Geostrophic Turbulence in the Ocean Mixed Layer

Raffaele Ferrari
EAPS

Collaborators:
Baylor Fox-Kemper, Glenn Flierl, and the CPT-EMILIE team
The goal of the CPT-EMILIE is to develop parameterizations of unresolved processes for models of the ocean mixed layer, a region key for coupled climate models:

- **Mesoscale turbulence** (10 km – 100 km)
- **Submesoscale turbulence** (100 m – 10 km)
- **Small-scale turbulent mixing** (10 cm – 100 m)

Subgrid-scale processes in ocean climate models:

- Small-scale mixing
- Submesoscale eddies
- Mesoscale eddies
- Large-scale circulation
**Interior ocean turbulence**

- **Traditional paradigm of ocean turbulence**
  - mesoscale eddies dominate along-isopycnal transport
  - submesoscale eddies are subdominant
  - small-scale mixing dominates cross-isopycnal transport

\[
\bar{b}_t + \bar{u} \cdot \nabla \bar{b} = - \nabla_b \cdot \bar{u}' \bar{b}' + \partial_z \kappa \bar{b}_z
\]

- **mesoscale**
- **turbulent mixing**
Mixed layer turbulence

• Paradigm of mixed layer turbulence
  • mesoscale eddies dominate horizontal transport
    - small Ro, large Ri and scales close to deformation radius
  • submesoscale eddies and small-scale mixing dominate vertical transport
    - large Ro and small Ri and scales close to mixed layer deformation radius

\[
\bar{b}_t + \bar{u} \cdot \nabla \bar{b} = - \nabla \bar{H} \cdot \bar{u}_H b' - \partial_z w'b' + \partial_z \kappa \bar{b}_z
\]

- mesoscale
- submesoscale
- boundary layer

Vorticity

Mixed layer turbulence
Outline of presentation

• Mesoscale and submesoscale turbulence in the mixed layer
  • effect of surface on ocean eddies
    • submesoscale frontogenesis
  • effect of weak mixed layer stratification on ocean eddies
    • submesoscale frontal instabilities

• Parameterization of submesoscale turbulence
  • scaling of vertical buoyancy fluxes
Part I. Effect of surface on mesoscale turbulence: frontogenesis

Horizontal SeaSoar section

Horizontal spectra

Ferrari and Rudnick, 2000
Mesoscale turbulence in the upper ocean

Shallow spectra at surface (z=–2 m)

Steep spectra in thermocline (z=–400 m)

Potential density
Kinetic energy

Klein et al., 2007
Quasi-geostrophic model of mesoscale turbulence

- Quasi-geostrophic approximation describes rotating stratified fluids with:
  - small Rossby number, $Ro = \frac{U}{f_0L} \ll 1$
  - motions with horizontal scales close to the deformation radius, $L \approx \frac{NH}{f_0}$
  - vertical stratification function of depth only, $N^2 = -\frac{g}{\rho_0} \partial_z \rho$

\[ b_T^t + J(\psi, b_T^t) = 0, \quad f_0 \partial_z \psi = b_T^t, \quad \text{at } z = 0 \]

\[ q_t + J(\psi, q) = 0, \quad \nabla^2 \psi + \partial_z \left( \frac{f_0^2}{N^2} \partial_z \psi \right) = q \]

\[ b_B^t + J(\psi, b_B^t) = 0, \quad f_0 \partial_z \psi = b_B^t, \quad \text{at } z = -H \]
Separating surface from boundary dynamics

\[ \psi = \psi^I + \psi^T + \psi^B \]

- **Interior dynamics** (quasi-geostrophy; Charney 1971)
- **Surface dynamics** (surface quasi-geostrophy; Blumen 1978)
- **Bottom dynamics** (surface quasi-geostrophy; Blumen 1978)

\[ q_t + J(\psi, q) = 0, \quad \nabla^2 \psi^I + \partial_z \left( \frac{f_0^2}{N^2} \partial_z \psi^I \right) = q \]

\[ f_0 \partial_z \psi^I = 0 \]

\[ N^2 q_t + J(\psi, q) = 0, \quad \nabla^2 \psi^T + \partial_z \left( \frac{f_0^2}{N^2} \partial_z \psi^T \right) = q = 0 \]

\[ f_0 \partial_z \psi^T = 0, \quad \text{at } z = 0 \]
Separating surface from boundary dynamics

\[ \psi = \psi^I + \psi^T + \psi^B \]

- Interior dynamics (quasi-geostrophy; Charney 1971)
- Surface dynamics (surface quasi-geostrophy; Blumen 1978)
- Bottom dynamics (surface quasi-geostrophy; Blumen 1978)

\[ b_t^T + J(\psi, b^T) = 0, \quad f_0 \partial_z \psi^T = b^T, \quad \text{at} \quad z = 0 \]

\[ \nabla^2 \psi^T + \partial_z \left( \frac{f_0^2}{N^2} \partial_z \psi^T \right) = 0 \]

\[ f_0 \partial_z \psi^T = 0, \quad \text{at} \quad z = -H \]
Ratio of kinetic and potential energies in the surface mixed layer

- **Surface modes** (Held surface QG theory)
  - equipartition between kinetic and potential energy at all scales
  - kinetic and potential energy spectra roll off as $k^{-5/3}$

$$KE = \frac{1}{2} \langle u^2 + v^2 \rangle \sim k^{-5/3} e^{-2Nkz/f}$$

$$PE = \frac{1}{2} \langle b^2 \rangle \sim k^{-5/3} e^{-2Nkz/f}$$

- **Interior modes** (Charney QG theory)
  - kinetic energy spectra roll off as $k^{-3}$
  - potential energy spectra roll off as $k^{-5}$
Mesoscale turbulence and surface frontogenesis

- Surface: submesoscales set rate of upwelling and restratification
- Interior: mesoscales set rate of upwelling and restratification

\[ f^2 \psi_{zz} + N^2 \psi_{yy} = -2v_y b_y \]
Part II. Effect of reduced stratification on mesoscale turbulence: frontal instabilities

- The mixed layer has weak vertical stratification (boundary layer mixing) and strong lateral gradients (frontogenesis)

- The ocean interior is has strong vertical stratification and weak lateral gradients

- Two types of baroclinic instability
Surface quasi-geostrophic model with a mixed layer

- Two layers with uniform potential vorticity
  - upper layer with weak stratification represents mixed layer
  - lower layer with stronger stratification represents upper ocean

\[
\nabla^2 \psi_m + \frac{f^2}{N_m^2} \frac{\partial^2 \psi_m}{\partial z^2} = 0
\]

\[
\nabla^2 \psi + \frac{f^2}{N^2} \frac{\partial^2 \psi}{\partial z^2} = 0
\]
Surface quasi-geostrophic model with a mixed layer

- Model configuration
  - prescribed meridional buoyancy gradients in thermal wind balance
  - linear bottom drag and hyperviscosity

\[ N^2 \]

\[ U(z) \]

\[ \nabla \cdot F^\dagger \nu \approx -\frac{1}{H_M L} \int_{-H_M L}^{-H} \nabla \cdot F^\dagger \nu \, dz \]

\[ F^\dagger b \approx -K_{hor} \nabla \bar{b} \]

\[ P = \nabla \cdot (\omega a b) = \langle P \rangle = \partial \langle f + \zeta \rangle / \partial z \]

\[ \eta = f g' (\psi_m(x, y, z = -h) - \psi(x, y, z = -h)) \]

\[ \Box \]

Bottom Ekman drag
Surface QG model with ML: frontogenesis and submesoscale instabilities

\[ \theta_1 \text{ (surface)} \quad \theta_2 \text{ (interface)} \quad \theta_3 \text{ (bottom)} \]

Linear stability analysis

Surface EKE spectrum

\[ k^{-2.5/3} \]

Growth rate

Spectral density

\[ 10^{-10} \]
Surface QG model without ML: mesoscale frontogenesis

\[ \theta_1 \text{ (surface)} \]

\[ \theta_2 \text{ (interface)} \]

\[ \theta_3 \text{ (bottom)} \]

Linear stability analysis

Surface EKE spectrum

\[ k^{-2, -5/3} \]
Vertical buoyancy flux at the ML base

- Vertical buoyancy flux is positive $\Rightarrow$ restratification
- Submesoscale instabilities at surface dominate the buoyancy flux

SQG model with ML
(meso and submesoscale eddies)

SQG model without ML
(mesoscale eddies)
Energetics of surface QG with a ML

- Forward flux at the mesoscale
- Forward flux at the submesoscale only with the ML
Part III. Scaling laws for submesoscale vertical fluxes

\[
\bar{b}_t + \bar{u}\cdot \nabla \bar{b} = -\nabla H \cdot \bar{u}_u \cdot \bar{H} b' \\
\bar{H} b' \quad \text{mesoscale} \\
\partial_z \bar{w} \cdot \bar{b}' \quad \text{submesoscale}
\]

(1)

Heat Flux \( z \) = \(- \lambda (T - T_{atm}) \) (2)

Surface cooling (3)

\(+\) Penetrative heating \( z \) (4)

\( \langle \bar{w}' \bar{b}' \rangle \) (5)

\( \langle \bar{w}' \bar{q}' \rangle \) (6)

\( f^{-1} H^2 |\nabla H \bar{b}|^2 \) (7)

\( f^{-1} \nabla H \bar{b} \cdot \tau \) (8)

(9)

Restratification by free frontal instabilities

Fox-Kemper and Ferrari, 2007
Re-destratification by wind-driven frontal instabilities

\[ \bar{b} + \bar{u} \cdot \nabla \bar{b} = - \nabla H \cdot \bar{u} \bar{u}^\prime \]

Heat Flux \((z)\) = \(- \lambda (T - T_{atm})\)

Surface cooling

Penetrative heating \((z)\)

\[ \langle w'b' \rangle \]

\[ \langle w'q' \rangle \]

\[ f^{-1} \nabla_H \bar{b} \cdot \tau \]

Thomas and Lee, 2006

Thomas and Ferrari, 2008
Conclusions

• Mesoscale eddies drive surface frontogenesis

• Submesoscale instabilities develop along surface fronts that
  • act to restratify the ocean surface mixed layers
  • drive strong upwelling/downwelling of tracers

• Scaling laws have been derived
  • for the combination of frontogenesis and submesoscale
    instabilities
  • for the effect of winds blowing over the surface fronts
Mesoscale straining generates ML fronts through frontogenesis.

Frontal instabilities develop along ML fronts.

Capet et al., 2007
Submesoscale instabilities and ML restratification

Submesoscale vertical fluxes
- co-located with fronts
- mostly positive => restratification
- largest at wiggly fronts
- increase with resolution

\[
\bar{b}_t + \bar{u} \cdot \nabla \bar{b} = - \nabla H \cdot u' H b' \tag{1}
\]

Heat Flux \( (z) \) = \(- \lambda (T - T_{am}) \) (2)

Surface cooling (3)

Penetration heating \( (z) \) (4)

\( \langle w'b' \rangle \) (5)

\( \langle w'q' \rangle \) (6)

\( f^{-1/2} |\nabla H \bar{b}|^2 \) (7)

\( f^{-1} \nabla H \bar{b} \cdot \tau \) (8)

\( w'b' \) (10)

\( |\nabla H b| \) (11)
Submesoscale instabilities and ML restratification

Submesoscale vertical fluxes
- co-located with fronts
- mostly positive => restratification
- largest at wiggly fronts
- increase with resolution
Submesoscale instabilities and ML restratification
Submesoscale instabilities and ML restratification
Surface quasi-geostrophic model with a mixed layer

- Two layers with uniform potential vorticity
- Boundary condition of no vertical velocity at top and bottom
- Matching conditions at the interface

\[
\frac{D_m b_1}{Dt} = \frac{D_m}{Dt} \frac{\partial \psi_m}{\partial z} = 0
\]

\[
\frac{D_m \eta}{Dt} = -w = \frac{f}{N_m^2} \frac{D_m}{Dt} \frac{\partial \psi_m}{\partial z}
\]

\[
\frac{D \eta}{Dt} = -w = \frac{f}{N^2} \frac{D_m}{Dt} \frac{\partial \psi_m}{\partial z}
\]

\[
\frac{D b_3}{Dt} = \frac{D}{Dt} \frac{\partial \psi}{\partial z} = 0
\]
Surface quasi-geostrophic model with a mixed layer

- Two layers with uniform potential vorticity
- boundary condition of no vertical velocity at top and bottom
- matching conditions at the interface

\[
\frac{Dm b_1}{Dt} = \frac{Dm}{Dt} \frac{\partial \psi_m}{\partial z} = 0
\]

\[
\frac{Db_2}{Dt} = \frac{D}{Dt} \left( \frac{\partial \psi_m}{\partial z} - \frac{N_m^2 \partial \psi}{N^2 \partial z} \right) = 0
\]

\[
\frac{Db_3}{Dt} = \frac{D}{Dt} \frac{\partial \psi}{\partial z} = 0
\]
Submesoscale heat fluxes

JAN Vertical Eq. Heat Flux from SSHA (W/m^2)

Vertical Eq. Heat Flux from SSHA (W/m^2)
Observations of submesoscale instabilities

- Spirals in the sea (Munk '01)
- Frontal instabilities in the California Current System (Flament et al., '85)

Figure 1. A pair of interconnected spirals in the Mediterranean Sea south of Crete. This vortex pair has a clearly visible stagnation point between the two spirals, the cores of which are aligned with the preconditioning wind field. 7 October 1984.

Enhanced AVHRR image showing submesoscale features in the California Current system related to an upwelling filament.
Surface QG model with ML

\[
\begin{align*}
\theta_1 \text{ (surface)} & \quad \theta_2 \text{ (interface)} & \quad \theta_3 \text{ (bottom)}
\end{align*}
\]

Linear stability analysis

Surface buoyancy spectrum

\[ k^{-5/3} \]
Surface QG model with ML: development of submesoscale instabilities

- Are submesoscale baroclinic instabilities suppressed by mesoscale strain (Bishop, 1993)?
- Mesoscale strain generates fronts at the surface, \((u, v) = (ax, -ay)\)
- Submesoscale waves are stretched by strain \((k, l) = (k_0 e^{at}, l_0 e^{at})\)
- Instabilities are modulated not suppressed by mesoscale strain
Surface QG model without ML: only mesoscale instabilities
Surface QG model with ML and no deep shear: only submesoscale instabilities
Typical Ocean Stratification Permits
Two Types of Baroclinic Instability

Mesoscale and Submesoscale (Boccaletti et al., 2007)

Mesoscale Eddies: $O(100 \text{ km})$
Submesoscale Mixed Layer Eddies: $O(1 \text{ km})$, $O(1 \text{ day})$
To what extent can uncertainties in model projections due to climate system feedbacks be reduced? (Climate Change Science Program)

- Imperfect or **missing parameterizations** of unresolved processes are a major source of model error and uncertainty

**CPT Framework**

- **Focus**
  - upper ocean dynamics
- **Goal**
  - develop parameterizations for key subgrid-scale processes
- **Team members**
  - PI: Raffaele Ferrari
  - Process modelers and theoreticians: Flierl, Fox-Kemper, Marshall, McWilliams, Tandon, Thomas, Vallis
  - Observationalists: Rudnick, Speer
  - Modeling centers: GFDL and NCAR
Upper ocean dynamics in climate models

- Large-scale ocean circulation
- Mesoscale turbulence (10 km – 100 km)
- Submesoscale turbulence (100 m – 10 km)
- Small-scale turbulent mixing (10 cm – 100 m)