Galerkin truncation, hyperviscosity and bottlenecks

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Hyperviscous equations

Burgers $\partial_t v + v\partial_x v = -\mu {k_{\mathrm{G}}}^{-2\alpha}(-\partial_x^2)^\alpha v$

$\mu > 0, \quad k_{\mathrm{G}} > 0, \ \alpha = \text{dissipativity. Here} \quad \alpha > 1.$

N-S $\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p - \mu {k_{\mathrm{G}}}^{-2\alpha}(-\nabla^2)^\alpha \boldsymbol{v}, \quad \nabla \cdot \boldsymbol{v} = 0$

Dissipation rate $\mu(k/k_{\mathrm{G}})^{2\alpha} \to 0 \text{ or } \infty \text{ when } \alpha \to \infty$

Abstract form $\partial_t v = B(v, v) + L_\alpha v$

Galerkin truncation $\partial_t u = P_{k_{\mathrm{G}}} B(u, u), \qquad u_o = P_{k_{\mathrm{G}}} v_0$

Projector $P_{k_{\mathrm{G}}}$: low-pass filter at wavenumber k_{G}

Large dissipativity limit and thermalization

For $\alpha \to \infty$, and fixed μ and k_G, the solution of the hyperdissipative equations tend to the solution of the Galerkin-truncated equations

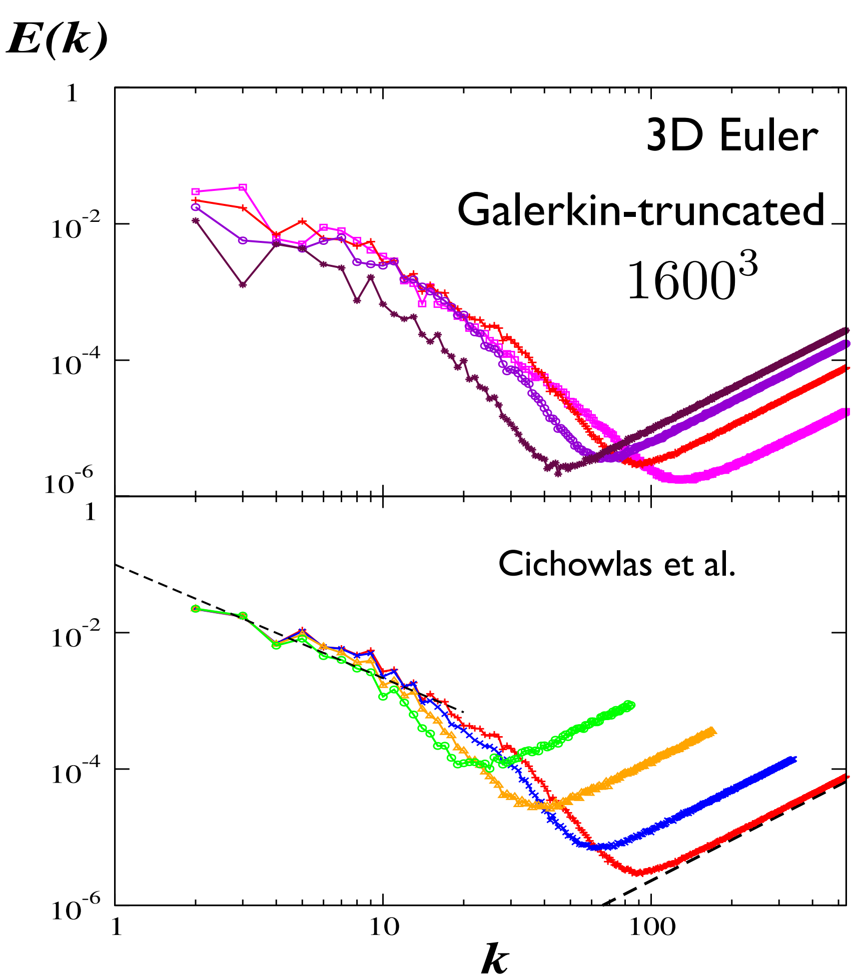
[8] We may regard this as the introduction of infinite damping (infinite resistance) for the degrees of freedom removed.

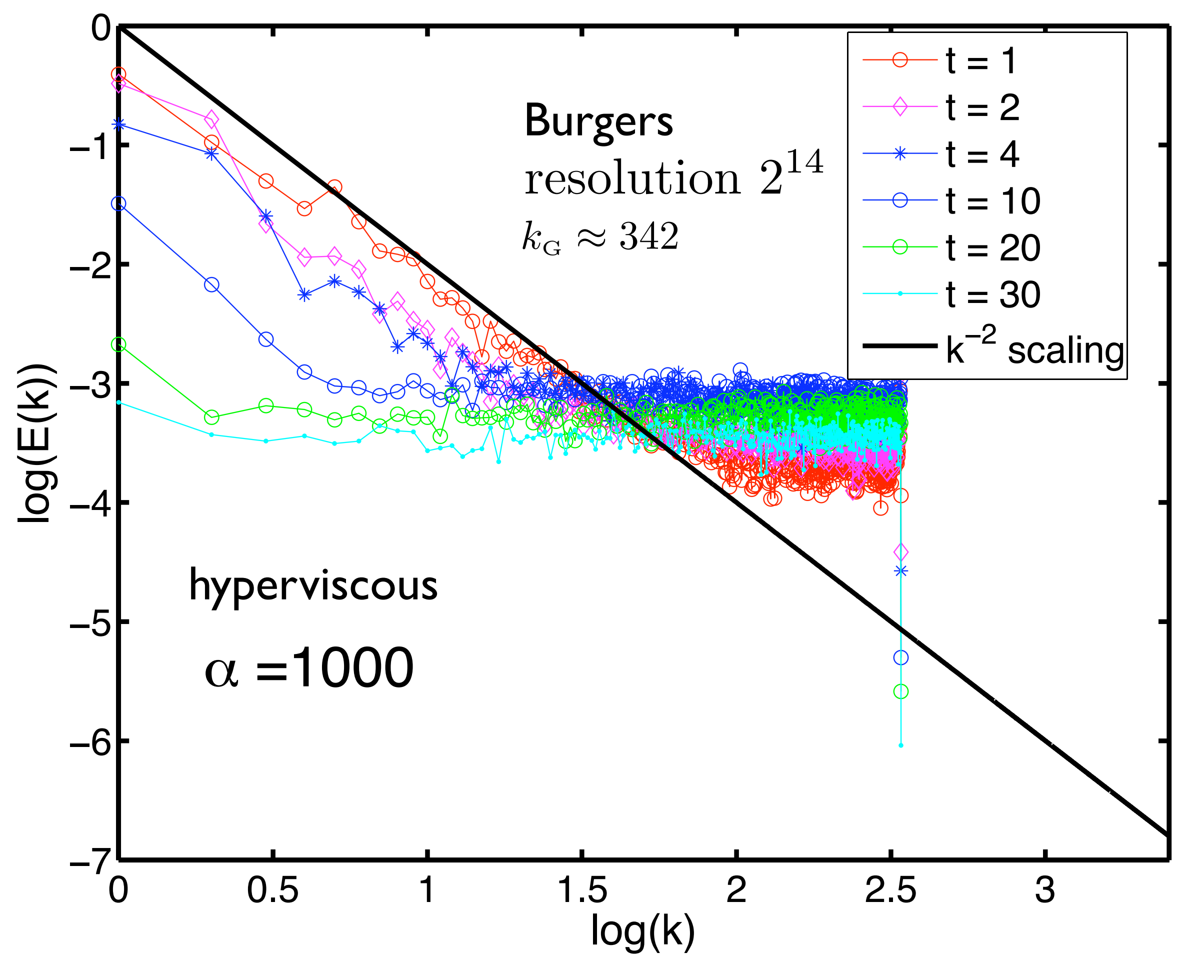
True for: Burgers, Navier-Sokes, MHD, DIA and EDQNM.
False for: MRCM and resonant wave interaction theory.

Galerkin-truncation ⇒ thermalization (Lee, 1952; Hopf, 1952; Kraichnan, 1958)

Galerkin-truncated Burgers first studied by Majda and Timofeyev 2000

Galerkin-truncated 3D incompressible Euler first studied at high resolution by Cichowlas, Bonaiti, Debbasch and Brachet 2005





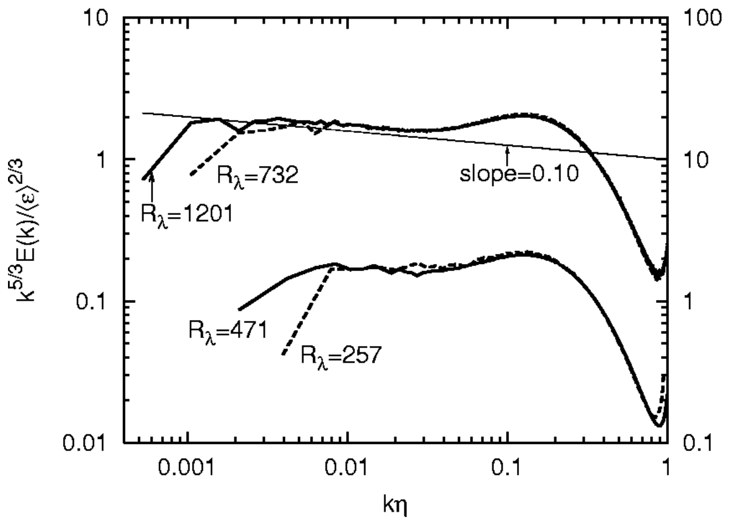
S.S. Ray

Bottleneck, thermalization, depletion of intermittency, etc

- Large α produces a huge thermalized bottleneck

- The standard $\alpha = 1$ bottleneck may be viewed as an *aborted thermalization*

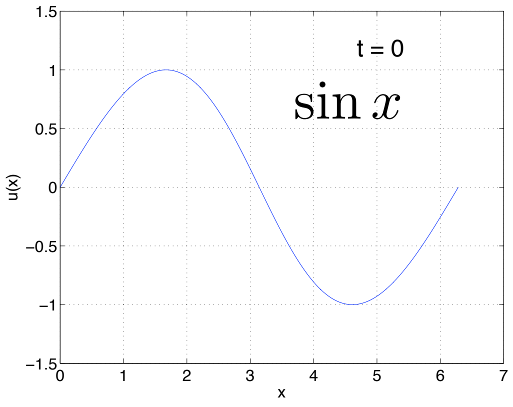
Kaneda et al. 2003 (Earth Simulator). Compensated energy spectrum

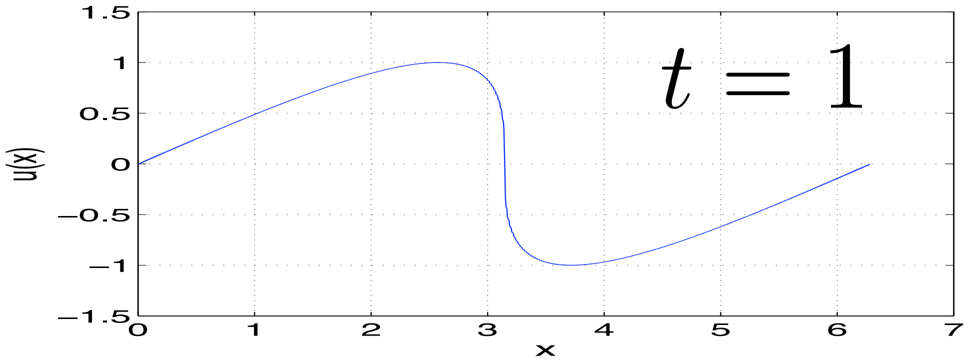


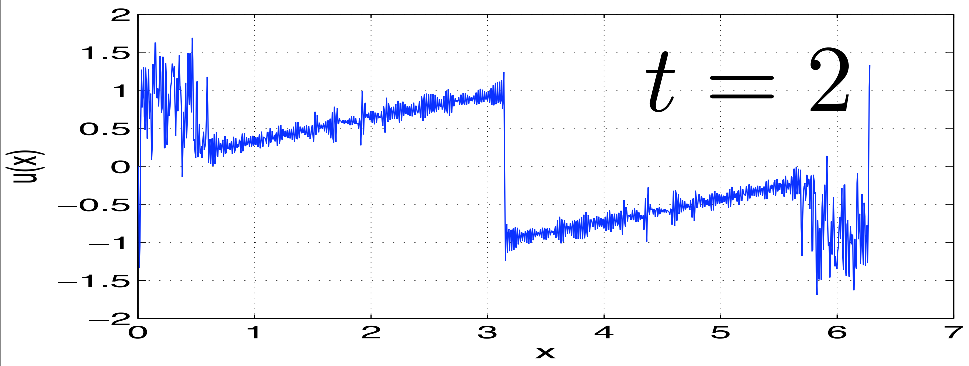
- Thermalization is accompanied by Gaussianization and isotropization

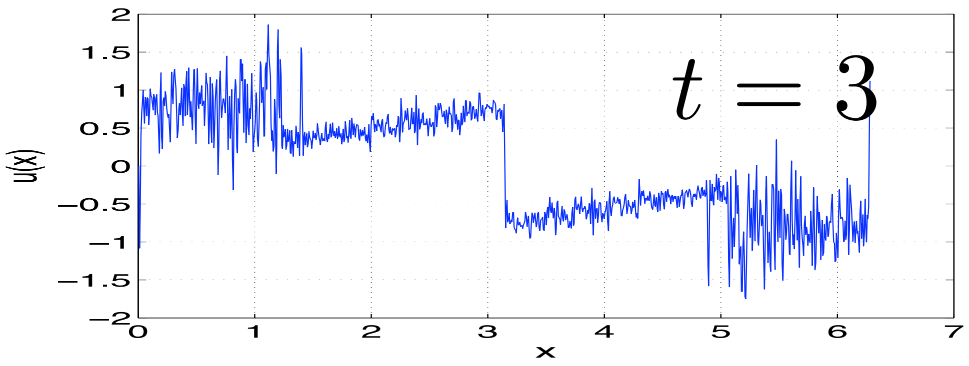
- Spurious effects are expected: depletion of intermittency and enhanced vertical transport

Evolution of Galerkin-truncated Burgers









The shock acts as a black hole

S.S. Ray $\quad$ resolution 2^{10} $\quad$ $k_{\mathrm{G}} = 342$