Zonal jet formation and Equatorial Super-rotation in the Destabilization of Short Mixed Rossby-Gravity Waves

Mark Fruman

Lien Hua, Richard Schopp

LPO, Ifremer

Brest, France

Turbulent Theory and Modeling Workshop; Boulder, Co

February 27$^{\mathrm{th}}$ - 29$^{\mathrm{th}}$, 2008

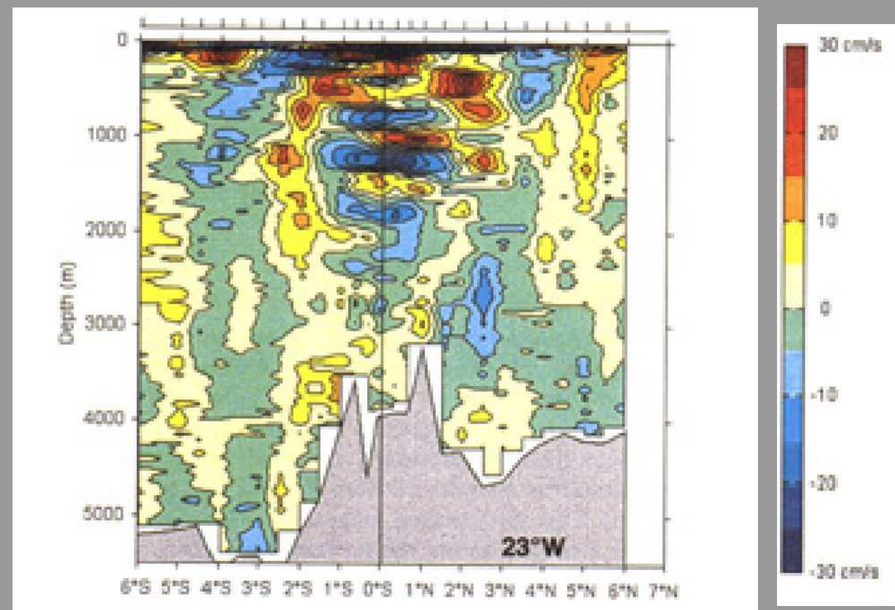
OUTLINE

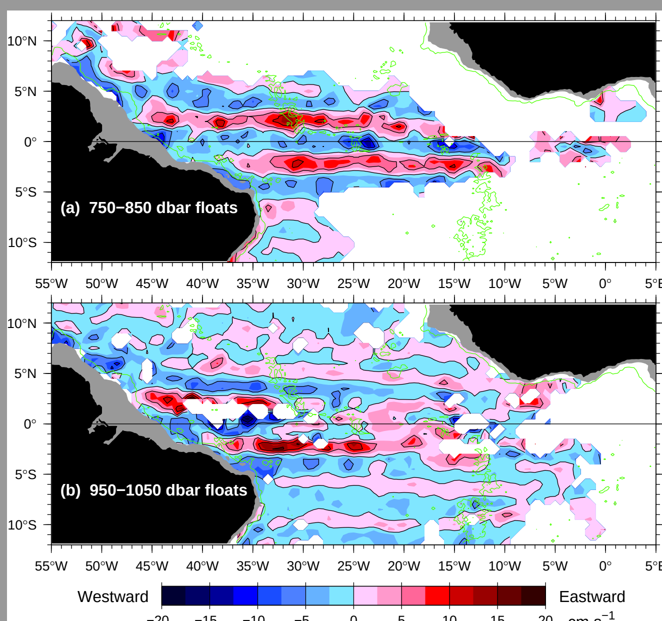
1. Equatorial zonal jets
2. Destabilization of short westward MRG wave
3. Inertial Instability and PV Homogenization
4. Super-rotation and the non-traditional Coriolis force

Gouriou et al. 2001 $\Longrightarrow$

– instantaneous merid. sect. 23° W

– strongly barotropic jets



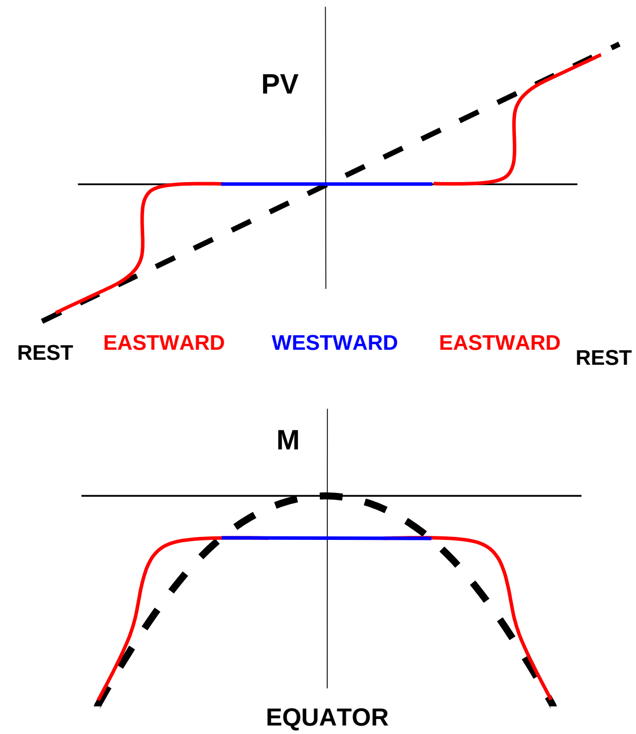


$\Longleftarrow$ Ollitraut et al. 2006

– zonal currents near 1000 m depth

– multiple, equally spaced jets

- Westward flow at equator and eastward jets at $\pm 2^{\circ}$ latitude (Gouriou et al. 2001)
- often corresponds to angular momentum M (and hence PV) homogenized about equator surrounded by PV barriers.

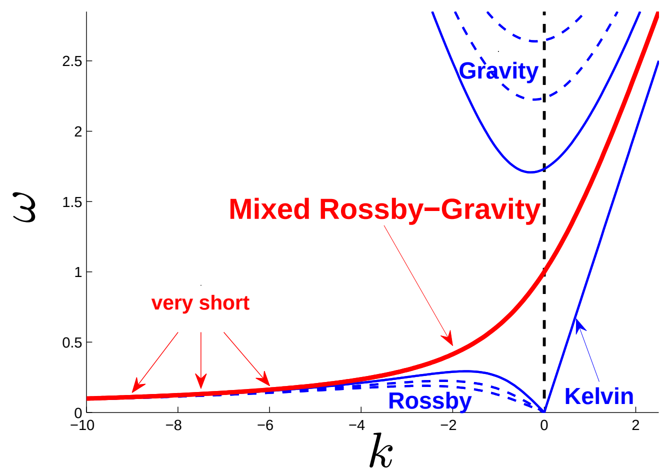


- Propose mechanism for generating jets by barotropic instability of short equatorial waves
- Also explains "equatorial deep jets" (d'Orgeville et al. (2006), Hua et al. (2008))

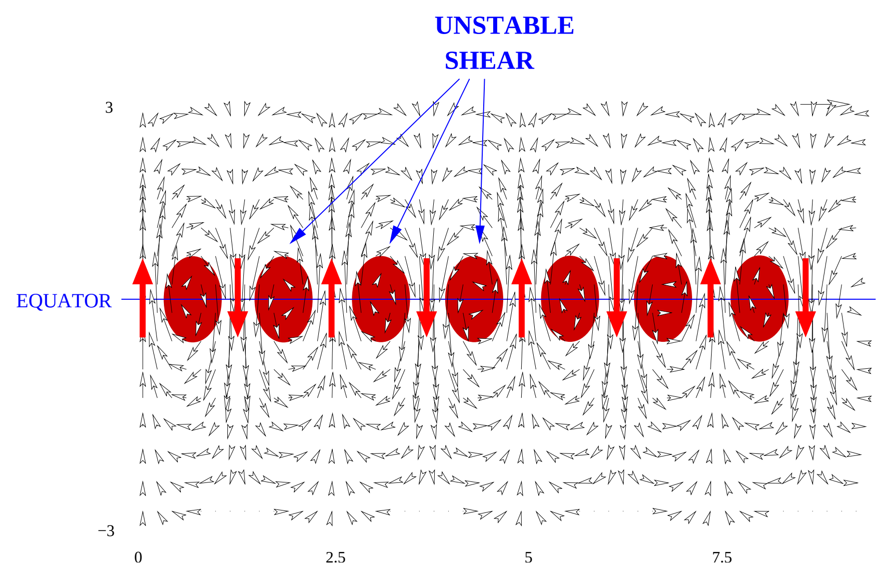
2. Destabilization of short westward MRG wave

MRG wave in large negative k limit: $\omega \sim -k^{-1}$

DISPERSION DIAGRAM FOR EQUATORIALLY TRAPPED WAVES



MRG WAVE VELOCITY FIELD



- Cases considered: $-6 < k < -16$
- Oceanic vertical mode 1 $\Rightarrow$ 50 - 200 day period, 1.2 $^\circ$ - 3° wavelength.

Linear Theory in $k \ll -1$ limit:

Scale length and time like $(x, y) = k^{-1}(\xi, \eta)$ and $t = (kV_0)^{-1}\tau$.

Velocity components of basic state wave:

$$\begin{aligned}
V &\sim V_0 \exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\cos(\xi), \\
U &\sim k^{-1}V_0\frac{\eta}{k^2}\exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\sin(\xi), \\
W &\sim k^{-2}V_0\left(\frac{-\eta^2}{k^2}\right)\exp\left(\frac{-\eta^2}{2k^2}\right)\sin(z)\cos(\xi), \\
Z &\sim -kV_0\exp\left(\frac{-\eta^2}{2k^2}\right)\cos(z)\cos(\xi),
\end{aligned}$$

Look for horizontally non-divergent perturbations $u' = -\psi'_\eta$, $v' = \psi'_\xi$.

Perturbation vorticity equation is approximately

$$\boxed{\tilde{\zeta}'_\tau + V\tilde{\zeta}'_\eta + k^{-1}Z_\xi u' = \nabla^2\psi'_\tau + \cos(z)\exp\left(\tfrac{-\eta^2}{2k^2}\right)\cos(\xi)\left(\nabla^2\psi'_\eta + \psi'_\eta\right) = 0}$$

Next order in k^{-1}: time dep. of MRG wave, advection by U, and β effect.

Case 1: η small, $\exp\left(\frac{-\eta^2}{2k^2}\right) \approx 1$

Special case of problem of Gill (1974) for barotropic Rossby wave.

Approximate solution by truncated series:

$$\psi' = \mathrm{Re}\left[e^{\mu\cos(z)\tau} e^{il\eta}\left(\psi'_{-1}e^{-i\xi} + \psi'_0 + \psi'_1 e^{i\xi}\right)\right]$$

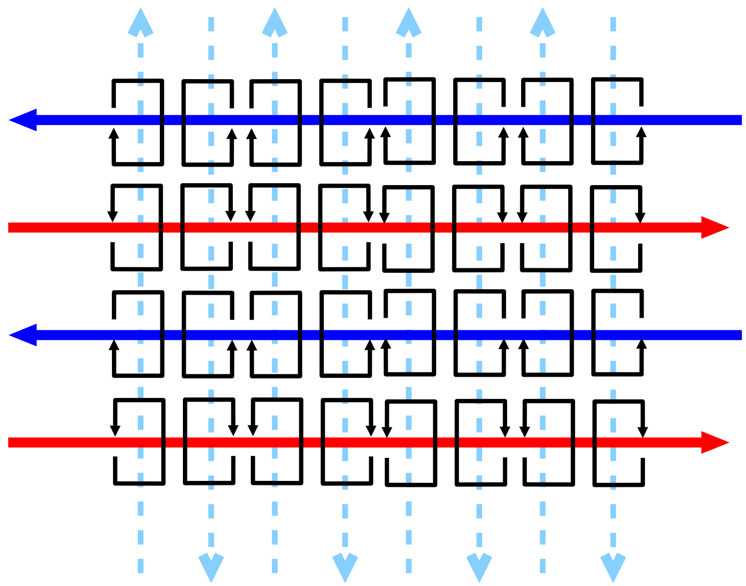
ψ'_0 is zonal jet, $\psi'_{\pm 1}$ zonally short wave.

Instability if eigenvalue

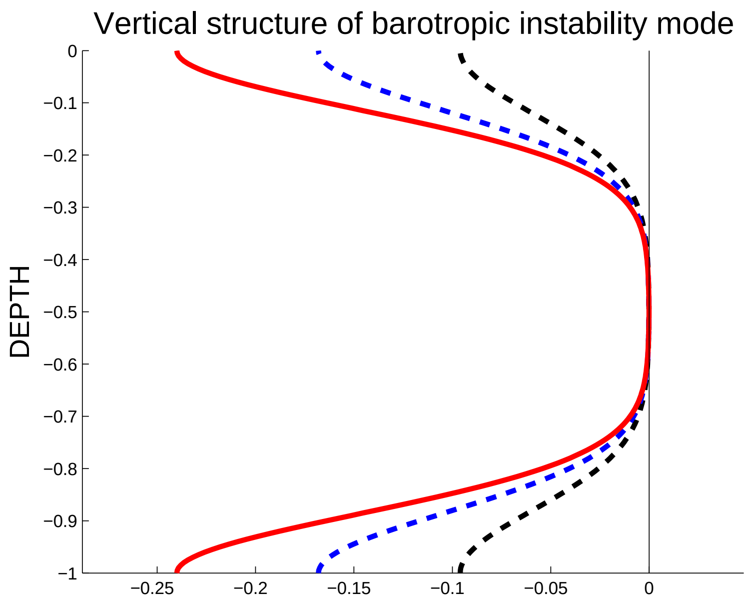
$$\mu = \frac{l}{\sqrt{2}}\sqrt{\frac{1-l^2}{l^2+1}} > 0$$

$\Rightarrow$ Instability if $|l| < 1 \Rightarrow$ perturbation of larger scale than original wave (expected on grounds of simultaneous conservation of energy and enstrophy). Fastest growth rate for $\boxed{l = \sqrt{\sqrt{2}-1} \approx 0.64}$.

Horizontal structure



Vertical structure



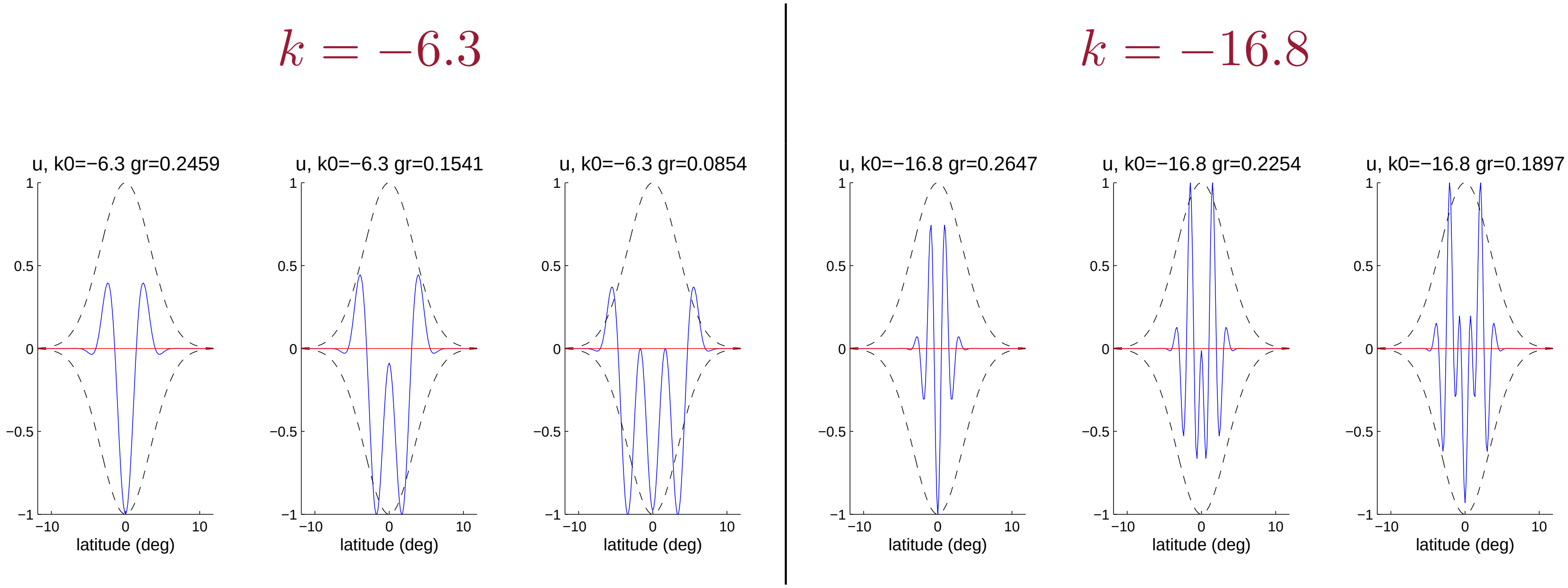
Zonal jets alternating in direction in latitude, and small scale cells stirring within jet centres.

Case 2: Solution valid on $-\infty < \eta < \infty$

Again look for truncated series solution:

$$\psi' = \mathrm{Re}\left[e^{\mu\cos(z)\tau}\left(\psi'_{-1}(\eta)e^{-i\xi} + \psi'_0(\eta) + \psi'_1(\eta)e^{i\xi}\right)\right]$$

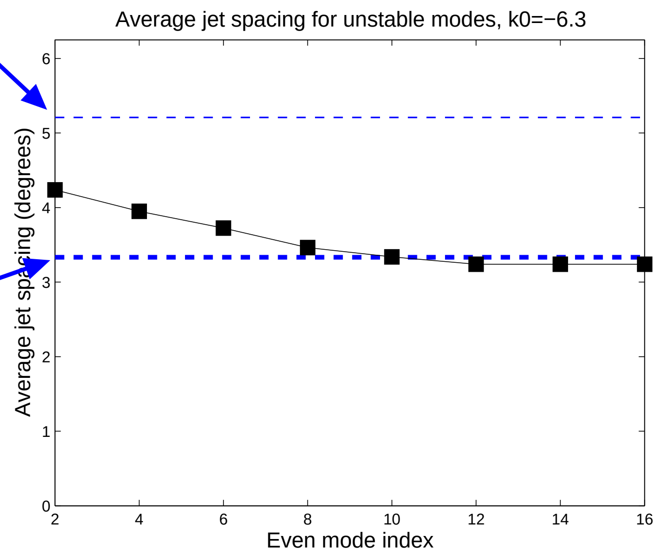
Solve numerically using finite difference method. Structure of $\psi'_0(\eta)$ for fastest growing eigenmodes:



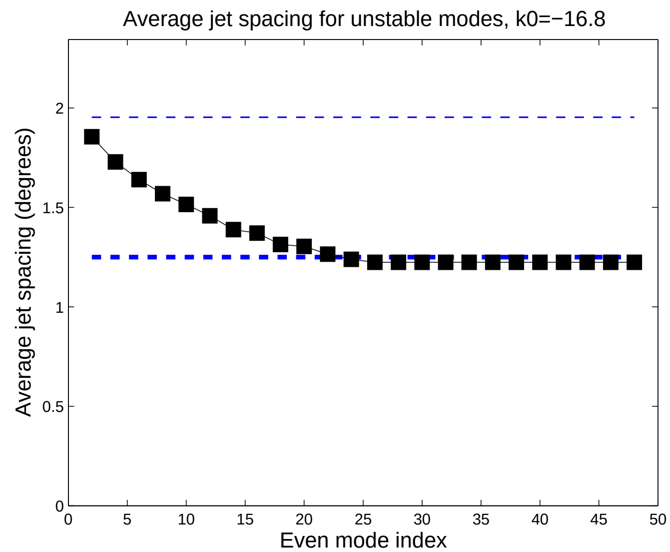
Average jet spacing:

$V = \mathrm{const}$

MRG wavelength



$k = -6.3$



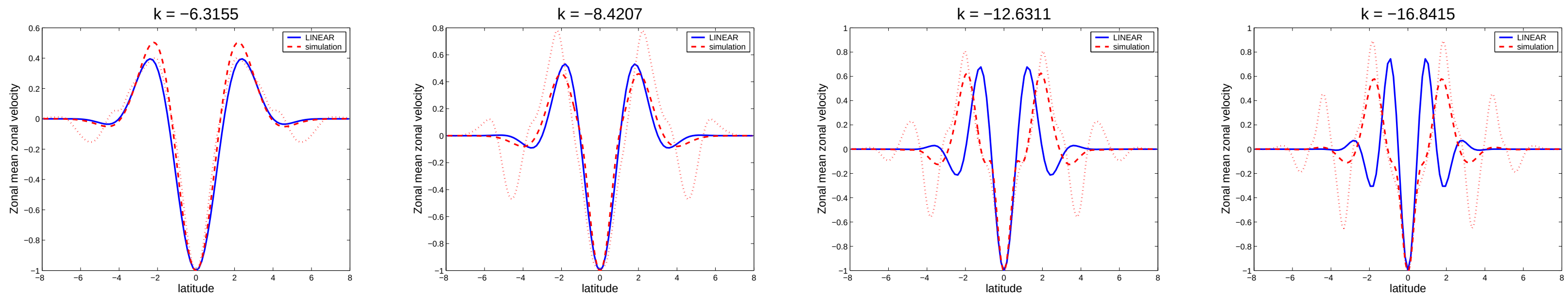
$k = -16.8$

- Jet spacing wider than wavelength of MRG wave
- Spacing less than for plane barotropic wave.

Simulations at $0.1^\circ \times 0.1^\circ$, 100-200 vertical levels using ROMS model:

Amplitude 0.36 cm, $k = -6.3$ (3.3° wavelength)

Comparison with simulations

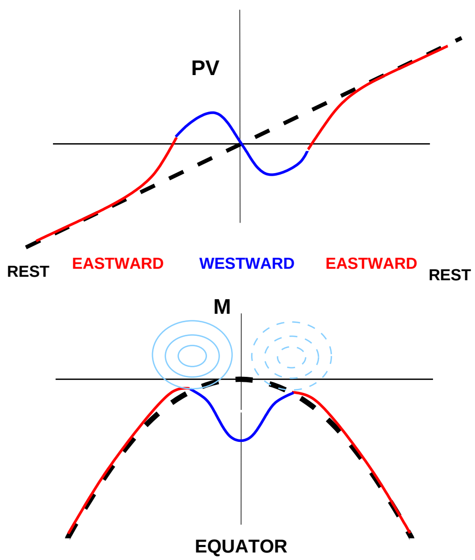


- Simulations even (in latitude) because odd modes are inertially unstable
- Equatorial jet always westward. Probably due to mixing of planetary angular momentum by short modes (which are strongest at equator) biasing simulations towards westward flow at equator.
- Extra-equatorial jet positioning poleward of in most unstable linear mode : barotropic stability?

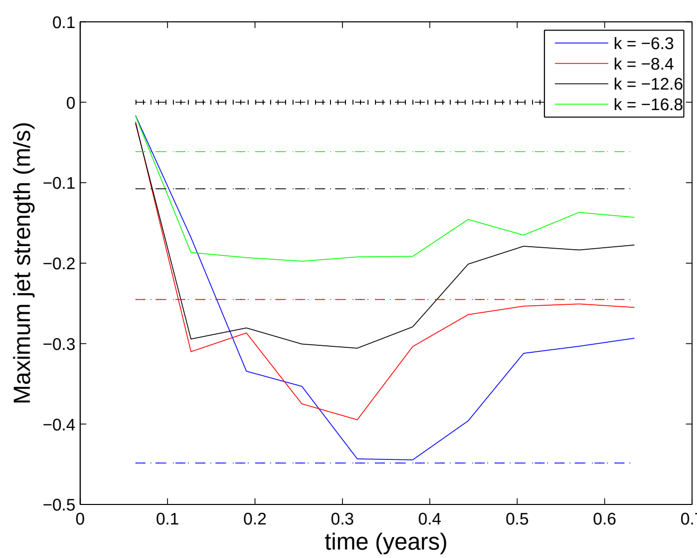
3. Inertial Instability and PV Homogenization

- inertial instability occurs when angular momentum increases poleward, i.e. if $f(PV) < 0$
- linear mode will have curvature greater than β at some moment, at which point flow will become inertially unstable

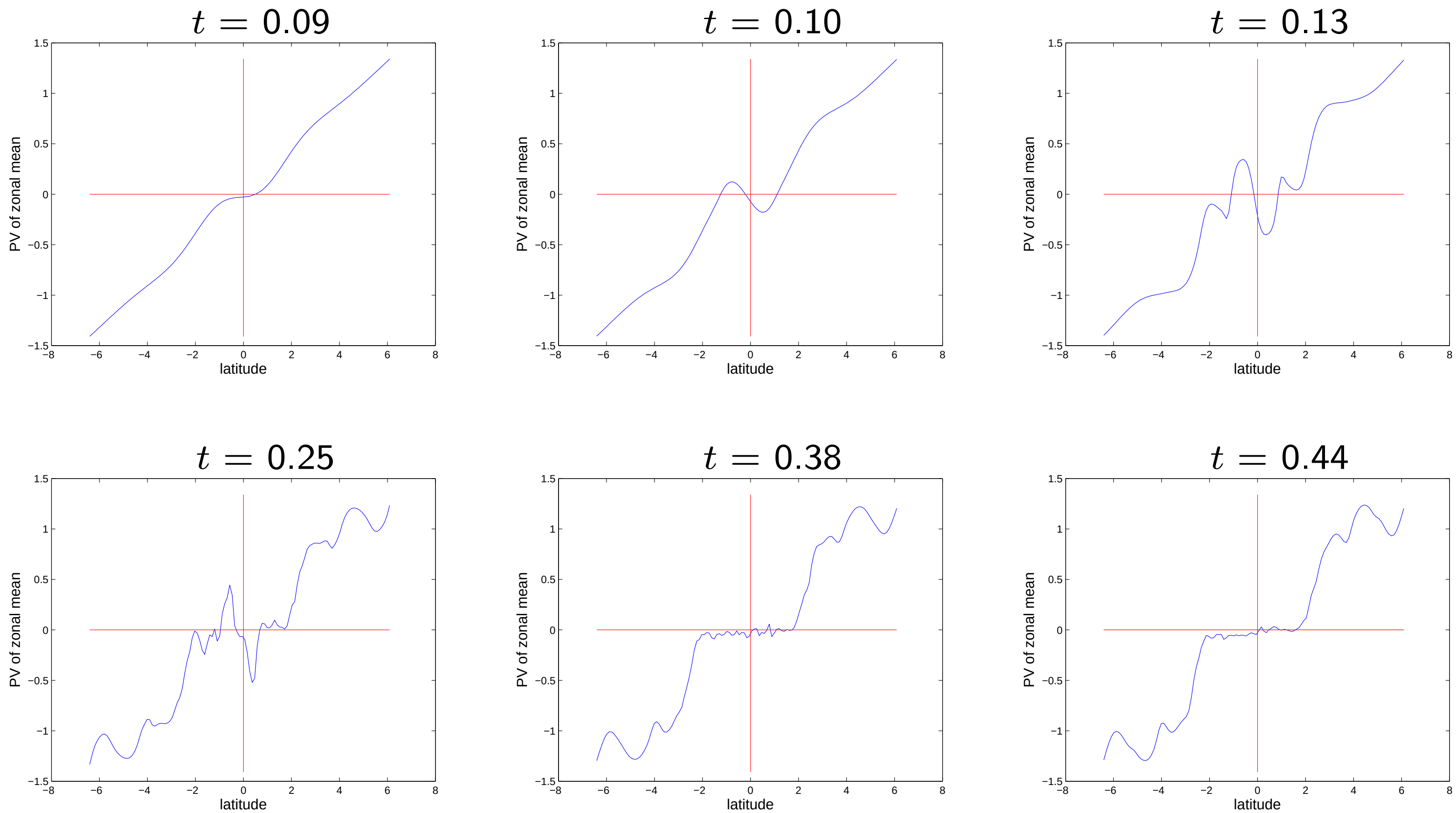
"Curvature" inertial instability



Jet amplitude and inertial instability thresholds of linear modes:



Time evolution of zonal mean PV:

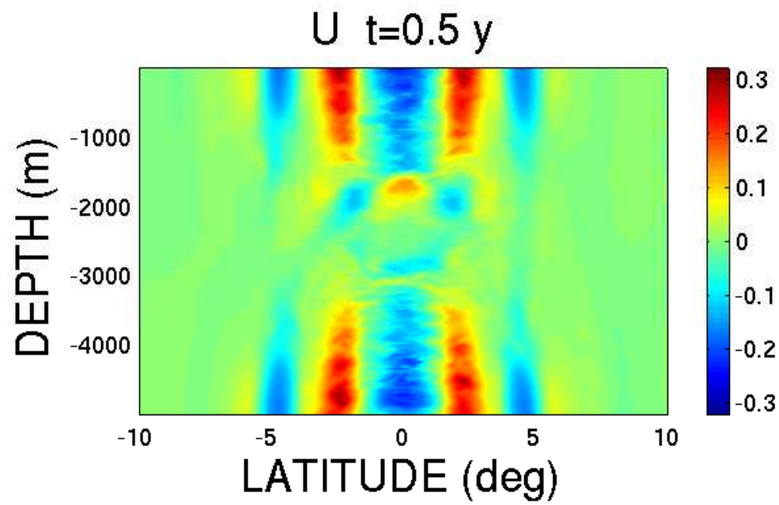


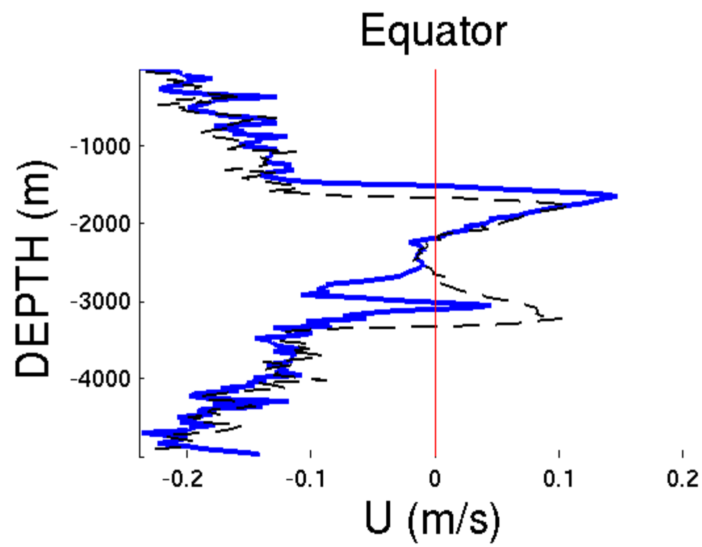
Note equatorially symmetric inertial instability ($t > 0.10$) and subsequent adjustment to uniform zero PV over interval wider than initially unstable interval (implies a widening and/or weakening of westward jet).

4. Super-rotation and the non-traditional Coriolis force

Amplitude 0.36 cm, $k = -6.3$ (3.3° wavelength)

- Clue may be effect on super-rotating jet of non-traditional Coriolis force terms (much greater than $\mathcal{O}(\Omega/N)$).
- What is reason for asymmetric vertical redistribution of angular momentum $M_\gamma \equiv U - \frac{1}{2}\beta y^2 + \gamma z$?
- Explanation in terms of lateral mixing of PV extending beyond unstable zone in both depth and latitude?





Summary

- Zonal jets with realistic spatial scales can be generated by the destabilization of mixed Rossby-gravity waves of frequencies consistent with forcing on equatorial ocean.
- Considerations of inertial stability and planetary potential vorticity mixing explain sign and position of equatorial jet.
- Shape of equatorial westward jet well described by truncated Floquet solution to linearized barotropic vorticity equation.
- Equatorial jet saturates at lower amplitude for more unstable initial waves – consistent with saturation due to inertial instability.
- Super-rotating equatorial jets form for very short waves, phenomenon particularly sensitive to traditional Coriolis force approximation.