Reduced Equations for Langmuir Turbulence

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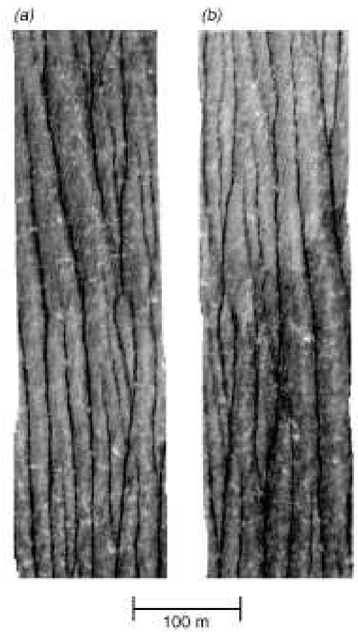
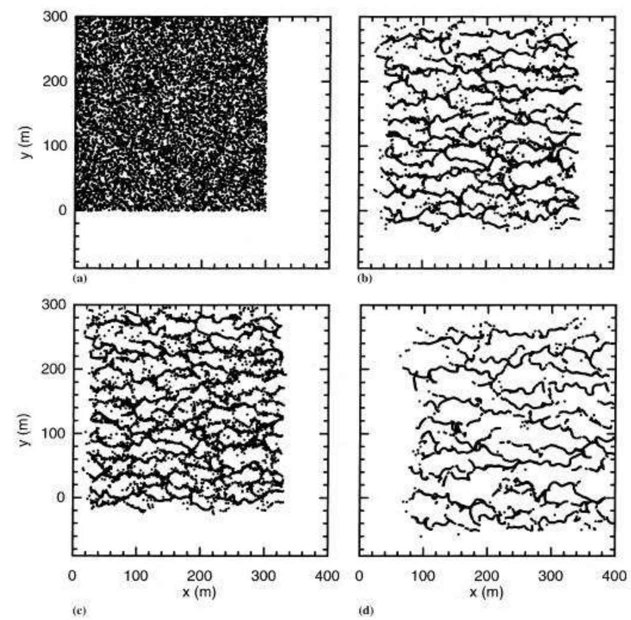
Langmuir Circulation (LC) Windrows





–A. Szeri (1996) –G. Marmorino –McWilliams *et al.* (1997)

Related Work – Theory and Simulation

Quasi-Laminar Simulations of 2D Craik–Leibovich (CL) Equations

- Li & Garrett (*J. Mar. Res.* 1993, *JPO* 1995,1997)
- Gnanadesikan & Weller (*JPO* 1995)

Weakly Nonlinear 2D and 3D Investigations

- 2D: Leibovich, Lele & Moroz (*JFM* 1989)
- 3D: Bhaskaran & Leibovich (*Phys. Fluids*, 2002)

Small Wavenumber Finite-Amplitude 3D Investigations

- Cox & Leibovich (*Phys. Fluids*, 1997)

Simulations of Full 3D Craik–Leibovich (CL) Equations

- DNS: Tandon & Leibovich (*JGR*, 1995)
- LES: Skyllingstad & Denbo (*JGR*, 1995), McWilliams *et al.* (*JFM*, 1997), Tejada-Martinez & Grosch (*JFM*, 2007)

Goals and Motivation

Objective Obtain reduced PDE model capable of describing coarse-grained, strongly anisotropic but otherwise turbulent LC dynamics.

Motivation

- Secondary stability analysis by Tandon & Leibovich (*JPO*, 1995)
- Reduced PDEs for rapidly-rotating thermal convection by Julien, Knobloch & Werne (*Theoret. Comput. Fluid Dyn.*, 1998), Sprague *et al.* (*JFM*, 2006)

Purpose

- Reveal dominant 3D physics.
- Enable simpler (e.g. upper-bound) analysis.
- Less expensive numerical simulations for multi-scale process studies.
- Incorporation into formal multiscale numerical scheme.

Isotropically Scaled CL Equations

- Consider full 3D, isotropically-scaled CL equations, where two parameters $R_* \equiv u_* H/\nu_e$, $La_t = \sqrt{u_*/u_{s0}}$ replace single parameter $La \equiv La_t R_*^{-3/2}$:

$$\boxed{\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \frac{1}{La_t^2}(\mathbf{U}_s \times \boldsymbol{\omega}) + \frac{1}{R_*}\nabla^2 \mathbf{u}}$$

- Two turbulence regimes:

Shear flow turbulence regime: $La_t \gg 1$ with $R_* \gg 1$.

Langmuir turbulence regime: $La_t = O(0.1)$ with $La \ll 1$.

- Motivates consideration of formal limit $La_t \to 0$ with R_* fixed or $R_* \to \infty$:

2D dynamics: $\Omega \neq 0$, $\partial_x \Omega = 0$, u–fluctuations $\ll$ (v, w)–fluctuations.

Anisotropic Velocity Scalings

- Employ anisotropic velocity scales to capture nonlinear, spatially anisotropic reduced dynamics:

$$\boxed{\begin{array}{c} L_x = H, \quad (L_y, L_z) = H, \quad \mathcal{T} = H/\mathcal{V} \\ \mathcal{U} = u_* R_*, \quad (\mathcal{V}, \mathcal{W}) = \sqrt{\mathcal{U} u_{s_0}}, \quad \mathcal{P} = \rho \mathcal{V}^2 \end{array}}$$

- In essence, perturbing off of strictly 2D [$\partial(\cdot)/\partial x = 0$] problem.

- Identify $\varepsilon \equiv \mathcal{U}/\mathcal{W} = R_*^{1/2} La_t$ (cf. Tejada-Martinez & Grosch 2007).

Rescaled CL Equations in Strong Wave–Forcing Limit

$$
\begin{aligned}
\partial_t u + \varepsilon\, u \partial_x u + (\mathbf{v}_\perp \cdot \nabla_\perp)\, u &= -\varepsilon^{-1} \partial_x P + \varepsilon R_*^{-2} \left[\partial_x^2 + \nabla_\perp^2\right] u \\
\partial_t \mathbf{v}_\perp + \varepsilon\, u \partial_x \mathbf{v}_\perp + (\mathbf{v}_\perp \cdot \nabla_\perp)\, \mathbf{v}_\perp &= -\nabla_\perp P + U_s \left(\nabla_\perp u - \varepsilon^{-1} \partial_x \mathbf{v}_\perp\right) \\
&\quad + \varepsilon R_*^{-2} \left[\partial_x^2 + \nabla_\perp^2\right] \mathbf{v}_\perp \\
\varepsilon\, \partial_x u + \nabla_\perp \cdot \mathbf{v}_\perp &= 0
\end{aligned}
$$

- Wind stress BC: $\partial_z u = 1$ along $z = 0, -1$.
- x-invariance at leading-order: $\partial_x P = \partial_x v = \partial_x w = 0$ and $\nabla_\perp \cdot \mathbf{v}_\perp = 0$.

Multiple Scale Expansion

1. Limit process: $\varepsilon \to 0$, i.e. $La_t \to 0$, $R_* = La_t^{-2\alpha/(1-\alpha)}$, $0 \le \alpha < 1/2$.

2. Introduce slow x scale: $X \equiv \varepsilon x$ so that $\partial_x \to \partial_x + \varepsilon \partial_X$.

3. Expand fields:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,X,y,z,t) + \varepsilon u_1(x,X,y,z,t) + \ldots \\
\mathbf{v}_\perp(x,y,z,t) &= \mathbf{v}_{0\perp}(X,y,z,t) + \varepsilon \mathbf{v}_{1\perp}(x,X,y,z,t) + \ldots \\
P(x,y,z,t) &= P_0(X,y,z,t) + \varepsilon P_1(x,X,y,z,t) + \ldots
\end{aligned}
$$

4. Substitute into PDEs, collect terms of like order and **average** over fast x.

5. Obtain closed set of equations for $\bar{u}_0 \equiv U(X,y,z,t)$, $\mathbf{v}_{0\perp} \equiv \mathbf{V}_\perp(X,y,z,t)$ and $P_0 \equiv \Pi(X,y,z,t)$.

Reduced PDEs

- Define:
$$D_t^{\perp}(\cdot) \;\equiv\; \partial_t(\cdot)+(\mathbf{V}_{\perp}\cdot\nabla_{\perp})(\cdot) \equiv \partial_t(\cdot)+J[(\cdot),\psi],$$
where $J[(\cdot),\psi] = \partial_z\psi\partial_y(\cdot) - \partial_y\psi\partial_z(\cdot)$.

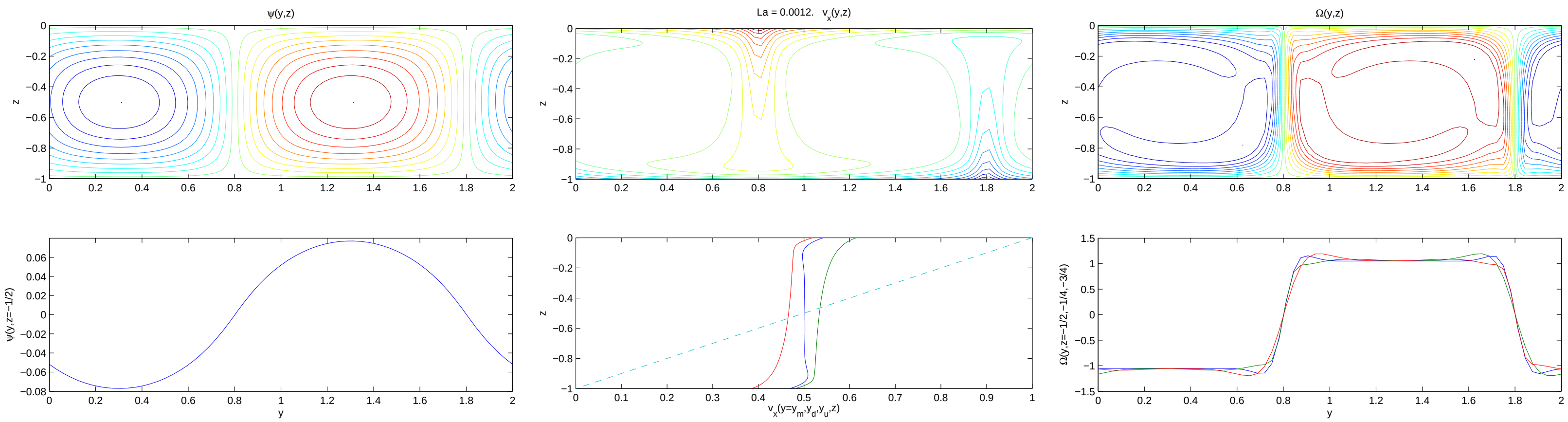
- Reduced dynamics governed by:

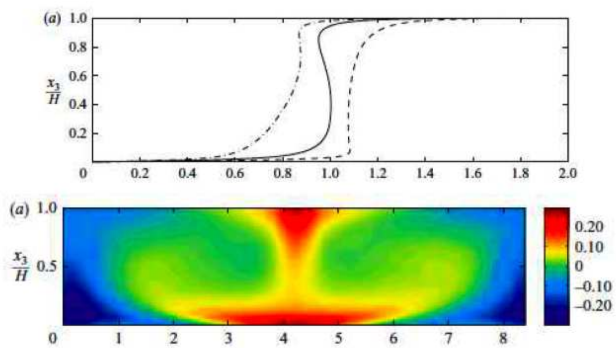
$$\boxed{\begin{aligned} D_t^{\perp}U &= -\partial_X \Pi + La\nabla_{\perp}^2 U \\ D_t^{\perp}\Omega + U_s(z)\partial_X\Omega &= U_s'(z)(\partial_X V - \partial_y U) + La\nabla_{\perp}^2\Omega \\ \nabla_{\perp}^2\Pi &= 2J[\partial_y\psi,\partial_z\psi] + \nabla_{\perp}\cdot(U_s(z)\nabla_{\perp}U) + U_s'(z)\partial_X(\partial_y\psi) \\ \nabla_{\perp}^2\psi &= -\Omega, \quad \mathbf{V}_{\perp} \equiv \nabla_{\perp}\times\psi\hat{\imath} \end{aligned}}$$

- Fast x averaged BCs along $z = 0, -1$: $\partial_z U = 1$, $\Omega = 0$, $\psi = 0$.
- Advection by U and stretching of Ω are subdominant processes.

Strongly Nonlinear, Strictly 2D Convective States

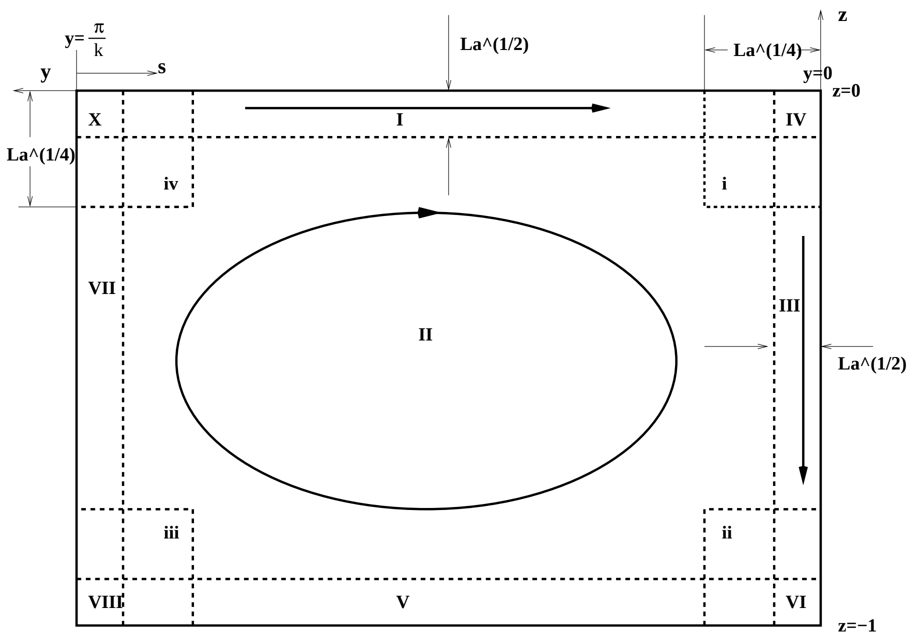
$\psi(y,z)$ $\quad$ $U(y,z)$ $\quad$ $\Omega(y,z)$

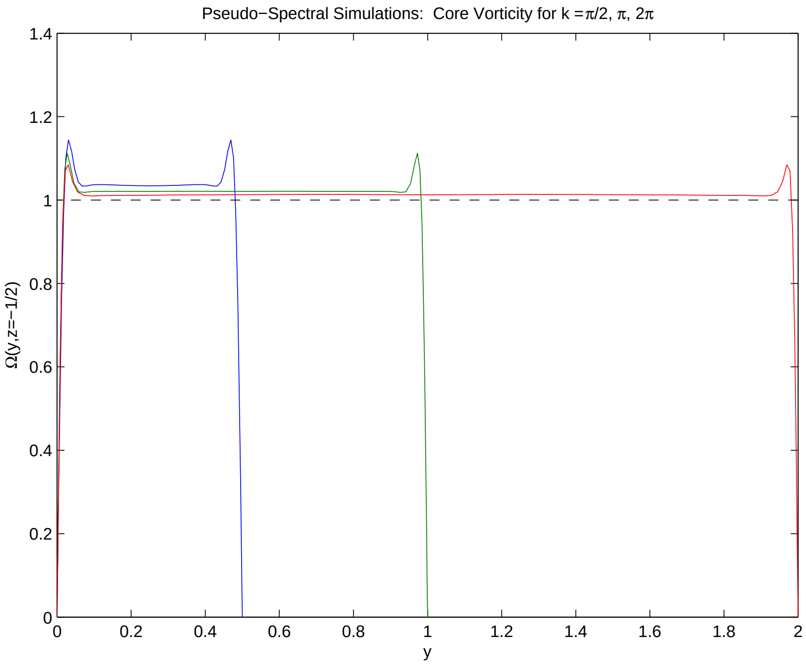




- Steady-state $U(y,z)$ profiles show excellent qualitative agreement with $x{-}t$ averaged LES profiles of Tejada–Martinez & Grosch (2007).

Matched Asymptotic Analysis

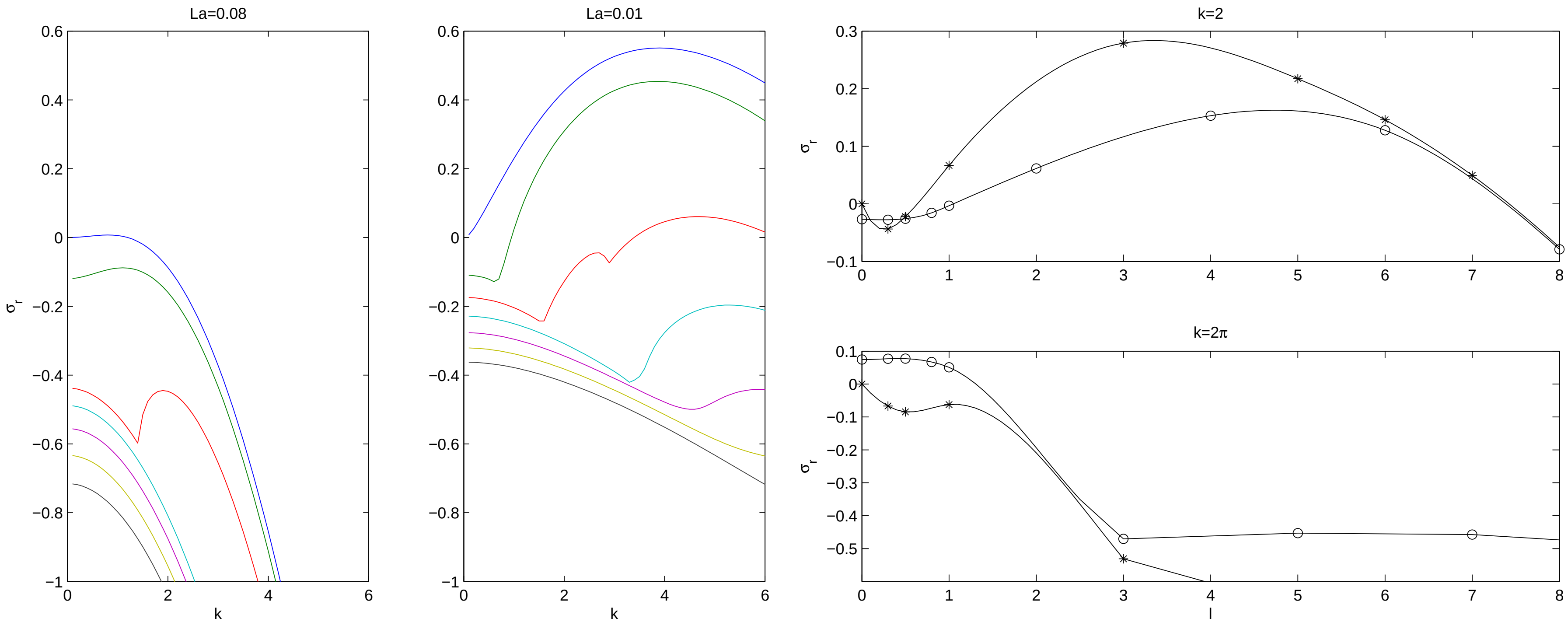




- Asymptotic analysis predicts core vorticity $|\bar{\Omega}| \sim 1 \;\; \forall\, k$ as $La \to 0$.
- With $\psi(y,z)$ known, boundary/interior layer problems linearize and entire solution can be approximated asymptotically.

† G. P. Chini. Strongly nonlinear Langmuir circulation and Rayleigh–Bénard convection. Submitted to the *Journal of Fluid Mechanics*.

Reduced PDEs — Linear and Secondary Stability Results



† G. Chini, K. Julien, E. Knobloch. An asymptotically reduced model of Langmuir turbulence. Submitted to *Geophysical and Astrophysical Fluid Dynamics*.

Conclusions

- Derived reduced PDEs for anisotropic Langmuir turbulence in strong wave-forcing limit that capture dominant linear and secondary instabilities.

- Ongoing investigations:
  - time-dependent simulations of reduced system.
  - incorporation of stratification (interactions b/w LC and internal waves) and rotation.
  - exploration of scalings yielding complementary reduced PDE models of Langmuir turbulence.
  - application of methodology to other shear-flow instability phenomena.

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Future Directions — Submesoscale–Mesoscale Interactions

- Investigate multiscale interactions b/w mixed-layer LC and submeso- and mesoscale eddies.
- Develop modulation (or homogenization) theory for asymptotic strongly-nonlinear 2D LC solutions.
- Construction of a hierarchy (algebraic, ODE, reduced PDE) of LC vertical-flux parameterizations for use in OGCMs.