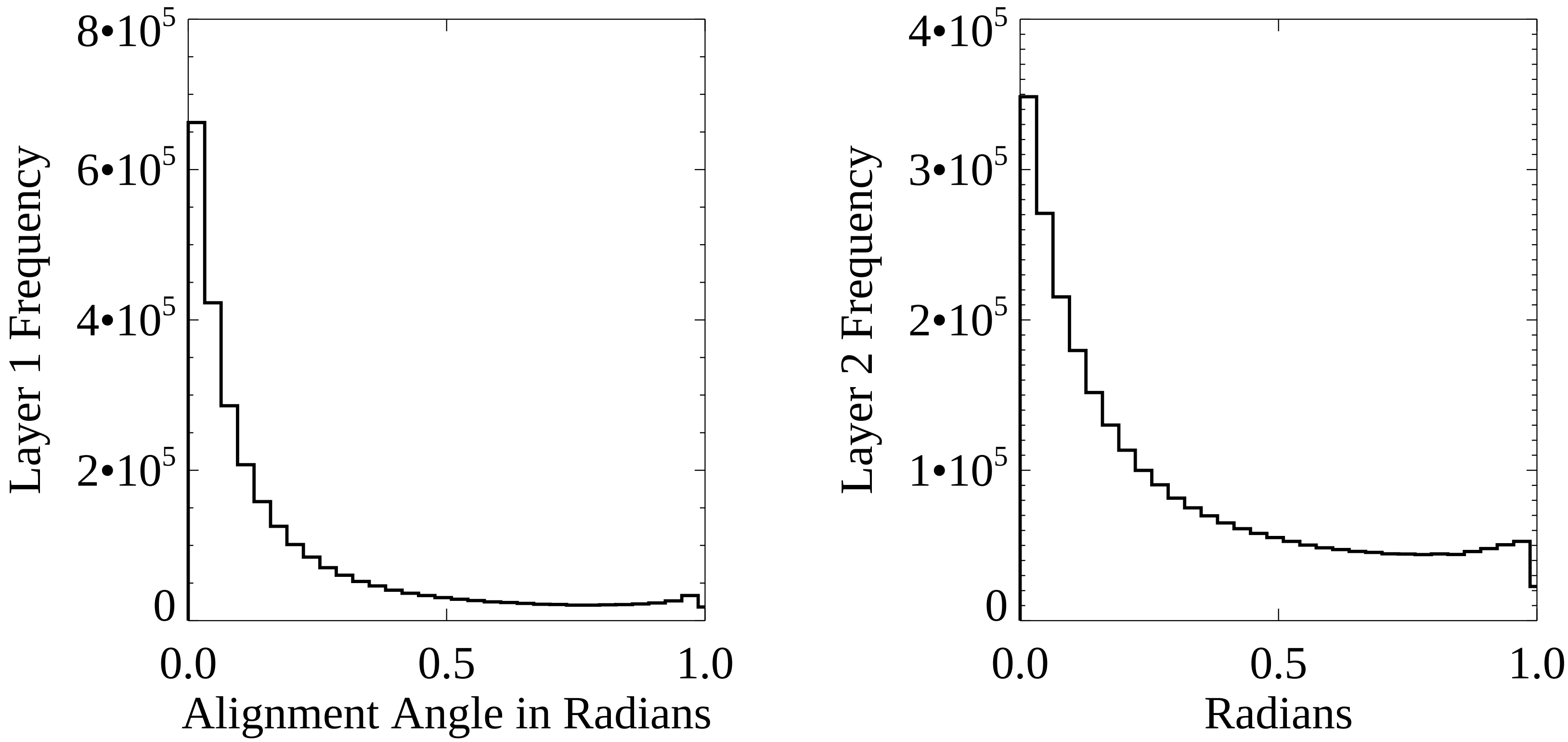
$$(\mathbf{u}_l q_l)_l - \mathbf{u}_{ll} q_{ll} =$$

$$= \int d\mathbf{x}'\, G(\mathbf{x}-\mathbf{x}') \left(\mathbf{u}_l(\mathbf{x}) + (x'-x)_j \frac{\partial u_{li}}{\partial x_j}(\mathbf{x}) \right) \left(q_l(\mathbf{x}) + (x'-x)_j \frac{\partial q_l}{\partial x_j}(\mathbf{x}) \right)$$

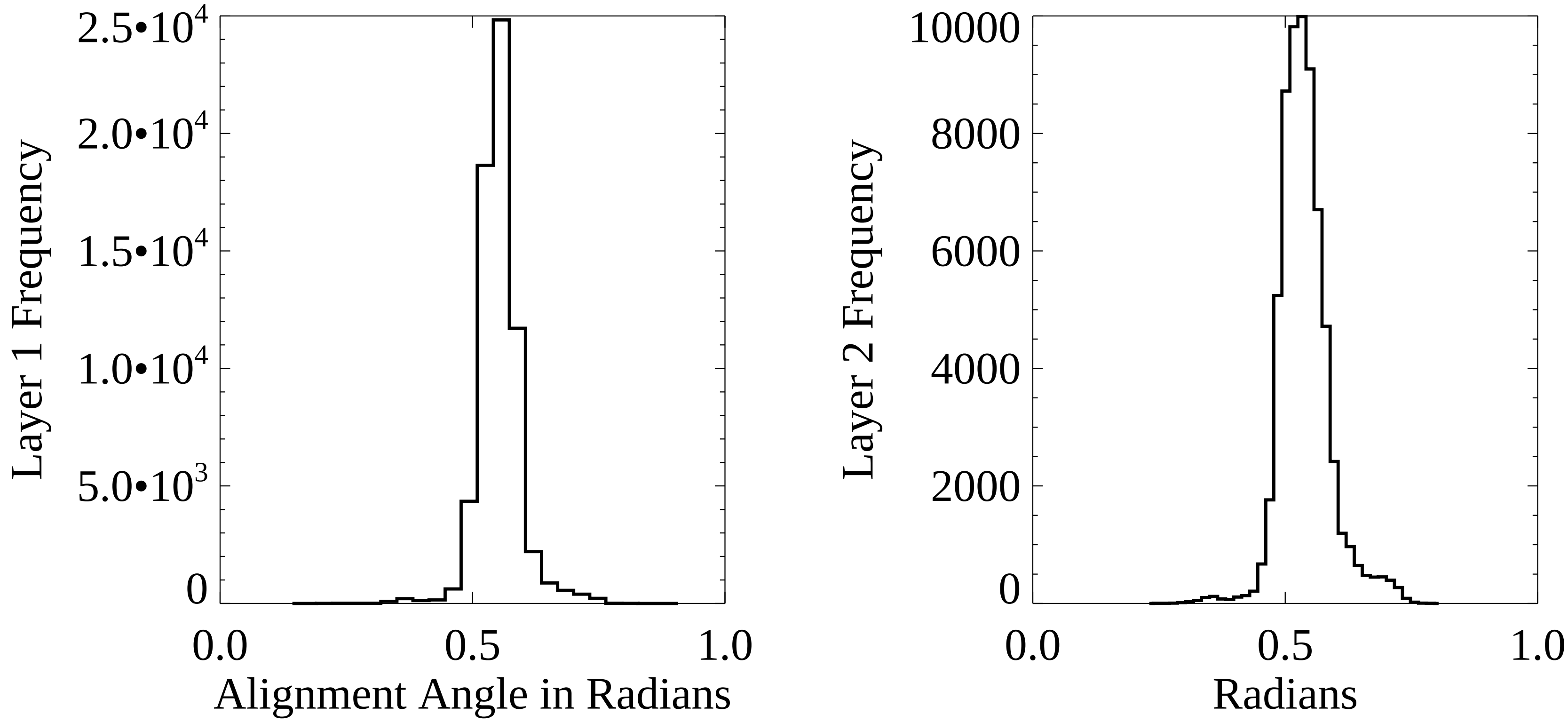
$$- \int d\mathbf{x}'\, G(\mathbf{x}-\mathbf{x}') \left(\mathbf{u}_l(\mathbf{x}) + (x'-x)_j \frac{\partial u_{li}}{\partial x_j}(\mathbf{x}) \right) *$$

$$\int d\mathbf{x}'\, G(\mathbf{x}-\mathbf{x}') \left(q_l(\mathbf{x}) + (x'-x)_j \frac{\partial q_l}{\partial x_j}(\mathbf{x}) \right)$$

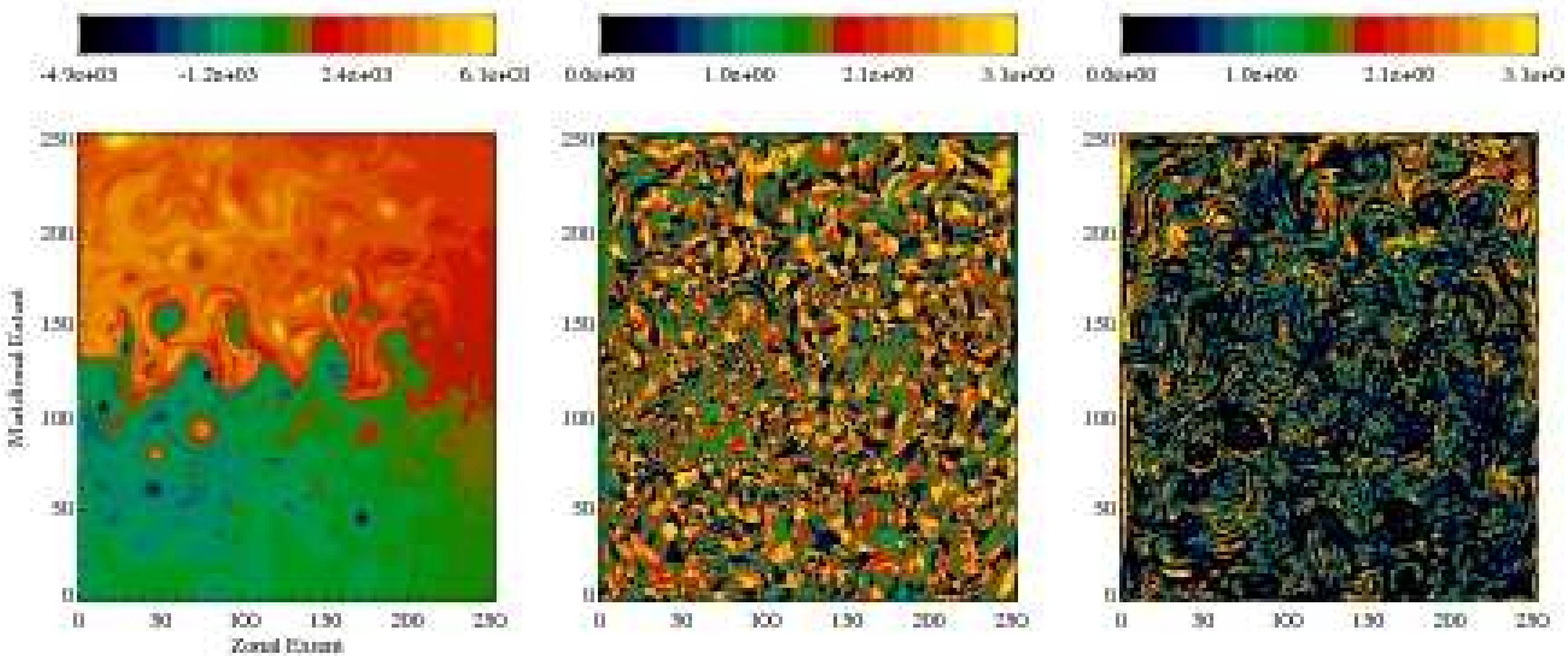
$$= C_{2l}\, \frac{\partial u_{li}}{\partial x_j} \frac{\partial q_l}{\partial x_j} = C_{2l}\, \nabla \mathbf{u}_l \cdot \nabla q_l$$



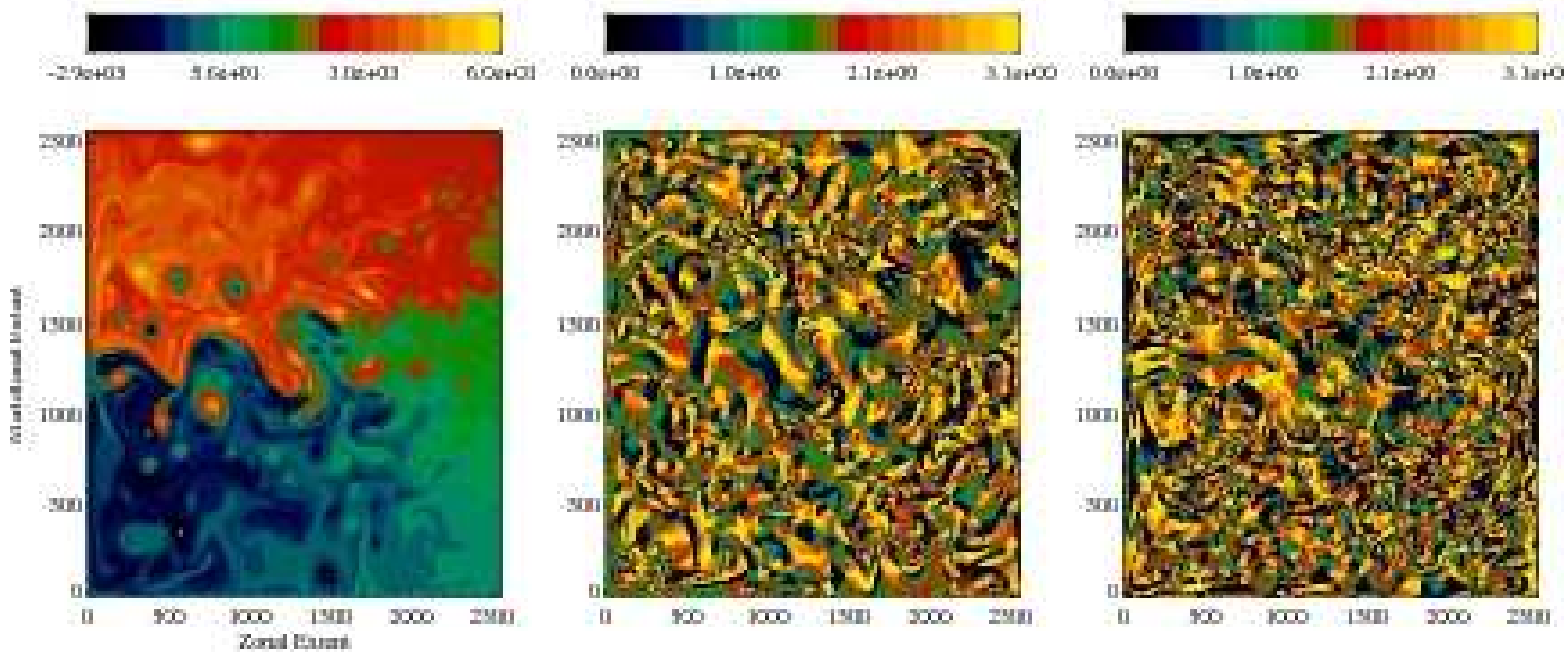
Distribution of angle between sub-filter pv flux and $\nabla \mathbf{u}_l \cdot \nabla q_l$. The peaking of the angle at 0 implies close alignment of the two vectors. That this angle is a random variable is also evident. Hence a putative eddy parameterization based on $\nabla \mathbf{u}_l \cdot \nabla q_l$ would ideally have a stochastic aspect to it.



Distribution of angle between eddy-flux of pv ($\overline{\mathbf{u}'q'}$) and $\nabla\overline{\mathbf{u}} \cdot \nabla\overline{q}$. Much unlike in the scale-decomposition approach, in the Reynolds decomposition approach, this alignment is particularly insightful.



Left: Instantaneous potential vorticity. Center: Spatial distribution of angle between sub-filter pv flux and ∇q_l. Right: Angle between sub-filter pv flux and $\nabla \mathbf{u}_l \cdot \nabla q_l$. An almost equitable distribution of green-blue-blacks and red-yellows in the center plot indicates poor alignment between the eddy-pv flux and the large-scale gradient. On the other hand, a predominance of blue-blacks in the plot on the right indicates strong alignment of the eddy flux with the nonlinear combination of large-scale gradients considered.



Left: Instantaneous potential vorticity. Center: Spatial distribution of angle between Reynolds eddy-flux of pv and $\nabla\overline{q}$. Right: Angle between Reynolds eddy-flux of pv and $\nabla\overline{\mathbf{u}}\cdot\nabla\overline{q}$.An almost equitable distribution of green-blue-blacks and red-yellows in the center and right plots indicates poor alignment of the eddy-pv flux with either of the two objects considered.

Potential Vorticity (Left) and Circulation (Right) in Top Layer

Less Inertial

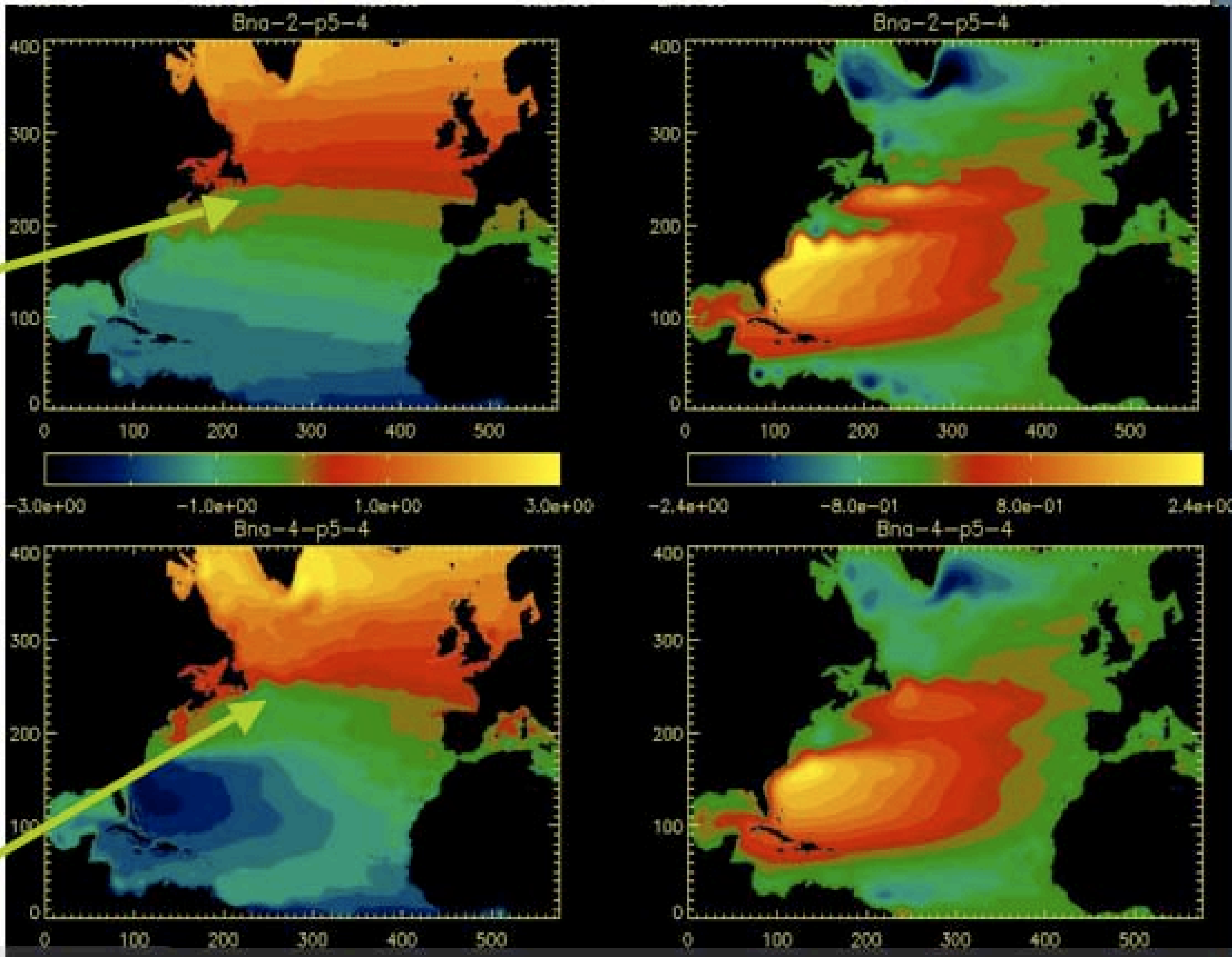
- Poor Separation
- Anticyclonic Recirc
- Weaker NRG

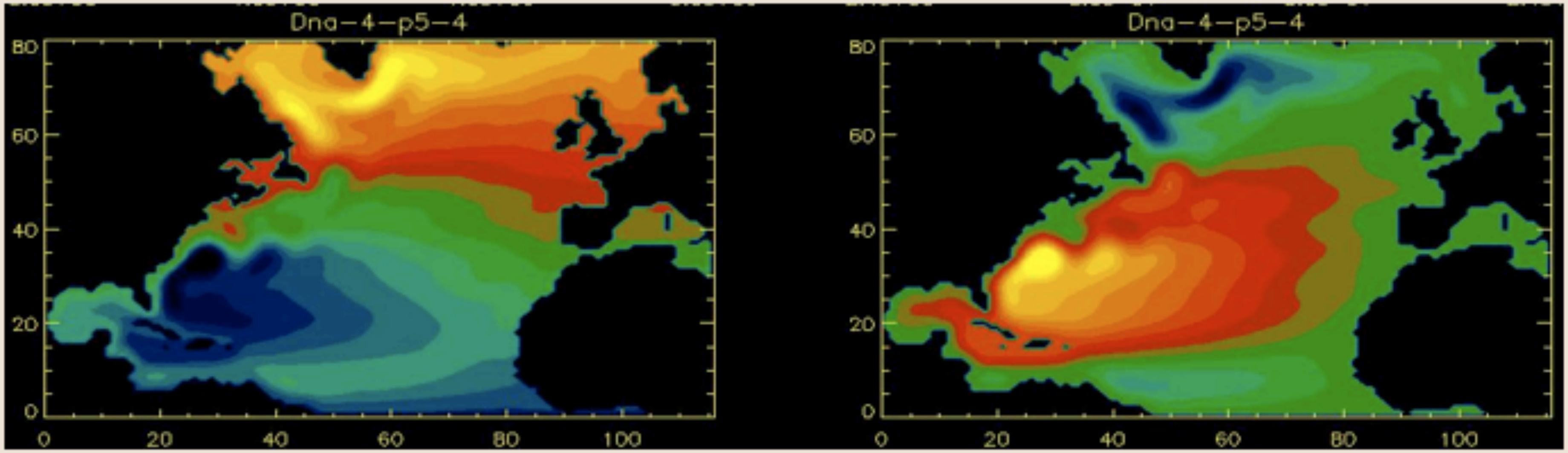
Weak eddying across this front does not aid in transporting high pv waters into NRG

More Inertial

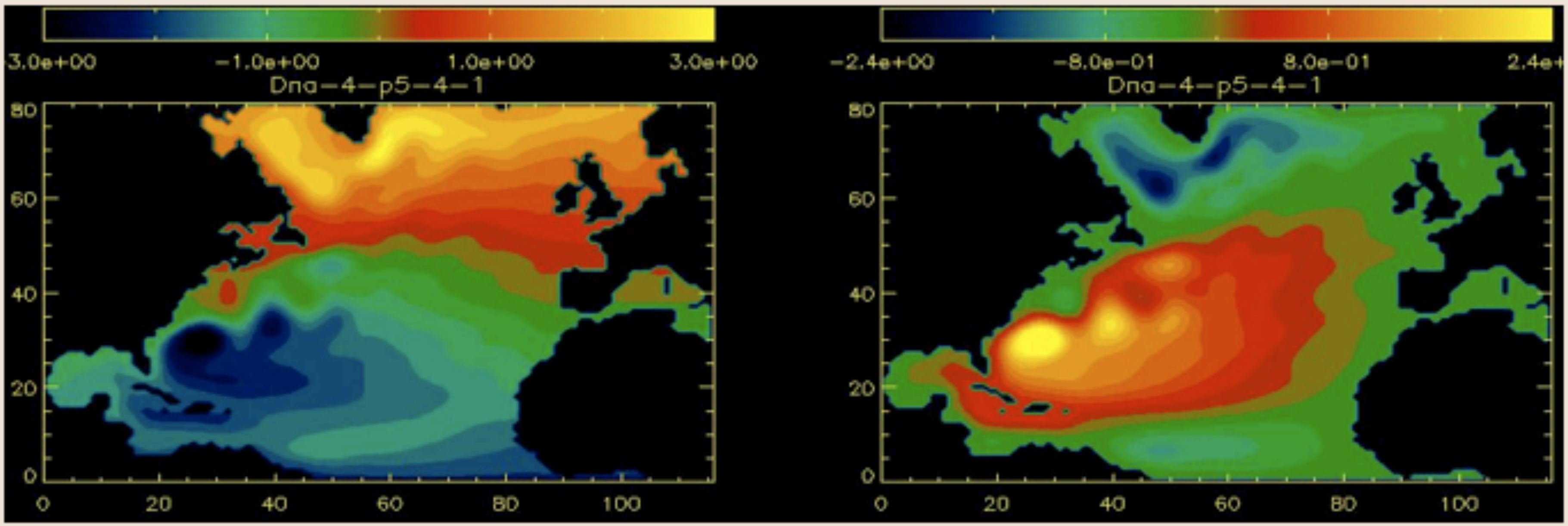
- Better Separation
- Stronger NRG
- Closed pv contours

Strong eddying here transports high pv eddies into NRG





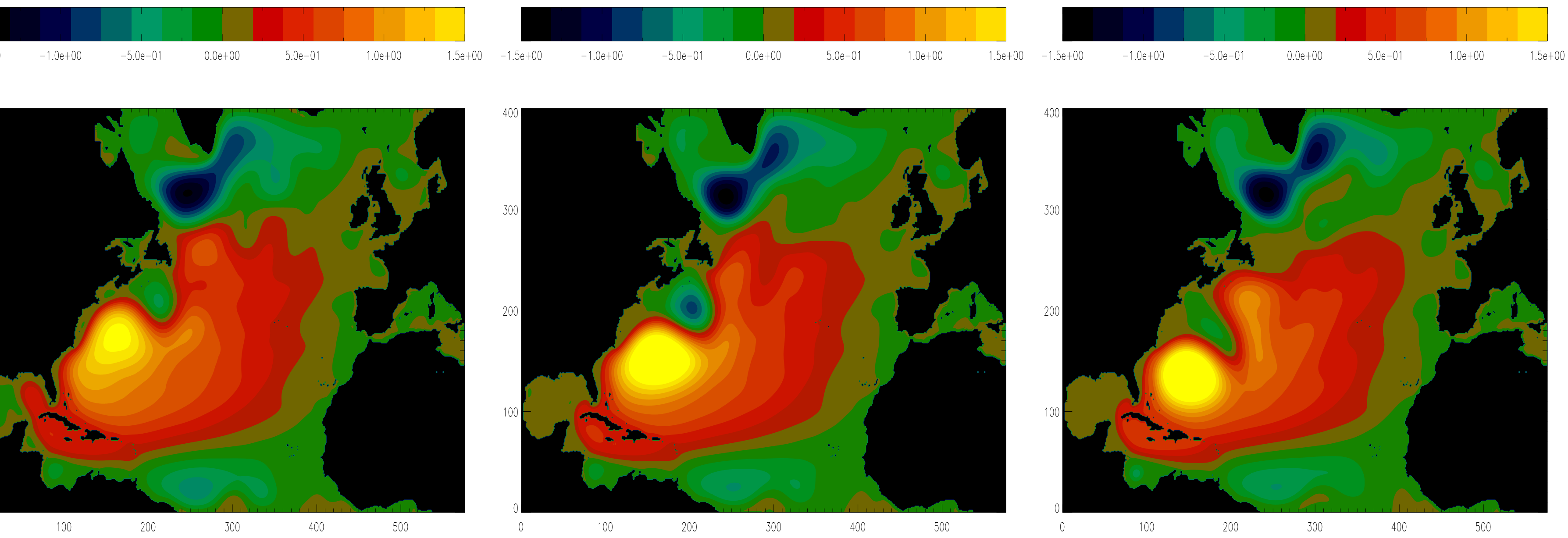
With a 5x reduced resolution, separation incorrect (less inertial)



Nonlinear-Gradient-based subgrid model Improves separation!

Gulf Stream Separation

Barotropic Streamfunction(Flat-bottom)



Conclusion

- Reynolds Average Based Modeling vs. LES
  - Intution for developing models of turbulence in ocean flows largely based on Reynolds Averaging
  - With affordable higher-resolutions,scale-decomposition/LES ideas are increasingly relevant
  - LES differs from Reynolds Average based models in modeling only unresolved scales
  - However, OGCMs (mean equations) are such that scale-decomposition motivated models can be easily implemented. Requires appropriate (re-)interpretation of model variables.

- Orientation of eddy-pv flux
  - Fwd. Cascade of Enstrophy requires net downgradient component
  - Backscatter is almost as important as Damping
  - Local Correlation with Gradient Very Poor
  - With scale-decomposition, good correlation with $\nabla \mathbf{u}_l \cdot \nabla q_l$
  - Doesn't hold for Reynolds decomposition
- Scalar vs. Tensor eddy-viscosity, nonlinear gradient model, numerics